Geometry

Chapter 12: Circles

Chapter Focus

Develop and apply the properties of lines and angles that intersect circles.

Analyze the properties of circles in the coordinate plane and use them to solve real-world problems.

Section 1: Lines and Arcs in Circles

12-1 Lines That Intersect Circles – G.C.2

12-2 Arcs and Chords – G.C.2

12-3 Sector Area and Arc Length – G.C.5

Section 2: Angles and Segments in Circles

12-4 Inscribed Angles – G.C.2

12-5 Angle Relationships in Circles – G.C.2

12-6 Segment Relationships in Circles – G.C.2

12-7 Circles in the Coordinate Plane – G.GPE.1

Previous Knowledge Needed

Used the fundamental vocabulary of circles.

Developed and applied formulas for the area and circumference of circles.

Used circles to solve problems.

Cadet's Study

Solving problems involving circles.

Finding lengths, angles, measures and areas associated with circles.

Applying circle theorems to solve a wide range of problems.

Skills Learned

To Use the Pythagorean Theorem to derive the Distance Formula.

Use the distance formula to derive the general form of an equation for a circle.

Find arc length for different arcs in a circle.

Find area of a sector for different angles in a circle.

To calculate distances inside and outside a circle.

REACHING ALL LEARNERS – Differentiated Instruction for students with

Non-Proficient	**Proficient**	**Mastered**	**English Language Learner**
☐ Multiple Representations	☐ Multiple Representations	☐ Multiple Representations	☐ Multiple Representations
☐ Practice A WORKSHEET	☐ Practice B WORKSHEET	☐ Practice C WORKSHEET	☐ Practice A, B, or C WORKSHEET
☐ Reteach WORKSHEET		☐ Challenge WORKSHEET	☐ *Success for Every Learner*
☐ Homework Help Online Textbook online	☐ Homework Help Online Textbook online	☐ Homework Help Online Textbook Online	☐ Homework Help Online Textbook Online
☐ *Lesson Tutorial Video* In class/at home	☐ *Lesson Tutorial Video* In Class/at home	☐ *Lesson Tutorial Video* In Class/at Home	☐ *Lesson Tutorial Video* In class/at home
☐ Reading Strategies WORKSHEET	☐ Problem Solving WORKSHEET	☐ Problem Solving WORKSHEET	☐ Reading Strategies WORKSHEET
☐ Questioning Strategies Ask more questions to gain deeper understanding	☐ Critical Thinking Problems from text book		
☐ *IDEA Works!* Pull items from this resource to help cadets			

ASSESSMENT

☐ Lesson Quiz Transparencies for end of class check for understanding

☐ End of Chapter Test. Use appropriate test for class or for cadet needs. Can select multiple assessments for a particular class based on cadet needs.

Alternate Materials – IDEA Works, Are You Ready – Intervention and Enrichment, Ready To Go On – Intervention and Enrichment, Success for Every Learner, Online videos – cadets do not like these videos but they are an option to use at home when they have missed class or are making up work at home.

Questioning Strategies – these questions are generic and can/should be modified depending on the lesson. Use with whole class discussions as well as small group or individual remediation.

1. Making sense of problems and persevere in solving them.
    a. What is this problem asking?
    b. How could you start this problem?
    c. How could you make this problem easier to solve?
    d. How is this way of solving the problem like/different from yours? Reference text book example or another cadet's solution.
    e. Does you plan make sense? Why or why not?
    f. What tools/manipulatives might help you?
    g. What are you having trouble with?
    h. How can you check this?
2. Reason abstractly and quantitatively.
    a. What does the number represent in the problem?
    b. How can you represent the problem with symbols and numbers?
    c. Create a representation of the problem.

3. Construct viable arguments and critique the reasoning of others (involve cadet in arguments and critiques)?
    a. How is your answer different than the one before us?
    b. How can you prove that your answer is correct?
    c. What math language will help you prove your answer?
    d. What examples could prove or disprove your argument?
    e. What do you think about the other methods argument?
    f. What is wrong with the other methods thinking?
    g. What questions do you have for me or others working the problem?
4. Model with Mathematics (involving real world situations to the task).
    a. Write a number sentence to describe the situation.
    b. What do you already know about solving this problem?
    c. What connections do you see?
    d. Why do the results make sense?
    e. Is this working or do you need to change your model?
5. Use appropriate tools strategically.
    a. How could you use manipulatives or a drawing to show your thinking?
    b. Which tool/manipulative would be best for this problem?
    c. What other resources could help you solve this problem?

6. Attention to precision.
    a. What does this word mean?
    b. Explain what you did to solve the problem?
    c. Compare your answer to another cadet's answer?
    d. What labels could you use?
    e. How do you know your answer is accurate?
    f. Did you use the most efficient way to solve the problem?
7. Look for and make use of structure (move cadet from the general to the specific).
    a. What does this happen?
    b. How is this related to the other problem?
    c. Why is this important to the problem?
    d. What do you know about something else that you can apply to this situation?
    e. How can you use what you know to explain why this works?
    f. What patterns do you see?
8. Look for and express regularity in repeating reasoning (move cadet from specific to general)?
    a. What generalizations can you make?
    b. Can you find a shortcut to solve the problem? How would your shortcut make the problem easier?
    c. How could this problem help you solve another problem?

Geometry

Chapter 12: Circles

Lesson 1 - Lines That Intersect Circles

Standards

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.C.4 – Construct a tangent line from a point outside a given circle to the circle.

Objectives

Identify tangents, secants and chords.

Use properties of tangents to solve problems.

Materials

Blank paper, compass, Text book, ToolBox, overhead and transparencies, Questioning Strategies, Remediation Materials, workbook

Vocabulary

Interior of a circle – points inside a circle

Exterior of a circle – points outside a circle

Chord – segment – endpoints lie on a circle

Secant – line that intersects circle at two points

Tangent – line that intersects circle at one point called a point of tangency.

Point of tangency – The point of intersection of a circle or sphere with a tangent line or plane.

Circles – a set of all points that are a fixed distance from a given point called the center of the circle.

Congruent – radii are equal

Concentric – coplanar with same center

Tangent – coplanar with intersection at one point

Common tangent – a line tangent to two circles

Formulas

None

Symbols

None

Postulates

None

Theorems

Theorem – If 2 segments are tangent to a circle from the same external point, then the segments are congruent.

Theorem – If a line is tangent to a circle, then it is perpendicular to the radius drawn to the tangent point.

Who Uses This?

Scientists use to solve problems pertaining to the earth.

Discussion – first day

Do vocabulary discussing words and drawing a picture representation of each.

Review example 1 from book. Discuss that there can be 2 identifications for one segment. Example would be a chord that goes through the center is the diameter. Ask what is the relationship between a chord and a diameter? How are they alike and how are they different?

Have cadets do Check-It-Out #1

Write the first theorem in ToolBox.

Review example 4 from book. Can we conclude that the two segments that are tangent to the circle are congruent? What if they start from the same point? What if they start at different points?

At this point, may need to review solving equations with a variable on both sides.

Guided Practice – first day

Check it out #4a and b

Practice

None for day 1

Homework

None for Day 1

Discussion – second day

Review vocabulary.

Questions – What is the relationship between a chord and the diameter?

What do you know about two tangents to a circle that start at the same point?

How many radii are there in a circle?

Review example 2 – find length of radius – if on a coordinate graph – count the squares. If not on a coordinate graph use the points and the distance formula. Review the formula. Find the tangent line and then find the equation of the line. May have to review how to come up with the equation by using the slope-point-formula. How do you know which axis to use – x or y? Discuss how to tell the difference.

Activity – have cadets draw a circle on a piece of plane paper. Make sure the center is identified. Select a point on the circle and draw a radius to that point. Draw a line tangent to the point you selected on the circle. What do you notice about the angle the tangent line makes with the radius? Discuss cadet's thoughts. Have cadets look at other cadets work to see if there is a patterns going on. Should see that the angle is 90 degrees. Draw several examples on the board that are not right angles to show that not all lines intersecting the point on the circle are perpendicular to the radius.

Discuss and write second theorem in ToolBox.

Review Pythagorean Theorem by looking at example 3. Make sure to point out that feet need to be converted to miles. Review how to do that conversion.

Guided Practice – second day

Check it out #3 – remind cadets to convert feet into miles. This problem is very confusing to cadets as they think that mountains do not extend into the center of the earth. Need to encourage them that they are using the center of the earth to measure the height of the mountain. The sketch is 3D and cadets need to be encouraged to imagine the 3D coming off the paper.

Page 797, # 9 – have cadets work on it and then review.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better

understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 11 – 28, 31 – 35, 38 – 40

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets - Problems 11 – 41

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets – Problems 11 – 43

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 14, 15, 16, 20, 26 – checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

Page 797 – 13, 16, 18-22, 26, 27, 31, 32, 33 – review each of these after cadets have had time to do them.

Homework

Page 73 and 157 in workbook. May be replaced with different work based on needs. Replaced by additional resources listed above.

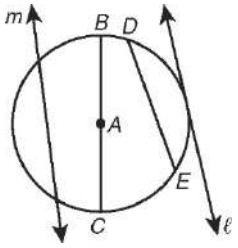
12-1 Practice A

Lines That Intersect Circles

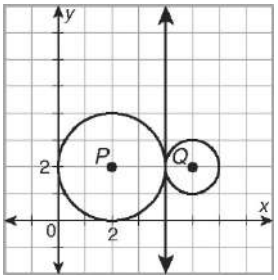
For Exercises 1–5, match the letter of the part of the figure to the names. Use each letter once.

1. chord _____
2. tangent _____
3. radius _____
4. secant _____
5. diameter _____

A. $\overline{AB}$
B. l
C. m
D. $\overline{BC}$
E. $\overline{DE}$



Use the figure for Exercises 6–8.

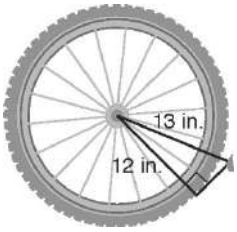


6. radius of $\odot P$ _____ radius of $\odot Q$ _____
7. coordinates of the point of tangency (_____, _____)
8. equation of the tangent line at the point of tangency

_____

Fill in the blanks to complete each theorem.

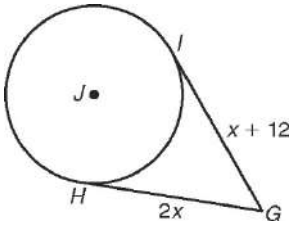
9. If a line is perpendicular to a radius of a circle at a point on the circle, then the line is _____ to the circle.
10. If two segments are tangent to a circle from the same external point, then the segments are _____.
11. If a line is tangent to a circle, then it is _____ to the radius drawn to the point of tangency.
12. Amiko is riding her bike on a wet street. As the bike wheel spins, water drops are sprayed off tangent to the wheel. Amiko's bike wheels have a radius of 12 inches. Use the Pythagorean Theorem to find the distance the water drops have been sprayed when they are 13 inches from the center of the wheel.



_____

In Exercises 13 and 14, $\overline{GH}$ and $\overline{GI}$ are tangent to $\odot J$. Find GH.

13.



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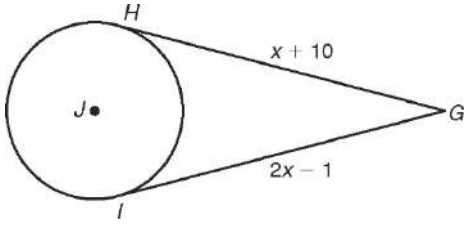
14.



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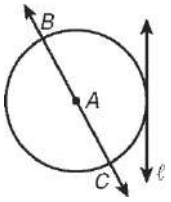
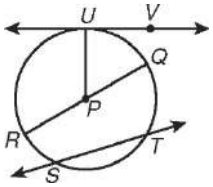
12-1 Practice B

Lines That Intersect Circles

Identify each line or segment that intersects each circle.

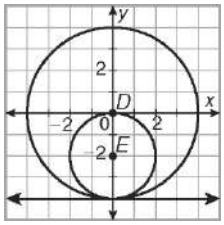
1. 

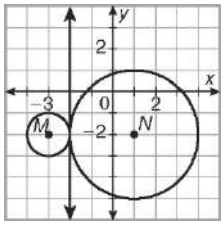
2. 

_____ _____

_____ _____

Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at this point.

3. 

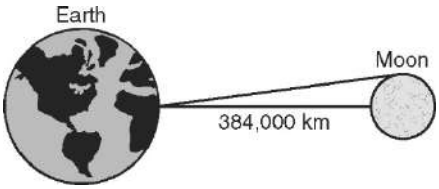
4. 

_____ _____

_____ _____

_____ _____

5. The Moon's orbit is not exactly circular, but the average distance from its surface to Earth's surface is 384,000 kilometers. The diameter of the Moon is 3476 kilometers. Find the distance from the surface of Earth to the visible edge of the Moon if the Moon is directly above the observer. Round to the nearest kilometer. (*Note:* The figure is not drawn to scale.)



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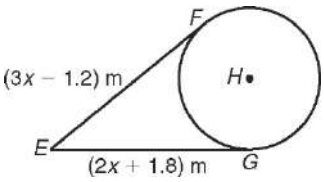
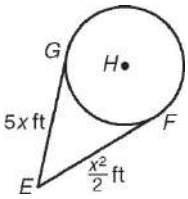
In Exercises 6 and 7, $\overline{EF}$ and $\overline{EG}$ are tangent to ⊙*H*. Find *EF*.

6. 

7. 

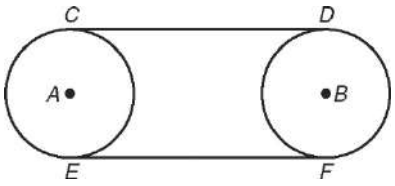
_____ _____

12-1 Practice C

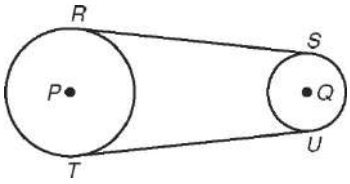
Lines That Intersect Circles

Write paragraph proofs for Exercises 1–3.

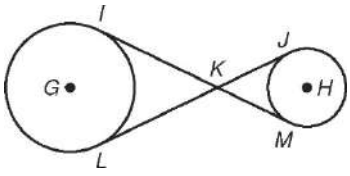
1. **Given:** ⊙ *A* and ⊙ *B* with congruent radii. $\overline{CD}$ and $\overline{EF}$ are common tangent segments.
   Prove: $\overline{CD} \cong \overline{EF}$ (*Hint:* Draw $\overline{AB}$. Use properties of quadrilaterals to show that *CD* = *AB* = *EF*.)



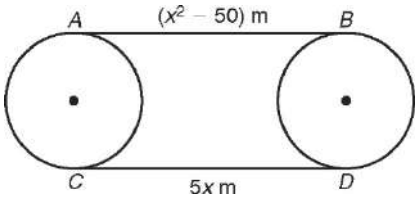
2. **Given:** ⊙ *P* and ⊙ *Q* with different radii. $\overline{RS}$ and $\overline{TU}$ are common tangent segments. $\overline{RS}$ and $\overline{TU}$ are not parallel.
   Prove: $\overline{RS} \cong \overline{TU}$



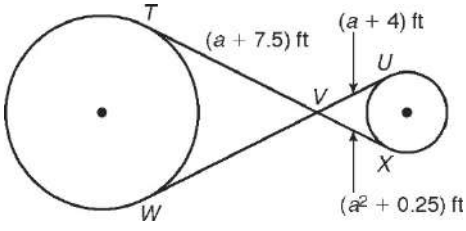
3. **Given:** ⊙ *G* and ⊙ *H*. $\overline{IM}$ and $\overline{JL}$ are common interior tangent segments.
   Prove: $\overline{IM} \cong \overline{JL}$



Assume the segments that appear to be tangent are tangent. Find each length.

4. 

CD _____

5. 

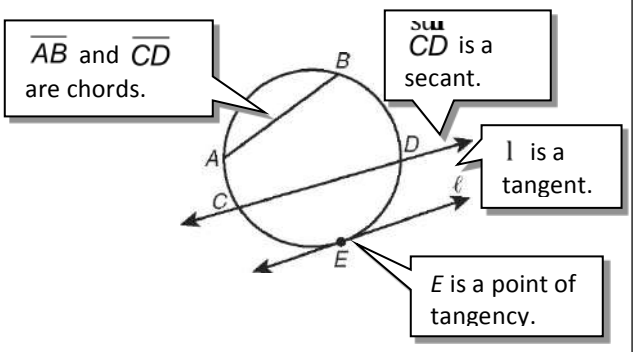
UW _____

12-1 Reteach

Lines That Intersect Circles

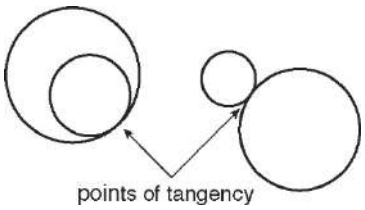
Lines and Segments That Intersect Circles

- A **chord** is a segment whose endpoints lie on a circle.
- A **secant** is a line that intersects a circle at two points.
- A **tangent** is a line in the same plane as a circle that intersects the circle at exactly one point, called the **point of tangency.**
- Radii and diameters also intersect circles.

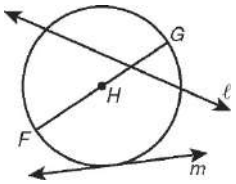


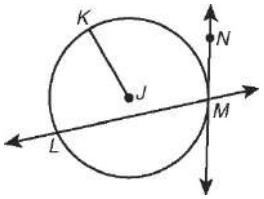
Tangent Circles

Two coplanar circles that intersect at exactly one point are called **tangent circles**.

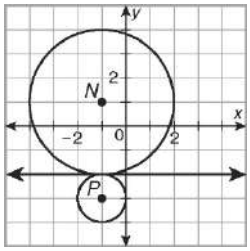


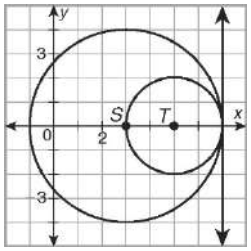
Identify each line or segment that intersects each circle.

1. 

2. 

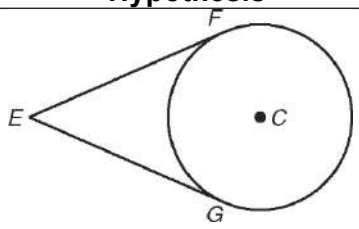
Find the length of each radius. Identify the point of tangency and write the equation of the tangent line at that point.

3. 

4. 

12-1 Reteach

Lines That Intersect Circles *continued*

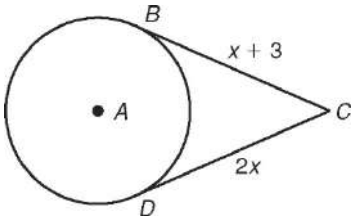
Theorem	Hypothesis	Conclusion
If two segments are tangent to a circle from the same external point, then the segments are congruent.	 $\overline{EF}$ and $\overline{EG}$ are tangent to $\odot C$.	$\overline{EF} \cong \overline{EG}$

In the figure above, $EF = 2y$ and $EG = y + 8$. Find EF.

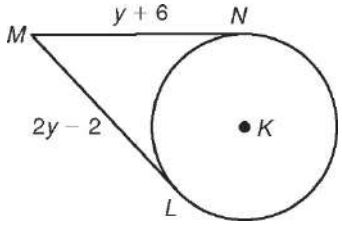
$EF = EG$	2 segs. tangent to $\odot$ from same ext. pt. $\rightarrow$ segs. $\cong$.
$2y = y + 8$	Substitute $2y$ for EF and $y + 8$ for EG.
$y = 8$	Subtract y from each side.
$EF = 2(8)$	$EF = 2y$; substitute 8 for y.
$= 16$	Simplify.

The segments in each figure are tangent to the circle.
Find each length.

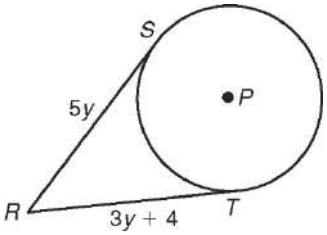
5. *BC*



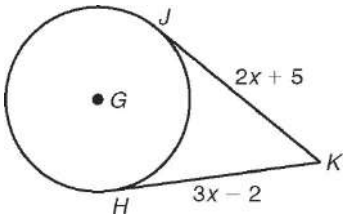
6. *LM*



7. *RS*



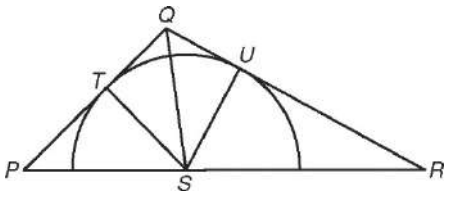
8. *JK*



12-1 Challenge

Lines That Intersect Circles

In $\triangle PQR$, in which $PQ = 15$, $QR = 22$, and $PR = 30$, a semicircle is drawn so that its diameter lies on $\overline{PR}$ and it is tangent to $\overline{QP}$ and $\overline{QR}$. If S is the center of the circle, find the measure of $\overline{PS}$ to the nearest hundredth.



Draw radii $\overline{ST}$ and $\overline{SU}$ to the points of tangency to $\overline{PQ}$ and $\overline{QR}$. Draw $\overline{QS}$.

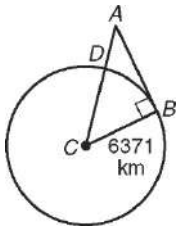
Statements	Reasons
1. _____	1. All radii have equal measures.
2. $m\angle QTS = m\angle QUS = 90°$	2. _____
3. $\overline{QT} \cong \overline{QU}$	3. _____
4. $\overline{QS} \cong \overline{QS}$	4. _____
5. $\triangle QTS \cong \triangle QUS$	5. _____
6. $\angle UQS \cong \angle TQS$	6. _____
7. _____	7. Triangle Angle Bisector Theorem

Let $PS = x$. Then _____ and $x =$ _____.

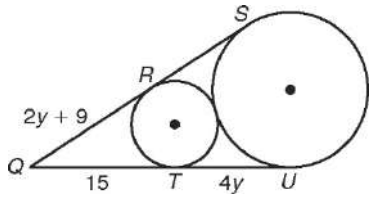
12-1 Problem Solving

Lines That Intersect Circles

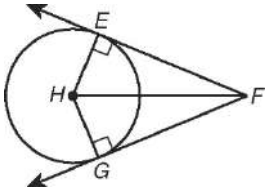
1. The cruising altitude of a commercial airplane is about 9000 meters. Use the diagram to find *AB*, the distance from an airplane at cruising altitude to Earth's horizon. Round to the nearest kilometer.



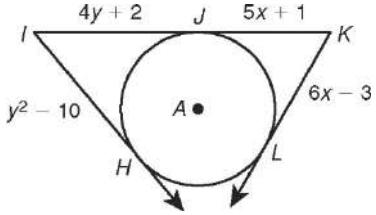
2. In the figure, segments that appear to be tangent are tangent. Find *QS*.



3. The area of ⊙*H* is 100π, and *HF* = 26 centimeters. What is the perimeter of quadrilateral *EFGH*?

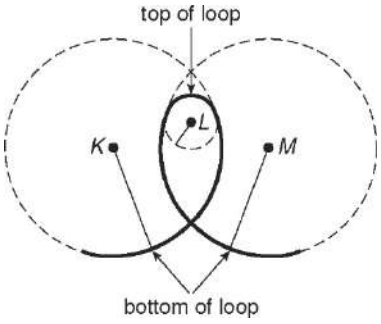


4. $\overleftrightarrow{IH}$, $\overline{IK}$, and $\overleftrightarrow{KL}$ are tangent to ⊙*A*. What is *IK*?



Choose the best answer.

5. A teardrop-shaped roller coaster loop is a section of a spiral in which the radius is constantly changing. The radius at the bottom of the loop is much larger than the radius at the top of the loop, as shown in the figure. Which is a true statement?



A ⊙*K* and ⊙*M* have two points of tangency.

B ⊙*K*, ⊙*L*, and ⊙*M* have one point of tangency.

C ⊙*L* is internally tangent to ⊙*K* and ⊙*M*.

D ⊙*L* is externally tangent to ⊙*K* and ⊙*M*.

6. ⊙ *G* has center (2, 5) and radius 3.
⊙ *H* has center (2, 0). If the circles are tangent, which line could be tangent to both circles?

F $x = 2$ H $y = 2$

G $x = 0$ J $y = 5$

7. The Hubble Space Telescope orbits 353 miles above Earth, and Earth's radius is about 3960 miles. Which is closest to the distance from the telescope to Earth's horizon?

A 1634 mi C 3976 mi

B 1709 mi D 5855 mi

12-1 Reading Strategies

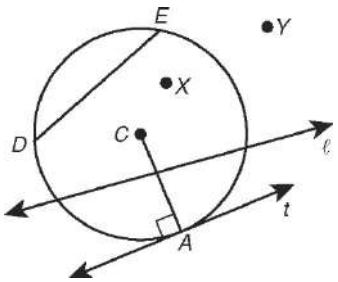
Focus on Vocabulary

The diagram below describes vocabulary words that are used with circles.

A **secant** is a line that intersects a circle at two points. Line l is a secant.

A **chord** is a segment whose endpoints lie on a circle. $\overline{DE}$ is a chord.

A **tangent** is a line in the same plane as the circle that intersects it at exactly one point. Line *t* is a tangent.



The **interior** of a circle is the set of all points inside the circle. Point *X* is in the interior of the circle.

The **point of tangency** is the point where the tangent and the circle intersect. Point *A* is a point of

The **exterior** of a circle is the set of all points outside the circle. Point *Y* is in the exterior of the circle.

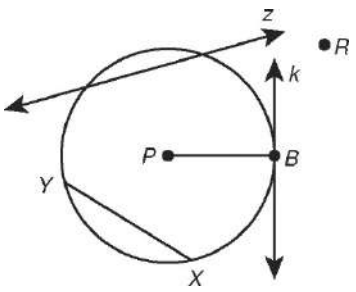
A tangent line is **perpendicular** to the **radius** of a circle drawn to the point of tangency. $\overline{CA} \perp$ line *t*
A line that is perpendicular to the radius of a circle at a point on the circle is a tangent line to the circle.

Answer the following.

1. The _____ of a circle is the set of all points inside the circle.
2. A _____ is a line that intersects a circle at two points.
3. Look at circle *C* above. Why is line *t* not a secant?

_____

Use circle *P* to identify each line, segment, or point.



4. secant line _____
5. point of tangency _____
6. tangent line _____
7. chord _____
8. a point in the exterior of the circle _____
9. a point in the interior of the circle _____

Geometry

Chapter 12: Circle

Lesson 2 - Arcs and Chords

Standards

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Objectives

Apply properties of arcs.

Apply properties of chords.

Materials

Protractor, compass, text book, Toolbox, How To Make A Circle Graph sheet, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

Central angle – an angle whose vertex is the center of the circle

Arc – an unbroken part of a circle consisting of two endpoints and all point between.

Minor Arc – an arc whose points are on or in the interior of the central angle.

Major Arc – an arc whose points are on or in the exterior of the central angle.

Semi-Circle – an arc of a circle whose endpoints lie on the diameter

Adjacent arcs – arcs that intersect at exactly one point on the same circle.

Congruent Arcs – Two arcs that are in the same or congruent circles and have the same measure.

Formulas

m of arc AC = m<ABC

m of arc ADC = 360 – m<ABC

m of arc EFG = 180 when segment EG is the diameter

m of arc ABC = m of arc AB + m of arc BC

Symbols

None

Postulates

Arc Addition Postulate – the measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Theorems

12-2-2 - In a circle or congruent circles:

1) congruent central angles have congruent chords

2) congruent chords have congruent arcs

3) congruent arcs have congruent central angles

12-2-3 – In a circle, if a radius or diameter is perpendicular to a chord, then it bisects the chord and its arc.

12-2-4 – In a circle, the perpendicular bisector of a chord is a radius or diameter.

Who Uses This?

Market analysts use circle graphs to compare sales of different products.

Comparisons of parts to the whole.

Discussion

Review how to create a circle graph. Have cadets write the steps in their ToolBox. Make sure to cover that 360 degrees is a circle and is used when calculating the angle measurement.

Guided Practice

Have cadets make a circle graph from data provided. What each cadet as they work. Make sure the percentages are correct and that they are graphing the angle and not the percentage column.

When all graphs look reasonable, look at Example 1 and discuss all of the areas. Cadets will do Check-It-Out 1.

Update the ToolBox with vocabulary for the chapter.

Review Example 2 discussing how two arcs total the entire arc and how it is like a log that has two different lengths that total the entire log.

Cadets will work on Check-it-Out 2.

Write Congruence Theorems in ToolBox and discuss how each one of them just makes sense when we read it. Look at Example 3. Discuss once again how to do the math – solving equations with variables on both sides. Have cadets do Check-It-Out 3.

Look at the relationships between radii and chords and then look at Example 4. Review again the squaring and square rooting of numbers. Cadets will do Check-it-Out 4.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 19 – 40, 45, 47 – 50

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 19 – 51

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 19 – 44, 46 – 53

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 20, 26, 30, 32, 38, 40 – checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

If this is a review and cadets have it, then do the following: Page 806 – 807 problems #5-35 – odd problems

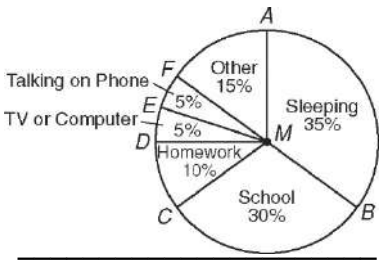
Homework

Page 74 and 158 May be replaced with different work based on needs. Replaced by additional resources listed above.

12-2 Practice A

Arcs and Chords

The circle graph shows the number of hours Rae spends on each activity in a typical weekday. Use the graph to find each of the following.



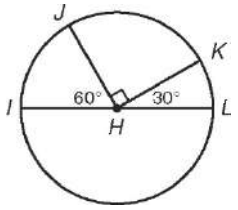
1. m∠*AMD* = _____
2. m∠*DMB* = _____
3. $m\overset{a}{BC}$ = _____
4. $m\overset{a}{CBA}$ = _____

In Exercises 5–10, fill in the blanks to complete each postulate or theorem.

5. In a circle or congruent circles, congruent central angles have congruent _____.
6. In a circle or congruent circles, congruent _____ have congruent arcs.
7. The measure of an arc formed by two _____ arcs is the sum of the measures of the two arcs.
8. In a circle, the _____ of a chord is a radius (or diameter).
9. In a circle or congruent circles, congruent arcs have congruent _____.
10. In a circle, if the _____ is perpendicular to a chord, then it bisects the chord and its arc.

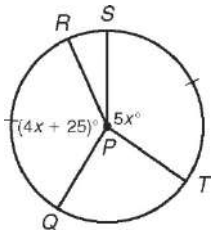
Find each measure.

11. $m\overset{\frown}{JK}$ = _____



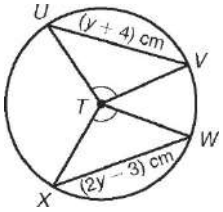
12. $m\overset{a}{JIL}$ = _____

13.



$m\overset{a}{QR} = m\overset{a}{ST}$. Find m∠*QPR*._____

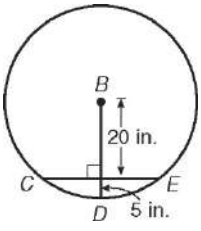
14.



∠*UTV* ≅ ∠*XTW*. Find *WX*._____

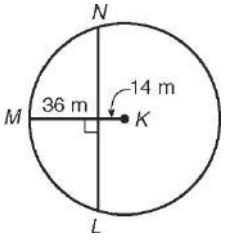
Find the length of each chord. (*Hint:* Use the Pythagorean Theorem to find half the chord length, and then double that to get the answer.)

15.



CE = _____

16.

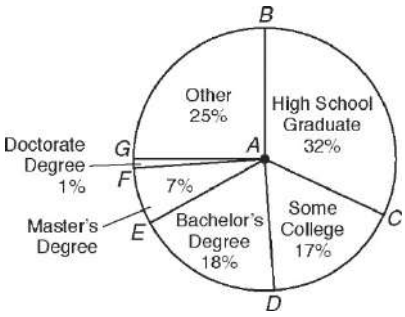


LN = _____

12-2 Practice B

Arcs and Chords

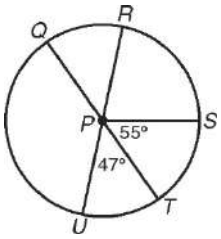
The circle graph shows data collected by the U.S. Census Bureau in 2004 on the highest completed educational level for people 25 and older. Use the graph to find each of the following. Round to the nearest tenth if necessary.



1. m∠*CAB* _____
2. m∠*DAG* _____
3. m∠*EAC* _____
4. m$\overset{\frown}{BG}$ _____
5. m$\overset{\frown}{GF}$ _____
6. m$\overset{\frown}{BDE}$ _____

Find each measure.

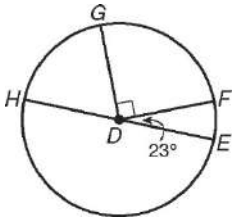
7.



m$\overset{\frown}{QS}$ _____

m$\overset{\frown}{RQT}$ _____

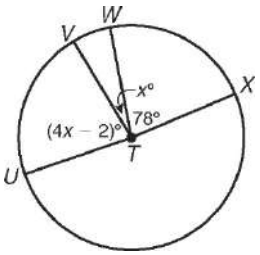
8.



m$\overset{\frown}{HG}$ _____

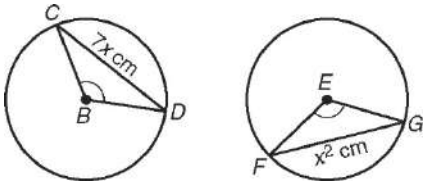
m$\overset{\frown}{FEH}$ _____

9.



Find m∠*UTW*. _____

10.

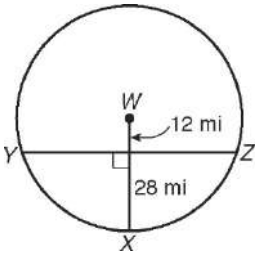


e *L* ≅ e *E*, and ∠*CBD* ≅ ∠*FEG*.

Find *FG*. _____

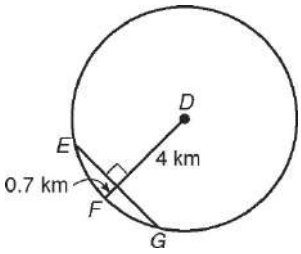
Find each length. Round to the nearest tenth.

11.



ZY _____

12.

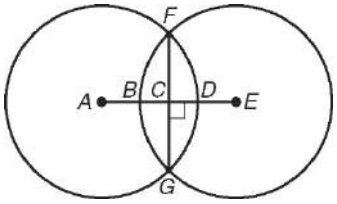


EG _____

12-2 Practice C

Arcs and Chords

Write proofs for Exercises 1 and 2.

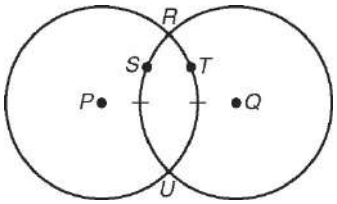


1. **Given:** $\overline{AC} \cong \overline{EC}, \overline{AE} \perp \overline{FG}$

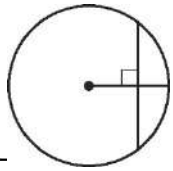
   Prove: $\angle A \cong \angle E$

2. **Given:** $\overset{\frown}{RSU} \cong \overline{RTU}$

   Prove: $\angle P \cong \angle Q$



Give the degree measure of the arc intercepted by the chord described in Exercises 3–8. The figure is given for reference. Round to the nearest tenth if necessary.



3. a chord congruent to the radius ___
4. a chord one-third the length of the radius _____
5. a chord congruent to the segment from the center to the chord _____
6. a chord twice the length of the segment from the center to the chord _____
7. a chord one-fourth the length of the circumference _____
8. a chord $\frac{1}{\pi}$ multiplied by the length of the circumference _____

Find the length of a chord that intercepts an arc of each given measure. Give your answer in terms of the radius *r*. Round to the nearest tenth.

9. 10° _____ 10. 45° _____ 11. 136° _____

12-2 Reteach

Arcs and Chords

Arcs and Their Measure

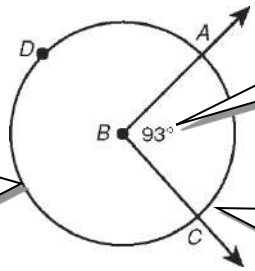
- A **central angle** is an angle whose vertex is the center of a circle.
- An **arc** is an unbroken part of a circle consisting of two points on a circle and all the points on the circle between them.

$\overset{\circ}{A}DC$ is a **major arc.**

$m\overset{\circ}{A}DC = 360° - m\angle ABC$

$= 360° - 93°$

$= 267°$



$\angle ABC$ is a central angle.

$\overset{a}{A}C$ is a **minor arc**

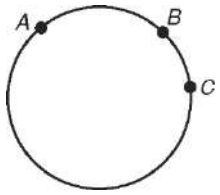
$m\overset{a}{A}C = m\angle ABC = 93°$.

- If the endpoints of an arc lie on a diameter, the arc is a semicircle and its measure is 180°.

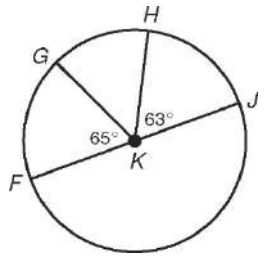
Arc Addition Postulate

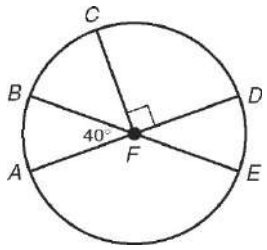
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$m\overset{\circ}{A}BC = m\overset{a}{A}B + m\overset{a}{B}C$



Find each measure.





1. $m\overset{a}{H}J$ _____

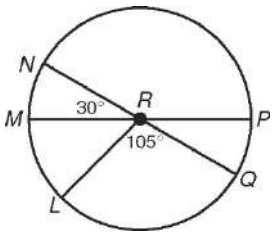
2. $m\overset{\circ}{F}GH$ _____

3. $m\overset{\circ}{C}DE$ _____

4. $m\overset{\circ}{B}CD$ _____

5. $m\overset{\circ}{L}MN$ _____

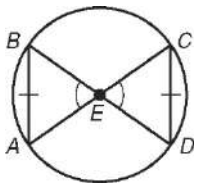
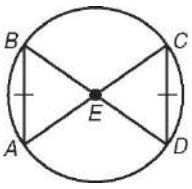
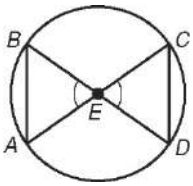
6. $m\overset{\circ}{L}NP$ _____



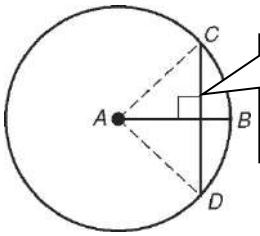
12-2 Reteach

Arcs and Chords *continued*

Congruent arcs are arcs that have the same measure.

Congruent Arcs, Chords, and Central Angles		
 If $m\angle BEA \cong m\angle CED$, then $\overline{BA} \cong \overline{CD}$.	 If $\overline{BA} \cong \overline{CD}$, then $\overset{a}{BA} \cong \overset{a}{CD}$.	 If $\overset{a}{BA} \cong \overset{a}{CD}$, then $m\angle BEA \cong m\angle CED$.
Congruent central angles have congruent chords.	Congruent chords have congruent arcs.	Congruent arcs have congruent central angles.

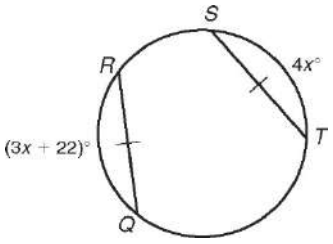
In a circle, if a radius or diameter is perpendicular to a chord, then it bisects the chord and its arc.



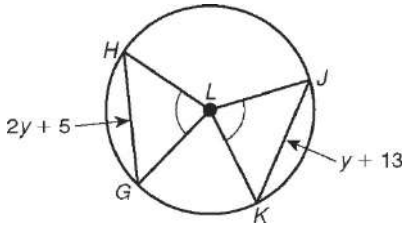
Since $\overline{AB} \perp \overline{CD}$, $\overline{AB}$ bisects $\overline{CD}$ and $\overset{a}{CD}$.

Find each measure.

7. $\overline{QR} \cong \overline{ST}$. Find $m\overset{a}{QR}$.

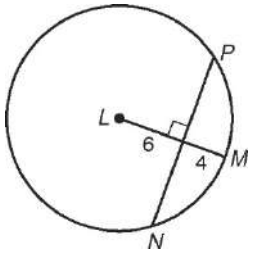


8. $\angle HLG \cong \angle KLJ$. Find GH.

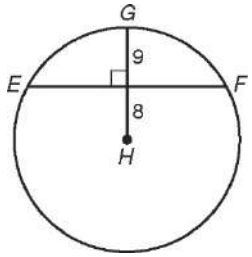


Find each length to the nearest tenth.

9. NP



10. EF

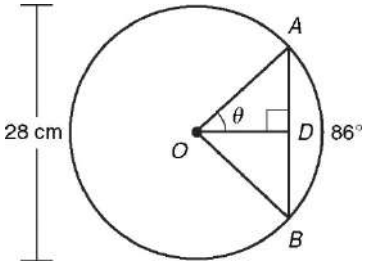


12-2 Challenge

Revisiting Chords of Circles

In the figure at right, the diameter of circle *O* is 28 centimeters.

The chord $\overline{AB}$ intercepts an arc whose measure is 86°. From your previous study of circles, you know that you can find the length of the intercepted arc, $\overset{\frown}{AB}$. In Exercises 1–5, you will see how your knowledge of trigonometry makes it possible for you to also find the length of the chord.



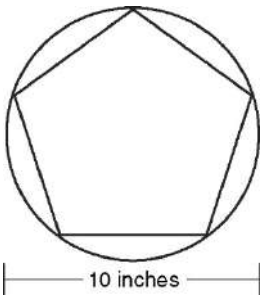
Using the figure above, find each measure.

1. $m\angle AOB$ _____
2. $m\angle OAB$ _____
3. θ _____
4. OA _____
5. a. Using appropriate measures from Exercises 1–4, write a trigonometric equation that can be used to find AD. _____
   b. Solve your equation from part **a**. Round to the nearest tenth. _____
   c. What is the length of $\overline{AB}$? _____

Find the length of a chord, $\overline{AB}$, that is in a circle of diameter d and that intercepts an arc, $\overset{\frown}{AB}$, of the given degree measure. Round your answers to the nearest tenth.

6. $d = 4$ inches, $m\overset{\frown}{AB} = 58°$ $AB \approx$ _____
7. $d = 3$ meters, $m\overset{\frown}{AB} = 162°$ $AB \approx$ _____
8. $d = 2\frac{1}{2}$ feet, $m\overset{\frown}{AB} = 60°$ $AB \approx$ _____
9. Devise a formula that can be used to find the length, l, of a chord in a circle of diameter d, given the degree measure, n, of its intercepted arc, where $0° < n < 180°$. _____

In the figure at right, a regular pentagon is inscribed in a circle of diameter 10 inches. Find each measure.



10. the length of one side of the pentagon _____
11. the perimeter of the pentagon _____
12. the length of an apothem of the pentagon _____
13. the area of the pentagon _____
14. Devise a formula that can be used to find the area, A, of a regular n-gon given the diameter, d, of its circumscribed circle. _____

12-2 Problem Solving

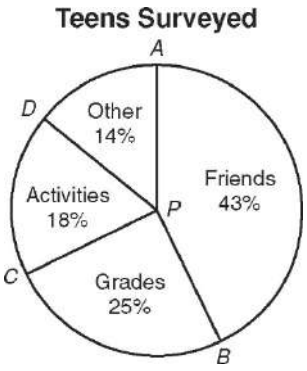
Arcs and Chords

1. Circle D has center $(-2, -7)$ and radius 7. What is the measure, in degrees, of the major arc that passes through points $H(-2, 0)$, $J(5, -7)$, and $K(-9, -7)$?

2. A circle graph is composed of sectors with central angles that measure $3x°$, $3x°$, $4x°$, and $5x°$. What is the measure, in degrees, of the smallest minor arcs?

Use the following information for Exercises 3 and 4.

The circle graph shows the results of a survey in which teens were asked what says the most about them at school. Find each of the following.



3. $m\overset{a}{A}B$

4. $m\angle APC$

Choose the best answer.

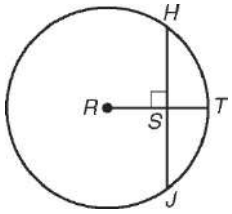
5. Students were asked to name their favorite cafeteria food. The results of the survey are shown in the table. In a circle graph showing these results, which is closest to the measure of the central angle for the section representing chicken tenders?

Favorite Lunch	Number of Students
Pizza	108
Chicken tenders	75
Taco salad	90
Other	54

A 21° C 83°

B 75° D 270°

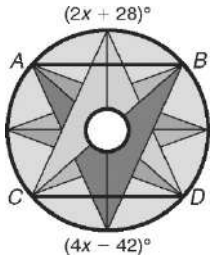
6. The diameter of $\odot R$ is 15 units, and $HJ = 12$ units. What is the length of $\overline{ST}$?



F 2.1 units H 4.5 units

G 3 units J 9.6 units

7. In the stained glass window, $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$. What is $m\overset{a}{C}BD$?



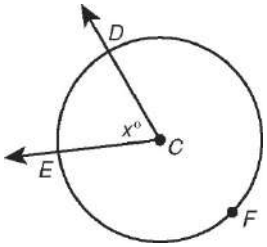
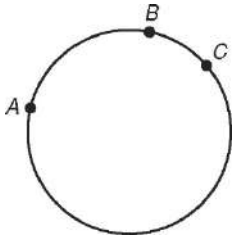
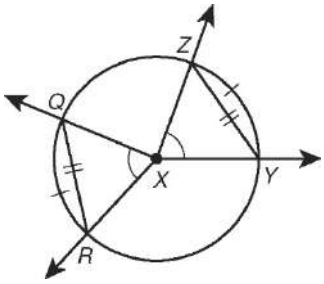
A 35° C 98°

B 70° D 262°

12-2 Reading Strategies

Use a Table

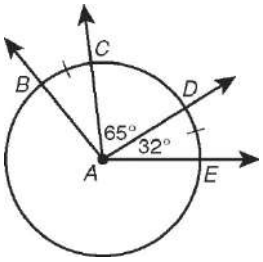
The table below shows some of the relationships among arcs, chords, and central angles.

Words	Diagram	Mathematical Symbols
A minor arc is equal to the measure of its central angle.		$m\overset{\frown}{DE} = m\angle DCE = x°$
A major arc is equal to 360° minus the measure of its central angle.		$m\overset{\frown}{DFE} = 360° - m\angle DCE$ $= 360° - x°$
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.		$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$
Congruent central angles have congruent chords.		$\overline{RQ} \cong \overline{YZ}$
Congruent chords have congruent arcs.		$\overset{\frown}{RQ} \cong \overset{\frown}{YZ}$
Congruent arcs have congruent central angles.		$\angle QXR \cong \angle ZXY$

Answer the following.

1. The measure of a central angle is 60°. What is the measure of its minor arc? _____
2. What will be the sum of a central angle's minor arc and major arc? _____
3. Congruent _____ have congruent chords.

Use circle *A* to find each measure.



4. $m\overset{\frown}{DE}$ _____
5. $m\overset{\frown}{CBE}$ _____
6. $m\overset{\frown}{EBD}$ _____
7. $m\overset{\frown}{CBD}$ _____
8. $m\angle CAB$ _____
9. $m\overset{\frown}{CD}$ _____

Geometry

Chapter 12: Circle

Lesson 3 - Sector Area and Arc Length

Standards

G.C.5 - Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Objectives

Find the area of sectors.

Find arc lengths.

Materials

Calculator, ToolBox, Text Book, paper, pencil, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

Sector of a circle – a region bounded by two radii of the circle and their intercepted arc.

Segment of a circle – region bounded by an arc and its chord.

Arc length – distance along an arc measured in linear units.

Formulas

Area of a Sector – $A = \pi r^2(m/360)$

Area of a Segment - $A = \pi r^2(m/360) – 1/2bh$

Arc length – $L = 2\pi r(m/360)$

Symbols

None

Postulates

None

Theorems

None

Who Uses This?

Farmers use irrigation radii to calculate areas of sectors.

Discussion

Update ToolBox with vocabulary. Go over formulas carefully and explain why the 360 degrees. Most cadets seem to be confused about "the fraction" so be sure to go over that it is the ratio or the portion of the complete circle that the arc takes up. Refer it a slice of pizza – cadets get that analogy.

Review example 1 with cadets. May need to keep reviewing the fraction idea. Have cadets work the Check It Out #1.

Review Example 2. This is another example of why we would need to know the amount of arc for a circle – farming and irrigation. Have cadets work the Check-it-Out #2.

Review Example 4. Be sure to show that the line drawn is the radius but so are the other segments from the center to the points on the circle. Notice that the radius is not squared – Why is this? Have cadets work the Check-It-Out #4.

Guided Practice

Check work from the Check-It-Out problems. If still need some help, go over problems 2, 3, 4, 5, 9, 10 as needed.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 12 – 34

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 35

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 37

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 15, 18, 20, 21, 24 – checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

Problems 12,13,14,16, 19, 20 Extra credit #22

Homework

Page 75 and 159 May be replaced with different work based on needs. Replaced by additional resources listed above.

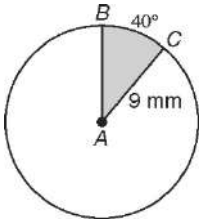
12-3 Practice A

Sector Area and Arc Length

In Exercises 1 and 2, fill in the blanks to complete each formula.

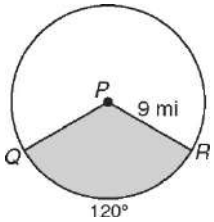
1. The area of a sector of a circle with radius r and central angle $m°$ is $A =$ _____.
2. The length of an arc with central angle $m°$ on a circle with radius r is $L =$ _____.

Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.

3. 

sector *BAC*

_____

4. 

sector *QPR*

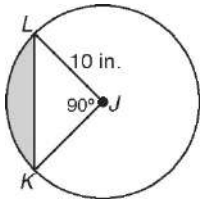
_____

Different animals have different fields of view. Humans can generally see a 180° arc in front of them. Horses can see a 215° arc. A horse and rider are in heavy fog, so they can see for only 25 yards in any direction. Round your answers to Exercises 5 and 6 to the nearest square yard.

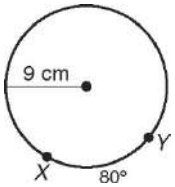
5. Find the area of the rider's field of view. _____
6. Find the area of the horse's field of view. _____

Complete Exercises 7–9 to find the area of segment *KJL*.

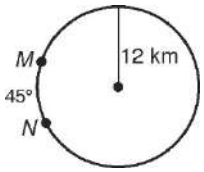
7. Find the area of sector *KJL*. Give your answer in terms of π. _____
8. Find the area of $\triangle KJL$. _____
9. Subtract the area of $\triangle KJL$ from the area of sector *KJL* to find the area of segment *KJL*. Round to the nearest hundredth. _____



Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.

10. 

$\overset{\frown}{XY}$ _____

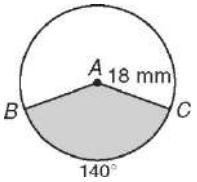
11. 

$\overset{\frown}{MN}$ _____

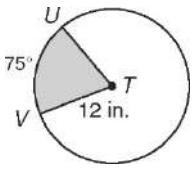
12-3 Practice B

Sector Area and Arc Length

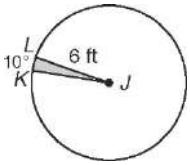
Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.

1. 

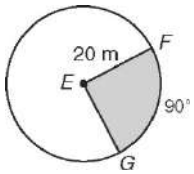
sector *BAC* _____

2. 

sector *UTV* _____

3. 

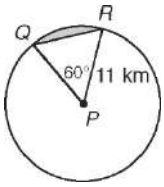
sector *KJL* _____

4. 

sector *FEG* _____

5. The speedometer needle in Ignacio's car is 2 inches long. The needle sweeps out a 130° sector during acceleration from 0 to 60 mi/h. Find the area of this sector. Round to the nearest hundredth. _____

Find the area of each segment to the nearest hundredth.

6. 

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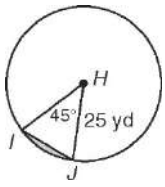
7. 

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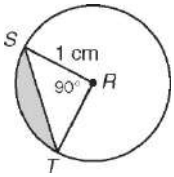
8. 

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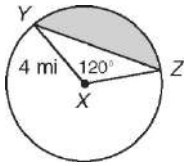
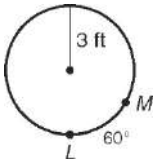
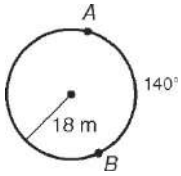
9. 

_____

Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.

10. 

_____

11. 

_____

12. an arc with measure 45° in a circle with radius 2 mi _____

13. an arc with measure 120° in a circle with radius 15 mm _____

12-3 Practice C

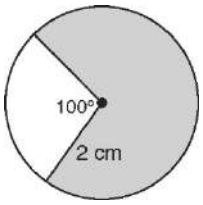
Sector Area and Arc Length

1. Find the measure of a central angle in a circle so that the segment has half the area of the sector. First derive an equation, and then use trial and error to estimate the measure of the central angle to within 1 degree. Explain your answer.

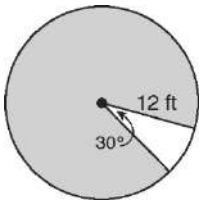
2. The circumference of a circle is 18π m. Find the central angle of a sector of the circle whose area is 40.5π m^2. _____

Find the shaded area of each figure. Round to the nearest hundredth.

3. _____



4. _____



5. Find the measure of the central angle of an arc so that the length of the arc is equal to the radius of the circle. Round to the nearest tenth. Explain your answer.

Angela is wrapping 1 meter of twine around a spool with a 2-centimeter diameter. The spool is thin and accommodates only one wrap of twine before the twine stacks on top of itself. The twine has a diameter of $\frac{1}{2}$ cm.

6. Find how many complete times Angela will wrap the twine around the spool. _____

7. Find the percentage of a complete circle that the last wrapping of the twine will make. Round to the nearest tenth. _____

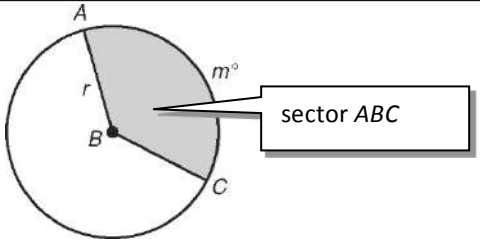
12-3 Reteach

Sector Area and Arc Length

Sector of a Circle

A **sector of a circle** is a region bounded by two radii of the circle and their intercepted arc.

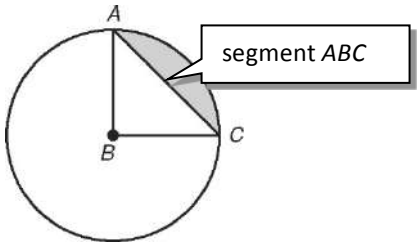
The area of a sector of a circle is given by the formula $A = \pi r^2 \left(\frac{m^\circ}{360^\circ}\right)$.



Segment of a Circle

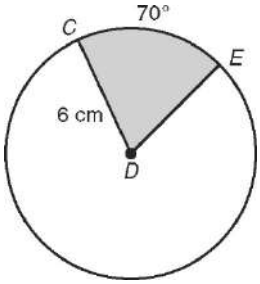
A **segment of a circle** is a region bounded by an arc and its chord.

area of segment *ABC* = area of sector *ABC* − area of ΔABC

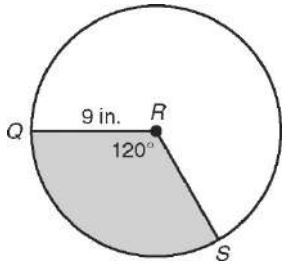


Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.

1. sector *CDE*

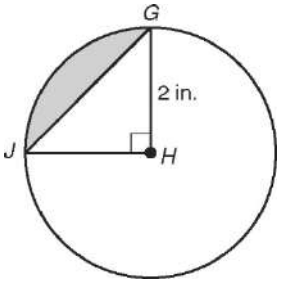


2. sector *QRS*

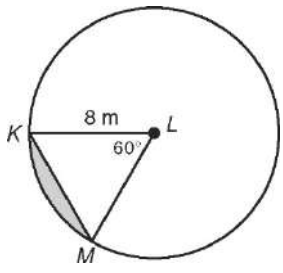


Find the area of each segment to the nearest hundredth.

3.



4.



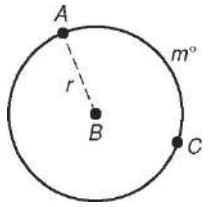
12-3 Reteach

Sector Area and Arc Length *continued*

Arc Length

Arc length is the distance along an arc measured in linear units.

The arc length of a circle is given by the formula $L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$.



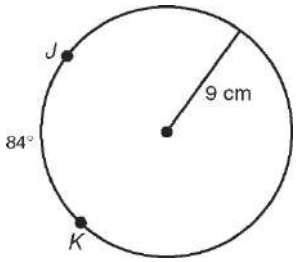
Find the arc length of $\overset{\frown}{JK}$.

$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$ Formula for arc length

$= 2\pi(9\text{ cm})\left(\frac{84^\circ}{360^\circ}\right)$ Substitute 9 cm for r and 84° for $m°$.

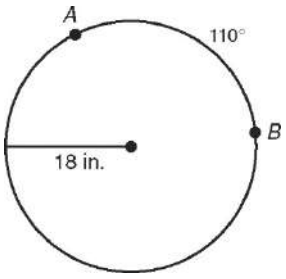
$= \frac{21}{5}\pi\text{ cm}$ Simplify.

≈ 13.19 cm Round to the nearest hundredth.

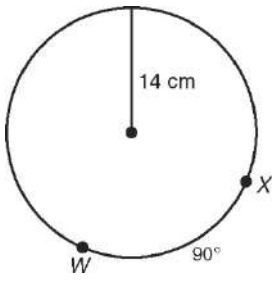


Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.

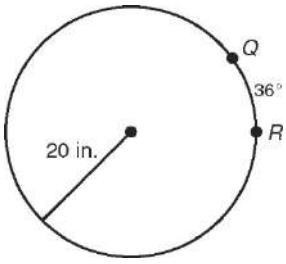
5. $\overset{\frown}{AB}$



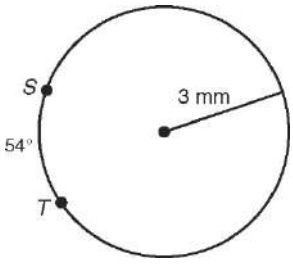
6. $\overset{\frown}{WX}$



7. $\overset{\frown}{QR}$



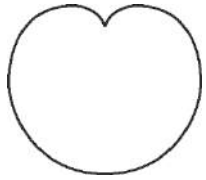
8. $\overset{\frown}{ST}$



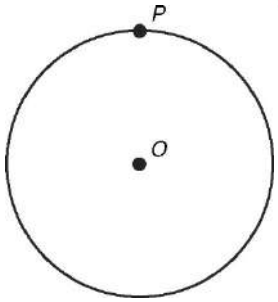
12-3 Challenge

Investigating Cardioids

The unusual curve at right is called a **cardioid.** The cardioid derives its name from its resemblance to the classic shape of a heart. A surprising fact about the cardioid is that it can be generated by constructing a set of circles that satisfies a certain set of conditions.



1. In the figure at right, *P* is a point on circle *O*. Use a compass and straightedge to perform the following construction:

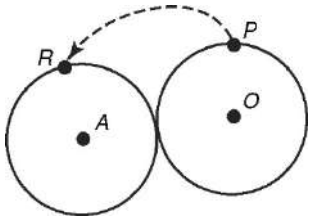


   a. Divide circle *O* into 24 congruent arcs, with point *P* being the common endpoint of two of the arcs. (*Hint:* Begin by constructing six congruent arcs.)

   b. Place the metal tip of the compass at any endpoint of an arc *except point P*. Open the compass so that the pencil tip is at point *P* and draw a circle.

   c. Repeat part **b** for the other 22 points from part **a.** The outline of the figure that results will approximate a cardioid.

2. In the figure at right, circle *A* is congruent to circle *O* and is being rolled counterclockwise around it. Point *R* is on circle *A*; its path is shown by the dashed arrow. After one complete revolution of circle *A*, the path of point *R* will trace a cardioid. The length of this cardioid is a whole-number multiple of the diameter of circle *O*. Make a conjecture about the value of that whole number. (*Hint:* Use your drawing from Exercise 1, or act out the roll, using two coins of the same type.)



_____

_____

3. Shade your cardioid construction from Exercise 1 to create the figure shown at right.

4. Research the meaning of the term *nephroid*. Find out how to construct a nephroid by using circles that satisfy a given set of conditions. Perform the construction on a separate sheet of paper. Also, find how the nephroid compares to the cardioid in terms of rolling one circle around another, and make a report of your findings on a separate sheet of paper.

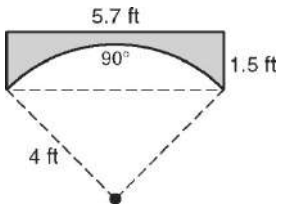
12-3 Problem Solving

Sector Area and Arc Length

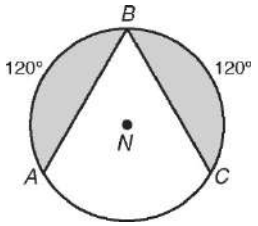
1. A circle with a radius of 20 centimeters has a sector that has an arc measure of 105°. What is the area of the sector? Round to the nearest tenth.

2. A sector whose central angle measures 72° has an area of 16.2π square feet. What is the radius of the circle?

3. The archway below is to be painted. What is the area of the archway to the nearest tenth?

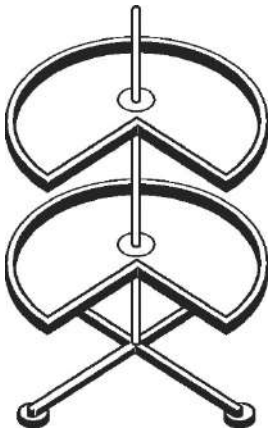


4. Circle *N* has a circumference of 16π millimeters. What is the area of the shaded region to the nearest tenth?



Choose the best answer.

5. The circular shelves in diagram are each 28 inches in diameter. The "cut-out" portion of each shelf is 90°. Approximately how much shelf paper is needed to cover both shelves?



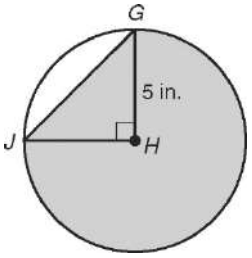
A 154 in^2

B 308 in^2

C 462 in^2

D 924 in^2

6. Find the area of the shaded region. Round to the nearest tenth.



F 8.2 in^2

G 19.6 in^2

H 71.4 in^2

J 78.5 in^2

7. A semicircular garden with a diameter of 6 feet is to have 2 inches of mulch spread over it. To the nearest tenth, what is the volume of mulch that is needed?

A 2.4 ft^3

B 4.8 ft^3

C 14.1 ft^3

D 28.3 ft^3

8. A round cheesecake 12 inches in diameter and 3 inches high is cut into 8 equal-sized pieces. If five pieces have been taken, what is the approximate volume of the cheesecake that remains?

F 42.4 in^3

G 70.7 in^3

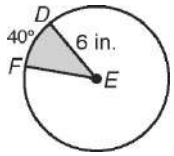
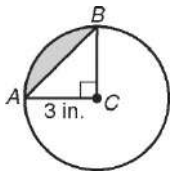
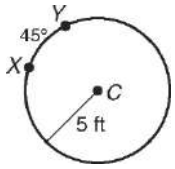
H 127.2 in^3

J 212.1 in^3

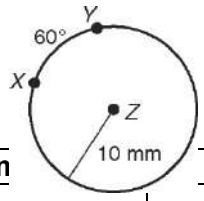
12-3 Reading Strategies

Use a Formula

The table below shows you how to use formulas for sector area and arc length.

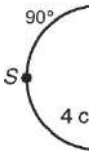
Area of a Sector	Area of a Segm	Arc Length
$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right)$	$A = \pi r^2\left(\frac{m^\circ}{360^\circ}\right) - \frac{1}{2}bh$	$L = 2\pi r\left(\frac{m^\circ}{360^\circ}\right)$
		
Find the area of sector *DEF*.	Find the area of segment *ACB*.	Find the length of $\overset{\frown}{XY}$.
$A = \pi(6)^2\left(\frac{40^\circ}{360^\circ}\right)$	$A = \pi(3)^2\left(\frac{90^\circ}{360^\circ}\right) - \frac{1}{2}(3)(3)$	$L = 2\pi(5)\left(\frac{45^\circ}{360^\circ}\right)$
$= 36\pi\left(\frac{1}{9}\right)$	$= 9\pi\left(\frac{1}{4}\right) - 4.5$	$= 10\pi\left(\frac{1}{8}\right)$
$= 4\pi \text{ cm}^2$	$= 2.25\pi - 4.5$	$= \frac{5\pi}{4} \text{ ft}$
$\approx 12.57 \text{ cm}^2$	$\approx 2.57 \text{ in}^2$	$\approx 3.93 \text{ ft}$

Find each arc length. Give your answer in terms of π and rounded to the nearest hundredth.



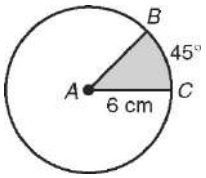
5. $\overset{\frown}{XY}$

_____



_____

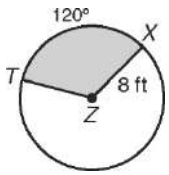
Find the area of each sector. Give your answer in terms of π and rounded to the nearest hundredth.



1. sector *BAC*

_____

_____

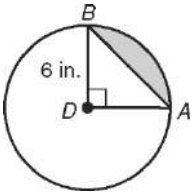


2. sector *TZX*

_____

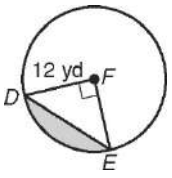
_____

Find the area of each segment. Round your answer to the nearest hundredth.



3. segment *BDA*

_____



4. segment *DFE*

_____

Geometry

Chapter 12: Circle

Lesson 4 - Inscribed Angles

Standards

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.). Copying a segment; copying an angle; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G.C.3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.CO.13 – Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.

Objectives

Find the measure of an inscribed angle.

Use inscribed angles and their properties to solve problems.

Materials

ToolBox, Pencil, Paper, Text Book, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

Inscribed angle – an angle whose vertex is on a circle and whose sides contain chords of the circle. The measure of the angle is half the measure of the arc.

Intercepted arc – consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them.

Subtends – a chord or arc will subtend an angle if its endpoints lie on the sides of the angle.

Formulas

Inscribed angle – m<ABC = ½ m of arc AC

Symbols

None

Postulates

None

Theorems

12-4-1 - Inscribed angle theory - The measure of an inscribed angle is half the measure of its intercepted arc.

12-4-2 – If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent.

12-4-3 - An inscribed angle subtends a semicircle if and only if the angle is a right angle.

12-4-4 - If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Who Uses This?

You can use inscribed angles to find measures of angles in string art.

Discussion

Update ToolBox with vocabulary, theorems and formulas.

Work through example 1. Be real careful in helping cadets find the arc that is listed. Remind them that multiplying by ½ is the same as dividing by 2. Have cadets work Check-It-Out 1 and monitor cadets carefully. Review the answers.

Work through example 2. Before working through it, discuss the picture and the way the circle is represented. Have cadets work the Check-It-Out 2 and monitor carefully. This picture confuses many cadets so make sure each has the proper perspective on the picture.

Example 3 involves variables so make sure to review how to solve equations. Work through the example making sure cadets follow which angle is being used. Make clear the difference between finding the value of the variable and finding the measure of the angle. The values of the variable will have no units on it. The measure of the angle will have the units of degrees. Have cadets work on the Check-It-Out #3.

Example 4 is good for working more with solving equations. Stress once again that when working the problem, they are a where of which angle they are using and which they are solving for. Review Check-It-Out 4.

Guided Practice

Have cadets select from problems 2-11 that they would like to see worked through. Pick one from each example section and work through them. Cadets that have it may start on the classwork.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better

understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 12 – 30, 33, 35, 36, 39 – 42

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 44

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 47

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 13, 16, 18, 22, 26 – checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

Homework

Page 76 and 160 May be replaced with different work based on needs. Replaced by additional resources listed above.

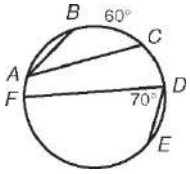
12-4 Practice A

Inscribed Angles

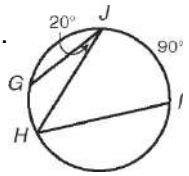
In Exercises 1–4, fill in the blanks to complete each theorem.

1. If a quadrilateral is inscribed in a circle, then its opposite angles are _____.
2. If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are _____.
3. The measure of an inscribed angle is _____ the measure of its intercepted arc.
4. An inscribed angle subtends a semicircle if and only if the angle is a _____.

Find each measure.

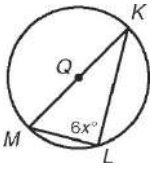
5. 

m∠*BAC* = _____
m$\overset{a}{FE}$ = _____

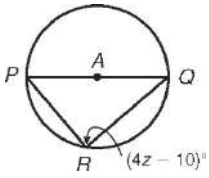
6. 

m∠*IHJ* = _____
m$\overset{o}{GH}$ = _____

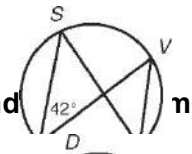
Find each value.

7. 

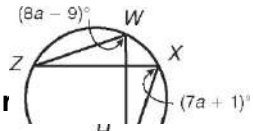
x = _____

8. 

z = _____

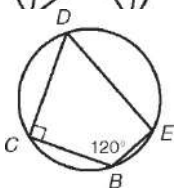
9. 

m∠*VUS* = _____

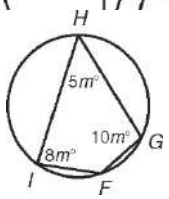
10. 

m∠*ZWY* = _____

Find measures of each inscribed quadr

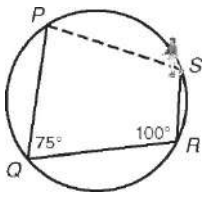
11. 

m∠*B* = _____
m∠*C* = _____
m∠*D* = _____
m∠*E* = _____

12. 

m∠*F* = _____
m∠*G* = _____
m∠*H* = _____
m∠*I* = _____

13. Iyla has not learned how to stop on ice skates yet, so she just skates straight across the circular rink until she hits a wall. She starts at *P*, turns 75° at *Q*, and turns 100° at *R*. Find how many degrees Iyla will turn at *S* to get back to her starting point.

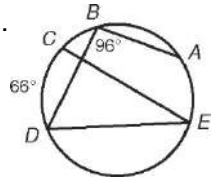


_____

12-4 Practice B

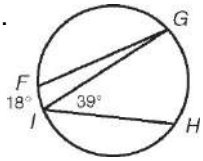
Inscribed Angles

Find each measure.

1. 

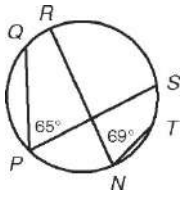
m∠*CED* = _____

m$\overset{\frown}{DEA}$ = _____

2. 

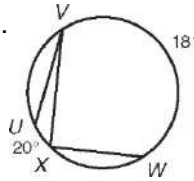
m∠*FGI* = _____

m$\overset{\frown}{GH}$ = _____

3. 

m$\overset{\frown}{QRS}$ = _____

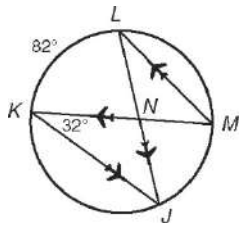
m$\overset{\frown}{TSR}$ = _____

4. 

m∠*XVU* = _____

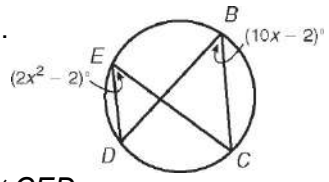
m∠*VXW* = _____

5. A circular radar screen in an air traffic control tower shows these flight paths. Find m∠*LNK*.

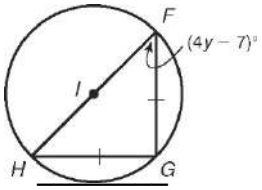


_____

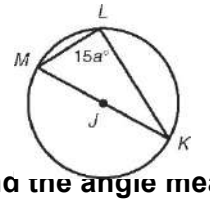
Find each value.

6. 

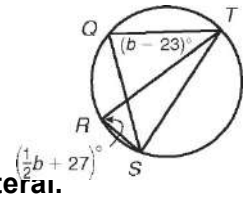
m∠*CED* = _____

7. 

_____

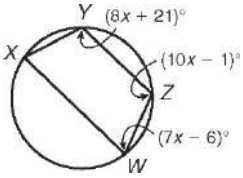
8. 

a = _____

9. 

m∠*SRT* = _____

Find the angle measures of each inscribed quadrilateral.

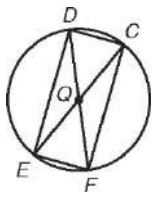
10. 

m∠*X* = _____

m∠*Y* = _____

m∠*Z* = _____

m∠*W* = _____

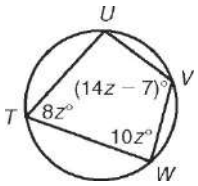
11. 

m∠*C* = _____

m∠*D* = _____

m∠*E* = _____

m∠*F* = _____

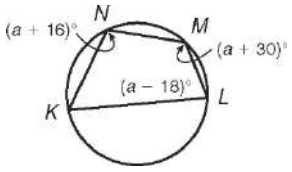
12. 

m∠*T* = _____

m∠*U* = _____

m∠*V* = _____

m∠*W* = _____

13. 

m∠*K* = _____

m∠*L* = _____

m∠*M* = _____

m∠*N* = _____

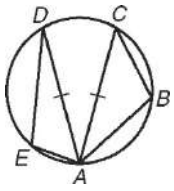
12-4 Practice C

Inscribed Angles

Write paragraph proofs for Exercises 1 and 2.

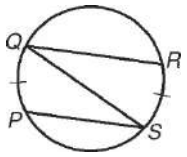
1. **Given:** $\overline{AC} \cong \overline{AD}$

   Prove: $\angle ABC \cong \angle AED$



2. **Given:** $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

   Prove: $\overline{QR} \parallel \overline{PS}$



For each quadrilateral described, tell whether it can be inscribed in a circle. If so, describe a method for doing so using a compass and straightedge, and draw an example.

3. a parallelogram that is not a rectangle or a square

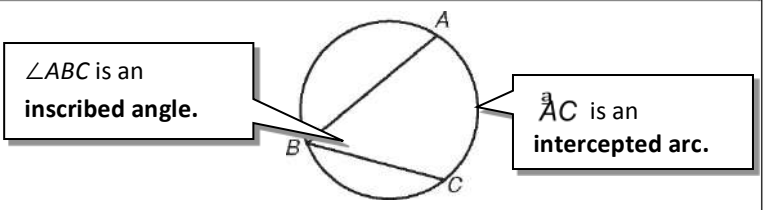
4. a kite

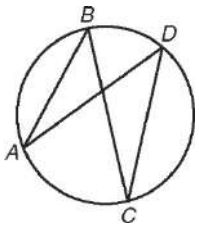
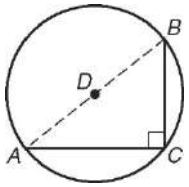
5. a trapezoid

6. a rhombus that is not a square

12-4 Reteach

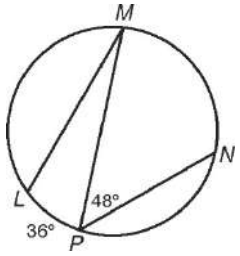
Inscribed Angles

Inscribed Angle Theorem	
The measure of an inscribed angle is half the measure of its intercepted arc.	 $m\angle ABC = \frac{1}{2}m\overset{a}{AC}$

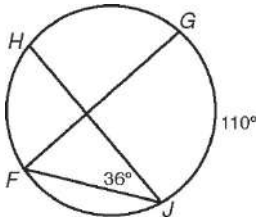
Inscribed Angles	
If inscribed angles of a circle intercept the same arc, then the angles are congruent. $\angle ABC$ and $\angle ADC$ intercept $\overset{a}{AC}$, so $\angle ABC \cong \angle ADC$.	
An inscribed angle subtends a semicircle if and only if the angle is a right angle.	

Find each measure.

1. $m\angle LMP$ and $m\overset{o}{MN}$

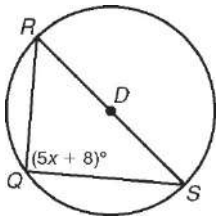


2. $m\angle GFJ$ and $m\overset{a}{FH}$

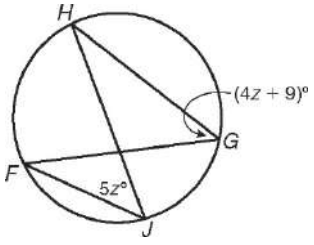


Find each value.

3. x

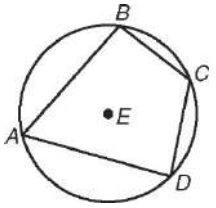


4. $m\angle FJH$



12-4 Reteach

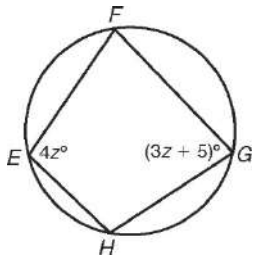
Inscribed Angles *continued*

Inscribed Angle Theorem		
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.		∠A and ∠C are supplementary. ∠*B* and ∠*D* are supplementary. *ABCD* is inscribed in ⊙ *E*.

Find m∠*G*.

Step 1 Find the value of *z*.

m∠*E* + m∠*G* = 180°	*EFGH* is inscribed in a circle.
4*z* + 3*z* + 5 = 180	Substitute the given values.
7*z* = 175	Simplify.
z = 25	Divide both sides by 7.



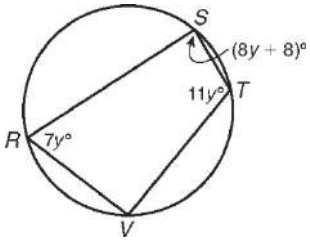
Step 2 Find the measure of ∠*G*.

m∠*G* = 3*z* + 5

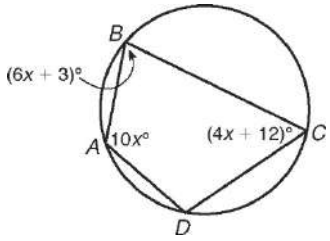
= 3(25) + 5 = 80° Substitute 25 for *z*.

Find the angle measures of each quadrilateral.

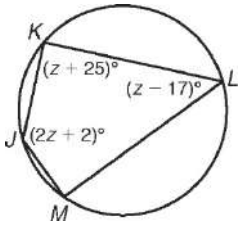
5. *RSTV*



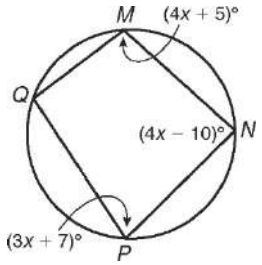
6. *ABCD*



7. *JKLM*



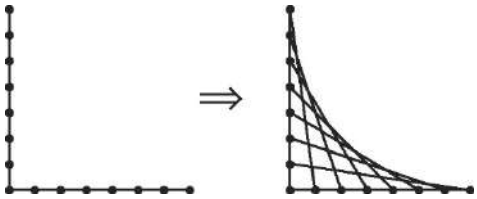
8. *MNPQ*



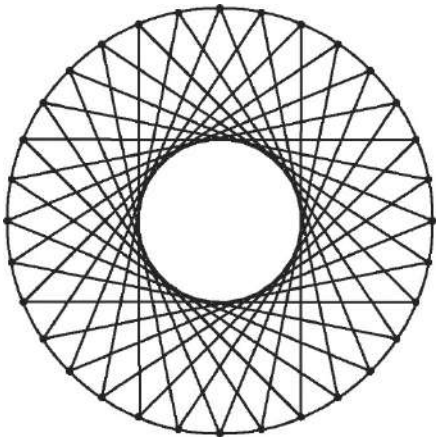
12-4 Challenge

Line Designs

Line designs are sets of segments arranged in a pattern that creates the illusion of a curve. Different figures can be used as the basis of a line design. For instance, the simple line design at right is formed by connecting equally spaced *anchor points* on the sides of a right angle. On this page, you will investigate line designs that are based on circles.



In the design at right, 32 anchor points are equally spaced on a circle. Segments connect each anchor point to the two anchor points that are exactly 12 spaces away from it. There is an illusion of a circle within the circle.



1. Fill in the blanks to make a true statement:

   Each segment that joins two anchor points is a(n) _____of the circle. Two segments that share a common anchor point form a(n) _____ angle of the circle.

2. Let x represent the degree measure of each angle formed by two segments with a common endpoint.

   a. What is the value of x in the figure above? _____

   b. Suppose that the segments connected each anchor point to the two anchor points exactly 10 spaces away from it. What is value of x? _____

   c. Let n represent the number of spaces between two anchor points that are connected by a segment. Write an expression for x. (*Hint:* Be sure to account for values of n that are greater than 16.) _____

   d. What restrictions must be placed on the value of n in part **c**? _____

   e. Let p represent the number of equally spaced anchor points, and let n represent the number of spaces between two anchor points connected by a segment. Write an expression for x. State any restrictions on n.

   _____

   f. Find values of n and p, $p \neq 32$, for which $x = 30$.

   _____

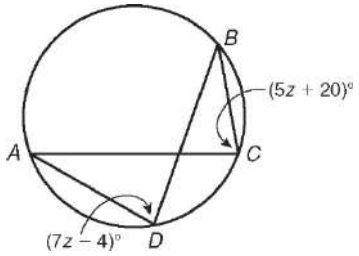
   _____

3. The type of line design that you investigated on this page is just one of countless types of line designs based on circles. Using the library or the Internet as a resource, find a different type of circle line design and re-create it on a separate paper, using only a compass and straightedge.

12-4 Problem Solving

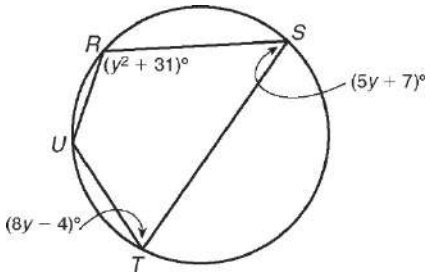
Inscribed Angles

1. Find $m\overset{\frown}{AB}$.



_____

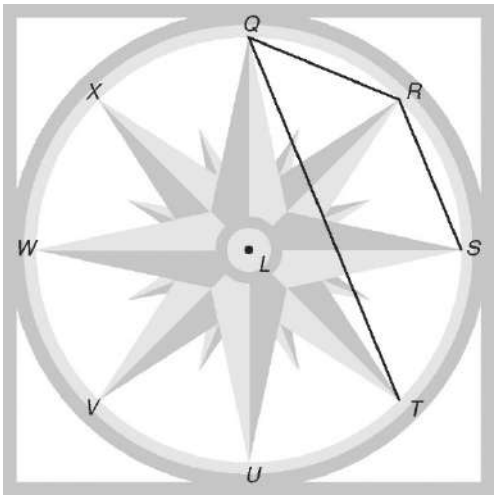
2. Find the angle measures of *RSTU*.



_____

Choose the best answer.

Use the diagram of a floor tile for Exercises 3 and 4. Points *Q*, *R*, *S*, *T*, *U*, *V*, *W*, and *X* are equally spaced around ⊙ *L*.



3. Find $m\angle RQT$.

   A 15° C 45°

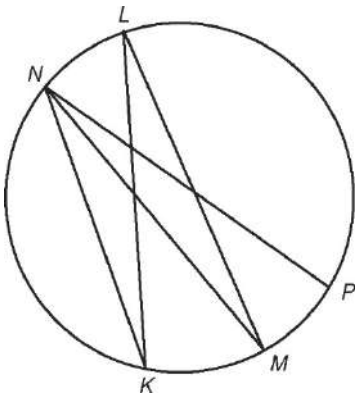
   B 30° D 60°

4. Find $m\angle QRS$.

   F 67.5° H 180°

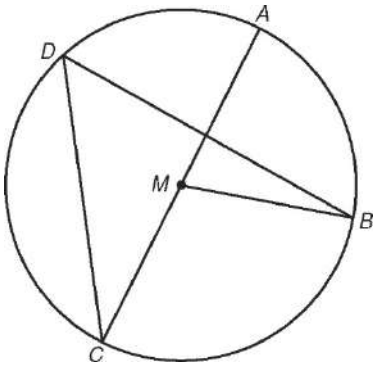
   G 135° J 270°

5. If $m\angle KLM = 20°$ and $m\overset{\circ}{MP} = 30°$, what is $m\angle KNP$?



A 25°
B 35°
C 50°
D 70°

6. In $\odot M$, $m\angle AMB = 74°$. What is $m\angle CDB$?

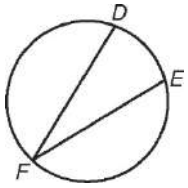
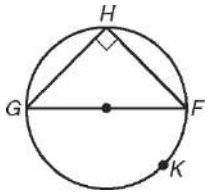
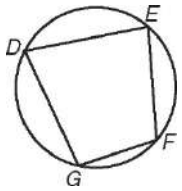


F 37°
G 53°
H 74°
J 106°

12-4 Reading Strategies

Use a Table

The table below shows properties of inscribed angles.

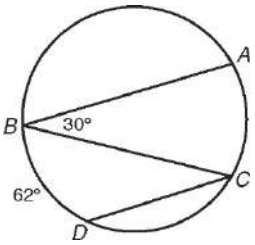
Words	Diagram	Mathematical Symbols
The measure of an angle inscribed in a circle is half the measure of the intercepted arc.		$m\angle DFE = \frac{1}{2}m\overset{\frown}{DE}$
An inscribed angle intercepts a semicircle if and only if the angle is a right angle.		$m\angle GHF = 90°$ $\overset{\frown}{GKF}$ is a semicircle (180°)
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.		$m\angle D + m\angle F = 180°$ $m\angle E + m\angle G = 180°$

Answer the following.

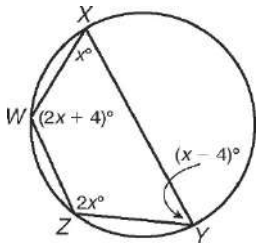
1. The measure of an angle inscribed in a circle is _____ the measure of the intercepted arc.
2. Quadrilateral *ABCD* is inscribed in a circle. Write two equations that show the relationships of the angles of the quadrilateral.

_____

Find each measure.



3. $m\angle BCD$ _____
4. $m\overset{\frown}{AC}$ _____



5. $m\angle W$ _____
6. $m\angle X$ _____
7. $m\angle Y$ _____
8. $m\angle Z$ _____

Geometry

Chapter 12: Circle

Lesson 5 - Angle Relationships in Circles

Standards

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Objectives

Find the measures of angles formed by lines that intersect circles.

Use angle measures to solve problems.

Materials

ToolBox, Pencil, Papers, Text Book, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

None

Formulas

m<AC = 1/2m of arc AB (B is tangent point) (on a circle)

m<1 = ½(m of arc AB + m of arc CD) (AD and BC are secants) (inside a circle)

m<1 = ½(m of arc AD – m of arc BD) (tangent and secant) (outside a circle)

Symbols

None

Postulates

None

Theorems

12-5-1 – If a tangent and a secant or chord intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.

12-5-2 – If two secants or chords intersect in the interior of a circle, then the measure of each angle formed is half the sum of the measures of its intercepted arcs.

12-5-3 – If a tangent and a secant, two tangents, or two secants intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.

Who Uses This?

Circles and angles help optometrists correct vision problems.

Discussion

ToolBox Update: Make a new page and label it 'Angle Relationships in Circles'. Have cadets write the chart in their ToolBox – this is the best example of the three angles that we will be looking at.

Review the first theorem and work through Example 1. This should be review so cadets should be able to work through Check-It-Out 1. The second theorem is also a good review as Example 2 is gone over. Cadets should be able to do the Check-It-Out #2 with any problem. Review quickly with them. The third theorem is confusing as presented in the book because it looks like there are three formulas to use. There is only one so help cadets see that there is in fact only one formula. Work through the Example 3. Cadets should have no problem since there is only one formula. Have them work through Check-It-Out #3.

Example 5 is a good example for having cadets pick out different arcs and different angles. Have cadets pick out the arcs and angles but do not necessarily work through the problem.

Guided Practice

Work problems 1 – 15 with cadets. Have them pick one from each example that they would like to see worked through.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 16 – 34, 39 – 44

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 16 – 45

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 16 – 48

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 16, 20, 24, 26, 28, 30– checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

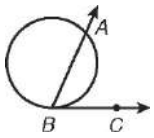
Homework

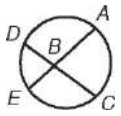
Page 77 and 161 May be replaced with different work based on needs. Replaced by additional resources listed above.

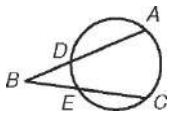
12-5 Practice A

Angle Relationships in Circles

In Exercises 1–3, match the letter of the drawing to the formula for finding the measure of the angle.

1. $m\angle ABC = \frac{1}{2}\left(m\overset{a}{A}C + m\overset{a}{D}E\right)$ _____ A. 

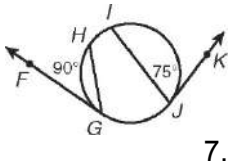
2. $m\angle ABC = \frac{1}{2}\left(m\overset{a}{A}C - m\overset{a}{D}E\right)$ _____ B. 

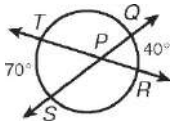
3. $m\angle ABC = \frac{1}{2}m\overset{a}{A}B$ _____ C. 

Find each measure.

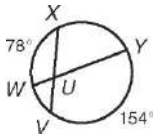
4. $m\angle FGH =$ _____

5. $m\angle \overset{\frown}{IJ} =$ _____



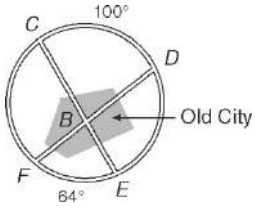
6. 

$m\angle QPR =$ _____

7. 

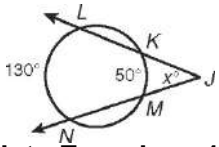
$m\angle YUV =$ _____

8. Some cities in Europe are thousands of years old. Often the small center of the old city is surrounded by a newer "ring road" that allows traffic to bypass the old streets. The figure shows a circular ring road and two roads that provide access to the old city. Find $m\angle CBD$.

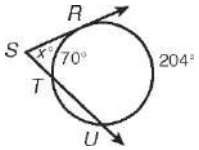


_____

Find the value of *x*.

9. 

_____

10. 

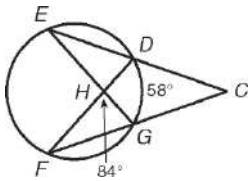
_____

Complete Exercises 11–13 in order to find $m\angle ECF$.

11. Find $m\angle DHG$. (*Hint:* $\overline{DF}$ is a straight segment.) _____

12. Find $m\overset{a}{E}F$. _____

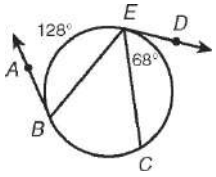
13. Find $m\angle ECF$. _____



12-5 Practice B

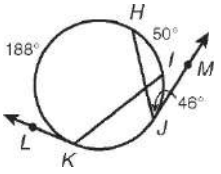
Angle Relationships in Circles

Find each measure.

1. 

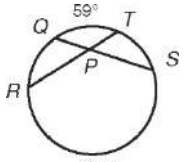
m∠*ABE* = _____

m$\overset{\frown}{BC}$ = _____

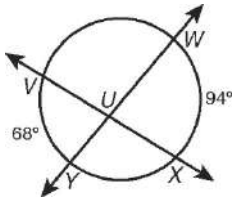
2. 

m∠*LKI* = _____

m$\overset{\frown}{IJ}$ = _____

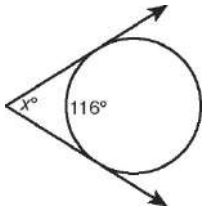
3. 

m∠*RPS* = _____

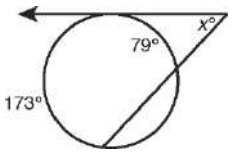
4. 

m∠*YUX* = _____

Find the value of *x*.

5. 

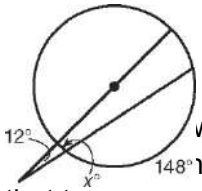
_____

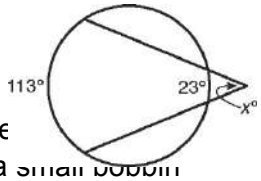
6. 

_____

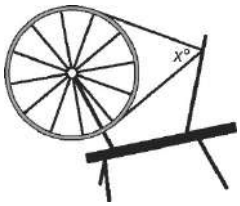
7. _____

8. _____

9. 

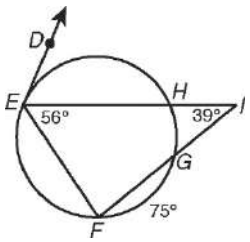


ws a spinning wheel. The large whe
n a foot trundle. A belt attaches to a small bobbin that turns very quickly. The bobbin twists raw materials into thread, twine, or yarn. Each pair of spokes intercepts a 30° arc. Find the value of *x*.



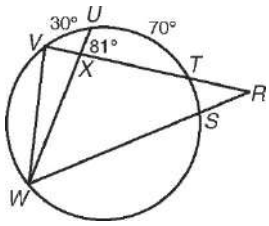
_____

Find each measure.

10. 

m∠*DEI* = _____

m$\overset{\frown}{EF}$ = _____

11. 

m∠*WVR* = _____

m$\overset{\frown}{TUW}$ = _____

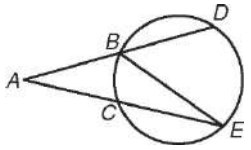
12-5 Practice C

Angle Relationships in Circles

Write paragraph proofs for Exercises 1–3.

1. **Given:** $\overline{AB} \cong \overline{EB}$

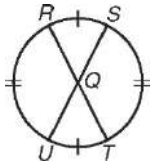
   Prove: $m\overset{\frown}{DE} = 2m\overset{\frown}{BC}$



2. **Given:** $\overset{\frown}{RS} \cong \overset{\frown}{TU}, \overset{\frown}{RU} \cong \overset{\frown}{ST}$

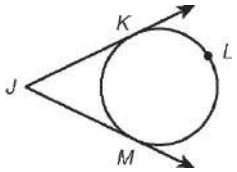
   Prove: Q is the center of the circle.

   (*Hint:* Show that Q is equidistant from three points on the circle.)



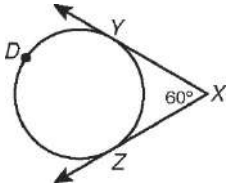
3. **Given:** $\overset{\longleftrightarrow}{JK}$ and $\overset{\longleftrightarrow}{JM}$ are tangent to the circle.

   Prove: $m\overset{\frown}{KM} < 180°$ (*Hint:* Use an indirect proof and consider two cases.)

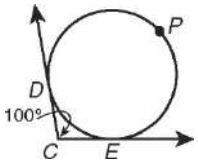


Find the measure of the minor arc intercepted by the two tangents.

4. _____

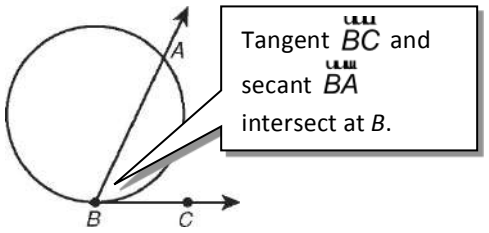
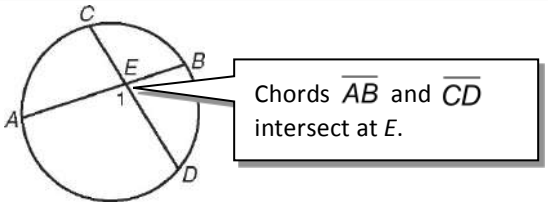


5. _____



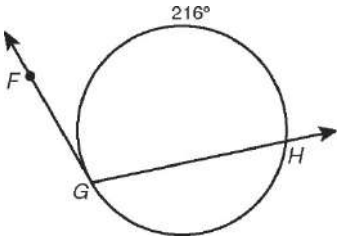
12-5 Reteach

Angle Relationships in Circles

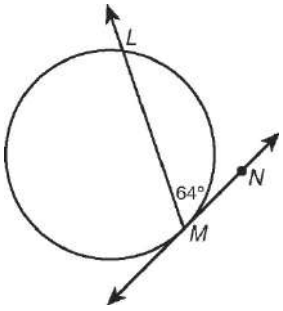
If a tangent and a secant (or chord) intersect on a circle at the point of tangency, then the measure of the angle formed is half the measure of its intercepted arc.	 $m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$
If two secants or chords intersect in the interior of a circle, then the measure of the angle formed is half the sum of the measures of its intercepted arcs.	 $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AD} + m\overset{\frown}{BC}\right)$

Find each measure.

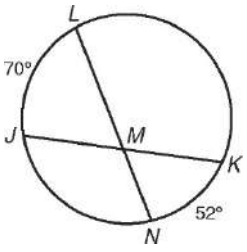
1. m∠*FGH*



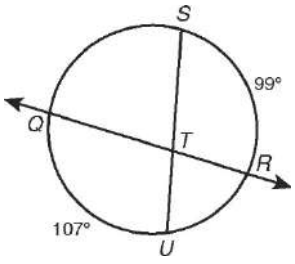
2. $m\overset{\frown}{LM}$



3. m∠*JML*



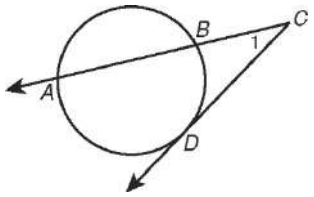
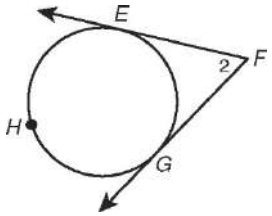
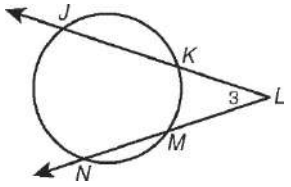
4. m∠*STR*



12-5 Reteach

Angle Relationships in Circles *continued*

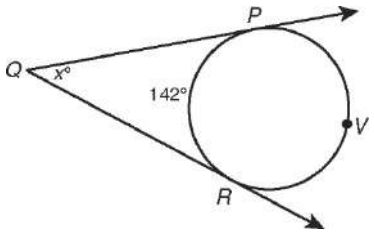
If two segments intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of its intercepted arcs.

A Tangent and a Secant	Two Tangents	Two Secants
		
$m\angle 1 = \frac{1}{2}\left(m\overset{a}{AD} - m\overset{a}{BD}\right)$	$m\angle 2 = \frac{1}{2}\left(m\overset{o}{EHG} - m\overset{a}{EG}\right)$	$m\angle 3 = \frac{1}{2}\left(m\overset{a}{JN} - m\overset{o}{KM}\right)$

Find the value of *x*.

Since $m\overset{o}{PVR} + m\overset{a}{PR} = 360°$, $m\overset{o}{PVR} + 142° = 360°$,

and $m\overset{o}{PVR} = 218°$.



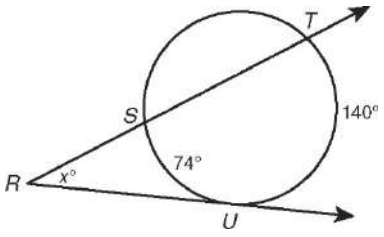
$$x° = \frac{1}{2}\left(m\overset{o}{PVR} - m\overset{a}{PR}\right)$$

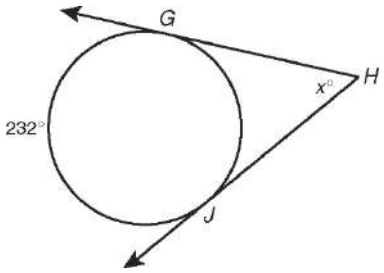
$$= \frac{1}{2}(218° - 142°)$$

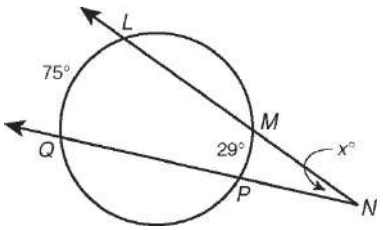
$$x° = 38°$$

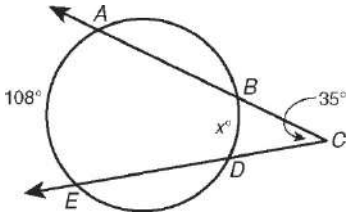
$$x = 38$$

Find the value of *x*.

5. 

6. 

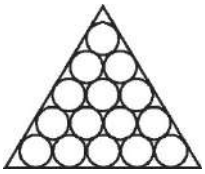
7. 

8. 

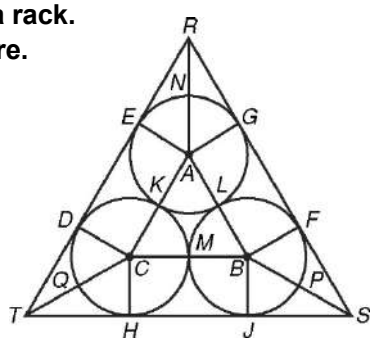
12-5 Challenge

Racking Billiard Balls

A regulation pocket billiard ball is a perfect sphere with a diameter of 2.25 inches, and a tolerance of 0.005 inch. At the start of a game of pocket billiards, the 15 balls must be arranged in five rows in a triangular rack as shown at right. On this page, you will see how the properties of circles determine the shape and size of the rack.



Below at right is a figure depicting just two rows of billiard balls in a rack. On a separate sheet of paper, justify each statement about this figure.



1. $\triangle ABC$ is an equilateral triangle.
2. $ACDE$ is a rectangle.
3. $m\overset{a}{KL} = 60°$
4. $m\overset{a}{EK} = m\overset{a}{GL} = 90°$
5. $m\overset{a}{ENG} = 120°$
6. $m\angle ERG = 60°$
7. $\triangle AER \cong \triangle AGR$
8. $m\angle ERA = 30°$
9. $AC = 2.25$ inches
10. $ED = 2.25$ inches
11. $AE = 1.125$ inches
12. $ER = (1.125)\sqrt{3}$ inches
13. $TD = (1.125)\sqrt{3}$ inches
14. $TR = \left(2.25 + 2.25\sqrt{3}\right)$ inches ≈ 6.1 inches

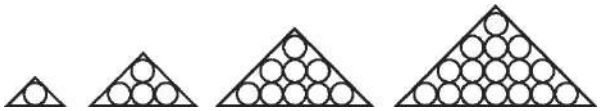
Following similar reasoning, $m\angle HTD = m\angle FSJ = 60°$ and $RS = ST \approx 6.1$ inches. So a triangular rack for two rows of pocket billiard balls would be an equilateral triangle with sides that are each slightly longer than 6.1 inches in length.

Suppose that a rack shaped like an equilateral triangle encloses the given number of pocket billiard balls. Find the length of each side of the rack. (*Hint:* How many rows of balls will there be?)

15. 6 balls _____
16. 10 balls _____
17. 15 balls _____
18. Write an expression for the length, in inches, of each side of the equilateral triangular rack that would enclose n rows of pocket billiard balls in the manner shown above.

    _____

19. Suppose that billiard balls of diameter d inches were racked in the pattern shown at right. Describe the rack that would enclose n rows of billiard balls in this way.



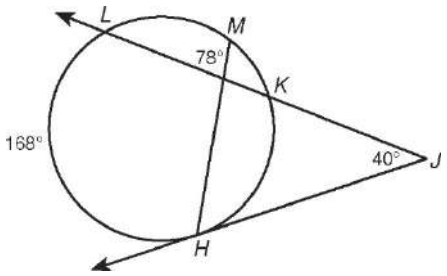
    _____

    _____

12-5 Problem Solving

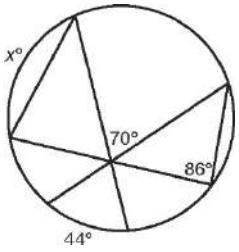
Angle Relationships in Circles

1. What is $m\overset{\frown}{LM}$?



_____

2.An artist painted the design shown below. What is the value of *x*?



_____

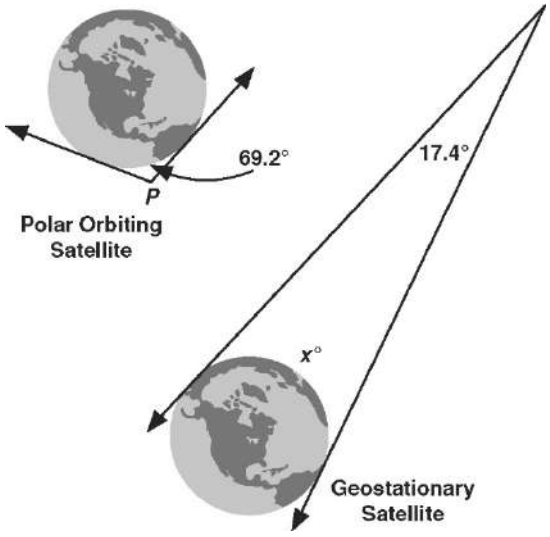
For Exercises 3 and 4, use the diagrams.

3. A polar orbiting satellite is about 850 kilometers above Earth. About 69.2 arc degrees of the planet are visible to a camera in the satellite. What is m∠*P*?

_____

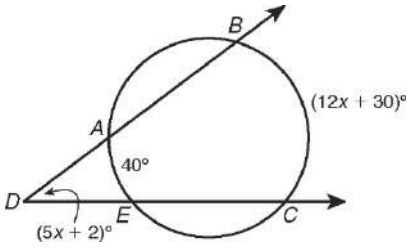
4. A geostationary satellite is about 35,800 kilometers above Earth. How many arc degrees of the planet are visible to a camera in the satellite?

_____



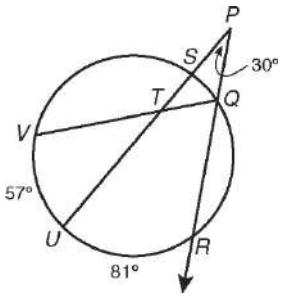
Choose the best answer.

5. What is m∠*ADE*?



A 7°
B 33°
C 37°
D 114°

6. Find m∠*VTU*.

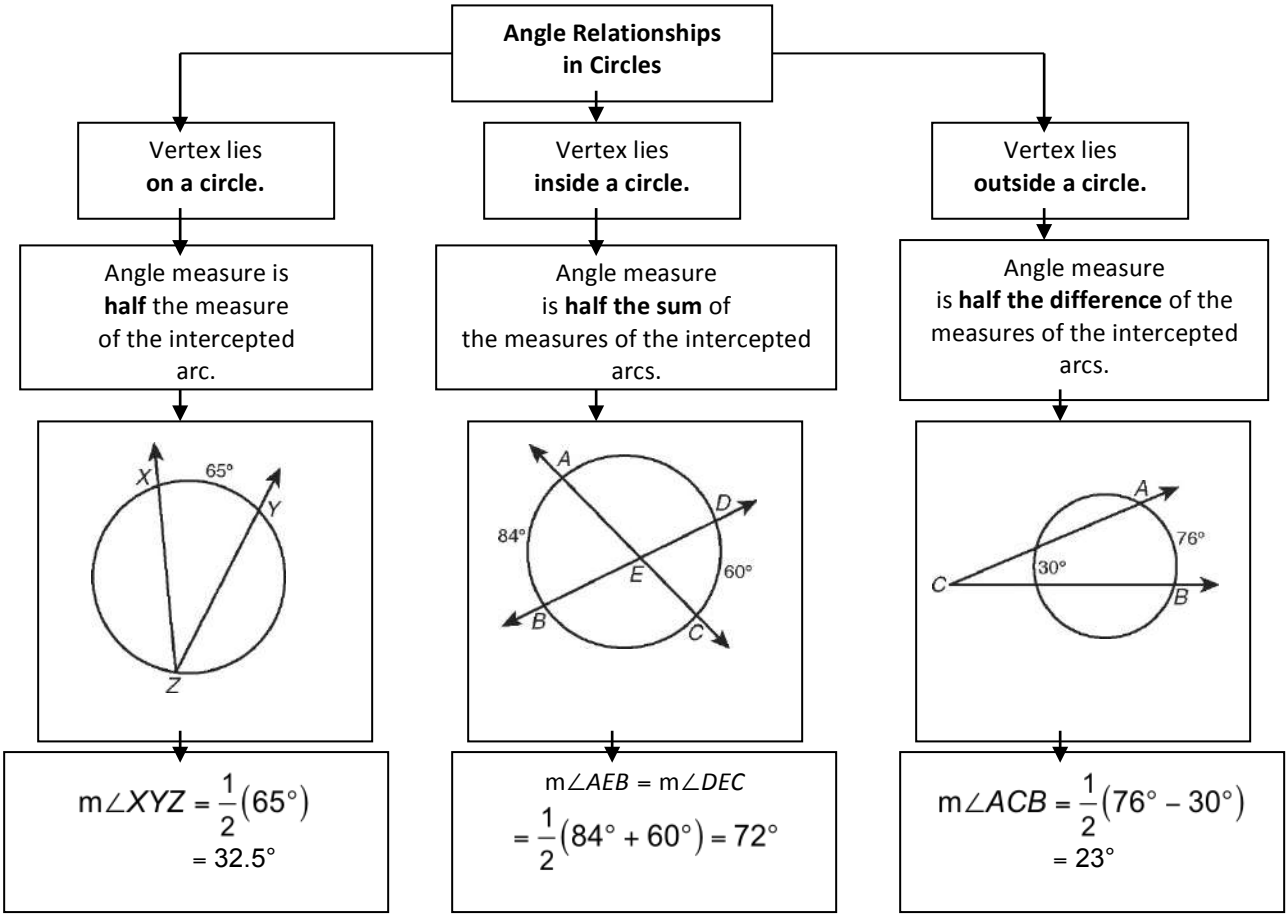


F 21°
G 29°
H 36°
J 39°

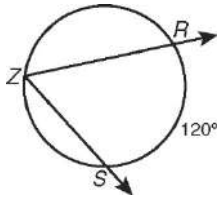
12-5 Reading Strategies

Use a Graphic Aid

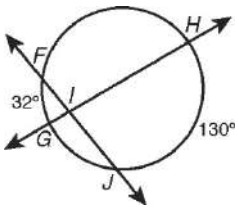
The graphic aid below summarizes angle relationships in circles.



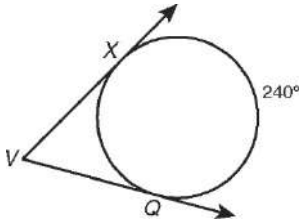
Find each measure.



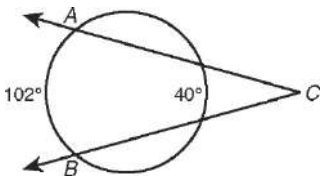
1. $m\angle RZS$ _____



2. $m\angle HIJ$ _____



3. $m\angle XVQ$ _____



4. $m\angle ACB$ _____

Geometry

Chapter 12: Circle

Lesson 6 - Segment Relationships in Circles

Standards

G.C.2 - Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Objectives

Find the lengths of segments formed by lines that intersect circles.

Use the lengths of segments in circles to solve problems.

Materials

ToolBox, pencil, paper, Text Book, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

Secant segment – A segment of a secant with at least one endpoint on the circle.

External secant segment - A segment of a secant that lies in the exterior of the circle with one endpoint on the circle.

Tangent Segment – A segment of a tangent with one endpoint on the circle.

Formulas

Two chords – AE X EB + CE X ED Chords AB and CD intersect at E

Two secants - AE X BE = CE X DE Secants AE and CE intersect at E

Secant and tangent – AC X BC = DC^2 – Secant AC and tangent DC

Symbols

None

Postulates

None

Theorems

12-6-1 – Chord-Chord Product Theorem – If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.

12-6-2 – Secant-Secant product Theorem – If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

12-6-3 - Secant-Tangent Product Theorem – If a secant and a tangent intersect in the exterior of a circle, then the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared.

Who Uses This?

Archaeologists use facts about segments in circles to help them understand ancient objects.

Discussion

Cadets have a difficult time with this lesson because there is a switch from angles to segments. Take time to be sure cadets have made that transition.

Present the first theorem and have cadets write it in their ToolBox. Work through the example 1 problem with cadets making sure that they are looking at the lengths of the segments and not the angles. Have cadets work through Check-It-Out #1.

Example #2 is great for giving cadets an example of how to use this theorem in the real world. Work through the example and then have them try the Check-It-Out #2.

Look at the next theorem and be sure that cadets understand that secants can extend outside the circle. Some have the impression that secants cannot intersect, so make sure everyone understands the model with the secants. Work through Example #3. Change the numbers in the problem and work another example of this theorem before having cadets work Check-It-Out #3.

The next theorem is the secant and a tangent. Write the theorem in the ToolBox and then discuss the example. Make sure to help cadets see the difference between each example in the way the secant, chord and tangent look when involved with the circle. Work example #4. May need to review once more about finding the square root to get the final answer. Have cadets work Check-it-Out #4.

Guided Practice

Pick some problems from 2-11 and work them until cadets feel they have the process and then they can work on the classwork. Number 5 is a great problem to work through.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 12 – 28, 32 – 35

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 12 – 36

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 12 – 36, 28 – 39

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 12, 14, 15, 16, 20, 22– checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

Homework

Page 78 and 162 May be replaced with different work based on needs. Replaced by additional resources listed above.

12-6 Practice A

Segment Relationships in Circles

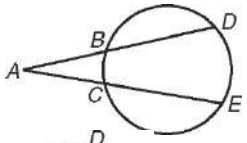
In Exercises 1–3, match the letter of the drawing to the formula that relates the lengths of the segments in the drawing.

1. $AC^2 = AB(AD)$ _____

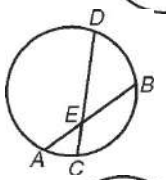
2. $AE(BE) = CE(DE)$ _____

3. $AB(AD) = AC(AE)$ _____

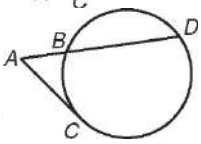
A.



B.

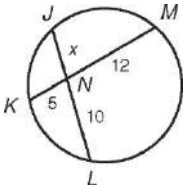


C.



Find the value of the variable and the length of each

4.



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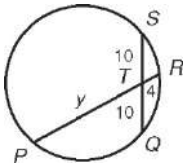
5.



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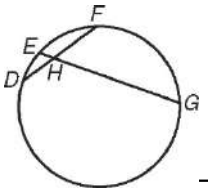
6. Henri is riding a carousel at an amusement park. Devon, Emile, Francis, and George are looking on from around the edge of the carousel. At the moment shown in the figure, Devon is 2.5 meters from Henri, Emile is 1 meter from Henri, and Francis is 3.5 meters from Henri. Find the distance from Emile to George.



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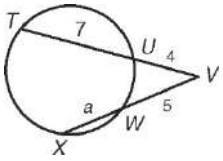
Find the value of the variable and the length of each secant segment.

7.



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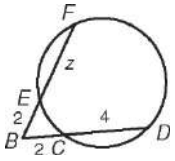
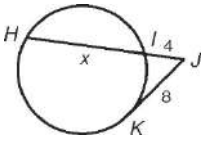
8.



_____

Find the value of the variable.

9.



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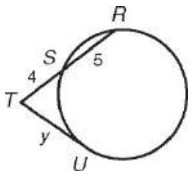
10.



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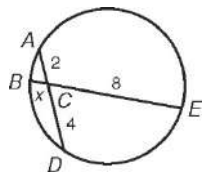
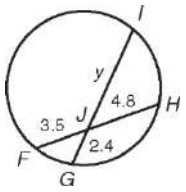
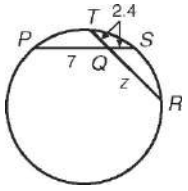
12-6 Practice B

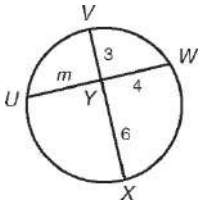
Segment Relationships in Circles

Find the value of the variable and the length of each chord.

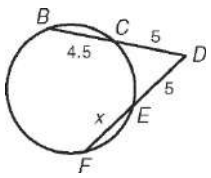
1. 

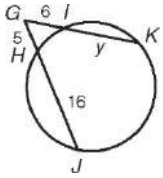
2. 

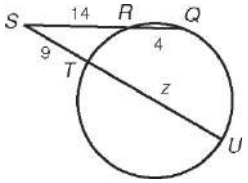
3. 

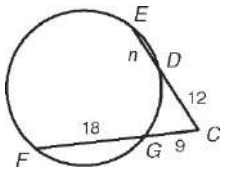
4. 

Find the value of the variable and the length of each secant segment.

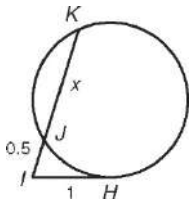
5. 

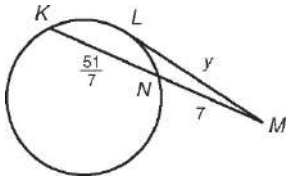
6. 

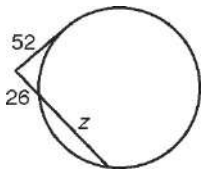
7. 

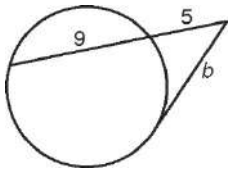
8. 

Find the value of the variable. Give answers in simplest radical form if necessary.

9. 

10. 

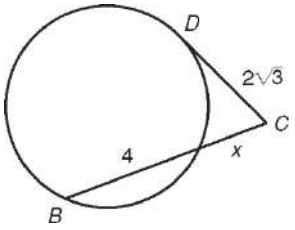
11. 

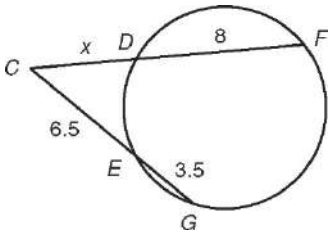
12. 

12-6 Practice C

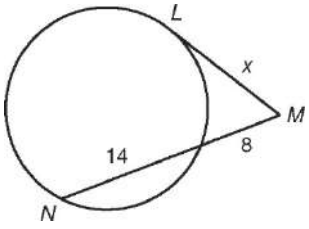
Segment Relationships in Circles

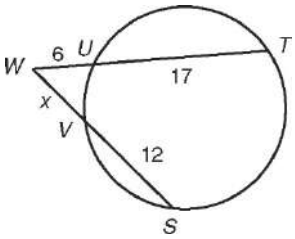
Find the value of *x*. Round to the nearest tenth if necessary.

1. 

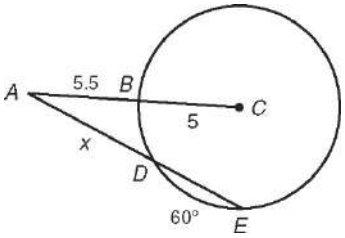
2. 

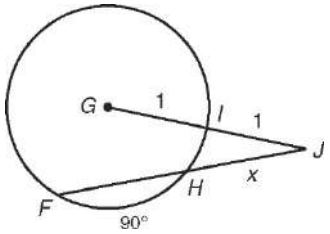
_____ _____

3. 

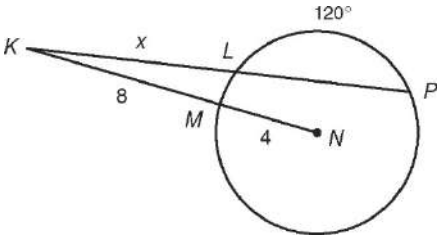
4. 

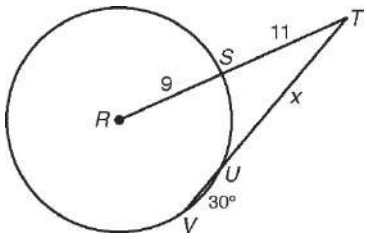
_____ _____

5. 

6. 

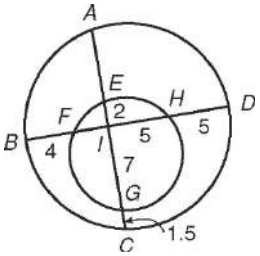
_____ _____

7. 

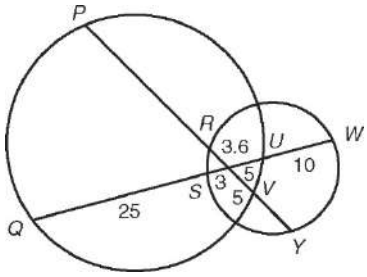
8. 

_____ _____

Find each length.

9. 

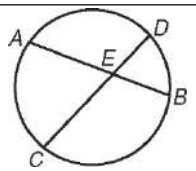
AC = _____ *BD* = _____

10. 

PY = _____

12-6 Reteach

Segment Relationships in Circles

Chord-Chord Product Theorem	
If two chords intersect in the interior of a circle, then the products of the lengths of the segments of the chords are equal.	 $AE \cdot EB = CE \cdot ED$

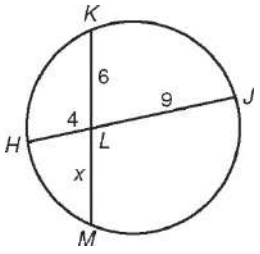
Find the value of *x* and the length of each chord.

$HL \cdot LJ = KL \cdot LM$	Chord-Chord Product Thm.
$4 \cdot 9 = 6 \cdot x$	$HL = 4$, $LJ = 9$, $KL = 6$, $LM = x$
$36 = 6x$	Simplify.
$6 = x$	Divide each side by 6.

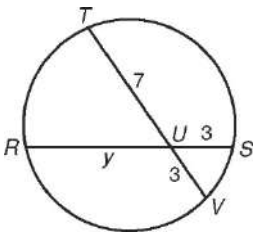
$HJ = 4 + 9 = 13$

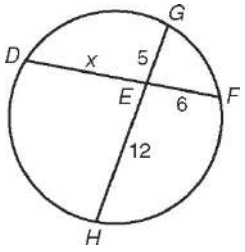
$KM = 6 + x$

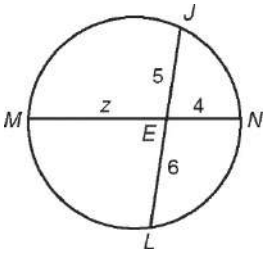
$= 6 + 6 = 12$

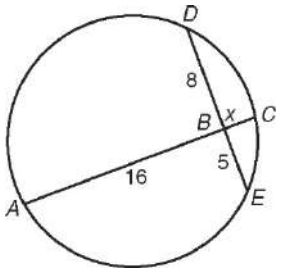


Find the value of the variable and the length of each chord.

1. 

2. 

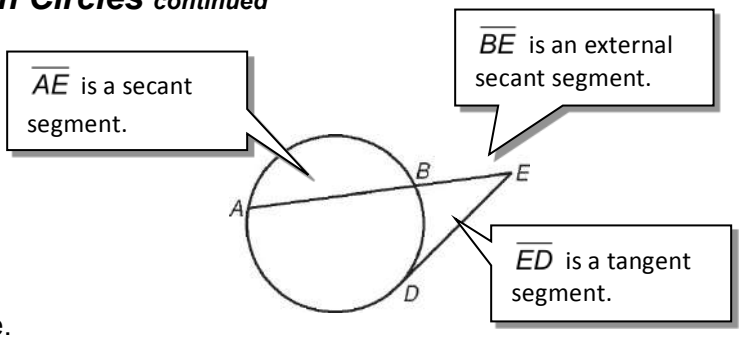
3. 

4. 

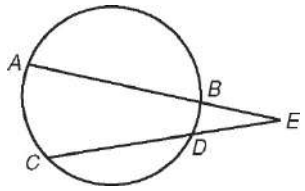
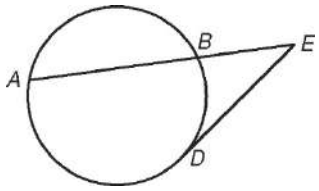
12-6 Reteach

Segment Relationships in Circles *continued*

- A **secant segment** is a segment of a secant with at least one endpoint on the circle.
- An **external secant segment** is the part of the secant segment that lies in the exterior of the circle.
- A **tangent segment** is a segment of a tangent with one endpoint on the circle.

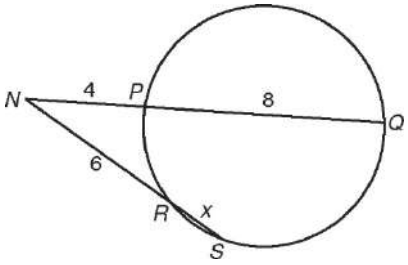


If two segments intersect outside a circle, the following theorems are true.

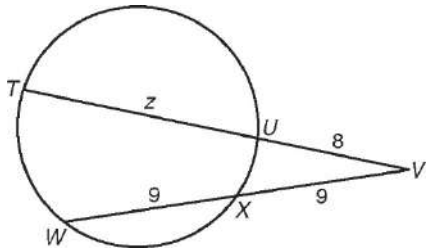
Secant-Secant Product Theorem The product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment. whole · outside = whole · outside $AE \cdot BE = CE \cdot DE$	
Secant-Tangent Product Theorem The product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. whole · outside = tangent^2 $AE \cdot BE = DE^2$	

Find the value of the variable and the length of each secant segment.

5.

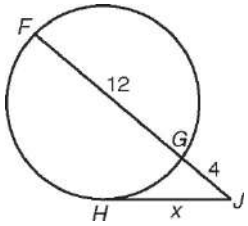


6.

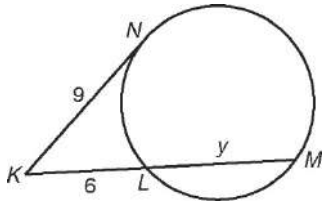


Find the value of the variable.

7.



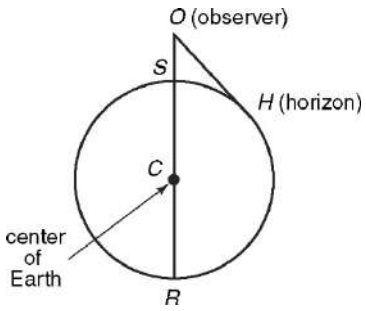
8.



12-6 Challenge

Finding the Distance to the Horizon

For an observer at a point *O* above Earth, the horizon is the place where Earth appears to "meet the sky." The higher above Earth's surface the observer is, the farther away the horizon appears to be. It may surprise you to learn that you can calculate this distance to the horizon by applying your knowledge of tangents and secants.



Refer to the diagram of Earth at right.

1. Name the segment that represents each measure.

   a. the diameter of Earth _____

   b. the observer's altitude above Earth's surface _____

   c. the distance the observer can see to the horizon _____

2. Justify the following equation: $(OH)^2 = OR \cdot OS$

   _____

   _____

When the observer's altitude above Earth's surface is small relative to the diameter of Earth, you can replace *OR* with *RS* in the equation from Exercise 2. Then, since the diameter of Earth is approximately 7920 miles, you obtain the formula for *OH* shown at right. In this formula, the unit for both *OH* and *OS* is miles.

$$(OH)^2 = OR \cdot OS$$
$$(OH)^2 \approx RS \cdot OS$$
$$(OH)^2 \approx 7920 \cdot OS$$
$$OH \approx \sqrt{7920 \cdot OS}$$

Use the formula above to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.

3. 2.5 miles _____

4. 30,000 feet _____

5. Rewrite the formula above so that you can input *OS* as a number of feet and find the distance to the horizon in *miles.*

   $\left(Hint: OH \text{ miles} = \sqrt{7920 \text{ miles} \cdot \frac{1 \text{ mile}}{? \text{ feet}} \cdot OS \text{ feet}} \right)$ _____

Use your formula from Exercise 5 to find the distance to the horizon for each altitude. Assume that it is a clear day and that the view is not obstructed. Round answers to the nearest tenth of a mile.

6. 10 feet _____

7. 200 feet _____

Find the altitude above Earth's surface that an observer must attain in order to see the given distance to the horizon. Round answers to the nearest tenth.

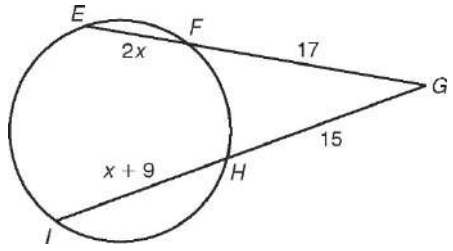
8. 1 mile _____

9. 300 miles _____

12-6 Problem Solving

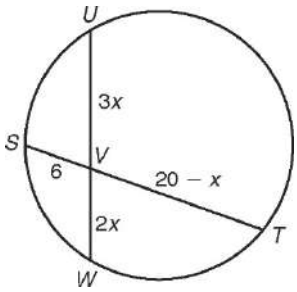
Segment Relationships in Circles

1. Find *EG* to the nearest tenth.



_____

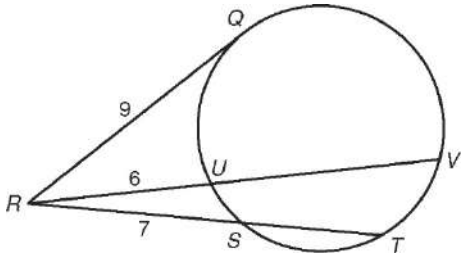
2. _____What is the length of $\overline{UW}$?



_____

Choose the best answer.

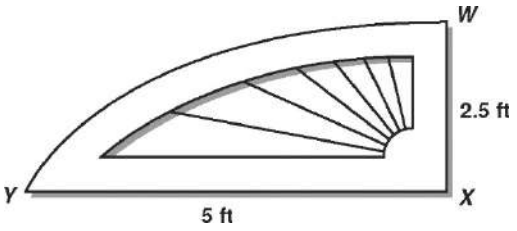
3. Which of these is closest to the length of $\overline{ST}$?



A 4.6 C 7.5

B 5.4 D 11.6

4. The figure is a "quarter" wood arch used in architecture. $\overline{WX}$ is the perpendicular bisector of the chord containing $\overline{YX}$. Find the diameter of the circle containing the arc.

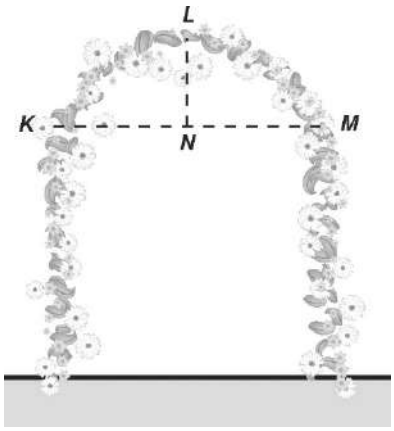


A 5 ft
B 8.5 ft
C 10 ft
D 12.5 ft 5

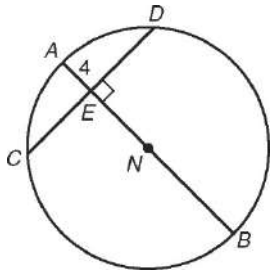
5.Floral archways like the one shown below are going to be used for the prom. $\overline{LN}$ is the perpendicular bisector of $\overline{KM}$.

$KM = 6$ feet and $LN = 2$ feet. What is the diameter of the circle that contains $\overset{\frown}{KM}$?

F 4.5 ft
G 5.5 ft
H 6.5 ft
J 8 ft



6. In $\odot N$, $CD = 18$. Find the radius of the circle to the nearest tenth.



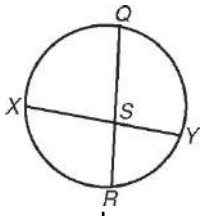
F 12.1
G 16.3
H 20.3
J 24.3

12-6 Reading Strategies

Use a Model

The models below show segment relationships in circles.

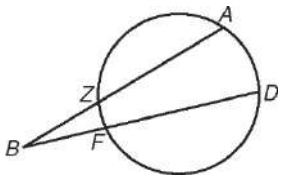
Chord-Chord



Chords $\overline{XY}$ and $\overline{QR}$ intersect at S.

$RS \cdot SQ = XS \cdot SY$

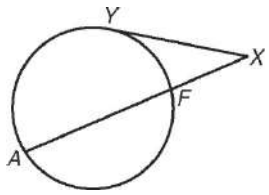
Secant-Secant



Secants $\overline{AB}$ and $\overline{DB}$ intersect at B.

$AB \cdot ZB = DB \cdot FB$

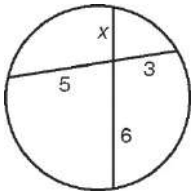
Secant-Tangent



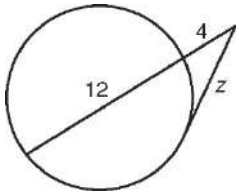
Secant $\overline{AX}$ and tangent $\overline{YX}$ intersect at X.

$AX \cdot FX = YX^2$

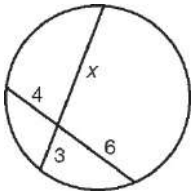
Find the value of each variable.



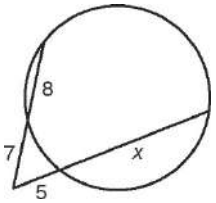
1. _____



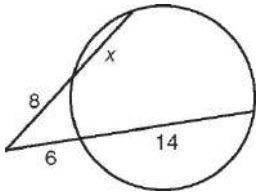
2. _____



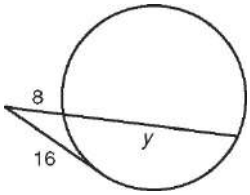
3. _____



4. _____



5. _____



6. _____

Geometry

Chapter 12: Circle

Lesson 7 - Circles in the Coordinate Plane

Standards

G.GPE.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Objectives

Write equations and graph circles in the coordinate plane.

Use the equation and graph of a circle to solve problems

Materials

Graph paper, pencil, Text Book, ToolBox, Compass, straight edge, overhead and transparencies, Remediation materials, Questioning Strategies, Workbook

Vocabulary

None

Formulas

Equation of a circle – $(x - h)^2 + (y - k)^2 = r^2$

(h, k) is center of circle and r is the radius.

Symbols

None

Postulates

None

Theorems

12-7-1 – Equation of a Circle – The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

Who Uses This?

Meteorologists use circles and coordinates to plan the location of weather stations.

Discussion

Write the theorem for the equation of a circle. Discuss how the formula is derived from the distance formula that has been already studied. May need to review the fact that the opposite of square rooting is to square. This is a concept that cadets often forget.

Review Example #1. May have to spend some time discussing the variables and what they stand for in the circle. Work through carefully step by step. Work the Check-It-Out #1 and monitor cadets carefully. Make adjustments and reteach where appropriate.

Example #2 is graphing a circle on the coordinate graph. Review with the cadets how to create a table of values that they can then use as points to graph the circle. Make sure to review which is the x and which is the y axis. Work the two Check-It-Out #2 problems.

The graphing calculator (TI83 and higher) will not graph a circle. However, the cadet can solve the equation for y and then enter the equation in the calculator with a positive value and a second equation with a negative value and the result will be a circle.

Found many cadets find example #3 very confusing and then have problems working on other problems. You can present triangulation with cadets that have a good handle on the circle. There are many uses for this skill in today's world. Cell phone towers are just one. Tread here carefully so cadets are not over whelmed.

Guided Practice

Look at problems 1-9 and work those that cadets select. Once cadets think they can work on their own, they can start on the classwork problems.

Practice

Classwork (additional resources used for differentiation)**:**

Use **Question Strategies** while checking cadet's work as they are working. If major understanding issues – stop class and asked appropriately modified questions to help cadet's get a better understanding. If small group of cadet's are struggling, use appropriately modified questions to help them obtain a better understanding. When working with individual cadets, use modified questions and vocabulary review to help them over the problem areas. Assign problems based on class and/or individual cadet abilities.

Non-proficient Cadets – Problems 10 – 35, 37 – 40, 42 – 44

Additional Resources – Practice A, Reteach, Reading Strategies

Proficient Cadets – Problems 10 – 45

Additional Resources – Practice B, Reteach, Problem Solving

Mastered Cadets - Problems 10 – 47

Additional Resources – Practice C, Challenge, Problem Solving

Concept problem check – 11, 15, 18, 20, 24– checking these problems will give one a good idea if the lesson has been learned. If not, use reteach materials.

Homework

Page 79 and 163 May be replaced with different work based on needs. Replaced by additional resources listed above.

12-7 Practice A

Circles in the Coordinate Plane

1. Write the equation of a circle with center (h, k) and radius r. _____

Write the equation of each circle.

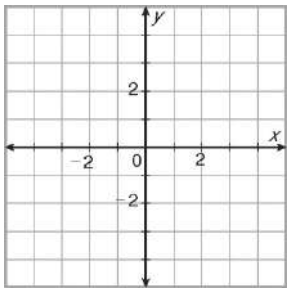
2. ⊙ A centered at the origin with radius 6 _____
3. ⊙ D with center $D(3, 3)$ and radius 2 _____
4. ⊙ L with center $L(-3, -3)$ and radius 1 _____
5. ⊙ M with center $M(0, -2)$ and radius 9 _____
6. ⊙ Q with center $Q(7, 0)$ and radius 3 _____

Complete Exercises 7 and 8 to write the equation of ⊙ F with center $F(2, -1)$ that passes through (10, 5).

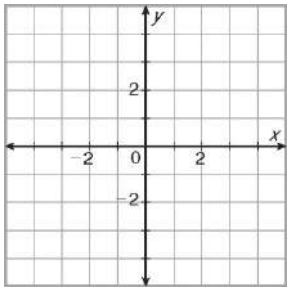
7. Use the distance formula with the two given points to find the radius of ⊙ F. _____
8. Write the equation of ⊙ F. _____

Graph each equation. First locate the center point, and use the radius to plot four points around the center that lie on the circle. Then draw a circle through the four points.

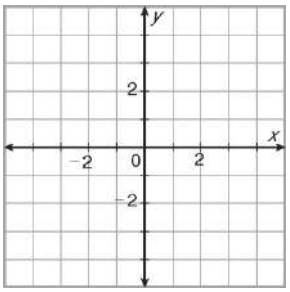
9. $x^2 + y^2 = 16$



10. $x^2 + y^2 = 4$



A county planning department is meeting to choose the location of a rural fire station. The fire station needs to be the same distance from each of the three towns it will serve. The towns are located at $A(-3, 2)$, $B(-3, -4)$, and $C(1, -4)$. Complete Exercises 11–13 in order to find the best location for the fire station.



11. Plot A, B, and C. Draw $\triangle ABC$.
12. Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$.
13. The intersection point of the perpendicular bisectors is the same distance from the three points. So it is the center of a circle that intersects A, B, and C. Find the coordinates where the fire station should be built. _____

12-7 Practice B

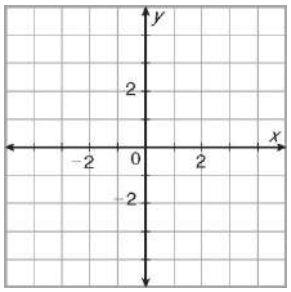
Circles in the Coordinate Plane

Write the equation of each circle.

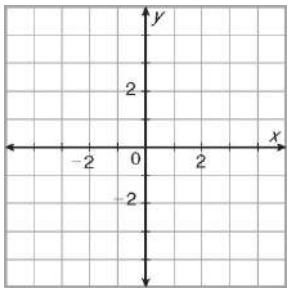
1. e X centered at the origin with radius 10 _____
2. e R with center $R(-1, 8)$ and radius 5 _____
3. e P with center $P(-5, -5)$ and radius $2\sqrt{5}$ _____
4. e O centered at the origin that passes through $(9, -2)$ _____
5. e B with center $B(0, -2)$ that passes through $(-6, 0)$ _____
6. e F with center $F(11, 4)$ that passes through $(-2, 5)$. _____

Graph each equation.

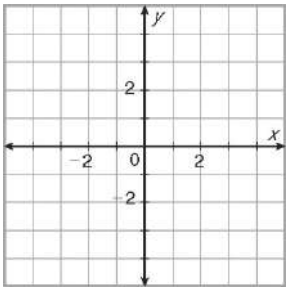
7. $x^2 + y^2 = 25$



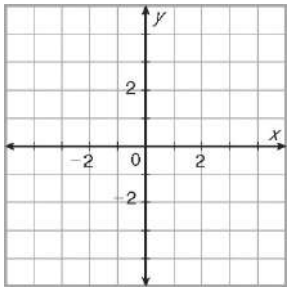
8. $(x + 2)^2 + (y - 1)^2 = 4$



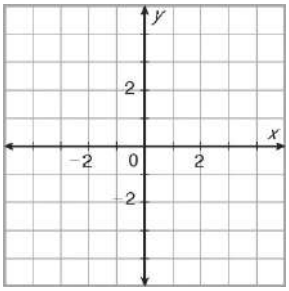
9. $x^2 + (y + 3)^2 = 1$



10. $(x - 1)^2 + (y - 1)^2 = 16$



Crater Lake in Oregon is a roughly circular lake. The lake basin formed about 7000 years ago when the top of a volcano exploded in an immense explosion. Hillman Peak, Garfield Peak, and Cloudcap are three mountain peaks on the rim of the lake. The peaks are located in a coordinate plane at $H(-4, 1)$, $G(-2, -3)$, and $C(5, -2)$.



11. Find the coordinates of the center of the lake.

_____

12. Each unit of the coordinate plane represents $\frac{3}{5}$ mile.

    Find the diameter of the lake. _____

12-7 Practice C

Circles in the Coordinate Plane

1. Points A, B, and C lie on the circumference of a circle. AB is twice the radius of the circle. Find $m\angle ACB$.

_____

2. Points A, B, and C lie on the circumference of a circle. The center of the circle lies in the exterior of $\triangle ABC$. Classify $\triangle ABC$ by its angles.

_____

Give answers in simplest radical form if necessary.

3. The points $X(3, 4)$ and $Y(9, 1)$ lie on the circumference of a circle. There is exactly 60° of arc between X and Y. Find the radius of the circle.

_____

4. Find the coordinates of all possible centers of the circle in Exercise 3.

_____

5. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the line $2x + y = 3$.

_____

6. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the line $y = \frac{4}{3}x - \frac{17}{3}$.

_____

7. Describe the relationship between the circle and the line in Exercise 6.

_____

_____

8. Find the intersection point(s) of the circle $(x + 2)^2 + y^2 = 25$ and the circle $x^2 + y^2 = 9$.

_____

9. Describe the relationship between the two circles in Exercise 8.

_____

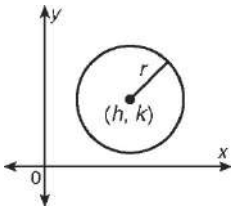
_____

12-7 Reteach

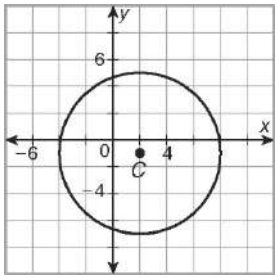
Circles in the Coordinate Plane

Equation of a Circle

The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.



Write the equation of ⊙ C with center $C(2, -1)$ and radius 6.



$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(x - 2)^2 + (y - (-1))^2 = 6^2$	Substitute 2 for h, −1 for k, and 6 for r.
$(x - 2)^2 + (y + 1)^2 = 36$	Simplify.

You can also write the equation of a circle if you know the center and one point on the circle.

Write the equation of ⊙ L that has center $L(3, 7)$ and passes through (1, 7).

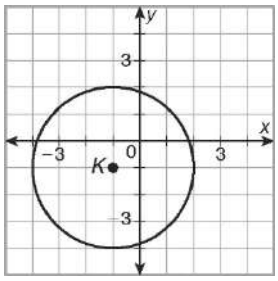
Step 1 Find the radius.

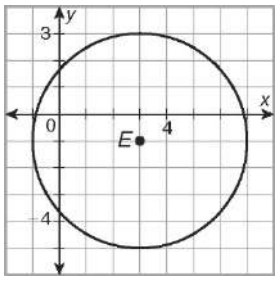
$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$r = \sqrt{(1 - 3)^2 + (7 - 7)^2}$	Substitution
$r = \sqrt{4} = 2$	Simplify.

Step 2 Use the equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(x - 3)^2 + (y - 7)^2 = 2^2$ $(h, k) = (3, 7)$	
$(x - 3)^2 + (y - 7)^2 = 4$	Simplify.

Write the equation of each circle.

1. 

2. 

_____ _____

3. ⊙ T with center $T(4, 5)$ and radius 8

4. ⊙ B that passes through (3, 6) and has center $B(-2, 6)$

_____ _____

12-7 Reteach

Circles in the Coordinate Plane ***continued***

You can use an equation to graph a circle by making a table or by identifying its center and radius.

Graph $(x - 1)^2 + (y + 4)^2 = 9$.

The equation of the given circle can be rewritten.

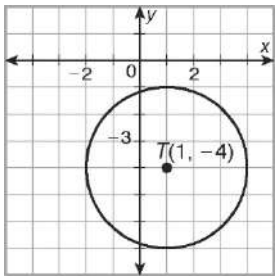
$(x - h)^2 + (y - k)^2 = r^2$

$\downarrow \quad \downarrow \quad \downarrow$

$(x - 1)^2 + (y - (-4))^2 = 3^2$

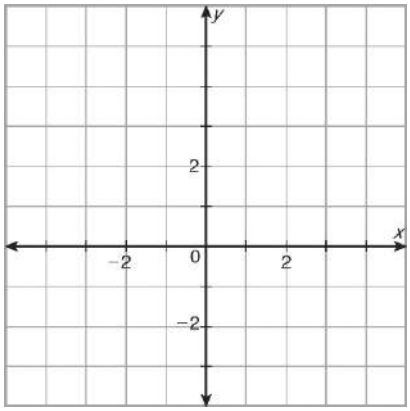
$h = 1$, $k = -4$, and $r = 3$

The center is at (h, k) or $(1, -4)$, and the radius is 3. Plot the point $(1, -4)$. Then graph a circle having this center and radius 3.

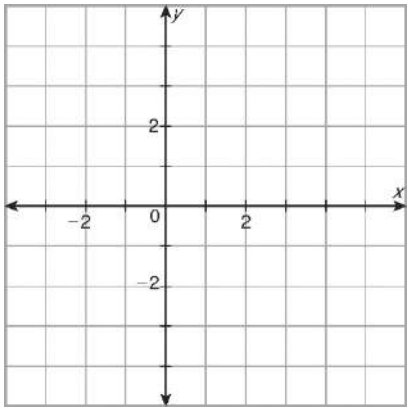


Graph each equation.

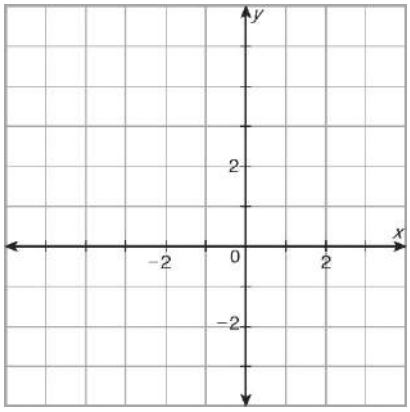
5. $(x - 1)^2 + (y - 2)^2 = 9$



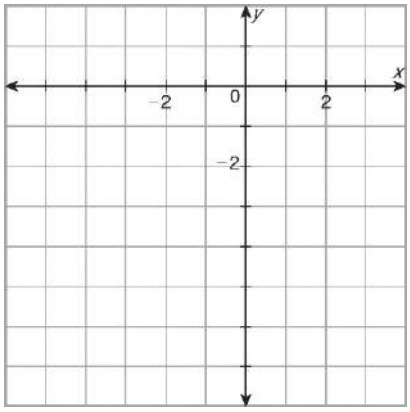
6. $(x - 3)^2 + (y + 1)^2 = 4$



7. $(x + 2)^2 + (y - 2)^2 = 9$



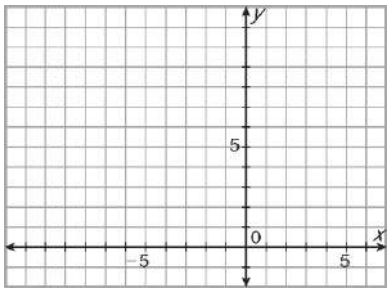
8. $(x + 1)^2 + (y + 3)^2 = 16$



12-7 Challenge

Circles in the Coordinate Plane

Find the area of the intersection of two circles. The equation for circle *A* is $(x - 2)^2 + (y - 3)^2 = 16$. The equation for circle *B* is $(x + 5)^2 + (y - 5)^2 = 36$.



1. Graph each circle in the coordinate grid. Shade the area of intersection of the two circles.
2. Draw $\overline{AB}$.
3. Find *AB* to the nearest whole number. _____
4. Label the points of intersection of the two circles *C* and *D*.
5. a. Solve each equation for *y*.

   _____

   b. Graph the positive square root equation for each curve on a graphing calculator and find the point of intersection, *C*. Round coordinates to the nearest hundredth. _____

   c. Graph the negative square root equation for each curve on a graphing calculator and find the point of intersection, *D*. Round coordinates to the nearest hundredth. _____
6. Draw radii $\overline{BC}$, $\overline{AC}$, $\overline{BD}$, and $\overline{AD}$.
7. Find *AC* and *BC*. _____
8. Find m∠*CBA*. Round to the nearest whole degree. (*Hint:* Use the Law of Cosines.) _____
9. Find m∠*CBD*. Round to the nearest whole degree. _____
10. Find m∠*CAB.* Round to the nearest whole degree. _____
11. Find m∠*CAD.* Round to the nearest whole degree. _____
12. Draw chord $\overline{CD}$.

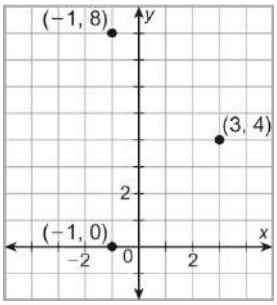
For Exercises 13–19, round to the nearest hundredth.

13. Find the area of sector *BCD*. _____
14. Find the area of △*BCD*. (*Hint:* Assume $\overline{BD}$ is the base of △*BCD*. Find the height of the triangle first. You know that $\overline{BC}$ and $\overline{BD}$ are both radii of circle *B*.) _____
15. Subtract the area of △*BCD* from the area of sector *BCD*. _____
16. Find the area of sector *ACD*. _____
17. Find the area of △*ACD*. (*Hint:* Assume $\overline{AD}$ is the base of △*ACD*. $\overline{AD}$ and $\overline{AC}$ are both radii of circle *A*.) _____
18. Subtract the area of △*ACD* from the area of sector *ACD*. _____
19. Find the sum of your answers to Exercises 15 and 18. This is the area of the intersection of the two circles. _____

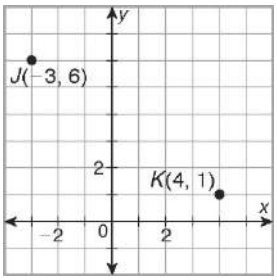
12-7 Problem Solving

Circles in the Coordinate Plane

1. Write the equation of the circle that contains the points graphed below.



2. Find the area of a circle that has center *J* and passes through *K*. Express your answer in terms of π.



Choose the best answer.

3. An English knot garden has hedges planted to form geometric shapes. A blueprint of a knot garden contains three circular hedges as described in the table. Flowers are to be planted in the space that is within all three circles. Which is a point that could be planted with flowers?

   A (7, 1) C (0, 5)

   B (5, 1) D (0, 0)

Circular Hedge	Center	Radius
A	(3, 2)	3 ft
B	(7, 2)	4 ft
C	(5, −1)	3 ft

4. Which of these circles intersects the circle that has center (0, 6) and radius 1?

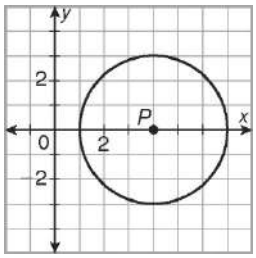
   F $(x - 5)^2 + (y + 3)^2 = 4$

   G $(x - 4)^2 + (y - 3)^2 = 9$

   H $(x + 5)^2 + (y + 1)^2 = 16$

   J $(x + 1)^2 + (y - 4)^2 = 4$

5. Which is an equation for a circle that has the same center as $\odot P$ but has a circumference that is four times as great?



   F $(x - 4)^2 + y^2 = 36$

   G $(x - 4)^2 + y^2 = 144$

   H $x^2 + (y - 4)^2 = 36$

   J $x^2 + (y - 4)^2 = 144$

6. The center of $\odot B$ is (9, 2), and the radius of the circle is 5 units. Which is a point on the circle?

   A (4, 2) C (9, 4)

   B (14, 0) D (9, –5)

7. The Maxair amusement park ride consists of a circular ring that holds 50 riders. Suppose that the center of the ride is at the origin and that one of the riders on the circular ring is at (16, 15.1). If one unit on the coordinate plane equals 1 foot, which is a close approximation of the circumference of the ride?

   A 22 ft C 138 ft

   B 44 ft D 1521 ft

12-7 Reading Strategies

Use a Concept Map

Use the concept map below to help you understand circles in the coordinate plane.

Circles in the Coordinate Plane

Equation

The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

Example

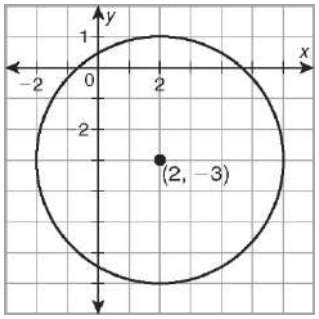
The equation of a circle with center (2, −3) and radius 4:

$(x - 2)^2 + (y - (-3))^2 = 4^2$

or

$(x - 2)^2 + (y + 3)^2 = 16$

Graph

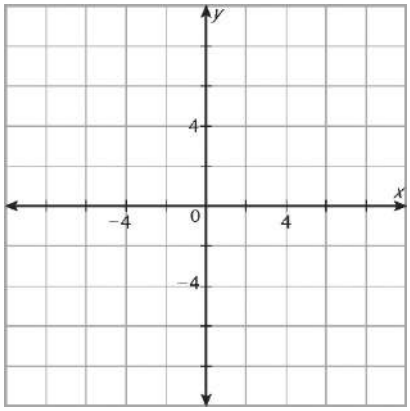


Write the equation of each circle.

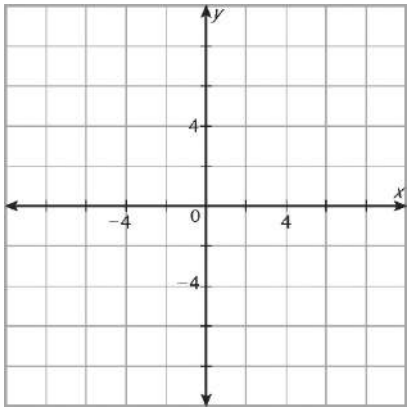
1. a circle with center (−2, 10) and radius 6 _____
2. a circle with center (0, 0) and radius 3 _____
3. a circle with center (8, 2) and radius $\sqrt{7}$ _____

Graph each circle.

4. $(x + 1)^2 + (y - 5)^2 = 25$



5. $(x - 4)^2 + (y - 2)^2 = 9$



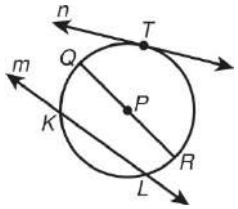
Quizzes to use to check for understanding before continuing.

Circles – Quiz for Sections 1-3

Section A Quiz

Choose the best answer.

Use the figure for Exercises 1–3.



1. Which is a chord of circle *P*?

   A line *m* C $\overline{PQ}$

   B line *n* D $\overline{QR}$

2. Which is a secant of circle *P*?

   F line *m* H $\overline{PQ}$

   G line *n* J $\overline{QR}$

3. Which is a tangent of circle *P*?

   A line *m* C $\overline{PQ}$

   B line *n* D $\overline{QR}$

4. The summit of Mt. McKinley is about 20,320 feet above sea level. Earth's radius is about 3950 miles. To the nearest mile, what is the distance from the summit to the horizon?

   F 67 mi H 1633 mi

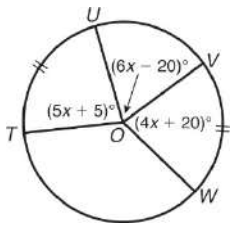
   G 174 mi J 3950 mi

5. The central angle of a circle measures 978 and intercepts a minor arc. What is the measure of its major arc?

   A 838 C 1878

   B 978 D 2638

6. What is $m\overset{a}{UV}$?



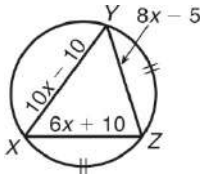
   F 708

   G 808

   H 1008

   J 1228

7. What is *XZ*?



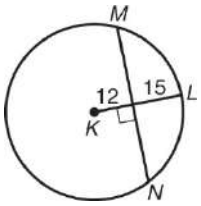
   A 10

   B 25

   C 55

   D 101.25

8. To the nearest tenth, what is *MN*?



   F 24.2

   G 27.0

   H 48.4

   J 54

9. The central angle of an arc measures 758. The radius of the circle measures 3 meters. What is the area of the sector to the nearest tenth of a square meter?

   A 0.6 m^2 C 5.9 m^2

   B 3.9 m^2 D 14.4 m^2

10. An arc measures 508 in a circle with a radius of 8 centimeters. What is the area of the sector to the nearest tenth of a square centimeter?

    F 7.0 cm^2 H 57.6 cm^2

    G 27.9 cm^2 J 174.5 cm^2

11. The central angle of an arc measures 128. The radius of its circle is 30 inches. What is the length of the arc to the nearest quarter inch?

    A 1 in. C $37\frac{1}{2}$ in.

    B $6\frac{1}{4}$ in. D $94\frac{1}{4}$ in.

12. An arc measures 1258 in a circle with a radius of 36 centimeters. What is the length of the arc to the nearest tenth?

    F 11.3 cm H 78.5 cm

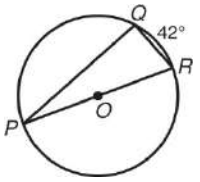
    G 12.5 cm J 1562.5 cm

Circles – Quiz for Sections 4-7

Section B Quiz

Choose the best answer.

1. What is $m\angle R$?



A 45°

B 69°

C 79.5°

D 90°

2. An artist used this design to create a stained glass window. If $m\angle M = 48°$ and $m\overset{\frown}{MN} = m\overset{\frown}{KN} = m\overset{\frown}{KL}$, what is $m\angle LKM$?

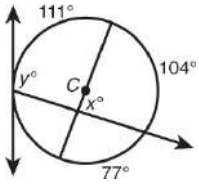
F 18°

G 36°

H 54°

J 72°

Use the figure for Exercises 3 and 4.



3. What is the value of x?

A 52

B 68

C 86

D 104

4. What is the value of y?

F 55.5

G 107.5

H 111

J 215

5. Which equation is of a circle that has a center at (3, −2) and a radius of 9?

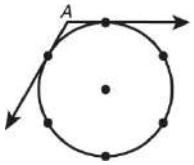
A $(x + 3)^2 + (y - 2)^2 = 9$

B $(x + 3)^2 + (y - 2)^2 = 81$

C $(x - 3)^2 + (y + 2)^2 = 9$

D $(x - 3)^2 + (y + 2)^2 = 81$

6. Six posts are evenly spaced along a circular wall encasing a fountain. Anna stands at point A, where she cannot see beyond the only two posts visible to her. What is $m\angle A$?



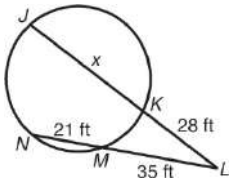
F 100°

G 120°

H 125°

J 150°

7. What is the value of x?



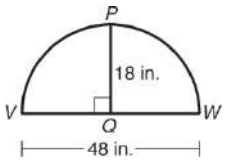
A 23 ft

B 28 ft

C 26.25 ft

D 42 ft

8. A plywood template for a kitchen breakfast bar is cut from a circle. $\overline{PQ}$ is the perpendicular bisector of $\overline{VW}$.



What was the radius of the circle?

F 14 in.

G 25 in.

H 30 in.

J 31 in.

9. A radio antenna is kept perpendicular to the ground by three wires of equal length. The wires touch the ground at three points on a circle whose center is at the base of the antenna. If the wires touch the ground at (9, −19), (−21, −19), and (14, 16), what are the coordinates of the base of the antenna?

A (−6, 1)

B (6, −1)

C (−4, 3)

D (4, −3)

Assessments for Chapter 12 – Circles

Select based on class or even at the individual cadet.

Circles – Optional Assessment

Performance Assessment

Purpose:

To assess student understanding of circles

Time:

15 minutes

Grouping:

pairs or small groups

Preparation Hints:

Review formulas pertaining to circles.

Overview:

Students identify angles, arcs, chords, secants, tangents, and radii in circles and match equations for finding their measures.

Introduce the Task:

Tell students they will be given seven circles for which they will identify the part of the specific figure that *x* represents. Tell them that if the value of *x* represents the measure of an arc, for example, they should further determine whether the arc is a major arc or a minor arc. If *x* represents the measure of an angle, students should determine whether it is a central angle or an inscribed angle. Then they should match the diagram to the equation that can be solved for *x*. They do not need to solve the equation. Inform them that the listed equations may be used more than once, exactly once, or not at all.

Performance Indicators:

_____ identifies tangents, secants, and chords

_____ uses correct formulas for tangents

_____ uses correct formulas relating to properties of arcs

_____ uses correct formulas relating to properties of chords

_____ uses correct formulas relating to properties of arc length

_____ uses correct formulas relating to properties of inscribed angles

_____ uses correct formulas relating to lines that intersect circles

Scoring Rubric:

Level 4: Student solves problems correctly and gives good explanations.

Level 3: Student solves problems but does not give satisfactory explanations.

Level 2: Student solves some problems but does not give satisfactory explanations.

Level 1: Student is not able to solve any of the problems.

Circles

Performance Assessment

In Exercises 1–7, demonstrate your knowledge by (a) naming the specific part of the figure that *x* represents, and (b) writing the equation that could be used to solve for *x*. Choose from the Equation Bank.

EQUATION BANK

$\frac{x+14}{2} = 24$

$\frac{x-14}{2} = 24$

$x = \frac{1}{2} \cdot 24$

$24 = 2\pi \cdot 10\left(\frac{x}{360}\right)$

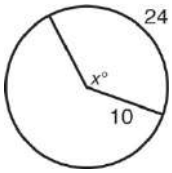
$x^2 = (x-10)^2 + 24^2$

$x + 24 = 360$

$x + 10 = 24$

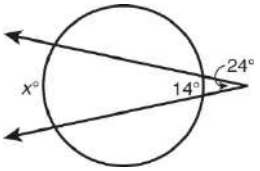
$x^2 = 24 \cdot 10$

$(2x-4)(x-4) = 24 \cdot 10$

1. 

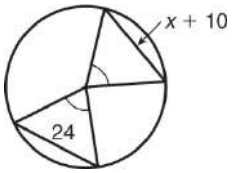
a. _____

b. _____

2. 

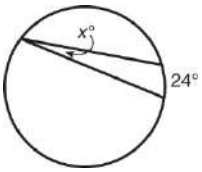
a. _____

b. _____

3. 

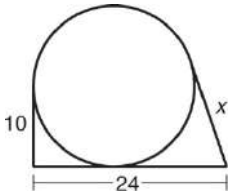
a. _____

b. _____

4. 

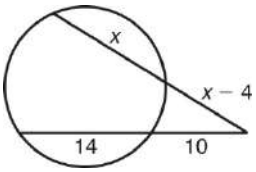
a. _____

b. _____

5. 

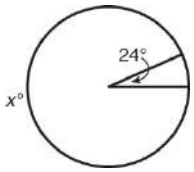
a. _____

b. _____

6. 

a. _____

b. _____

7. 

a. _____

b. _____

8. Suppose two lines intersect a circle. Explain how the location of their intersection helps to determine the angle measure in relation to the intercepted arc measures.

_____

_____

_____

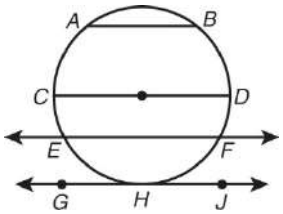
_____

Circles

Chapter 12 Test *Form A*

Circle the best answer.

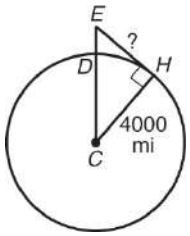
1. Which describes $\overset{\text{Scan}}{EF}$?



A chord C secant

B radius D tangent

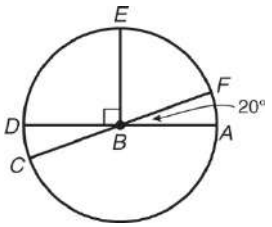
2. A plane is cruising at an altitude of 5.5 miles. Which equation can be used to find the distance from the plane to the horizon?



A $EC^2 = EH^2 + CH^2$

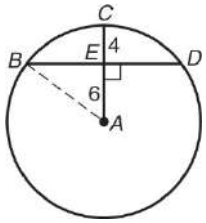
B $DC^2 = EH^2 + CH^2$

3. What is $m\overset{a}{EA}$?



A 70° B 90°

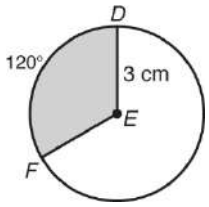
4. What is the length of $\overline{BD}$?



A 8 C 12

B 10 D 16

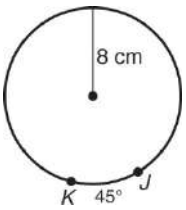
5. What is the area of sector *DEF* in terms of π?



A $\frac{\pi}{3}$ cm^2 C 2π cm^2

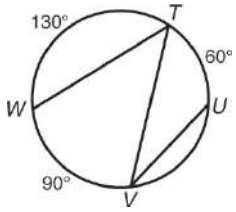
B π cm^2 D 3π cm^2

6. What is the length of $\overset{a}{JK}$?



A 2π cm B 8π cm

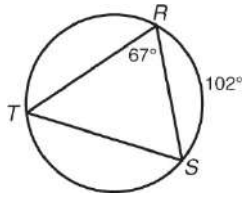
7. What is $m\angle WTV$?



A 30° C 60°

B 45° D 90°

8. What is $m\angle RST$?

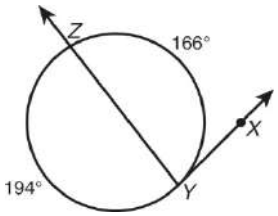


A 51° C 67°

B 62° D 102°

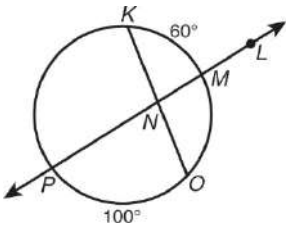
Chapter 12 Test *Form A continued*

9. What is m∠*XYZ*?



A 83° B 97°

10. What is m∠*KNL*?



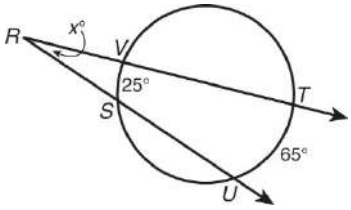
A 20°

B 30°

C 60°

D 80°

11. What is the value of *x*?



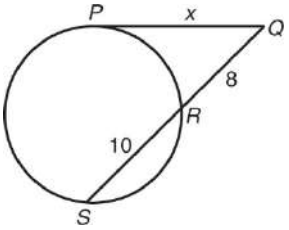
A 20

B 25

C 45

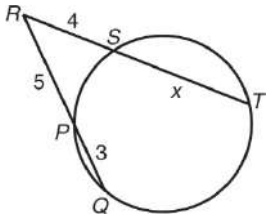
D 65

12. What is the value of *x*?



A 12 B $\sqrt{80}$

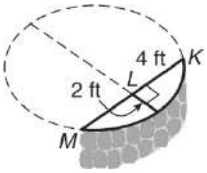
13. What is the value of *x*?



A 3.75 C 7.75

B 6 D 10

14. Archaeologists discovered a portion of a stone wall. To calculate its original diameter, they marked and measured chord $\overline{KM}$ and its perpendicular bisector. What was the diameter of the original circular wall?

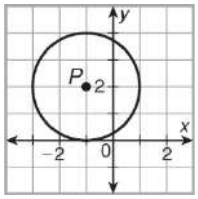


A 8 ft B 10 ft

15. What are the coordinates of the center of the circle $(x + 3)^2 + (y + 5)^2 = 144$?

A (3, 5) B (−3, −5)

16. Which is the equation for circle *P*?



A $(x - 1)^2 + (y - (-2))^2 = 4$

B $(x - 1)^2 + (y - 2)^2 = 4$

C $(x - (-1))^2 + (y - 2)^2 = 4$

D $(x - (-1))^2 + (y - (-2))^2 = 4$

17. Which are used to find the center of a circle drawn through three noncollinear points?

A perpendicular bisectors

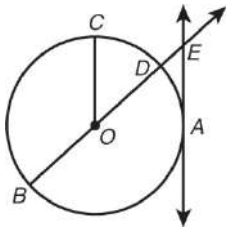
B altitudes

Circles

Chapter 12 Test *Form B*

Circle the best answer.

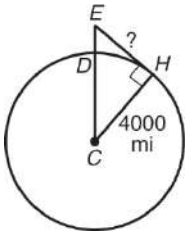
1. Which is a chord?



A $\overline{AE}$ C $\overline{BD}$

B $\overline{BE}$ D $\overline{OC}$

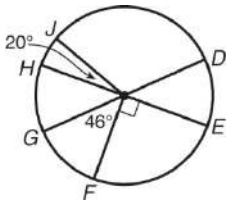
2. A plane is cruising at an altitude of 30,000 feet. What is the distance, to the nearest mile, from the plane to the horizon?



F 213 mi H 8,500,000 mi

G 4000 mi J Not here

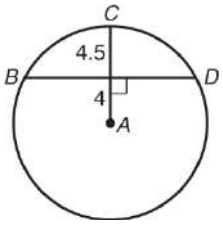
3. Which of these arcs has a measure of 134°?



A $\overset{\frown}{FJ}$ $\overset{\frown}{EG}$

B $\overset{\frown}{DF}$ $\overset{\frown}{DH}$

4. What is *BD*?



F 7.5 H 9.4

G 8.5 J 15

5. Which sector does NOT have an area of 3π?

A central angle 135°; radius $2\sqrt{2}$

B central angle 80°; radius 3

C central angle 67.5°; radius 4

D central angle 270°; diameter 4

6. Which arc has a length of 5π units?

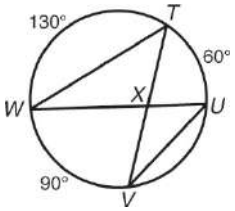
F arc measure 45°; radius 10

G central angle 90°; radius 10

H arc measure 90°; radius 5

J central angle 45°; diameter 20

7. What is m$\angle VXU$?



A 30° C 65°

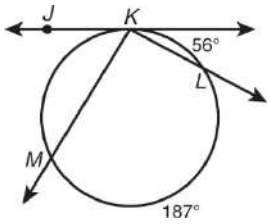
B 45° D 105°

8. Quadrilateral *PQRS* is inscribed in a circle. The ratio of m$\angle P$ to m$\angle R$ is 2 : 4. What is m$\angle R$?

F 30° H 120°

G 60° J Not here

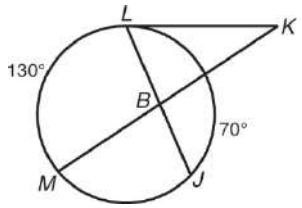
9. What is m$\angle JKM$?



A 28° C 90°

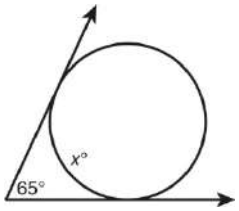
B 58.5° D 117°

10. What is m∠*JBM*?



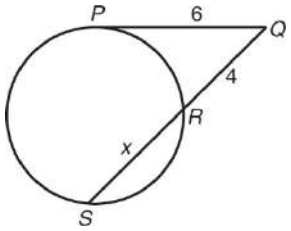
F 50°
G 80°
H 110°
J 150°

11. How many arc degrees are in the minor arc?



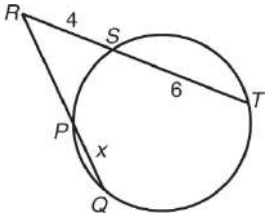
A 32.5°
B 65°
C 115°
D Not here

12. What is the length of $\overline{SQ}$?



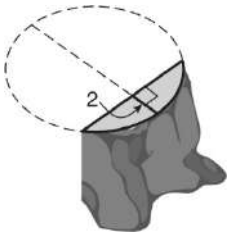
F 5
G 9
H 13
J 17

13. If $RQ = 8$, what is the length of $\overline{RP}$?



A 3
B 5
C 13.5
D Not here

14. Hikers came across a part of a redwood stump. If the length of the chord is 8 feet, what was the diameter of the tree?



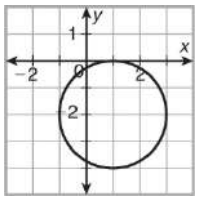
F 4 ft
G 5 ft
H 8 ft
J 10 ft

15. Which is the equation of a circle that passes through (2, 2) and is centered at (5, 6)?

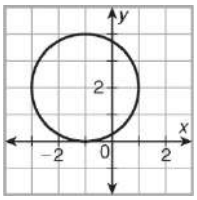
A $(x - 6)^2 + (y - 5)^2 = 25$
B $(x - 5)^2 + (y - 6)^2 = 5$
C $(x + 5)^2 + (y + 6)^2 = 25$
D $(x - 5)^2 + (y - 6)^2 = 25$

16. Which is the graph of $(x - 1)^2 + (y + 2)^2 = 4$?

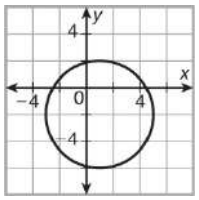
F



H



G



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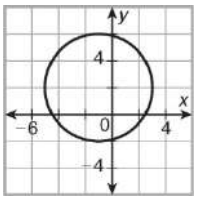


17. A hospital trauma center is going to be built equidistant from three cities. Positioned on a grid, the cities would be located at (1, 5), (2, −2), and (−6, −2). What are the coordinates of the location where the trauma center should be built?

A (−2, −1)
B (−2, 1)
C (2, −1)
D (2, 1)

Circles

Chapter 12 Test *Form C*

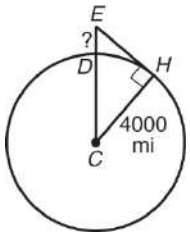
Circle the best answer.

1. Which is never a chord?

   I diameter II radius
   III secant IV tangent

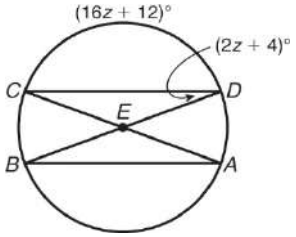
   A I and II C II and IV
   B III and IV D I, II, and III

2. A mountain climber is standing at the top of Mount Everest. The distance from the summit to the horizon is about 210 miles. About how high is Mount Everest?



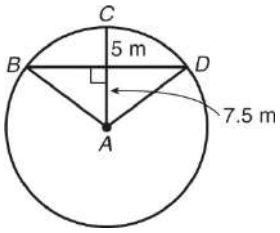
   F 5.5 mi H 210 mi
   G 11 mi J 8000 mi

3. Which is $m\overset{a}{BA}$?



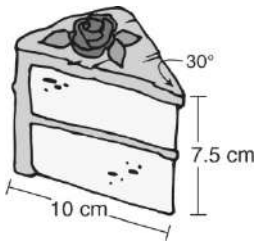
   A 80° C 128°
   B 120° D 140°

4. Which is the area of $\triangle ABD$?



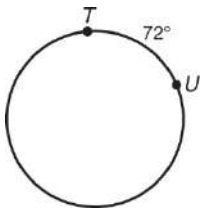
   F 10 m^2 H 75 m^2
   G 62.5 m^2 J 125 m^2

5. A slice of cake is a sector of a cylinder. To the nearest hundredth, what is the volume of the piece of cake? Use 3.14 for π.



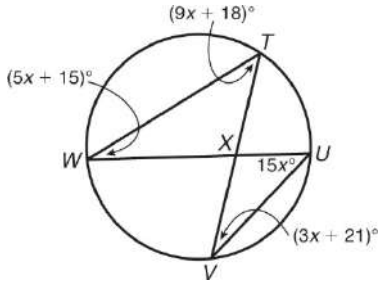
   A 26.17 cm^3 C 196.25 cm^3
   B 39.25 cm^3 D Not here

6. If the length of $\overset{a}{TU}$ is 6π, what is the radius of the circle?



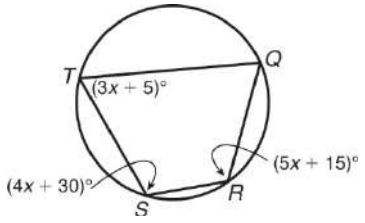
   F 2.4 H 15
   G 5.48 J 47.1

7. Which is $m\angle VXU$?



   A 105° C 122.5°
   B 120° D 126°

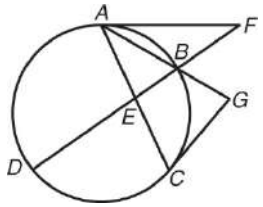
8. What is $m\angle TQR$?



   F 65° H 110°
   G 70° J 115°

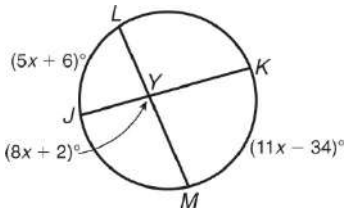
Chapter 12 Test *Form C continued*

9. If $m\angle FAE = 65°$, $m\angle AFD = 35°$, $m\overset{\frown}{AB} = 60°$, and $\overline{FA}$ and $\overline{GC}$ are tangent to the circle, what is $m\angle AGC$?



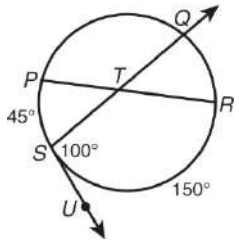
A 65° C 80°
B 70° D Not here

10. What is $m\angle LYK$?



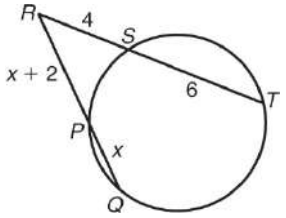
F 12 H 98
G 66 J Not here

11. What is $m\overset{\frown}{PQ}$?



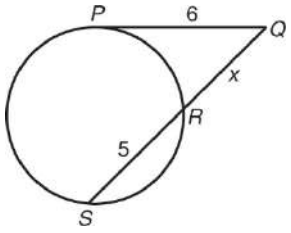
A 50° C 150°
B 115° D 200°

12. What is the length of $\overline{RP}$?



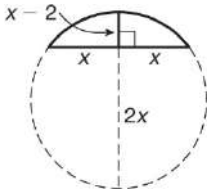
F 3 H 8
G 5 J Not here

13. What is the length of $\overline{SQ}$?



A 4 C 9
B −9 D Not here

14. What is the length of the diameter?



F 5 H 10
G 8 J Not here

15. For which value(s) of the constant k is the circle $x^2 + (y - k)^2 = 16$ tangent to the line $y = 3$?

A −1 only C ±1
B −1 and 7 D 1 and −7

16. Which is the equation of a circle that has a diameter with endpoints (1, 3) and (−3, 1)?

F $(x + 1)^2 + (y - 2)^2 = 10$
G $(x + 1)^2 + (y - 2)^2 = 20$
H $(x + 1)^2 + (y - 2)^2 = 5$
J $(x - 1)^2 + (y - 2)^2 = 5$

17. A new firehouse is being built equidistant from three other fire stations. Positioned on a grid, the current fire stations would be located at (3, 7), (−1, −1), and (−4, 8). What are the coordinates of the location where the new firehouse should be built?

A (−1, −4) C (1, −4)
B (−1, 4) D (1, 4)

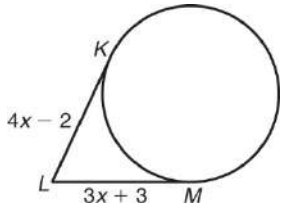
Circles

Chapter 12 Test *Form A*

1. Write *True* or *False.* A diameter is a chord of a circle.

_____

2. Find *LM*.



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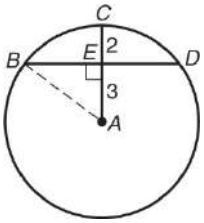
3. Find the measure of the major arc if its central angle is 358.

_____

4. Find *DB*.



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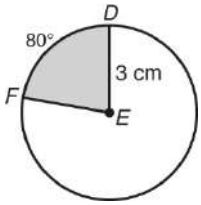
5. Find the area of sector *DEF*. Give your answer in terms of π.



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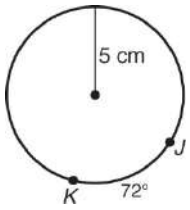
6. Find the length of $\overset{\frown}{JK}$. Give your answer in terms of π.



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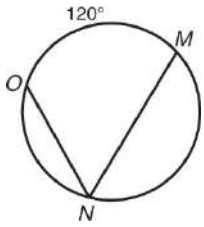
7. Find m∠*MNO*.



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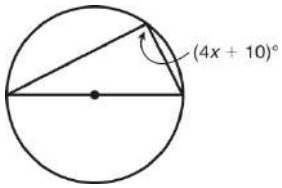
8. Find the value of *x*.



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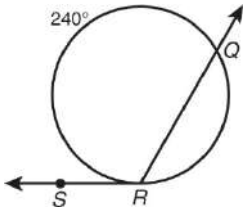
9. Find m∠*QRS*.



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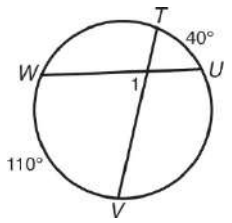
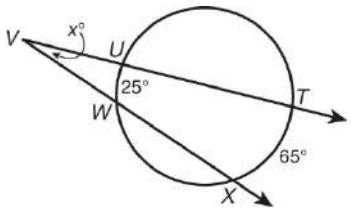
Circles

Chapter 12 Test *Form A continued*

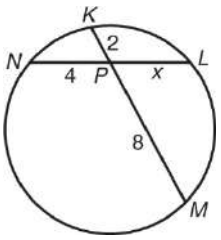
10. Find m∠1.



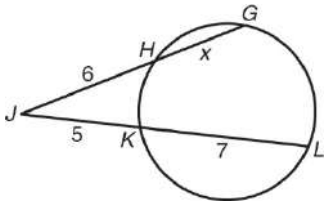
11. Find the value of *x*.



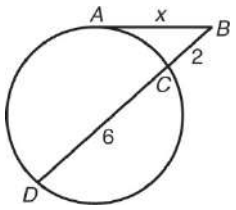
12. Find the length of $\overline{NL}$.



13. Find the length of $\overline{GJ}$.

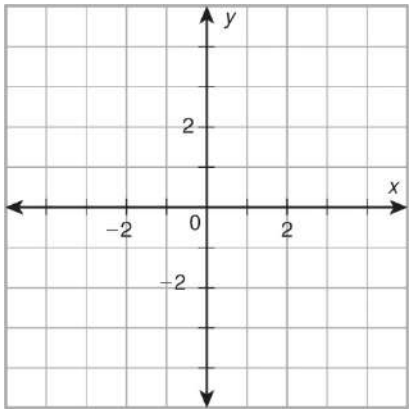


14. Find the value of *x*.



15. Identify the center and radius of the circle with the equation $(x - 3)^2 + (y - 5)^2 = 25$.

16. Graph $x^2 + y^2 = 16$.



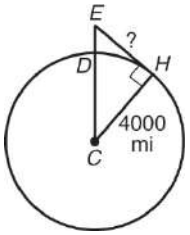
17. Write *True* or *False.* When finding a point equidistant from three noncollinear points, you need to find where the perpendicular bisectors of the segments connecting them intersect.

Circles

Chapter 12 Test *Form B*

1. Complete the sentence. A secant is a _____ in the plane of a circle that intersects the circle at exactly _____ points.

2. Mount McKinley in Alaska is North America's highest mountain. The mountain is 20,320 feet high. To the nearest mile, find the distance from the summit to the horizon at sea level.



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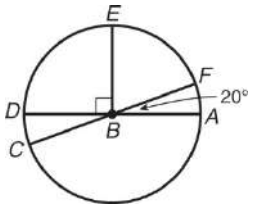
3. Find $m\overset{\frown}{CDE}$.



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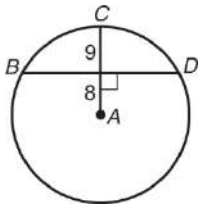
4. Find *BD*.



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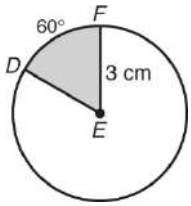
5. Find the area of sector *DEF*. Give your answer in terms of π and rounded to the nearest hundredth.



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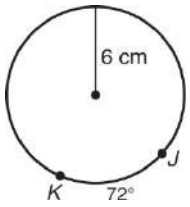
6. Find the length of $\overset{\frown}{JK}$. Give your answer in terms of π and rounded to the nearest hundredth.



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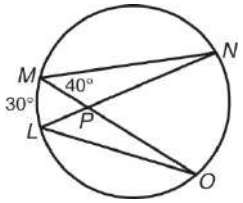
7. Find m∠*LPO*.



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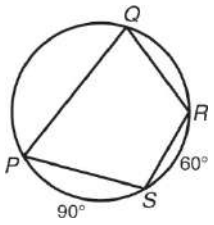
8. Find m∠*RSP*.



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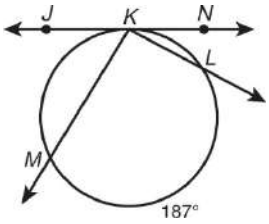
9. If m∠*JKM* = 58.58, find m∠*NKL*.



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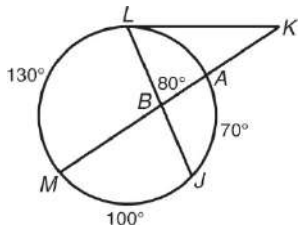
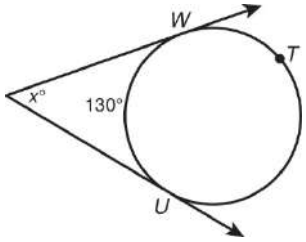
Circles

Chapter 12 Test *Form B continued*

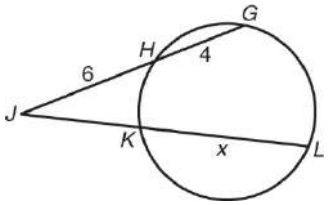
10. Find m∠*LKA*.



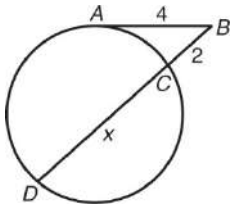
11. Find the value of *x*.



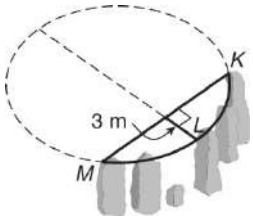
12. If *JL* = 12, find *KL*.



13. Find the length of $\overline{BD}$.

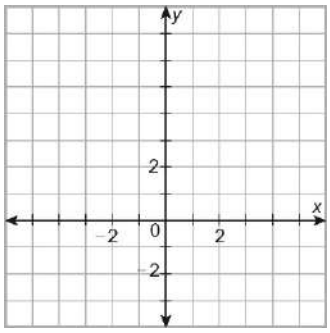


14. An arrangement of stones that formed an arc of a circle was discovered. If the chord is 12 meters, find the diameter of the completed circle.



15. Write the equation of $\odot B$ with center *B*(−2, 3) that passes through (1, 2).

16. Graph $(x + 1)^2 + (y - 2)^2 = 16$.

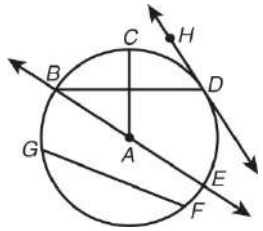


17. A new firehouse is being built equidistant from three other fire stations. Positioned on a grid, the current fire stations would be located at (2, 2), (3, −5), and (−5, −5). Find the coordinates of the new firehouse.

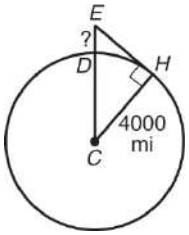
Circles

Chapter 12 Test *Form C*

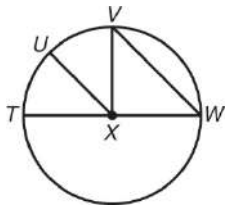
1. Classify the lines and segments that intersect e *A*.



2. Mount McKinley in Alaska is North America's highest mountain. The distance from the summit to the horizon is about 176 miles. To the nearest tenth of a mile, find the height of the mountain.

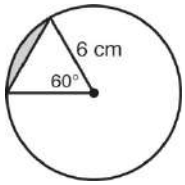


3. Given m∠*WVX* = 458 and $\overline{VW}$ P$\overline{UX}$, find m$\overset{a}{U}V$.



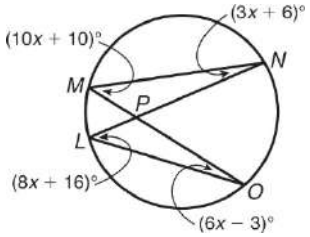
4. Write *True* or *False.* Chords equally distant from the center of a circle are congruent.

5. Find the area of the segment of the circle to the nearest hundredth.

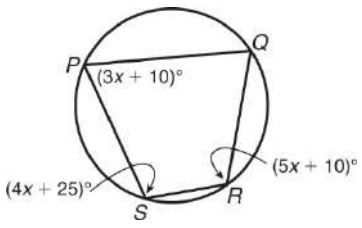


6. To the nearest degree, find the measure of the central angle for $\overset{a}{J}K$ if the length of $\overset{a}{J}K$ is 2.4 units and the radius is 6 units.

7. Find m∠*LPO*.



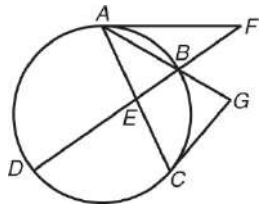
8. Find m∠*RSP*.



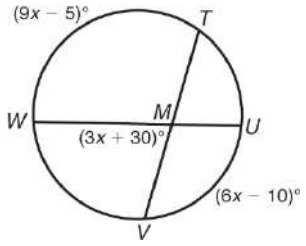
Circles

Chapter 12 Test *Form C continued*

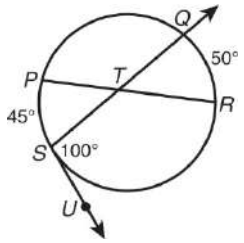
9. If $m\angle ACG = 65°$, $m\angle AGC = 80°$, $m\overset{\frown}{DC} = 100°$, $m\overset{\frown}{BC} = 70°$, and $\overline{FA}$ and $\overline{GC}$ are tangent to the circle, find $m\angle AFD$.



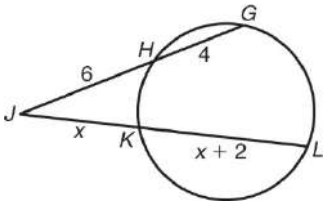
10. Find $m\angle TMU$.



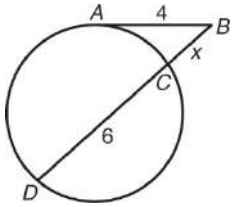
11. Find $m\overset{\frown}{SPQ}$.



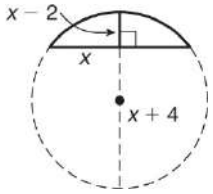
12. Find the length of $\overline{KL}$.



13. Find the length of $\overline{BD}$.

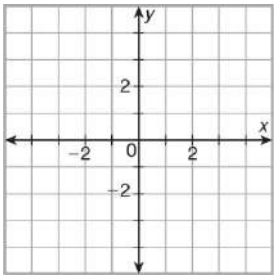


14. Find the diameter.



15. Write an equation for the locus of all points in the coordinate plane that are 5 units from (3, 4).

16. Graph a circle with a diameter of 4 units that is tangent to the line $y = 2$.



17. A hospital trauma center is going to be built equidistant from three cities. Positioned on a grid, the cities would be located at (3, −2), (−2, 3), and (−6, −5). What are the coordinates of the location where the trauma center should be bui