COMMON CORE ASSESSMENT COMPARISON FOR MATHEMATICS

STATISTICS GRADES 9–11

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Prepared by:

Delaware Department of Education
Accountability Resources Workgroup
401 Federal Street, Suite 2
Dover, DE 19901

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INTRODUCTION

The purpose of this document is to illustrate the differences between the Delaware Comprehensive Assessment System (DCAS) and the expectations of the next-generation Common Core State Standard (CCSS) assessment in Mathematics. A side-by-side comparison of the current design of an operational assessment item and the expectations for the content and rigor of a next-generation Common Core mathematical item are provided for each CCSS. The samples provided are designed to help Delaware's educators better understand the instructional shifts needed to meet the rigorous demands of the CCSS. This document does not represent the test specifications or blueprints for each grade level, for DCAS, or the next-generation assessment.

For mathematics, next-generation assessment items were selected for CCSS that represent the shift in content at the new grade level. Sites used to select the next-generation assessment items include:

- Smarter Balanced Assessment Consortium
- Partnership of Assessment of Readiness for College and Career
- Illustrative Mathematics
- Mathematics Assessment Project

Using released items from other states, a DCAS-like item, aligned to the same CCSS, was chosen. These examples emphasize the contrast in rigor between the previous Delaware standards, known as Grade-Level Expectations, and the Common Core State Standards.

Section 1, DCAS-Like and Next-Generation Assessment Comparison, includes content that is in the CCSS at a different "rigor" level. The examples are organized by the CCSS. For some standards, more than one example may be given to illustrate the different components of the standard. Additionally, each example identifies the standard and is separated into two parts. Part A is an example of a DCAS-like item, and Part B is an example of a next-generation item based on CCSS.

Section 2 includes at least one Performance Task that addresses multiple aspects of the CCSS (content and mathematical practices).

How to Use Various Aspects of This Document

- Analyze the way mathematics standards are conceptualized in each item or task.
- Identify the instructional shifts that need to occur to prepare students to address these more rigorous demands. Develop a plan to implement the necessary instructional changes.
- Notice how numbers (e.g., fractions instead of whole numbers) are used in the sample items.
- Recognize that the sample items and tasks are only one way of assessing the standard.
- Understand that the sample items and tasks do not represent a mini-version of the next-generation assessment.
- Instruction should address "focus," coherence," and "rigor" of mathematics concepts.
- Instruction should embed mathematical practices when teaching mathematical content.

- For grades K–5, calculators should not be used as the concepts of number sense and operations are fundamental to learning new mathematics content in grades 6–12.
- The next-generation assessment will be online and the scoring will be done electronically. It is important to note that students may not be asked to show their work and therefore will not be given partial credit. It is suggested when using items within this document in the classroom for formative assessments, it is good practice to have students demonstrate their methodology by showing or explaining their work.

Your feedback is welcome. Please do not hesitate to contact Katia Foret at katia.foret@doe.k12.de.us or Rita Fry at rita.fry@doe.k12.de.us with suggestions, questions, and/or concerns.

* The Smarter Balanced Assessment Consortium has a 30-item practice test available for each grade level (3-8 and 11) for mathematics and ELA (including reading, writing, listening, and research). These practice tests allow students to experience items that look and function like those being developed for the Smarter Balanced assessments. The practice test also includes performance tasks and is constructed to follow a test blueprint similar to the blueprint intended for the operational test. The Smarter Balanced site is located at: http://www.smarterbalanced.org/.

Priorities in Mathematics

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Common Core State Standards for Mathematical Practices

Mathematical Practices		Student Dispositions:	Teacher Actions to Engage Students in Practices:
Essential Processes for a Productive Math Thinker	**1. Make sense of problems and persevere in solving them**	▪ Have an understanding of the situation ▪ Use patience and persistence to solve problem ▪ Be able to use different strategies ▪ Use self-evaluation and redirections ▪ Communicate both verbally and written ▪ Be able to deduce what is a reasonable solution	▪ Provide open-ended and rich problems ▪ Ask probing questions ▪ Model multiple problem-solving strategies through Think-Aloud ▪ Promote and value discourse ▪ Integrate cross-curricular materials ▪ Promote collaboration ▪ Probe student responses (correct or incorrect) for understanding and multiple approaches ▪ Provide scaffolding when appropriate ▪ Provide a safe environment for learning from mistakes
	6. Attend to precision	▪ Communicate with precision—orally and written ▪ Use mathematics concepts and vocabulary appropriately ▪ State meaning of symbols and use them appropriately ▪ Attend to units/labeling/tools accurately ▪ Carefully formulate explanations and defend answers ▪ Calculate accurately and efficiently ▪ Formulate and make use of definitions with others ▪ Ensure reasonableness of answers ▪ Persevere through multiple-step problems	▪ Encourage students to think aloud ▪ Develop explicit instruction/teacher models of thinking aloud ▪ Include guided inquiry as teacher gives problem, students work together to solve problems, and debrief time for sharing and comparing strategies ▪ Use probing questions that target content of study ▪ Promote mathematical language ▪ Encourage students to identify errors when answers are wrong
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	▪ Create multiple representations ▪ Interpret problems in contexts ▪ Estimate first/answer reasonable ▪ Make connections ▪ Represent symbolically ▪ Talk about problems, real-life situations ▪ Attend to units ▪ Use context to think about a problem	▪ Develop opportunities for problem-solving strategies ▪ Give time for processing and discussing ▪ Tie content areas together to help make connections ▪ Give real-world situations ▪ Demonstrate thinking aloud for students' benefit ▪ Value invented strategies and representations ▪ More emphasis on the process instead of on the answer
	3. Construct viable arguments and critique the reasoning of others	▪ Ask questions ▪ Use examples and counter examples ▪ Reason inductively and make plausible arguments ▪ Use objects, drawings, diagrams, and actions ▪ Develop ideas about mathematics and support their reasoning ▪ Analyze others arguments ▪ Encourage the use of mathematics vocabulary	▪ Create a safe environment for risk-taking and critiquing with respect ▪ Provide complex, rigorous tasks that foster deep thinking ▪ Provide time for student discourse ▪ Plan effective questions and student grouping ▪ Probe students

Mathematical Practices		Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	▪ Realize that mathematics (numbers and symbols) is used to solve/work out real-life situations ▪ Analyze relationships to draw conclusions ▪ Interpret mathematical results in context ▪ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable—if not, go back and look for more information ▪ Make sense of the mathematics	▪ Allowing time for the process to take place (model, make graphs, etc.) ▪ Modeling desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ▪ Making appropriate tools available ▪ Creating an emotionally safe environment where risk-taking is valued ▪ Providing meaningful, real-world, authentic, performance-based tasks (non-traditional work problems) ▪ Promoting discourse and investigations
	5. Use appropriate tools strategically	▪ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base ten blocks, compass, protractor) ▪ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ▪ Compare the efficiency of different tools ▪ Recognize the usefulness and limitations of different tools	▪ Maintaining knowledge of appropriate tools ▪ Modeling effectively the tools available, their benefits, and limitations ▪ Modeling a situation where the decision needs to be made as to which tool should be used ▪ Comparing/contrasting effectiveness of tools ▪ Making available and encouraging use of a variety of tools
Seeing Structure and Generalizing	**7. Look for and make use of structure**	▪ Look for, interpret, and identify patterns and structures ▪ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ▪ Reflect and recognize various structures in mathematics ▪ Breakdown complex problems into simpler, more manageable chunks ▪ "Step back" or shift perspective ▪ Value multiple perspectives	▪ Being quiet and structuring opportunities for students to think aloud ▪ Facilitating learning by using open-ended questions to assist students in exploration ▪ Selecting tasks that allow students to discern structures or patterns to make connections ▪ Allowing time for student discussion and processing in place of fixed rules or definitions ▪ Fostering persistence/stamina in problem solving ▪ Allowing time for students to practice
	8. Look for and express regularity in repeated reasoning	▪ Identify patterns and make generalizations ▪ Continually evaluate reasonableness of intermediate results ▪ Maintain oversight of the process ▪ Search for and identify and use shortcuts	▪ Providing rich and varied tasks that allow students to generalize relationships and methods and build on prior mathematical knowledge ▪ Providing adequate time for exploration ▪ Providing time for dialogue, reflection, and peer collaboration ▪ Asking deliberate questions that enable students to reflect on their own thinking ▪ Creating strategic and intentional check-in points during student work time

For classroom posters depicting the Mathematical Practices, please see: http://seancarberry.cmswiki.wikispaces.net/file/detail/12-20math.docx

Interpreting Categorical and Quantitative Data (S.ID)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Summarize, represent, and interpret data on a single count or measurement variable.

9-11.S.ID.1 – Represent data with plots on the real number line (dot plots, histograms, and box plots).

DCAS-Like

1A

An election involving four candidates for mayor has been held. Of the following, which is the best way to present the percentage of votes each candidate received?

A. Circle graph

B. Box plot

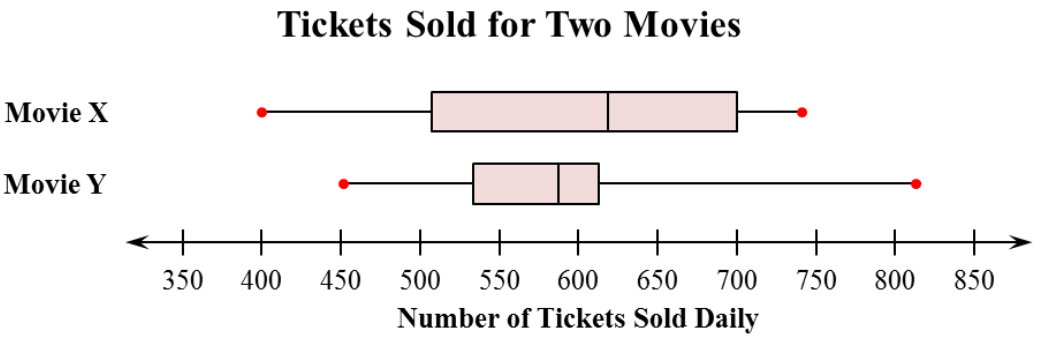
C. Scatterplot

D. Histogram

Next-Generation

1B

A movie theater recorded the number of tickets sold for two movies each day during one week. Box plots of the data are shown below.



Based on the box plot, determine whether each of the following statements is **True**, **False**, or **Cannot Be Determined** from the information given in the box plot.

	True	False	Cannot Be Determined
a. The mean number of tickets sold for Movie X is greater than the mean number sold for Movie Y.	○	○	○
b. The median number of tickets sold for Movie X is greater than the median number of tickets sold for Movie Y.	○	○	○
c. The interquartile range of the number of tickets sold for Movie X is greater than the interquartile range of the number of tickets sold for Movie Y.	○	○	○

9-11.S.ID.2 – Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

DCAS-Like

2A

Below are the scores for two different sections of a vocabulary quiz. Given that the distribution of scores follows a normal distribution, which section had a greater spread in the data?

Section 1:

16	10	19	18	17	18	14	16	16	15
13	12	15	12	18	20	10	15	11	18

Section 2:

11	11	16	14	15	11	10	18	17	19
9	10	9	14	10	19	9	9	15	17
12	10	12	11	14					

A. Section 1 had a greater spread

B. Both sections had no spread

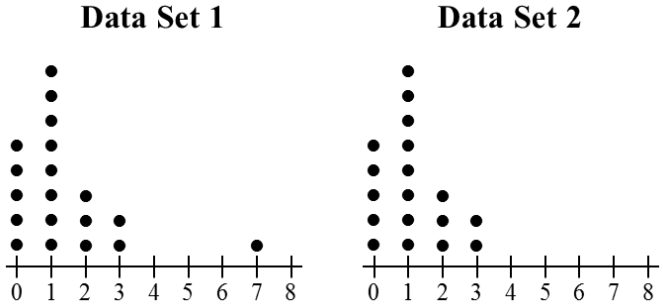
C. Both sections had equal spread

D. Section 2 had a greater spread

Next-Generation

2B

The frequency distributions of two data sets are shown in the dot plots below.



For each of the following statistics, determine whether the value of the statistic is greater for Data Set 1, equal for both data sets, or greater for Data Set 2.

	Greater for Data Set 1	Equal for Both Data Sets	Greater for Data Set 2
a. Mean	○	○	○
b. Median	○	○	○
c. Standard Deviation	○	○	○

9-11.S.ID.3 – Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

DCAS-Like

3A

Given the data points 18, 14, 12, 14, 11, 11, 19, 20, 16, and 11, which values would be considered outliers?

A. Outliers must be less than 4 or greater than 25

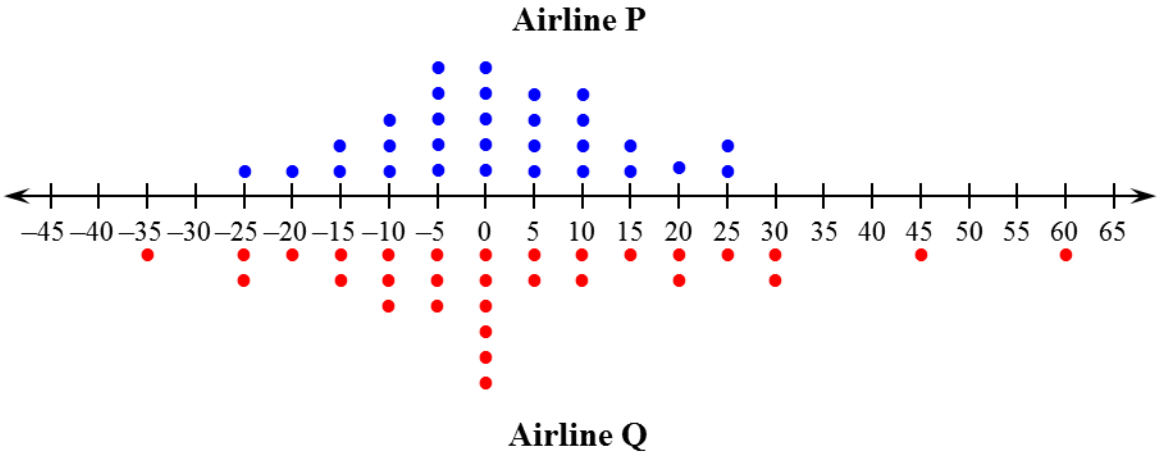
B. Outliers must be less than 11 or greater than 18

C. Outliers must be less than 11 or greater than 20

D. Outliers must be less than 0.5 or greater than 28.5

3B

The dot plots below compare the number of minutes 30 flights made by two airlines arrived before or after their scheduled arrival times.



- Negative numbers represent the minutes the flight arrived **before** its scheduled time.
- Positive numbers represent the minutes the flight arrived **after** its scheduled time.
- Zero indicates the flight arrived **at** its scheduled time.

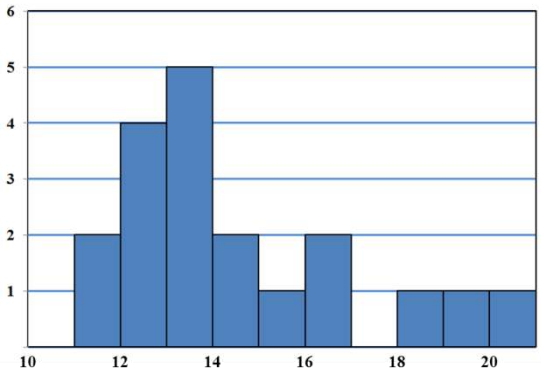
Based on these data, from which airline will you choose to buy your ticket? Use the ideas of center and spread to justify your choice.

9-11.S.ID.3 – Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

DCAS-Like

4A

Given the following histogram, how can we describe the shape of the data?



A. Skewed right

B. Skewed left

C. Symmetric

D. Constant

Next-Generation

4B

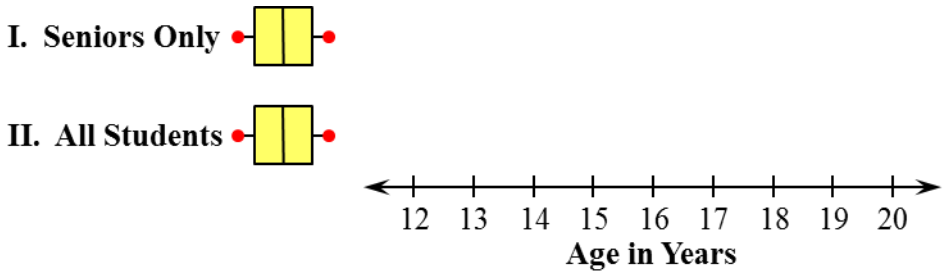
The ages of the students in a certain high school are to be graphed on a set of parallel box plots according to the following:

Set I: All seniors in the school (grade 12)
Set II: All students in the school (grades 9 through 12)

In the figure below, drag each of the two box plots into position above the number line to approximate the ages of the two sets of students. To do this:

- First move each box plot to an appropriate location according to its center.
- Then drag each endpoint to stretch the box plot to represent the spread.

Note: There are no outliers in either set.

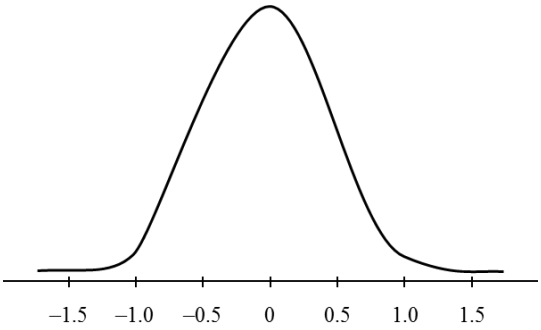


9-11.S.ID.4 – Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

DCAS-Like

5A

A clock manufacturer has found that the amount of time their clocks gain or lose per week is normally distributed with a mean of 0 minutes and a standard deviation of 0.5 minute, as shown below.



In a random sample of 1,500 of their clocks, which of the following is closest to the expected number of clocks that would gain or lose more than 1 minute per week?

A. 30

B. 50

C. 70

D. 90

Next-Generation

5B

Automobile manufacturers have to design the driver's seat area so that both tall and short adults can sit comfortably, reach all the controls and pedals, and see through the windshield. Suppose a new car is designed so that these conditions are met for people from 58 inches to 76 inches tall.

The heights of adult men in the United States are approximately normally distributed with a mean of 70 inches and a standard deviation of 3 inches. Heights of adult women are approximately normally distributed with a mean of 64.5 inches and a standard deviation of 2.5 inches.

a. What percentage of men in the United States is this car not designed to accommodate?

_____

b. What percentage of women in the United States is this car not designed to accommodate?

_____

Cluster: Summarize, represent, and interpret data on two categorical and quantitative variables.

9-11.S.ID.5 – Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

DCAS-Like

6A

The following table summarizes the number of students in a class that received different letter grades on 2 recent exams. The first exam is shown across the top row and is summarized by A_1, B_1, and C_1, and the second exam is in the first column, A_2, B_2, and C_2. What is the probability that a student gets an A on the first exam and a B on the second exam?

	A_1	B_1	C_1
A_2	2	7	1
B_2	2	10	3
C_2	1	3	1

A. 0.067

B. 0.1667

C. 0.333

D. 0.667

Next-Generation

6B

During one month, exactly half of the 180 babies born in a hospital were boys, and 40 of the babies weighted 4 kg or more. There were 26 baby boys who weighed 4 kg or more.

Using the information above, complete the following table.

	Less Than 4 kg	**4 kg orMore**
Boys		
Girls		

What percentage of the babies were girls weighing less than 4 kg? _____

9-11.S.ID.6 – Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*

b. Informally assess the fit of a function by plotting and analyzing residuals.

c. Fit a linear function for a scatter plot that suggests a linear association.

DCAS-Like

7A

It is thought that the score on a particular math test is dependent on the number of hours spent studying and that the equation used to describe the score is: $y = 50 + 3.5x$. A student who studied for 6 hours earned a score of 82. What is the residual for this score?

A. 11

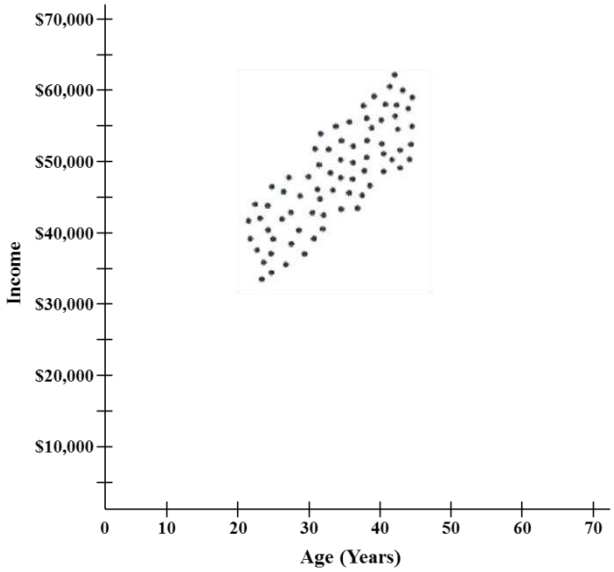
B. –11

C. 121

D. 10

7B

A random sample of graduates from a particular college program reported their ages and incomes in response to a survey. Each point of the scatterplot below represents the age and income of a different graduate.



a. Of the following equations, which best fits the data above? _____

1. $y = 1{,}000x$
2. $y = 1{,}000x + 15{,}000$
3. $y = 10{,}000x$
4. $y = 10{,}000x + 15{,}000$

b. Based on the data in the above scatterplot, predictions can be made about the income of a 35-year-old and the income of a 55-year-old. For which group is the prediction more likely to be accurate?

Cluster: Interpret linear models.

9-11.S.ID.7 – Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

DCAS-Like

8A

What does the intercept of a linear model represent?

A. The value of the independent variable when the dependent variable equals zero.

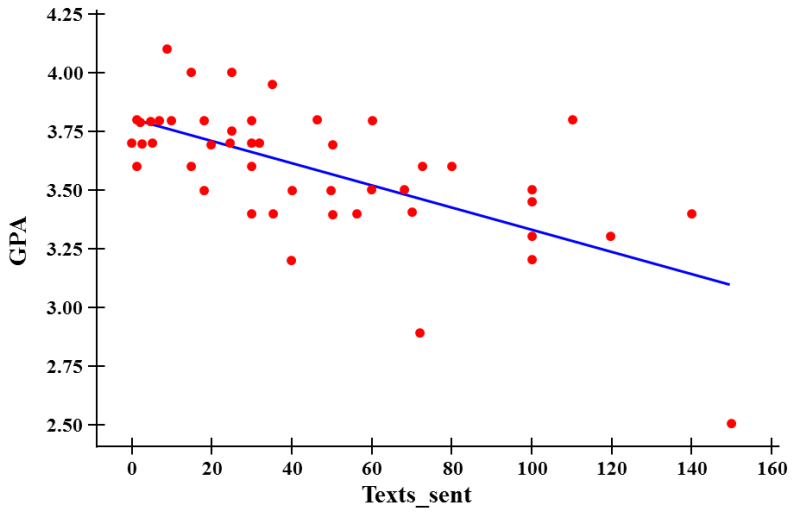
B. The value of the dependent variable when the independent variable equals zero.

C. The value of the independent variable when the dependent variable equals its maximum value.

D. The value of the independent variable when the dependent variable equals its minimum value.

8B

Meghan suspects that there is a relationship between the number of text messages high school students send and their academic achievement. To explore this, she asks a random sample of 52 students at her school how many text messages they sent yesterday and what their grade point average (GPA) was during the most recent marking period. Her data are summarized in the scatter plot below. The least squares regression line is also shown.



The equation of the least squares regression line is $\widehat{GPA} = 3.8 - 0.005(\text{Texts\_sent})$.

a. Interpret the quantity -0.005 in the context of these data.

b. Interpret the quantity 3.8 in the context of these data.

9-11.S.ID.8 – Compute (using technology) and interpret the correlation coefficient of a linear fit.

DCAS-Like

9A

Which calculator output shows the strongest linear relationship between x and y?

A. Lin Reg
$y = a + bx$
$a = 59.026$
$b = 6.767$
$r = 0.8643$

B. Lin Reg
$y = a + bx$
$a = 0.7$
$b = 24.2$
$r = 0.8361$

C. Lin Reg
$y = a + bx$
$a = 2.45$
$b = 0.95$
$r = 0.6022$

D. Lin Reg
$y = a + bx$
$a = -2.9$
$b = 24.1$
$r = -0.8924$

Next-Generation

9B

x	3	7	5	4	6	20	15	12
y	1	8	–1	5	10	18	20	–6

a. Use a graphing calculator to find the equation of the line of best fit for the data above.

_____

b. Is there a strong correlation between the data? ○ **Yes** ○ **No**

Explain.

9-11.S.ID.9 – Distinguish between correlation and causation.

DCAS-Like

10A

Why does correlation not imply causation?

A. Actually, correlation does imply causation.

B. Because we must take into account all possible variables when proving causation.

C. Because we must take into account all possible variables when proving correlation.

D. All of the above.

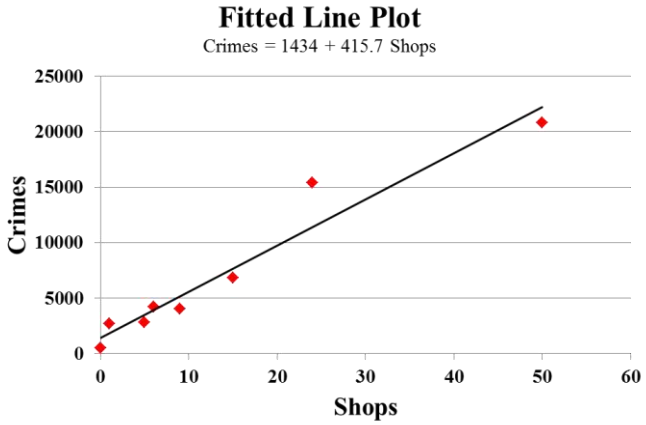
Next-Generation

10B

Many counties in the United States are governed by a county council. At public county council meetings, county residents are usually allowed to bring up issues of concern. At a recent public County Council meeting, one resident expressed concern that 3 new coffee shops from a popular company were planning to open in the county, and the resident believed that this would create an increase in property crimes in the county. (Property crimes include burglary, larceny-theft, motor vehicle theft, and arson—from http://www.fbi.gov/about-us/cjis/ucr/crime-in-the-u.s/2010/crime-in-the-u.s.-2010/property-crime).

To support this claim, the resident presented the following data and scatterplot (with the least-squares line shown) for 8 counties in the state:

County	Shops	Crimes
A	9	4000
B	1	2700
C	0	500
D	6	4200
E	15	6800
F	50	20800
G	5	2800
H	24	15400



The scatterplot shows a positive linear relationship between "Shops" (the number of coffee shops of this company in the county) and "Crimes" (the number of annual property crimes for the county). In other words, counties with more of these coffee shops tend to have more property crimes annually.

a. Does the relationship between Shops and Crimes appear to be linear? Would you consider the relationship between Shops and Crimes to be strong, moderate, or weak?

b. Compute the correlation coefficient. Does the value of the correlation coefficient support your choice in part a? Explain.

c. The equation of the least-squares line for these data is:

$$\text{Predicted Crimes} = 1434 + 415.7\ (\text{Shops})$$

Based on this line, what is the estimated number of additional annual property crimes for a given county that has 3 more coffee shops than another county?

d. Do these data support the claim that building 3 additional coffee shops will necessarily cause an increase in property crimes? What other variables might explain the positive relationship between the number of coffee shops for this company and the number of annual property crimes for these counties?

e. If the following two counties were added to the data set, would you still consider using a line to model the relationship? If not, what other types (forms) of model would you consider?

County	Shops	Crimes
I	25	36900
J	27	24100

Making Inferences and Justifying Conclusions (S.IC)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand and evaluate random processes underlying statistical experiments.

9-11.S.IC.1 – Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

DCAS-Like

11A

Your school's kitchen manager needs to find out how many packets of Choco-Chips will be sold per month in the cafeteria. Since these are a new chip brand, there is no prior information about Choco-Chips sales. Which of the following is the best way to find out?

A. Send a survey to every student in the school asking how many packets of Choco-Chips they plan to buy every month.

B. Buy a small amount for the first month, observe initial sales, and then make a judgment from there based on how many were bought and how to proceed.

C. Estimate how many of the other types of chip flavors are bought per month and using that information for Choco-Chips.

D. Ask the principal of the school.

Next-Generation

11B

The 54 students in one of several middle school classrooms were asked two questions about musical preferences: "Do you like rock?" "Do you like rap?" The responses are summarized in the table below.

		Likes Rap	
		Yes	**No**
Likes Rock	**Yes**	27	6
	No	4	17

a. Is this a random sample, one that fairly represents the opinions of all students in the middle school?

b. What percentage of the students in the classroom like rock?

c. Is there evidence in this sample of a positive association in this class between liking rock and liking rap? Justify your answer by pointing out a feature of the table that supports it.

d. Explain why the results for this classroom might not generalize to the entire middle school.

9-11.S.IC.2 – Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

DCAS-Like

12A

Your teacher claims that if you were to come up to the front of the class and select one of ten numbers, 1 through 10, randomly, his model would predict the numbers you will choose with statistically accurate significance. In fact, he is so confident he is willing to lower the p-value to 0.01. How good is his model of predicting your behavior?

Observed Value	Expected Value
1	4
3	1
5	7
2	2
7	6
1	4
10	1
4	10
8	4
4	3

A. His model works, even with $p = 0.01$

B. His model does not work since the values are too different to be due to chance alone

C. His model works but just barely

D. His model is so incorrect that it cannot be gauged

Next-Generation

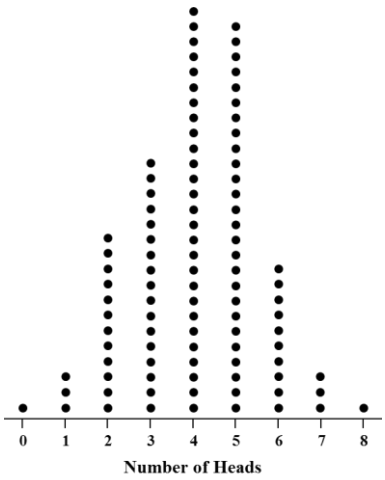
12B

Many researchers have studied chimpanzees to learn about their problem-solving skills. In 1978, researchers Premack and Woodruff published an article in Science magazine, reporting findings from a study on an adult chimpanzee named Sarah, who had been raised in captivity and had received extensive training using photos and symbols. In one experiment, Sarah was shown videotapes of eight different situations in which a human being was faced with a problem. After each videotape showing, Sarah was presented with two photographs, one of which depicted a possible solution to the problem. The researchers ensured that the order in which the photographs were presented was randomized (for example, the correct answer was not always presented first, etc.) and that the photographs had similar visuals (for example, similar colors, etc.) Of the eight problems, Sarah picked the photograph with the correct solution seven times.

Could Sarah have been merely guessing and just lucky with her responses, or is there evidence that Sarah does better than just guessing? Answer the following questions.

a. A student, James, decides to use simulation to investigate whether the study data provide evidence that Sarah was doing better than just randomly guessing, and so James tosses a coin 8 times and obtains 6 heads. Explain why James should repeat the process of tossing the coin 8 times and recording the number of heads, many times.

b. James repeats the process of 8 coin tosses 100 times, each time recording the number of heads on the 8 coin tosses. The following is a dot plot of his results.



Based on the above dot plot, what was the most common result for "number of heads" in 8 coin tosses?

_____

Why does this make sense?

c. Based on this dot plot, would you say that a score of 7 out of 8 would be unusual if Sarah has just been guessing?

○ **Yes** ○ **No** Why or why not?

d. Which of the following is a possible explanation for Sarah's performance?
    1. Sarah had been just guessing and got lucky with her responses.
    2. Sarah does better than just guessing.
    3. Both 1 and 2 are possible explanations.

e. Based on the simulation results, which of the following appears to be a *plausible* (likely) explanation for Sarah's performance?
    1. Sarah had been just guessing and got lucky with her responses.
    2. Sarah does better than just guessing.
    3. Both 1 and 2 are possible explanations.

f. Based on the results of this study, would it be reasonable to say that all chimpanzees do better than just guessing when faced with the kind of problems posed to Sarah? Explain why or why not.

Cluster: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

9-11.S.IC.3 – Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

DCAS-Like

13A

Jason decides to conduct an experiment to see which type of engine oil will make his car run better. Where does this experiment make a mistake with respect to bias?

- Step 1: Randomly pick four different types of engine oil from the local auto shop.
- Step 2: Change out the old oil as best you can to avoid residue.
- Step 3: Pour in the new oil, always the same amounts as the previous oil.
- Step 4: Drive around for the same distance and amount of time for each different type of oil.

A. Steps 1 and 2

B. Steps 2 and 3

C. Steps 3 and 4

D. Steps 1 and 4

Next-Generation

13B

A student interested in comparing the effect of different types of music on short-term memory conducted the following study: 80 volunteers were randomly assigned to one of two groups. The first group was given five minutes to memorize a list of words while listening to rap music. The second group was given the same task while listening to classical music. The number of words correctly recalled by each individual was then measured, and the results for the two groups were compared.

a. Is this an experimental study or an observational study? Justify your answer.

b. In the context of this study, explain why it is important that the subjects were randomly assigned to the two experimental groups (rap music and classical music).

9-11.S.IC.4 – Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

DCAS-Like

14A

A company specializing in building robots that clean your house has found that the average amount of time kids spend cleaning their houses is about 2 hours per week. If their sample size was 1000 randomly chosen kids and the standard deviation was 0.3 hours, what is the margin of error for a confidence interval of 95%?

A. 0.392

B. 0.018

C. 0.039

D. 0.185

Next-Generation

14B

Sometimes hotels, malls, banks, and other businesses will present a display of a large, clear container holding a large number of items and ask customers to estimate some aspect of the items in the container as a contest. In some cases, contestants are allowed to sample items from the jar, but usually contestants simply have to estimate based on visual inspection of the jar. A local bank is running such a contest, but one of the bank employees is concerned.

The bank has placed 1500 marbles in a very large, clear jar near the customer entrance. Since the bank's logo's colors are blue and white, some of the 1500 marbles are blue and the rest are white. In order to enter the contest, a customer must fill in an entry form with his/her estimate for the percentage of blue marbles in the jar and then place the entry form in a ballot box. A random drawing will be held and the first entry drawn that correctly estimates the percentage of blue marbles in the whole jar will receive a $100 gift certificate. The entry form says the following:

> ***Name:*** _____ ***Phone:*** _____
>
> ***In think that 1 out of every _____ marbles in this jar is blue.***
>
> ***(Fill in the blank with a "2", "3", "4", "5", or "6".)***

Note that for the ease of the contestants, the estimate is to be stated as "1 out of every 2" instead of "50%," "1 out of every 3" instead of "33.3%," and so on.

Now the concerned employee is fairly confident that the true proportion of blue marbles is 25% (1 out of every 4), but he has heard other employees (some of whom are responsible for the contest) state a true proportion value that is different. The employee is worried enough that he wants to investigate, but he certainly does not want to empty the jar and inspect all 1500 marbles! He decides to select a random sample of marbles from the jar and calculate the percentage of blue marbles in his sample. The percentage of blue marbles in the random sample will be his estimate for the actual percentage of blue marbles in the jar.

He selects a random sample of 5 marbles, and only 1 of the marbles is blue. Based on this sample which gives him an estimate of 20% (1 out of 5) blue marbles, the employee is concerned, but he decides to stick with his original claim of 25% blue marbles in the jar. However, he is now inspired to take even larger samples. He records his results in the table below.

Sample Number	Sample Size	Total Number of Blue Marbles in Sample	Percentage of Blue Marbles in Sample
1	5	1	20%
2			
3			
4			
5			
6			
7			
8			
9			

a. His second random sample consists of 12 marbles. Only 2 of the marbles are blue. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar or should he come up with a different estimate? Explain why you think this.

b. He then takes a random sample of 20 marbles (Sample 3). Five of the 20 marbles are blue. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar or should he come up with a different estimate? Explain why you think this.

c. He then takes a random sample of 32 marbles (Sample 4). Eight of the marbles are blue. Based on this sample, do you think the employee should stick with his original claim of 25% blue marbles in the jar or should he come up with a different estimate? Explain why you think this.

At this point, the employee feels compelled to talk to the bank manager who is responsible for the contest. The bank manager is a little surprised by the results, but she is not overly concerned. She is quite confident that the true proportion of blue marbles is 33.3%, or 1 in every 3, and she asks the concerned employee to go back and look at an even larger random sample of marbles.

Here are the results of some additional random samples.

- Sample 5 – sample size = 40, 13 blue
- Sample 6 – sample size = 55, 17 blue
- Sample 7 – sample size = 65, 21 blue
- Sample 8 – sample size = 75, 24 blue
- Sample 9 – sample size = 85, 27 blue

d. Based on the random sample of 85 marbles, and mindful that the correct, true proportion of blue marbles in the jar is 1 in 2, or 1 in 3, or 1 in 4, etc., do you think that the employee should challenge the bank manager's claim that 1 in every 3 marbles is blue? Explain why you think this.

9-11.S.IC.5 – Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

DCAS-Like

15A

The following data shows, in minutes, how long it takes a new drug to dissolve inside your stomach. A market competitor's data is shown as well. Is there a significant difference between the two drug manufacturers? Conduct a one-tail test with $a = 0.01$.

New Drug (Dissolving Time in Minutes)	**Competitor** (Dissolving Time in Minutes)
5	1
2	5
6	2
2	6
1	2
6	4
2	1
7	6
7	2
5	6
6	2
3	6
6	2
2	6
8	4
1	5
4	2

A. Yes, the p-value is below the significance value

B. No, the p-value is above the significance value

C. It is very close, they are both exactly the same

D. None of the above

15B

For a sample of 36 men, the mean head circumference if 57.5 cm with a standard deviation equal to 2.4 cm. For a sample of 36 women, the mean head circumference is 55.3 cm with a standard deviation equal to 1.8 cm.

a. To determine if the mean head circumference for men was greater than for women, what would be the null and alternative hypotheses?

b. Assuming the conditions for a t-test are met (and population variances are equal), calculate the test statistic.

c. Calculate the p-value.

d. Make a conclusion based on a 5% significance level and interpret the result in the context of the problem.

9-11.S.IC.6 – Evaluate reports based on data

DCAS-Like

16A

A study is done to determine which steroid cream is more effective for bug bites. If the only bug bites treated in this study were mosquito bites, which of the following is true?

A. The steroid cream that is found to be the best will work for all bug bites.

B. The steroid cream that is found to be the best will work only for mosquito bites.

C. The study will only be able to produce results concerning the effect of the steroid creams on mosquito bites.

D. The observational study is inherently biased.

Next-Generation

16B

Consider each of the following survey questions. For each one, explain any bias you can find. If you think the question is unbiased (or fair), explain why.

a. Do you agree that it is important to make "ending homelessness" a high priority?

b. Which of the following factor is the most important to address in order to slow global climate change?

- ○ Car emissions
- ○ Airplane emissions
- ○ Pollutants from private industry
- ○ Dependence on oil

c. How important is it that teacher salaries be raised?

Conditional Probability and the Rules of Probability (S.CP)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand independence and conditional probability and use them to interpret data.

9-11.S.CP.1 – Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").

DCAS-Like

17A

The set of all outcomes of a rolled die is {1, 2, 3, 4, 5, 6}. What is the complement of the subset {1, 2}?

A. {3, 4, 5, 6}

B. {1, 2}

C. {5, 6}

D. There is not enough information to determine.

Next-Generation

17B

In a survey of 250 people concerning how they obtain information about current events, 70 people read a newspaper each day, 130 watch news on television each day, and 30 read a newspaper and watch news on television each day.

a. If a person is selected at random from the group surveyed, what is the (empirical) probability that the person does not either read a newspaper or watch news on television each day?

    1. 80
    2. None of the answers given
    3. 0.32
    4. 0.68

b. If a person is selected at random from the group surveyed, what is the (empirical) probability that the person reads the newspaper but does not watch the news on television each day?

    1. 0.16
    2. 0.282
    3. None of the answers given
    4. 40

c. If a person is selected at random from the group surveyed, what is the (empirical) probability that the person does not read a newspaper each day?

    1. 0.4
    2. 0.72
    3. None of the answers given
    4. 180

9-11.S.CP.2 – Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

DCAS-Like

18A

Mary rolls two number cubes with sides numbered from 1 to 6.

If she rolls a 3 on one of the cubes, what is the probability that the sum of the numbers facing up on both cubes is greater than or equal to 5?

A. 0.62

B. 0.75

C. 0.83

D. 0.91

Next-Generation

18B

Determine if the following events are independent or not independent. $P(A)$ is the probability of event A occurring and $P(B)$ is the probability of event B occurring.

	Independent	**Not Independent**
a. $P(A) = 0.60$: you cleaned your room this morning. $P(B) = 0.1$: your mom is upset with you.	☐	☐
b. $P(A) = 0.55$: you enjoy strawberry cheesecake. $P(B) = 0.1$: you like strawberries.	☐	☐
c. $P(A) = 0.33$: you sleep on your left side at night. $P(B) = 0.25$: you snore so loudly that you can wake your neighbor's cat up.	☐	☐
d. $P(A) = 0.55$: you prefer Mario over Luigi. $P(B) = 0.45$: you prefer Luigi over Mario.	☐	☐
e. $P(A) = 0.30$: your cat woke you up this morning. $P(B) = 0.1$: your cat was plotting your demise.	☐	☐

9-11.S.CP.3 – Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given B is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

DCAS-Like

19A

For two events A and B, $P(A) = \frac{1}{2}$, and $P(B|A) = \frac{2}{3}$. What is $P(B)$?

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. There is not enough information to determine.

Next-Generation

19B

There are four red envelopes, four blue envelopes, and four $1 bills, which will be placed in four of the eight envelopes. Define the event *A* as "*you pick a lucky envelope (one that has a $1 bill in it)*" and event *B* as "*you pick a blue envelope*."

Suppose one $1 bill is placed in a blue envelope, and the three remaining $1 bills are placed in three red envelopes.

a. Write the following probability questions symbolically (using letters A and/or B).
   i. If you choose one envelope at random, what is the probability that you pick a lucky envelope?
   ii. If you know that the envelope you picked is blue, what is the probability that you picked a lucky envelope?
   iii. Are the events in item a. independent events?

b. Now suppose we redistributed the four $1 bills between two blue and two red envelopes.
   i. If you choose one envelope at random, what is the probability that you pick a lucky envelope?
   ii. If you know that the envelope you picked is blue, what is the probability that you picked a lucky envelope?
   iii. Are the events in item b. independent events?

9-11.S.CP.4 – Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

DCAS-Like

20A

A sample of 100 female politicians was asked, "Which ice cream flavor do you prefer: chocolate or vanilla?" The respondents were classified by their political parties: Party *X* or Party *Y*. The results are shown in the table below.

		Flavor Preference		
		Chocolate	***Vanilla***	***Total***
Political Party	Party *X*		?	40
	Party *Y*			60
	Total	50	50	100

If ice cream flavor preference is independent of political party, how many female politicians are in Party *X* and prefer vanilla?

A. 20

B. 30

C. 40

D. 50

20B

Jaime randomly surveyed some students at his school to see what they thought of a possible increase to the length of the school day. The results of his survey are shown in the table below.

Lengthening School Day Survey

Grade	In Favor	Opposed	Undecided
9	12	6	9
10	15	3	11
11	8	12	10
12	5	16	9

Part A

A newspaper reporter will randomly select a grade 11 student from this survey to interview. What is the probability that the student selected is opposed to lengthening the school day? Show your work to support your answer.

Part B

The newspaper reporter would also like to interview a student in favor of lengthening the school day. If a student in favor is randomly selected, what is the probability that this student is also from grade 11? Show your work to support your answer.

9-11.S.CP.4 – Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

DCAS-Like

21A

Gender and Color of Puppies		
	Male	**Female**
Black	1	2
Brown	1	3

The table above shows the gender and color of 7 puppies. If a puppy selected at random from the group is brown, what is the probability it is a male?

A. $\frac{2}{7}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

21B

The table shows the color and style of the vehicles sold in one month at a car dealership.

		Color					
		Black	**Silver**	**Red**	**Tan**	**Other**	***Total***
Style of Vehicle	Truck	8	7	2	1	5	***23***
	SUV	7	15	5	12	15	***54***
	Sedan	12	10	6	21	8	***57***
	Sports Car	7	3	12	0	2	***24***
	Total	***34***	***35***	***25***	***34***	***30***	***158***

a. What percentage of the vehicles sold were silver SUVs?

b. What is the most common combination of vehicle style and color sold?

c. One salesman made the statement, "Over half of the sports cars we sold are red." Explain whether this statement is correct. Include a specific example to support your answer.

9-11.S.CP.5 – Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

DCAS-Like

22A

When looking at the association between the events "owns a car" and "owns a pet," if the events are independent, then the probability:

$P(\text{owns a pet}|\text{owns a car})$ is equal to _____.

A. $P(\text{owns a pet})$

B. $P(\text{owns a car})$

C. $P(\text{owns a pet}) \times P(\text{owns a car})$

D. $P(\text{owns a pet}) + P(\text{owns a car})$

22B

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Data on survival of passengers are summarized in the table below. We will use this data to investigate the validity of such claims. (Data source: http://www.encyclopedia-titanica.org/titanic-statistics.html)

	Survived	**Did Not Survive**	**Total**
First Class Passengers	201	123	324
Second Class Passengers	118	166	284
Third Class Passengers	181	528	709
Total	***500***	***817***	***1,317***

a. Are the events "passenger survived" and "passenger was in first class" independent events? Support your answer using appropriate probability calculations.

b. Are the events "passenger survived" and "passenger was in third class" independent events? Support your answer using appropriate probability calculations.

c. Did all passengers aboard the Titanic have the same probability of surviving? Support your answer using appropriate probability calculations.

Cluster: Use the rules of probability to compute probabilities of compound events in a uniform probability model.

9-11.S.CP.6 – Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.

DCAS-Like

23A

A new superman Master Card has been issued to 2000 customers. Of these customers, 1500 hold a Visa card, 500 hold an American Express card, and 40 hold a Visa card and an American Express card. What is the probability that a customer chosen at random holds a Visa card, given that the customer holds an American Express card?

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. $\frac{2}{25}$

D. $\frac{1}{50}$

Next-Generation

23B

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Data on survival of passengers are summarized in the table below. (Data source: http://www.encyclopedia-titanica.org/titanic-statistics.html)

	Survived	Did Not Survive	Total
First Class – Children	4	1	5
First Class – Women	139	4	143
First Class – Men	58	118	176
Second Class – Children	22	0	22
Second Class – Women	83	12	95
Second Class – Men	13	154	167
Third Class – Children	30	50	80
Third Class – Women	91	88	179
Third Class – Men	60	390	450
Total	***500***	***817***	***1,317***

a. Some believe that the rescue procedures favored the wealthier first class passengers. Did all passengers aboard the Titanic, regardless of class, have the same probability of surviving? Support your answer using appropriate probability calculations.

b. Others believe that the survival rates can be explained by the "women and children first" policy. Did all passengers aboard the Titanic, regardless of gender, have the same probability of surviving? Support your answer using appropriate probability calculations.

9-11.S.CP.7 – Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.

DCAS-Like

24A

The probabilities an adult male has high blood pressure and/or high cholesterol are given below.

		Blood Pressure	
		High	**Normal**
Cholesterol	**High**	0.10	0.20
	Normal	0.15	0.55

What is the probability a randomly selected adult male has high blood pressure or high cholesterol?

A. 0.075

B. 0.375

C. 0.45

D. 0.55

Next-Generation

24B

At Mom's diner, everyone drinks coffee. Let C equal the event that a randomly selected customer puts cream in their coffee. Let S equal the event that a randomly selected customer puts sugar in their coffee. Suppose that after years of collecting data, Mom has estimated the following probabilities:

$P(C) = 0.6$

$P(S) = 0.5$

$P(C \text{ or } S) = 0.7$

Estimate $P(C \text{ and } S)$. Interpret this value in the context of the problem.

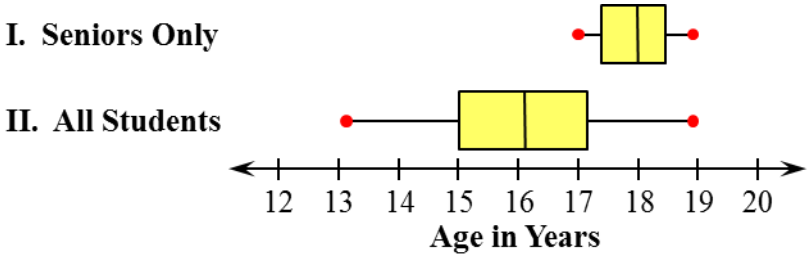
Answer Key and Item Rubrics

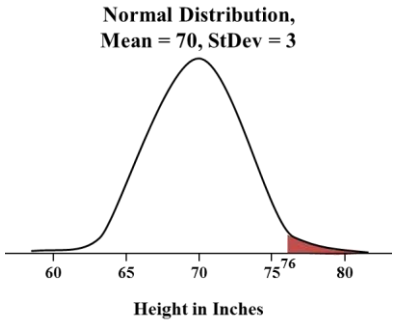
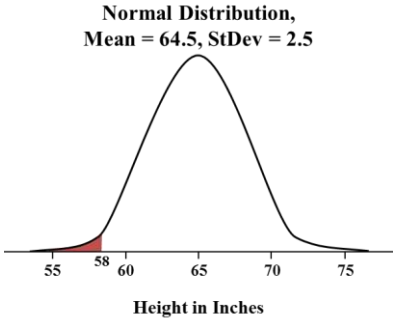
Interpreting Categorical and Quantitative Data (S.ID)

DCAS-Like Answer	Next-Generation Solution
1A: A (9-11.S.ID.1)	**1B:** ***Key:*** a. Cannot be determined (C) b. True (T) c. True (T) ***Scoring Rubric*** Responses to this item will receive 0-2 points based on the following: **2 points:** CTT – The student has a thorough understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student knows that the mean cannot be determined from the box plots and correctly compares the median and interquartile range for both data sets. **1 point:** TTT, FTT – The student has only a basic understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student correctly compares the median and interquartile range for both data sets but does not realize that the mean cannot be used to compare the data sets. **0 points:** All other possibilities. The student demonstrates inconsistent understanding of how to appropriately use the mean, median, and interquartile range to compare data in box plots. The student correctly compares either the median or the interquartile range of the two data sets. OR The student correctly compares neither the median nor the interquartile range of the two data sets.

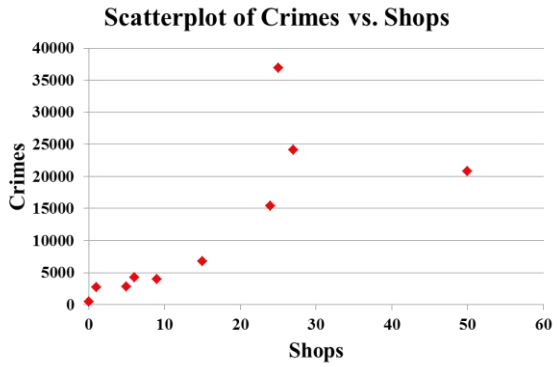
DCAS-Like Answer	Next-Generation Solution
2A: D (9-11.S.ID.2)	**2B:** ***Key*** a. Greater for Data Set 1 b. Equal for both data sets c. Greater for Data Set 1 ***Scoring Rubric*** Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a thorough understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student correctly indicates how the inclusion of the outlier affects both the measures of center (mean, median) and spread (standard deviation) **1 point:** The student has a basic understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student correctly identifies how the outlier affects the mean and median but not the standard deviation. OR The student correctly identifies how the outlier affects the standard deviation and the mean or median. **0 points:** The student has an inconsistent understanding of how to apply mathematical concepts and carry out mathematical procedures for comparing the center and spread of two different data sets, where one set contains an outlier. The student fails to correctly identify how the outlier affects the mean, median, and standard deviation. OR The student correctly identifies how the outlier affects the standard deviation but not the mean or median.
3A: D (9-11.S.ID.3)	**3B:** ***Sample Top-Score Response*** I would choose to buy the ticket from Airline P. Both airlines are likely to have an on-time arrival since they both have median values at 0. However, Airline Q has a much greater range in arrival times. Airline Q could arrive anywhere from 35 minutes early to 60 minutes late. For Airline P, this flight arrived within 10 minutes on either side of the scheduled arrival time two-thirds of the time, and for Airline Q, that number was only about one-half. For these reasons, I think Airline P is the better choice.

DCAS-Like Answer	Next-Generation Solution
	Scoring Rubric Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student chooses Airline P and clearly explains that both airlines have the same center but that Airline P has a smaller spread. **1 point:** The student has some understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student states that either airline could be chosen because they have the same median and does not address the issue of spread. OR The student states that both airlines have the same median and chooses Airline P but does not justify the choice based on spread. OR The student explains that Airline P would be the better choice based on the smaller spread but does not identify that both airlines have the same median. **0 points:** The student demonstrates an inconsistent understanding of how to make productive use of knowledge and problem-solving skills by comparing center and spread of two data sets using a graph and interpreting the results. The student does not state that the two airlines have the same median and that Airline Q has greater spread. The student either does not make a choice between the two airlines or makes a choice but does not defend it using center or variation.

DCAS-Like Answer	Next-Generation Solution
4A: A (9-11.S.ID.3)	**4B:** ***Sample Top-Score Response***  *Graphs should show:* Median of I > Median of II Range of I < Range of II Max of I ≤ Max of II ***Scoring Rubric for Multi-Part Items*** Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student accurately represents the median of Set I as greater than the median of Set II. The student also accurately represents the range of Set I as less than the range of Set II and represents the maximum of Set I as less than or equal to the maximum of Set II. **1 point:** The student has a basic understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student accurately represents the median of Set I as greater than the median of Set II. But the student misrepresents the relationship between the ranges of both sets or between the maximums of both sets. **0 points:** The student demonstrates an inconsistent understanding of how to apply the mathematical concepts of center and spread to compare data sets in context. The student does not accurately represent the median of Set I as greater than the median of Set II.

DCAS-Like Answer	Next-Generation Solution
5A: C (9-11.S.ID.4)	**5B:** a. For men, we want the percentage of the normal distribution with mean 70 and standard deviation 3 that is above 76 inches or below 58 inches. Since 58 is 4 standard deviations below 70, the percentage below 58 is insignificant, so all we need is the percentage above 76, which corresponds to the shaded region in the diagram below. The area of this region is 0.0228, so about 2.3% of adult men will not fit in this car.  b. For women, 76 inches is $\frac{76-64.5}{2.5} = 4.6$ standard deviations above the mean, so essentially 0% of women are too tall for the car. Thus, all we need is the percentage below 58 inches, which corresponds to the shaded region in the diagram below. The area of this region is 0.00466, so about 0.5% of adult women will not fit in this car. 

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>6A: A
(9-11.S.ID.5)</td><td>6B:
<table>
<tr><th></th><th>Less Than 4 kg</th><th>4 kg orMore</th></tr>
<tr><td>Boys</td><td>64</td><td>26</td></tr>
<tr><td>Girls</td><td>76</td><td>14</td></tr>
</table>
$\frac{76}{180} = 42.2\%$</td></tr>
<tr><td>7A: B
(9-11.S.ID.6)</td><td>7B:
a. Solution: 2
b. Solution: 35 year olds. The prediction for the 35 year old is more likely to be accurate because the age is contained within the interval of the data set. The age 55 is outside the interval of the data set, so any prediction for the income of a 55 year old would be an extrapolation.</td></tr>
<tr><td>8A: B
(9-11.S.ID.7)</td><td>8B:
a. Interpretation of the slope: For students at this school, the predicted GPA decreases by 0.005 for each additional text message sent. OR GPA decreases by 0.005, on average, for each additional text message sent.
b. Interpretation of intercept: The model predicts that students at this school who send no text messages have, on average, a GPA of 3.8.</td></tr>
<tr><td>9A: D
(9-11.S.ID.8)</td><td>9B:
a. $y = 0.9x - 1.1$
b. No, data is too far from line of best fit.</td></tr>
<tr><td>10A: B
(9-11.S.ID.9)</td><td>10B:
a. The relationship does appear to be linear. The relationship would be considered a strong and positive given how closely the points adhere to a line with positive slope.
b. $r = 0.968$ Since the pattern shown is one of very strong, positive, linear association, a correlation coefficient value near +1 is plausible.</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
	c. $415.7 \bullet 3 = 1247.1$ According to the model, the predicted increase in the number of annual property crimes for a county with 3 additional coffee shops would be 1247. d. Association, no matter how strong, does not necessarily imply causation. It is unlikely that building a new coffee shop would cause crime rates to increase, for such logic would imply that coffee drinkers engage in more criminal behavior than non-coffee drinkers, the coffee shop attracts criminals to the county, etc. From a perspective of context, students should consider other variables that may be responsible for the association (e.g., counties with higher populations or higher population density may have both more coffee shops and more property crimes). Depending upon student knowledge of experiments and observational studies, a discussion can occur reinforcing the risk associated with stating/implying causation based on data from an observational study. e. With the addition of the two observations, the scatterplot now displays a curved relationship with one outlier at (50, 20800). The scatterplot still shows a positive relationship between "Shops" (the number of coffee shops of this coffee shop chain in the county) and "Crimes" (the number of annual property crimes for the county in the previous year)—but the relationship no longer appears to be linear (or does not appear as linear as before). When only a few observations are used to assess a trend, sometimes just adding one or two points can change the appearance significantly. The new plot is shown below. This relationship might be modeled using a quadratic or an exponential curve. 

Making Inferences and Justifying Conclusions (S.IC)

DCAS-Like Answer	Next-Generation Solution
11A: B (9-11.S.IC.1)	**11B:** a. This is not a randomly selected sample that fairly represents the students in the school. See item d. for more details. b. $\frac{33}{54} = 61.1\%$ c. Yes, there is evidence of a positive association. Of those who like Rap, $\frac{27}{31} = 87.1\%$ like Rock, too. This means that the percentage of those who like Rock is higher among those who like Rap than among the entire sample. d. The sample is not necessarily a random sample. While it might be true that the association holds in other classes, we have no evidence of this. It is possible, for instance, that this was an unusual class at this school—maybe this class consisted entirely of music students, and their preferences would be different than in other classes or than in the entire school.
12A: B (9-11.S.IC.2)	**12B:** a. Because James needs to see what happens in the long-run on 8 coin tosses. What outcomes are more common? What outcomes are less common? How often do 7 heads in 8 coin tosses happen just by chance? b. Most common outcome is 4, which makes sense because that is what we expect would happen if Sarah was randomly picking a photograph. c. Yes, 7 is unusual (surprising) because on the 100 tosses, an outcome as or more extreme as 7 happened by chance only 4 times. d. Item c., both 1 and 2 are possible explanations. e. Item 2, Sarah does better than just guessing. f. The question of interest is about Sarah getting the answer right, rather than about all chimpanzees. Note that Sarah's trials are not a random sample from the population of all possible chimp responses.

DCAS-Like Answer	Next-Generation Solution
13A: C (9-11.S.IC.3)	**13B:** a. This is an experiment, because a treatment (type of music) was imposed on the subjects. b. We randomly assign subjects to groups in order to create two groups that are as similar as possible with respect to any variables that might influence the subjects' capacity for recalling words. That way, any differences we see in the mean number of words recalled can be attributed to either the type of music or to variation arising from random assignment. For example, if subjects were not assigned at random and were allowed to choose which music group they wanted to participate in, people who are easily distracted and may have more difficulty memorizing a list of words may tend to choose the classical music group because there are usually no lyrics that might distract in classical music.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>14A: B
(9-11.S.IC.4)</td><td>
14B
For questions a-d, students would complete the tale as shown below.
<table>
<tr><th>Sample Number</th><th>Sample Size</th><th>Total Number of Blue Marbles in Sample</th><th>Percentage of Blue Marbles in Sample</th></tr>
<tr><td>1</td><td>5</td><td>1</td><td>20.0%</td></tr>
<tr><td>2</td><td>12</td><td>2</td><td>16.7%</td></tr>
<tr><td>3</td><td>20</td><td>5</td><td>25.0%</td></tr>
<tr><td>4</td><td>32</td><td>8</td><td>25.0%</td></tr>
<tr><td>5</td><td>40</td><td>13</td><td>32.5%</td></tr>
<tr><td>6</td><td>55</td><td>17</td><td>30.9%</td></tr>
<tr><td>7</td><td>65</td><td>21</td><td>32.3%</td></tr>
<tr><td>8</td><td>75</td><td>24</td><td>32.0%</td></tr>
<tr><td>9</td><td>85</td><td>27</td><td>31.8%</td></tr>
</table>
For questions a-c, students are asked to consider if the claim of a population proportion of 25% blue marbles is viable. Generally speaking, the results of each of the samples in these questions (of size 12, 20, and 32 respectively) would not warrant abandoning a claim of a population proportion of 25% blue marbles. Note: a student may be swayed to dismiss that claim given the early samples ($n = 5$ and 12) which yield estimates of “1 in 5 blue” and “1 in 6 blue” respectively. However, later sampling ($n = 20$ and 32) could encourage the student to reconsider for similar reasons.

For question d, as the sample results yield estimates close to 33.3% (and far from 25%), students should communicate greater confidence that the population proportion is in fact 33.3% (as opposed to the “neighboring” choices of 25% and 50%).
</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
15A: B (9-11.S.IC.5)	**15B:** a. $Ho: \mu_M - \mu_F, Ha: \mu_M > \mu_F$ b. $t = 4.4, df = 70$ c. P-value $= P(t_{70} > 4.4) = 0.00002$ d. Since 0.00002 is less than 0.05, reject the null hypothesis. At the 5% significance level, there is sufficient statistical evidence to indicate that the mean male head circumference is greater the mean female head circumference.
16A: C (9-11.S.IC.6)	**16B:** a. The question implies that the questioner holds this opinion, thus biasing results. b. The question assumes that the respondent will think that one of the given factors is important and that it is important to slow global climate change, biasing results. c. The question implies that teacher salaries should be raised.

Conditional Probability and the Rules of Probability (S.CP)

DCAS-Like Answer	Next-Generation Solution
17A: A (9-11.S.CP.1)	**17B:** a. #3 b. #1 c. #2
18A: C (9-11.S.CP.2)	**18B:** a. Independent b. Not independent c. Not independent d. Not independent e. Independent
19A: D (9-11.S.CP.3)	**18D:** a. i. Out of 8 envelopes, 4 have \$1 bills in them. So the probability of picking a lucky envelope (with a \$1 bill) is $\frac{4}{8} = \frac{1}{2}$. Symbolically we write this as $P(A) = \frac{1}{2}$. ii. In this part, we only consider blue envelopes. Out of 4 blue envelopes, only 1 has a \$1 bill in it. So the probability of picking the lucky envelope is $\frac{1}{4}$. This is a conditional probability: the probability that the envelope is lucky given that the iii. Knowing that the envelope was blue (event B) changed the probability that the envelope was a lucky envelope (event B) from $\frac{1}{2}$ to $\frac{1}{4}$. Therefore A and B are not independent events. b. i. Out of 8 envelopes, 4 have \$1 bills in them. So the probability of picking a lucky envelope (with a \$1 bill) is $\frac{4}{8} = \frac{1}{2}$. Symbolically we write this as $P(A) = \frac{1}{2}$.

DCAS-Like Answer	Next-Generation Solution
	ii. In this part, we only consider blue envelopes. Out of 4 blue envelopes, 2 have $1 bills in them. So the probability of picking a lucky envelope is $\frac{2}{4} = \frac{1}{2}$. This is a conditional probability: the probability that the envelope is lucky given that the envelope is blue. Symbolically we write this as $P(A\|B) = \frac{1}{2}$. iii. Knowing that the envelope was blue (event B) did not change the probability that the envelope was a lucky envelope (event B). Therefore, A and B are independent events.
20A: A (9-11.S.CP.4)	**20B:** Each item is scored independently and will receive 1 point. Part A: 0.4 (or equivalent) Part B: 0.2 (or equivalent)
21A: C (9-11.S.CP.4)	**21B:** a. 4% b. Tan sedans c. The statement is incorrect. The percentage of sports cars sold that were red is 48%. This is less than half. *Points Assigned* a. 1 point b. 1 point for tan sedans c. 1 point for correctly identifying that the statement is incorrect with a complete justification

DCAS-Like Answer	Next-Generation Solution
22A: A (9-11.S.CP.5)	**22B:** a. We use the fact that two events A and B are independent, if $P(A\|B) = P(A)$. In this case, we compare the conditional probability $P(\text{passenger survived}\|\text{passenger was in first class})$ with the probability $P(\text{passenger survived})$. The probability of surviving, given that the passenger was in first class, is the fraction of first class passengers who survived. That is, we restrict the sample space to only first class passengers to obtain: $P(\text{passenger survived}\|\text{passenger was in first class}) = \frac{201}{324} \approx 0.620$ The probability that the passenger survived is the number of all passengers who survived divided by the total number of passengers. That is, $P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$. Since $0.620 \neq 0.380$, the two given events are not independent. Moreover, we can say that being a passenger in first class increased the chances of surviving. Note that we could also compare $P(\text{passenger was in first class}\|\text{passenger survived}) = \frac{201}{500} \approx 0.402$ and $P(\text{passenger was in first class}) = \approx 0.246$. Again, since $0.402 \neq 0.246$ the two events are not independent. b. Using similar reasoning as in part a., we compare $P(\text{passenger survived}\|\text{passenger was in third class}) = \frac{181}{709} \approx 0.255$, and $P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$. Since $0.255 \neq 0.380$, the two given events are not independent. Moreover, we can see that being a passenger in third class decreased the chances of being rescued. c. One way to answer this question is to compare the probabilities of surviving for randomly chosen passengers in first, second, and third class, respectively. To do this, we calculate the following conditional probabilities:

DCAS-Like Answer	Next-Generation Solution
	▪ In part a. we calculated that $P(\text{passenger survived}\mid\text{passenger was in first class}) = \frac{201}{324} \approx 0.620$ ▪ The probability that the passenger survived, given that this passenger was in second class, is the fraction of passengers in second class who survived, that is $P(\text{passenger survived}\mid\text{passenger was in second class}) = \frac{118}{284} \approx 0.415$ ▪ In part b., we calculated that $P(\text{passenger survived}\mid\text{passenger was in third class}) = \frac{181}{709} \approx 0.255$ Comparing these probabilities we can say that not all passengers aboard the Titanic had the same chance of surviving. More precisely, the chance of surviving depended on the class, with the first class passengers having the greatest, and the third class passengers having the smallest chance of being rescued. Note that there are different probabilities we could use to answer this question (for example we could compare probability that a randomly selected passenger survived $P(\text{passenger survived}) = \frac{500}{1317} \approx 0.380$ with the conditional probability $P(\text{passenger survived}\mid\text{passenger was in third class}) = \frac{201}{324} \approx 0.620$. However, the conclusion should always be the same.

DCAS-Like Answer	Next-Generation Solution
23A: C (9-11.S.CP.6)	**23B:** a. First, we ignore the gender and compare the probability of surviving for a randomly chosen passenger in first class, to the probabilities of surviving for randomly selected second and third class passengers, respectively. To do this, we calculate the following conditional probabilities. ▪ The probability that the passenger survived, give n that this passenger was in first class, is the fraction of first class passengers who survived, that is $P(\text{passenger survived}\mid\text{passenger was in third class}) = \frac{201}{324} \approx 0.620$ ▪ The probability that the passenger survived, given that the passenger was in second class, is the fraction of second class passengers who survived, that is $P(\text{passenger survived}\mid\text{passenger was in second class}) = \frac{118}{284} \approx 0.415$ ▪ The probability that the passenger survived, given that the passenger was in third class, is the fraction of third class passengers who survived, that is $P(\text{passenger survived}\mid\text{passenger was in third class}) = \frac{181}{709} \approx 0.255$ b. Now we want to investigate if what appears to point to class discrimination could be explained in terms of gender of passengers. First we ignore the class and take and take into consideration only the gender of the passengers. We can calculate the following conditional probabilities to compare the probabilities or surviving for a randomly selected child, woman, and man. ▪ The probability that the passenger survived, given that the passenger was a child, is the fraction of children who survived, that is $P(\text{passenger survived}\mid\text{passenger was was a child}) = \frac{56}{107} \approx 0.523$ ▪ The probability that the passenger survived, given that the passenger was a woman, is the fraction of women who survived, that is $P(\text{passenger survived}\mid\text{passenger was was a woman}) = \frac{313}{417} \approx 0.751$ ▪ The probability that the passenger survived, given that the passenger was a man, is the fraction of men who survived, that is $P(\text{passenger survived}\mid\text{passenger was was a man}) = \frac{131}{793} \approx 0.165$

DCAS-Like Answer	Next-Generation Solution
	These probabilities suggest that gender was an important factor with rescue procedures, with both women and children having a larger chance of surviving than men. Now we look at gender distribution between the three classes. Since women and children had a large chance of surviving, w we can consider them together and calculate the following conditional probabilities: ▪ The probability that the passenger was a child or a woman, given that the passenger was in first class, is the fraction of first class passengers who were children or women, that is $P(\text{passenger was a child or a woman}\mid\text{passenger was in first class}) = \frac{148}{324} \approx 0.457$ ▪ The probability that the passenger was a child or a woman, given that the passenger was in second class, is the fraction of second class passengers who were children or women, that is $P(\text{passenger was a child or a woman}\mid\text{passenger was in second class}) = \frac{117}{284} \approx 0.412$ ▪ The probability that the passenger was a child or a woman, given that the passenger was in third class, is the fraction of third class passengers who were children or women, that is $P(\text{passenger was a child or a woman}\mid\text{passenger was in third class}) = \frac{259}{709} \approx 0.365$ Looking at these probabilities, we can see that there were larger proportions of children and women in first and second class than in third class. Now the question is if the difference in gender distribution together with different survival rates for different genders was the only reason to explain the different survival rates for different classes. If that were the case, that is, if class was not a factor in rescue procedures, then any child regardless of the class in which the child traveled would have roughly the same chance of surviving (≈ 0.523). The same should hold for all women and all men. Thus we compare the survival rates for passengers of the same gender but from different classes. First, consider the children: ▪ The probability that a child survived, given that the child was in first class: $P(\text{child survived}\mid\text{child was in first class}) = \frac{4}{5} \approx 0.800$ ▪ The probability that a child survived, given that the child was in second class: $P(\text{child survived}\mid\text{child was in second class}) = \frac{22}{22} \approx 1.0$

DCAS-Like Answer	Next-Generation Solution
	▪ The probability that a child survived, given that the child was in third class: $P(\text{child survived}\mid\text{child was in third class}) = \frac{30}{80} \approx 0.375$ We can see that the children in first and second class had a larger chance of surviving than the children in the third class. We can do similar calculations for women and men. ▪ The probability that a woman survived, given that the woman was in first class: $P(\text{woman survived}\mid\text{woman was in first class}) = \frac{139}{143} \approx 0.972$ ▪ The probability that a woman survived, given that the woman was in second class: $P(\text{woman survived}\mid\text{woman was in second class}) = \frac{83}{95} \approx 0.874$ ▪ The probability that a woman survived, given that the woman was in third class: $P(\text{woman survived}\mid\text{woman was in third class}) = \frac{91}{179} \approx 0.508$ ▪ The probability that a man survived, given that the man was in first class: $P(\text{man survived}\mid\text{man was in first class}) = \frac{58}{176} \approx 0.330$ ▪ The probability that a man survived, given that the man was in second class: $P(\text{man survived}\mid\text{man was in second class}) = \frac{13}{167} \approx 0.078$ ▪ The probability that a man survived, given that the man was in third class: $P(\text{man survived}\mid\text{man was in third class}) = \frac{60}{450} \approx 0.133$ The final conclusion: The survival rates for women (0.751) and children (0.523) were larger than for men (0.1651), which suggests that the rescue procedures favored women and children. However, a random passenger in first class of any gender had at least twice as large of a chance of surviving as a passenger of the same gender in third class. For example, 0.972 survival rate for women in first class compared to 0.508 survival rate for women in third class. Such discrepancy cannot be justified with different gender distribution between the three classes. Therefore, the given data also suggests that the rescue procedures favored the first class passengers.

DCAS-Like Answer	Next-Generation Solution
24A: C (9-11.S.CP.7)	**24B:** Using the addition rule $P(C \text{ or } S) = P(C) + P(S) - P(C \text{ and } S)$, it follows that $0.7 = 0.6 + 0.5 - P(C \text{ and } S)$ $P(C \text{ and } S) = 0.6 + 0.5 - 0.7$ $= 0.4$ The probability that a randomly selected customer at Mom's has both cream and sugar in his or her coffee is 0.4.