Common Core Assessment Comparison for Mathematics

Geometry
Grades 9–11

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INTRODUCTION

The purpose of this document is to illustrate the differences between the Delaware Comprehensive Assessment System (DCAS) and the expectations of the next-generation Common Core State Standard (CCSS) assessment in Mathematics. A side-by-side comparison of the current design of an operational assessment item and the expectations for the content and rigor of a next-generation Common Core mathematical item are provided for each CCSS. The samples provided are designed to help Delaware's educators better understand the instructional shifts needed to meet the rigorous demands of the CCSS. This document does not represent the test specifications or blueprints for each grade level, for DCAS, or the next-generation assessment.

For mathematics, next-generation assessment items were selected for CCSS that represent the shift in content at the new grade level. Sites used to select the next-generation assessment items include:

- Smarter Balanced Assessment Consortium
- Partnership of Assessment of Readiness for College and Career
- Illustrative Mathematics
- Mathematics Assessment Project

Using released items from other states, a DCAS-like item, aligned to the same CCSS, was chosen. These examples emphasize the contrast in rigor between the previous Delaware standards, known as Grade-Level Expectations, and the Common Core State Standards.

Section 1, DCAS-Like and Next-Generation Assessment Comparison, includes content that is in the CCSS at a different "rigor" level. The examples are organized by the CCSS. For some standards, more than one example may be given to illustrate the different components of the standard. Additionally, each example identifies the standard and is separated into two parts. Part A is an example of a DCAS-like item, and Part B is an example of a next-generation item based on CCSS.

Section 2 includes at least one Performance Task that addresses multiple aspects of the CCSS (content and mathematical practices).

How to Use Various Aspects of This Document

- Analyze the way mathematics standards are conceptualized in each item or task.
- Identify the instructional shifts that need to occur to prepare students to address these more rigorous demands. Develop a plan to implement the necessary instructional changes.
- Notice how numbers (e.g., fractions instead of whole numbers) are used in the sample items.
- Recognize that the sample items and tasks are only one way of assessing the standard.
- Understand that the sample items and tasks do not represent a mini-version of the next-generation assessment.
- Instruction should address "focus," coherence," and "rigor" of mathematics concepts.
- Instruction should embed mathematical practices when teaching mathematical content.

- For grades K–5, calculators should not be used as the concepts of number sense and operations are fundamental to learning new mathematics content in grades 6–12.
- The next-generation assessment will be online and the scoring will be done electronically. It is important to note that students may not be asked to show their work and therefore will not be given partial credit. It is suggested when using items within this document in the classroom for formative assessments, it is good practice to have students demonstrate their methodology by showing or explaining their work.

Your feedback is welcome. Please do not hesitate to contact Katia Foret at katia.foret@doe.k12.de.us or Rita Fry at rita.fry@doe.k12.de.us with suggestions, questions, and/or concerns.

* The Smarter Balanced Assessment Consortium has a 30-item practice test available for each grade level (3-8 and 11) for mathematics and ELA (including reading, writing, listening, and research). These practice tests allow students to experience items that look and function like those being developed for the Smarter Balanced assessments. The practice test also includes performance tasks and is constructed to follow a test blueprint similar to the blueprint intended for the operational test. The Smarter Balanced site is located at: http://www.smarterbalanced.org/.

Priorities in Mathematics

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Common Core State Standards for Mathematical Practices

	Mathematical Practices	Student Dispositions:	Teacher Actions to Engage Students in Practices:
Essential Processes for a Productive Math Thinker	**1. Make sense of problems and persevere in solving them**	▪ Have an understanding of the situation ▪ Use patience and persistence to solve problem ▪ Be able to use different strategies ▪ Use self-evaluation and redirections ▪ Communicate both verbally and written ▪ Be able to deduce what is a reasonable solution	▪ Provide open-ended and rich problems ▪ Ask probing questions ▪ Model multiple problem-solving strategies through Think-Aloud ▪ Promote and value discourse ▪ Integrate cross-curricular materials ▪ Promote collaboration ▪ Probe student responses (correct or incorrect) for understanding and multiple approaches ▪ Provide scaffolding when appropriate ▪ Provide a safe environment for learning from mistakes
	6. Attend to precision	▪ Communicate with precision—orally and written ▪ Use mathematics concepts and vocabulary appropriately ▪ State meaning of symbols and use them appropriately ▪ Attend to units/labeling/tools accurately ▪ Carefully formulate explanations and defend answers ▪ Calculate accurately and efficiently ▪ Formulate and make use of definitions with others ▪ Ensure reasonableness of answers ▪ Persevere through multiple-step problems	▪ Encourage students to think aloud ▪ Develop explicit instruction/teacher models of thinking aloud ▪ Include guided inquiry as teach gives problem, students work together to solve problems, and debrief time for sharing and comparing strategies ▪ Use probing questions that target content of study ▪ Promote mathematical language ▪ Encourage students to identify errors when answers are wrong
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	▪ Create multiple representations ▪ Interpret problems in contexts ▪ Estimate first/answer reasonable ▪ Make connections ▪ Represent symbolically ▪ Talk about problems, real-life situations ▪ Attend to units ▪ Use context to think about a problem	▪ Develop opportunities for problem-solving strategies ▪ Give time for processing and discussing ▪ Tie content areas together to help make connections ▪ Give real-world situations ▪ Demonstrate thinking aloud for students' benefit ▪ Value invented strategies and representations ▪ More emphasis on the process instead of on the answer
	3. Construct viable arguments and critique the reasoning of others	▪ Ask questions ▪ Use examples and counter examples ▪ Reason inductively and make plausible arguments ▪ Use objects, drawings, diagrams, and actions ▪ Develop ideas about mathematics and support their reasoning ▪ Analyze others arguments ▪ Encourage the use of mathematics vocabulary	▪ Create a safe environment for risk-taking and critiquing with respect ▪ Provide complex, rigorous tasks that foster deep thinking ▪ Provide time for student discourse ▪ Plan effective questions and student grouping ▪ Probe students

	Mathematical Practices	Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	▪ Realize that mathematics (numbers and symbols) is used to solve/work out real-life situations ▪ Analyze relationships to draw conclusions ▪ Interpret mathematical results in context ▪ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable—if not, go back and look for more information ▪ Make sense of the mathematics	▪ Allowing time for the process to take place (model, make graphs, etc.) ▪ Modeling desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ▪ Making appropriate tools available ▪ Creating an emotionally safe environment where risk-taking is valued ▪ Providing meaningful, real-world, authentic, performance-based tasks (non-traditional work problems) ▪ Promoting discourse and investigations
	5. Use appropriate tools strategically	▪ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base ten blocks, compass, protractor) ▪ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ▪ Compare the efficiency of different tools ▪ Recognize the usefulness and limitations of different tools	▪ Maintaining knowledge of appropriate tools ▪ Modeling effectively the tools available, their benefits, and limitations ▪ Modeling a situation where the decision needs to be made as to which tool should be used ▪ Comparing/contrasting effectiveness of tools ▪ Making available and encouraging use of a variety of tools
Seeing Structure and Generalizing	**7. Look for and make use of structure**	▪ Look for, interpret, and identify patterns and structures ▪ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ▪ Reflect and recognize various structures in mathematics ▪ Breakdown complex problems into simpler, more manageable chunks ▪ “Step back” or shift perspective ▪ Value multiple perspectives	▪ Being quiet and structuring opportunities for students to think aloud ▪ Facilitating learning by using open-ended questions to assist students in exploration ▪ Selecting tasks that allow students to discern structures or patterns to make connections ▪ Allowing time for student discussion and processing in place of fixed rules or definitions ▪ Fostering persistence/stamina in problem solving ▪ Allowing time for students to practice
	8. Look for and express regularity in repeated reasoning	▪ Identify patterns and make generalizations ▪ Continually evaluate reasonableness of intermediate results ▪ Maintain oversight of the process ▪ Search for and identify and use shortcuts	▪ Providing rich and varied tasks that allow students to generalize relationships and methods and build on prior mathematical knowledge ▪ Providing adequate time for exploration ▪ Providing time for dialogue, reflection, and peer collaboration ▪ Asking deliberate questions that enable students to reflect on their own thinking ▪ Creating strategic and intentional check-in points during student work time

For classroom posters depicting the Mathematical Practices, please see: http://seancarberry.cmswiki.wikispaces.net/file/detail/12-20math.docx

Congruence (G.CO)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Experiment with transformations in the plane.

9-11.G.CO.1 – Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

DCAS-Like

1A

A portion of a line bounded by two points is defined as: ___________________________

A. Arc

B. Axis

C. Ray

D. Segment

Next-Generation

1B

Match each vocabulary word with one precise definition.

_____ 1. Rhombus
_____ 2. Isosceles Triangle
_____ 3. Hexagon
_____ 4. Segment
_____ 5. Ray
_____ 6. Angle
_____ 7. Plane
_____ 8. Endpoint
_____ 9. Line
_____ 10. Vertex
_____ 11. Point
_____ 12. Parallel Lines
_____ 13. Perpendicular Lines
_____ 14. Skew Lines
_____ 15. Chord

a. A quadrilateral with 4 congruent sides
b. A quadrilateral with 4 right angles and 4 congruent sides
c. A triangle with no sides equal
d. A triangle with all 3 sides equal
e. A triangle with 2 sides equal
f. A 6-sided polygon
g. An 8-sided figure
h. A 7-sided figure
i. Flat surface
j. Point at the end of a ray or segment
k. Exact location in space
l. Two endpoints and all the points between them
m. Common endpoint of the rays of an angle
n. Extends forever in opposite direction
o. Two rays with a common endpoint
p. Part of a line with one endpoint
q. Lines in the same plane that never intersect
r. Lines not in the same plane
s. Lines that intersect at right angles
t. The line segment between two points on a given curve

9-11.G.CO.2 – Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

DCAS-Like

2A

The vertices of ΔABC are $A(2,1)$, $B(3,4)$, and $C(1,3)$. If ΔABC is translated 1 unit down and 3 units to the left to create ΔDEF, what are the coordinates of the vertices of ΔDEF?

A. $D(0,1), E(1,2), F(1,3)$

B. $D(0,-1), E(0,3), F(-2,-2)$

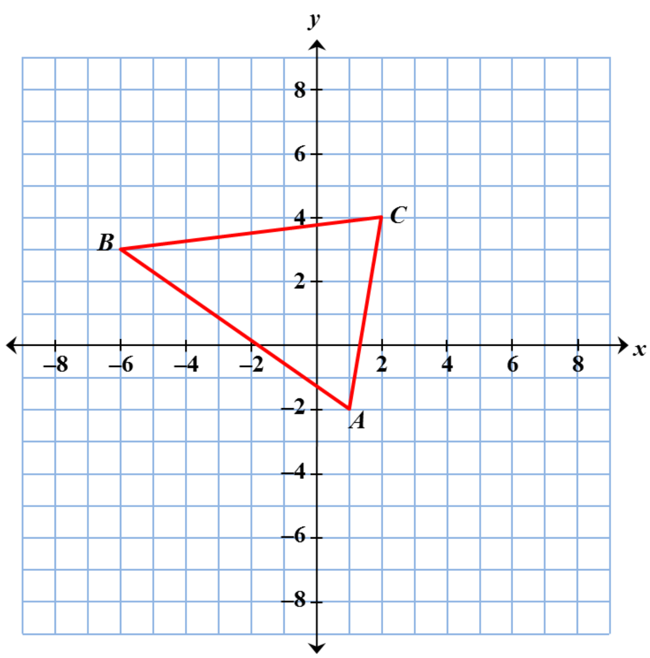
C. $D(-2,2), E(0,3), F(-1,)$

D. $D(-1,0), E(0,3), F(-2,2)$

Next-Generation

2B

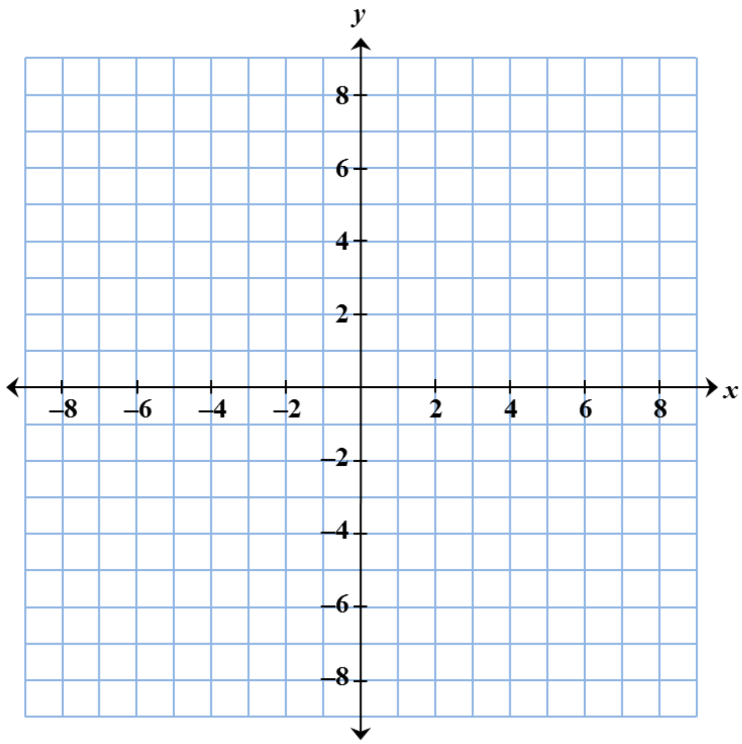
Given: ΔABC, shown on the coordinate plane below



a. If ΔABC is reflected over the x-axis to yield $\Delta A'B'C'$, what are the coordinates of the vertices of $\Delta A'B'C'$?

b. Using this reflection, write a general rule that will map ΔABC onto $\Delta A'B'C'$.

c. If ΔABC is translated 4 units to the left and 3 units down to yield ΔRST, draw ΔRST on the coordinate plane below.



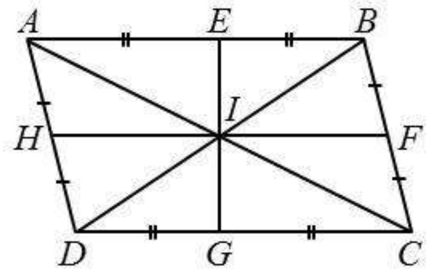
d. Using your translation, write a general rule that will map ΔABC onto ΔRST. Use words, numbers, and/or pictures to show your work.

9-11.G.CO.3 – Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

DCAS-Like

3A

Using the diagram below, which statement about parallelogram $ABCD$ must be true.



A. A reflection across $\overleftrightarrow{EG}$ carries parallelogram $ABCD$ onto itself.

B. A reflection across $\overleftrightarrow{HF}$ carries parallelogram $ABCD$ onto itself.

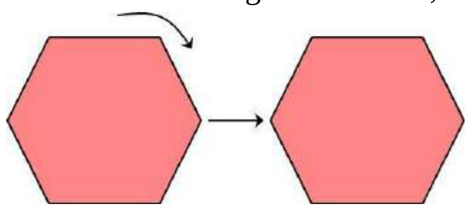
C. A rotation of 90° about I carries parallelogram $ABCD$ onto itself.

D. A rotation of 180° about I carries parallelogram $ABCD$ onto itself.

Next-Generation

3B

Evan has a regular hexagon. He applies rotations to the shape around the center of the hexagon. Each rotation that Evan performs carries the hexagon onto itself, as shown in the figure below.



a. What is the lowest degree of rotation greater than 0° that will carry the hexagon onto itself?

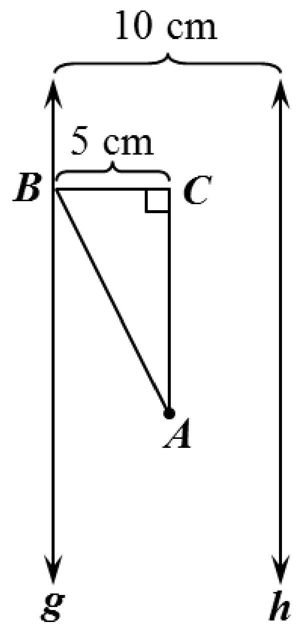
b. What are two other rotations less than 360° that will also carry the hexagon onto itself?

9-11.G.CO.4 – Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

DCAS-Like

4A

In the diagram, $g \parallel h$ and point B lies on line g.



The figure ABC is reflected across line g, and its image is reflected across line h. What is the distance from line g to the final image of point A?

A. 5 cm

B. 15 cm

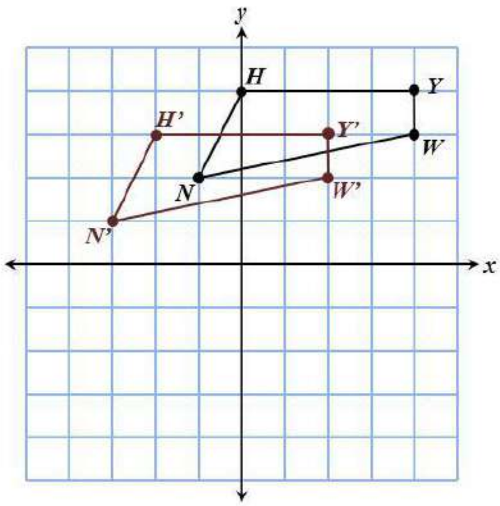
C. 20 cm

D. 25 cm

4B

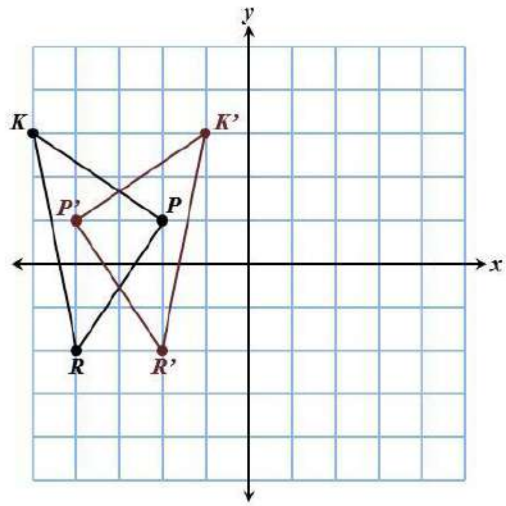
The graph of a figure and its image are shown below. Identify the transformation to map the image back onto the figure.

a.



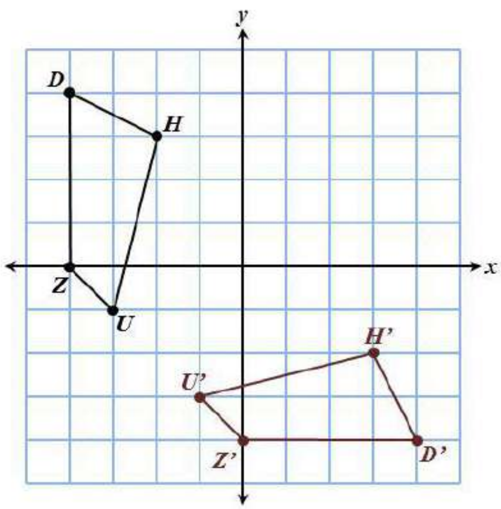
- O reflection
- O rotation
- O translation

b.



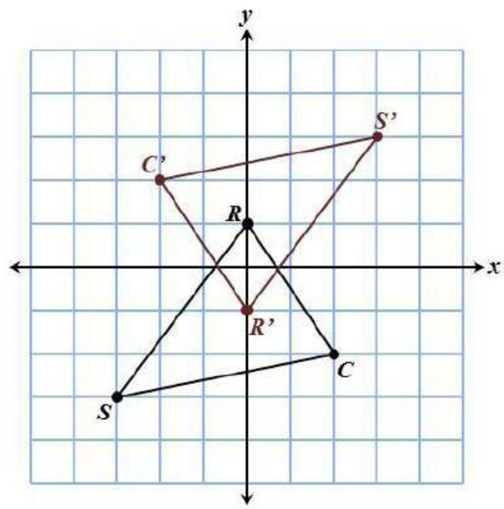
- O reflection
- O rotation
- O translation

c.



- O reflection
- O rotation
- O translation

d.



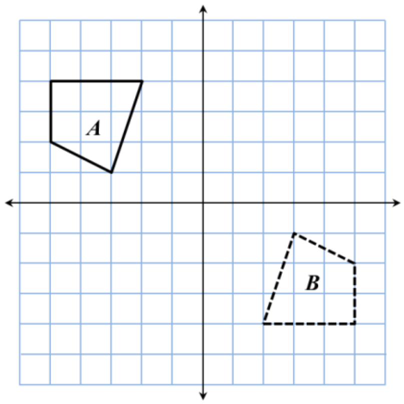
- O reflection
- O rotation
- O translation

9-11.G.CO.5 – Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

DCAS-Like

5A

Which transformation maps the solid figure *A* onto the dashed figure *B*?



A. Rotation 180° about the origin

B. Translation to the right and down

C. Reflection across the x-axis

D. Reflection across the y-axis

Next-Generation

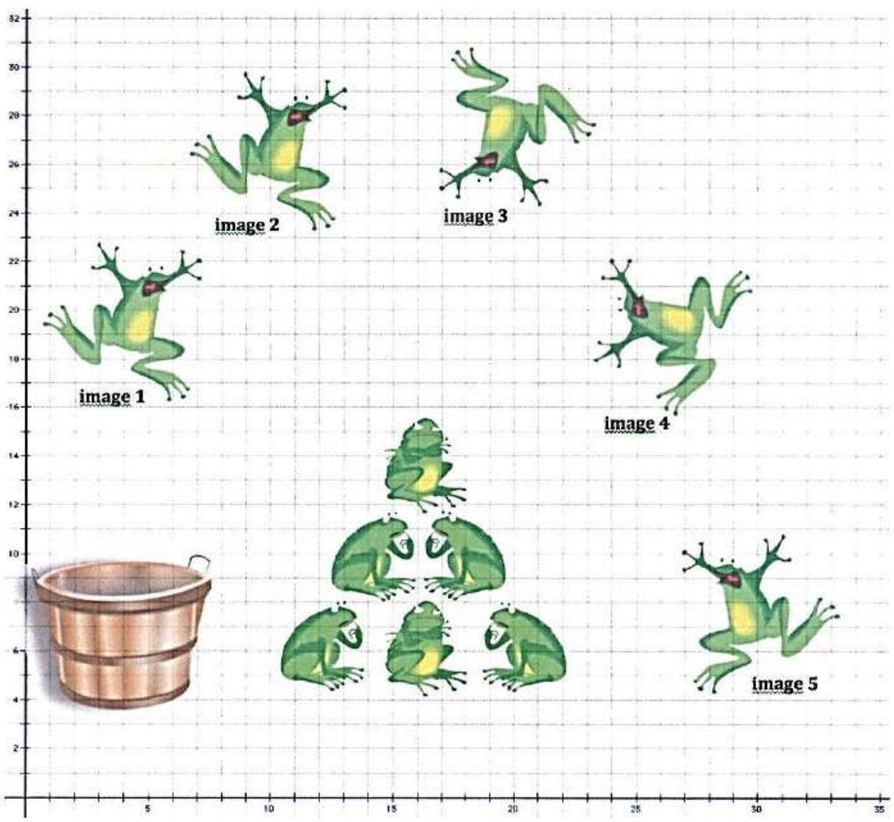
5B

Josh is animating a scene where a troupe of frogs is auditioning for the Animal Channel reality show, “The Bayou’s Got Talent.: In this scene the frogs are demonstrating their “leap frog” acrobatics act. Josh has completed a few key images in this segment, and now needs to describe the transformations that connect various images in the scene.

For each pre-image/image combination listed below, describe the transformation that moves the pre-image to the final image.

- If you decide the transformation is a **rotation**, you will need to give the **center of rotation**, the **direction** of the rotation (clockwise or counterclockwise), and the **measure of the angle** of rotations.
- If you decide the transformation is a **reflection**, you will need to give the **equation of the line** of reflection.
- If you decide the transformation is a **translation**, you will need to describe the **“rise” and “run”** between pre-image points and their corresponding image points.
- If you decide it takes a **combination of transformations** to get from the pre-image to the final image, **describe each transformation in the order** they would be completed.

Pre-image	Final Image	Description
Image 1	Image 2	
Image 2	Image 3	
Image 3	Image 4	
Image 1	Image 5	
Image 2	Image 4	



Cluster: Understand congruence in terms of rigid motions.

9-11.G.CO.6 – Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

DCAS-Like

6A

Figure 1 is reflected about the x-axis and then translated four units left. Which is the new figure?

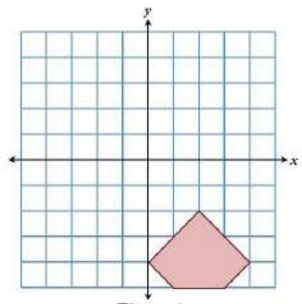


Figure 1

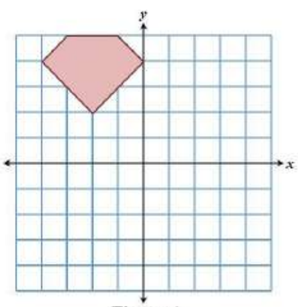


Figure A

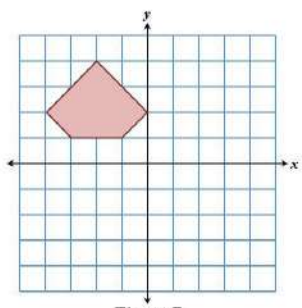


Figure B

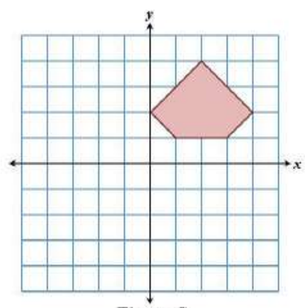


Figure C

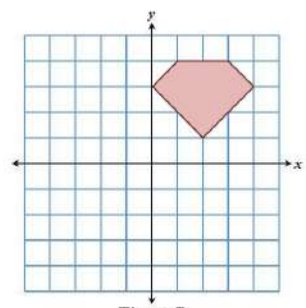


Figure D

A. Figure A

B. Figure B

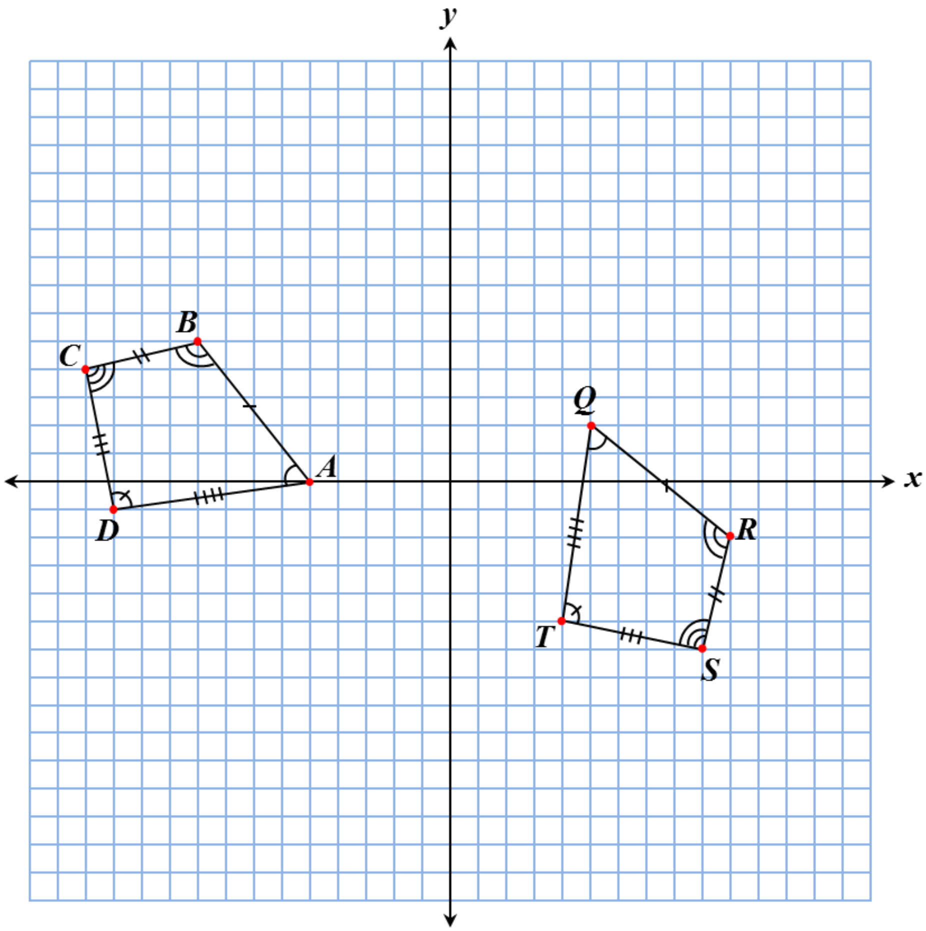
C. Figure C

D. Figure D

6B

The two quadrilaterals shown below, quadrilateral $ABCD$ and $QRST$ are congruent, with corresponding congruent parts marked in the diagrams.

Describe a sequence of rigid-motion transformations that will carry quadrilateral $ABCD$ onto quadrilateral $QRST$. Be very specific in describing the sequence and types of transformations you will use so that someone else could perform the same series of transformations.



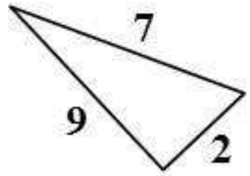
9-11.G.CO.7 – Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

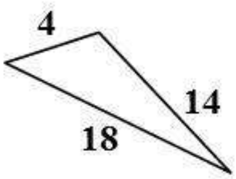
DCAS-Like

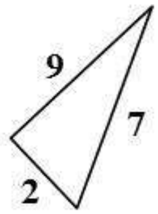
7A

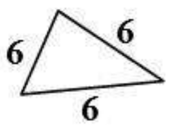
The triangle to the right can be subject to reflections, rotations, or translations. With which of the triangles can it coincide after a series of these transformations?

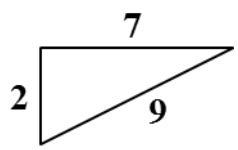
Figures are not necessarily drawn to scale.



A. 

B. 

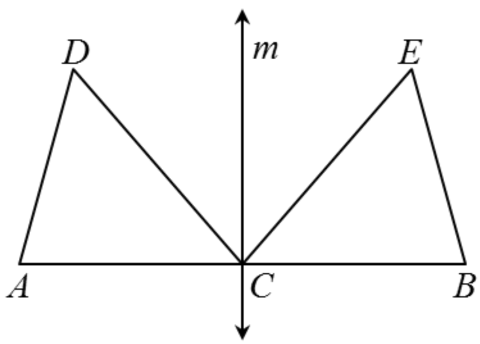
C. 

D. 

Next-Generation

7B

In the diagram, m is the perpendicular bisector of $\overline{AB}$ at C, and $r_m(D) = E$.



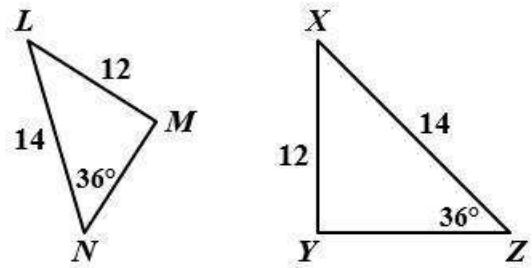
Prove $\Delta ADC \cong \Delta BEC$.

9-11.G.CO.8 – Understand congruence in terms of rigid motions. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

DCAS-Like

8A

Can we prove ΔLMN is congruent to ΔXYZ? Is so, which postulate can we use to do so?



A. Yes, we can prove they are congruent using the SAS postulate.

B. Yes, we can prove they are congruent using the ASA postulate.

C. No, we cannot prove they are congruent because ΔXYZ appears to be a right triangle and ΔLMN is not a right triangle.

D. No, we cannot prove they are congruent because none of the three postulates can be used.

8B

Consider a ΔABC that has been transformed through rigid motions and its image is compared to ΔXYZ. Determine if the given information is sufficient to draw the provided conclusion.

a.

Given	Conclusion
$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\angle C \cong \angle Z$	$\Delta ABC \cong \Delta XYZ$

○ True ○ False

b.

Given	Conclusion
$\angle A \cong \angle X$ $\angle B \cong \angle Y$ $\overline{BC} \cong \overline{YZ}$	$\Delta ABC \cong \Delta XYZ$

○ True ○ False

c.

Given	Conclusion
$\angle A \cong \angle X$ $\overline{AB} \cong \overline{XY}$ $\overline{BC} \cong \overline{YZ}$	$\Delta ABC \cong \Delta XYZ$

○ True ○ False

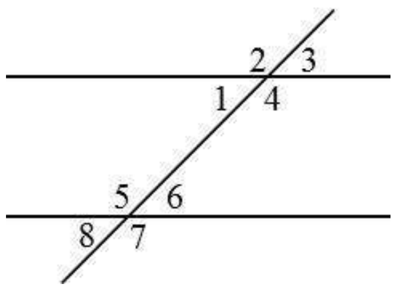
Cluster: Prove geometric theorems.

9-11.G.CO.9 – Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

DCAS-Like

9A

A transversal crosses two parallel lines. Which statement should be used to prove that the measures of angles 1 and 5 sum to 180°?



A. Angles 1 and 8 are congruent as corresponding angles; angles 5 and 8 form a linear pair.

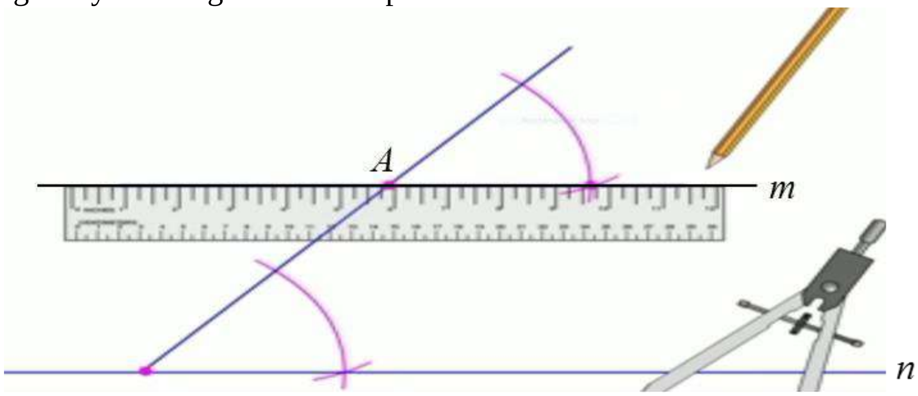
B. Angles 1 and 2 form a linear pair; angles 3 and 4 form a linear pair.

C. Angles 5 and 7 are congruent as vertical angles; angles 6 and 8 are congruent as vertical angles.

D. Angles 1 and 3 are congruent as vertical angles; angles 7 and 8 form a linear pair.

9B

Click on the play button to view an animation of the construction of a parallel line. The animation begins by showing a line and a point above the line.



The steps in the construction result in a line (*m)* through the given point (*A*) that is parallel to the given line (*n*).

Which statement justifies why the constructed line is parallel to the given line?

a. When two lines are each perpendicular to a third line, the lines are parallel.

b. When two lines are each parallel to a third line, the lines are parallel.

c. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel.

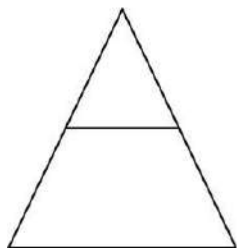
d. When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel.

9-11.G.CO.10 – Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

DCAS-Like

10A

If two angles of one triangle are congruent to two angles of another triangle, then the remaining angle in one triangle is congruent to the remaining angle in the other. Which of the following makes this statement true?



A. The definition of supplementary angles.

B. The angle sum theorem for triangles.

C. SSS postulate.

D. The definition of congruent triangles.

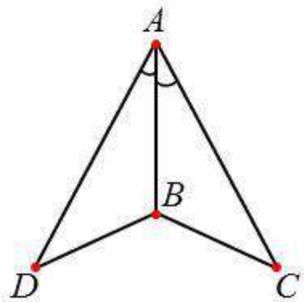
10B

For items a. and b., what additional information is required in order to prove the two triangles are congruent using the provided justification?

Use the set of choices in the box below. Select a side or angle and place it in the appropriate region. Only one side or angle can be placed in each region.

$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	$\overline{BC}$
$\overline{BD}$	$\overline{CD}$	$\overline{CE}$	$\overline{DE}$
$\angle ABC$	$\angle ABD$	$\angle ACB$	$\angle ADB$
$\angle BAC$	$\angle CDE$	$\angle CED$	$\angle DCE$

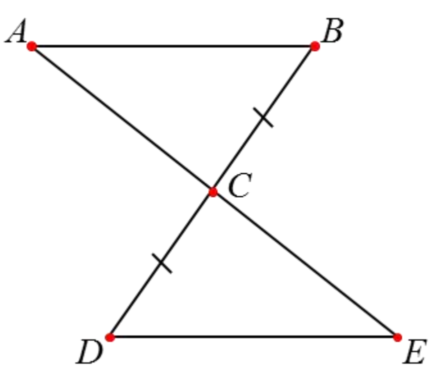
a.



ASA Postulate

$\square \cong \square$

b.



SAS Theorem

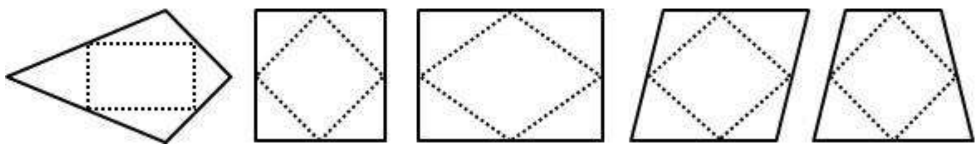
$\square \cong \square$

9-11.G.CO.10 – Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180 degrees; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

DCAS-Like

11A

In the five quadrilaterals shown below, the midpoints of the sides have been joined by dotted line segments. Which best describes the five dotted figures formed?



A. All are parallelograms.

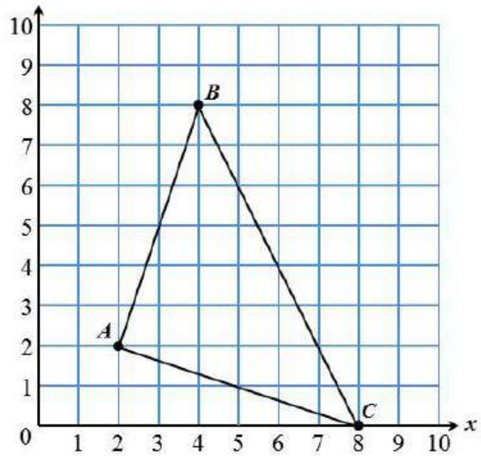
B. All are rectangles.

C. All are squares.

D. All are rhombuses.

11B

Triangle ABC is graphed below.



a. What are the midpoints of $\overline{AB}$ and $\overline{BC}$? Enter your answers as ordered pairs.

Midpoint of $\overline{AB}$ = ____________________

Midpoint of $\overline{BC}$ = ____________________

b. What is the length of $\overline{AC}$? Provide your answer as a radical.

Length of $\overline{AC}$ = ____________________

c. What is the slope of $\overline{AC}$?

Slope of $\overline{AC}$ = ____________________

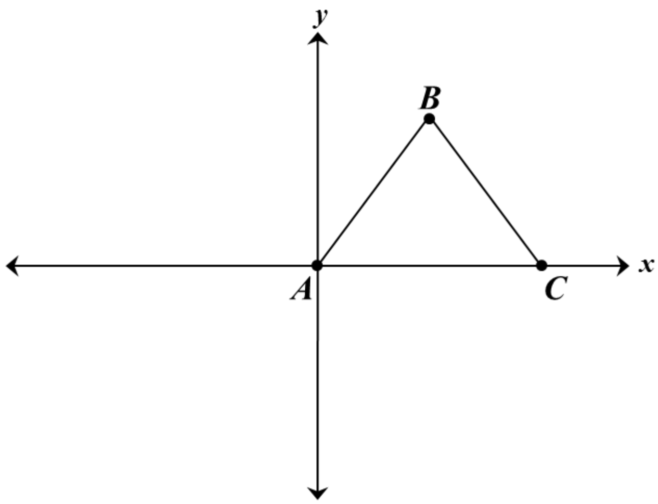
d. Darlene claims that the line segment connecting the midpoints of $\overline{AB}$ and $\overline{BC}$ is parallel to $\overline{AC}$ and is half the length of $\overline{AC}$. Explain whether Darlene is correct.

9-11.G.CO.11 – Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

DCAS-Like

12A

The diagram shows ΔABC.



Which statement would prove that ΔABC is a right triangle?

A. $(\text{slope } \overline{AB})(\text{slope } \overline{BC}) = 1$

B. $(\text{slope } \overline{AB})(\text{slope } \overline{BC}) = -1$

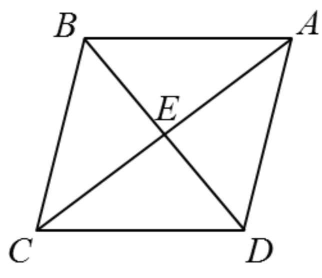
C. distance from A to B = distance from B to C

D. distance from A to B = – (distance from B to C)

Next-Generation

12B

Use the quadrilateral $ABCD$ to answer each part. $ABCD$ is a quadrilateral with $\overline{AB} \| \overline{CD}$ and $\overline{AD} \| \overline{BC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E.



a. $\Delta CBE \cong \Delta ABE$ ○ True ○ False

b. $\Delta ADE \cong \Delta ABE$ ○ True ○ False

c. $\Delta CDE \cong \Delta ABE$ ○ True ○ False

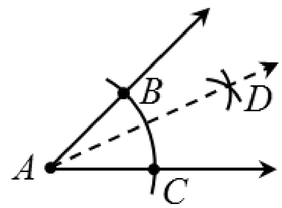
Cluster: Make geometric constructions.

9-11.G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

DCAS-Like

13A

What is the first step in constructing the angle bisector of angle A?



A. Draw ray $\overline{AD}$.

B. Draw a line segment connecting points B and C.

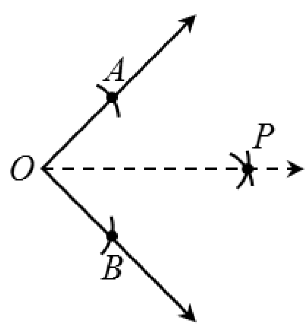
C. From points B and C, draw equal arcs that intersect at D.

D. From point A, draw an arc that intersects the sides of the angle at points B and C.

Next-Generation

13B

The figure below shows the construction of the angle bisector of $\angle AOB$ using a compass. Which of the following statements must always be true in the construction of the angle bisector? Select **Yes** or **No** for each statement.



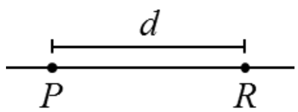
a. O$A = OB$ ○ Yes ○ No
b. $AP = BP$ ○ Yes ○ No
c. $AB = BP$ ○ Yes ○ No
d. $OB = BP$ ○ Yes ○ No

9-11.G.CO.13 – Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

DCAS-Like

14A

Carol is constructing an equilateral triangle with P and R being two of the vertices. She is going to use a compass to draw circles around P and R. What should the radius of the circles be?



A. d

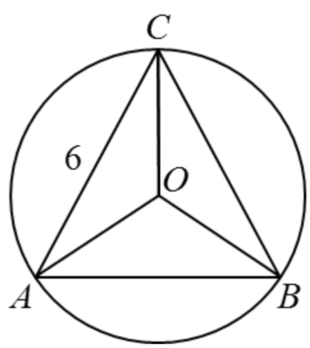
B. $2d$

C. $\frac{d}{2}$

D. d^2

Next-Generation

14B



What is the diameter of a circle with an inscribed equilateral triangle with side lengths of 6?

____

Similarity, Right Triangles, and Trigonometry (G.SRT)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand similarity in terms of similarity transformations.

9-11.G.SRT.1 – Verify experimentally the properties of dilations given by a center and a scale factor:

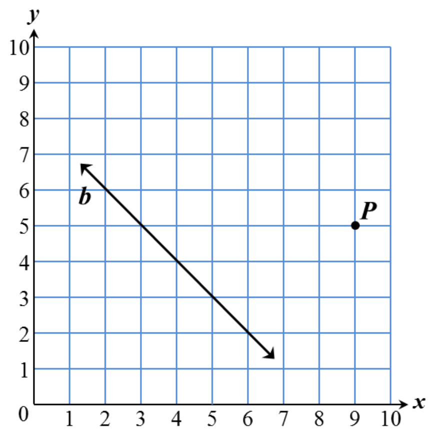
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

DCAS-Like

15A

Line b is defined by the equation $y = 8 - x$. If line b undergoes a dilation with a scale factor of 0.5 and center P, which equation will define the image of the line?



A. $y = 4 - x$

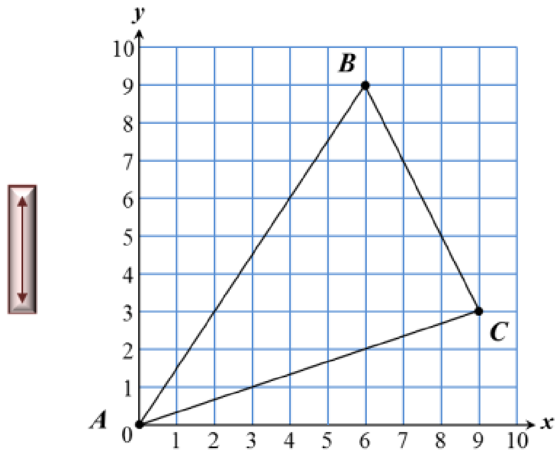
B. $y = 5 - x$

C. $y = 8 - x$

D. $y = 11 - x$

15B

The figure below correctly plots ΔABC at (0, 0), (6, 9), and (9, 3). If the center of dilation is the origin, use the connect line tool to plot this triangle dilated with a scale factor of two-thirds.

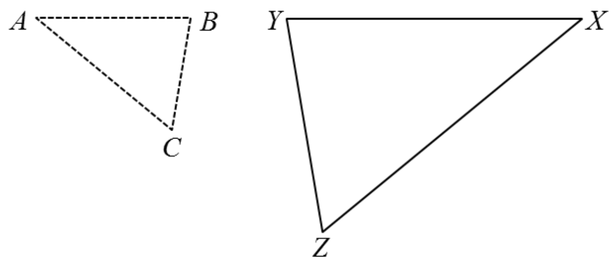


9-11.G.SRT.2 – Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

DCAS-Like

16A

Triangle ABC was reflected and dilated so that it coincides with triangle XYZ. How did this transformation affect the sides and angles of triangle ABC?



A. The side lengths and angle measures were multiplied by $\frac{XY}{AB}$.

B. The side lengths were multiplied by $\frac{XY}{AB}$, while the angle measures were preserved.

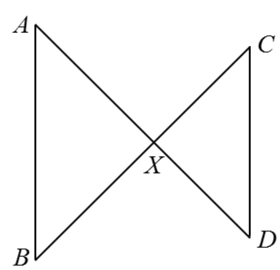
C. The angle measures were multiplied by $\frac{XY}{AB}$, while the side lengths were preserved.

D. The angle measures and side lengths were preserved.

Next-Generation

16B

In the picture below, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles AXB and CXD.



In items a-d below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to prove that the two triangles are similar.

a. The lengths AX and XD satisfy the equation $2AX = 3XD$. ○ Yes ○ No

b. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$. ○ Yes ○ No

c. Lines AB and CD are parallel. ○ Yes ○ No

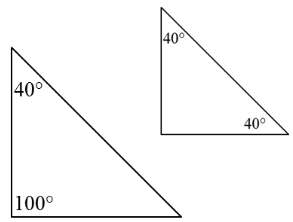
d. Angle XAB is congruent to angle XCD. ○ Yes ○ No

9-11.G.SRT.3 – Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

DCAS-Like

17A

As marked, by which method would it be possible to prove these triangles are similar (if possible)?



A. AA

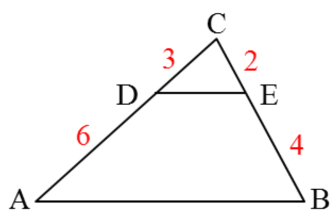
B. SSS

C. SAS

D. Not similar

Next-Generation

17B



Given: $AD = 6; DC = 3$

$BE = 4; EC = 2$

Prove: $\Delta CDE \sim \Delta CAB$

	Statements	Reasons
1		
2		
3		
4		
5		
6		
7		
8		

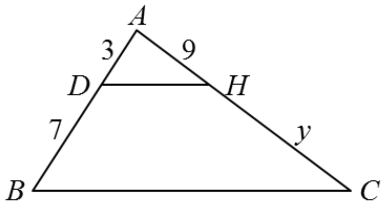
Cluster: Prove theorems involving similarity.

9-11.G.SRT.4 – Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

DCAS-Like

18A

In the diagram below $\overline{BC} \| \overline{DH}$.



What is the value of y?

A. 13

B. 19

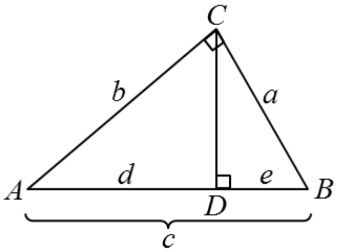
C. 21

D. 30

Next-Generation

18B

In the diagram, ΔABC is a right triangle with right angle C, and $\overline{CD}$ is an altitude of ΔABC.



Use the fact that $\Delta ABC \sim \Delta ACD \sim \Delta CBD$ to prove $a^2 + b^2 = c^2$

Statements	Reasons

9-11.G.SRT.5 – Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

DCAS-Like

19A

Right triangle PQR has sides of length 6 units, 8 units, and 10 units. The triangle is dilated by a scale factor of 4 about point Q. What is the area of triangle $P'Q'R'$?

A. 96 square units

B. 192 square units

C. 384 square units

D. 768 square units

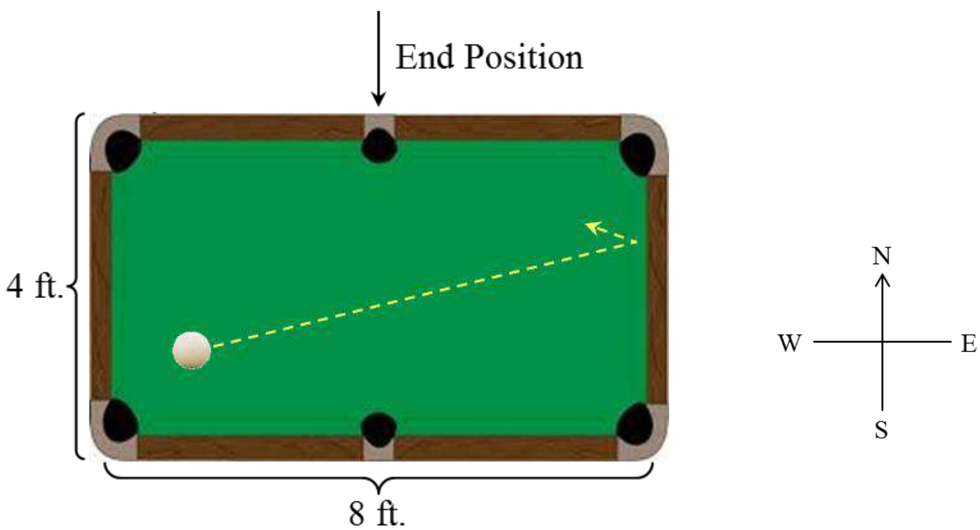
Next-Generation

19B

Pablo is practicing bank shots on a standard 4 ft.-by-8 ft. pool table that has a wall on each side, a pocket in each corners and a pocket at the midpoint of each 8-ft. side.

Pablo places the cue ball one foot away from the south wall of the table and one foot away from the west wall, as shown in the diagram below. He wants to bank the cue ball off of the east wall and into the pocket at the midpoint of the north wall.

At what point should the cue ball hit the east wall?



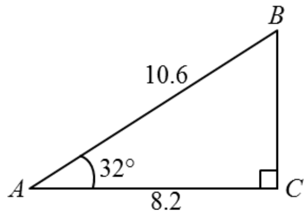
Cluster: Define trigonometric ratios and solve problems involving right triangles.

9-11.G.SRT.6 – Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

DCAS-Like

20A

Right triangle ABC is pictured below.



Which equation gives the correct value for BC?

A. $\sin 32° = \frac{BC}{8.2}$

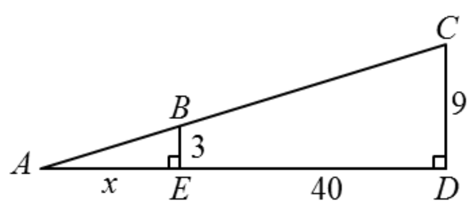
B. $\cos 32° = \frac{BC}{10.6}$

C. $\tan 58° = \frac{8.2}{BC}$

D. $\sin 58° = \frac{BC}{10.6}$

Next-Generation

20B



The figure above shows two right angles. The length of AE is x units and the length of DE is 40 units. Using the triangle above, find the following:

a. $x = \boxed{}$

b. $\sin A = \frac{\boxed{}}{\boxed{}}$

c. $\cos A = \frac{\boxed{}}{\boxed{}}$

d. $\tan A = \frac{\boxed{}}{\boxed{}}$

9-11.G.SRT.7 – Explain and use the relationship between the sine and cosine of complementary angles.

DCAS-Like

21A

In ΔABC where C is a right angle, $\sin A = \frac{\sqrt{7}}{4}$. What is $\cos B$?

A. $\frac{\sqrt{7}}{4}$

B. $\frac{\sqrt{7}}{3}$

C. $\frac{3}{4}$

D. $\frac{3}{\sqrt{7}}$

Next-Generation

21B

Determine whether each statement is **True** or **False**.

a. $\sin x° = \cos(90 - x)°$ ○ True ○ False

b. $\cos(90 - x)° = \cos x°$ ○ True ○ False

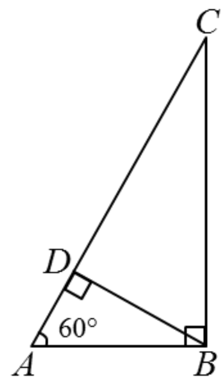
c. $\sin(90 - x)° = \cos x°$ ○ True ○ False

9-11.G.SRT.8 – Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

DCAS-Like

22A

In ΔABC below, $AC = 12$. What is the length of segment BD?



A. $3\sqrt{2}$

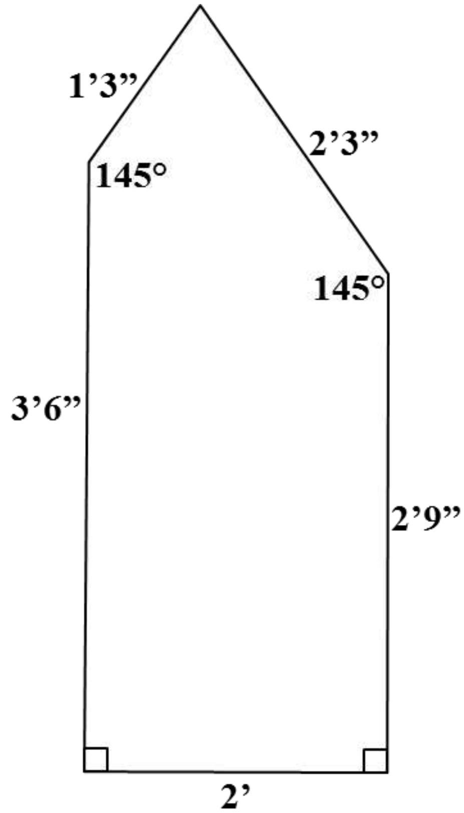
B. $3\sqrt{3}$

C 6

D. $6\sqrt{2}$

22B

The diagram shows a model of a closet floor on which Kim is laying carpet. (Measurements are approximate.)



a. What is the area of the closet? _______________

b. The carpet Kim is using is cut by the carpet store in rectangular pieces from a 4-foot wide roll. What is the shortest length of carpet Kim would need to cover the closet floor in a single piece? Justify your answer.

Length of carpet: _______________

Circles (G.C)

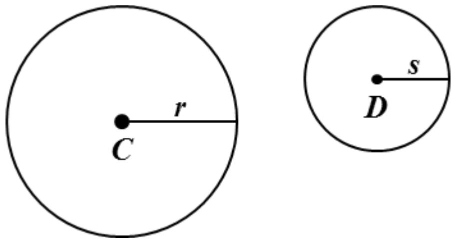
Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand and apply theorems about circles.

9-11.G.C.1 – Prove that all circles are similar.

DCAS-Like

23A



To show circle C is similar to circle D, one would have to translate circle C by the vector $\overrightarrow{CD}$. Then, circle C' would have to be dilated by what factor?

A. $s - r$

B. $s^2 - r^2$

C. $\frac{s}{r}$

D. $\frac{s^2}{r^2}$

Next-Generation

23B

Show that circle C with center $(-1, 2)$ and radius 3 is similar to circle D with center (3, 4) and radius 5, by finding the translation and enlargement ratio.

Translation: ________________

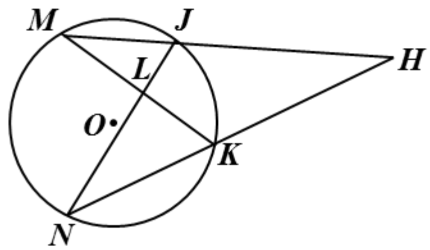
Enlargement Ratio: ________________

9-11.G.C.2 – Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

DCAS-Like

24A

In the figure below, $m\widehat{JK} = 66°$ and $m\widehat{MN} = 128°$.



What is $m\angle H$?

A. $31°$

B. $62°$

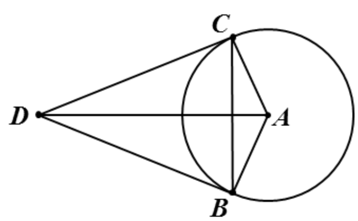
C. $64°$

D. $97°$

Next-Generation

24B

The segments $\overline{DB}$ and $\overline{DC}$ are tangent to circle A.



Using the above diagram, which of the following statements are **True** or **False**?

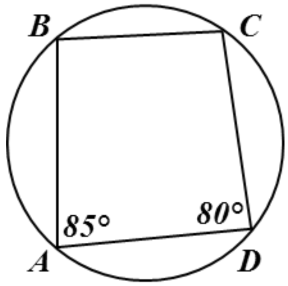
a. ΔBCD is a right triangle	O True	O False
b. $BD = CD$	O True	O False
c. $\frac{1}{2} m\widehat{BC} = m\angle BDC$	O True	O False

9-11.G.C.3 – Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

DCAS-Like

25A

Quadrilateral $ABCD$ is inscribed in a circle as shown in the diagram below.



If $m\angle A = 85°$ and $m\angle D = 80°$, what is $m\angle B$?

A. 80°

B. 85°

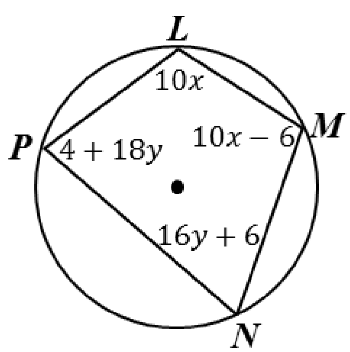
C. 95°

D. 100°

Next-Generation

25B

Solve for x and y. What are the measures of the angles of the quadrilateral?



$x =$ ________________ $y =$ ________________

$\angle L =$ ________________ $\angle M =$ ________________

$\angle N =$ ________________ $\angle P =$ ________________

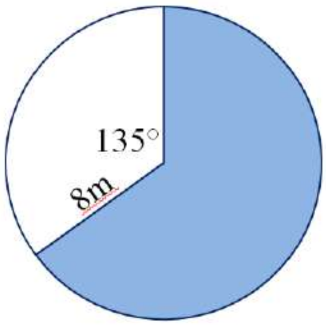
Cluster: Find arc lengths and areas of sectors of circles.

9-11.G.C.5 – Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

DCAS-Like

26A

What is the area of the shaded sector?



A. 5π square meters

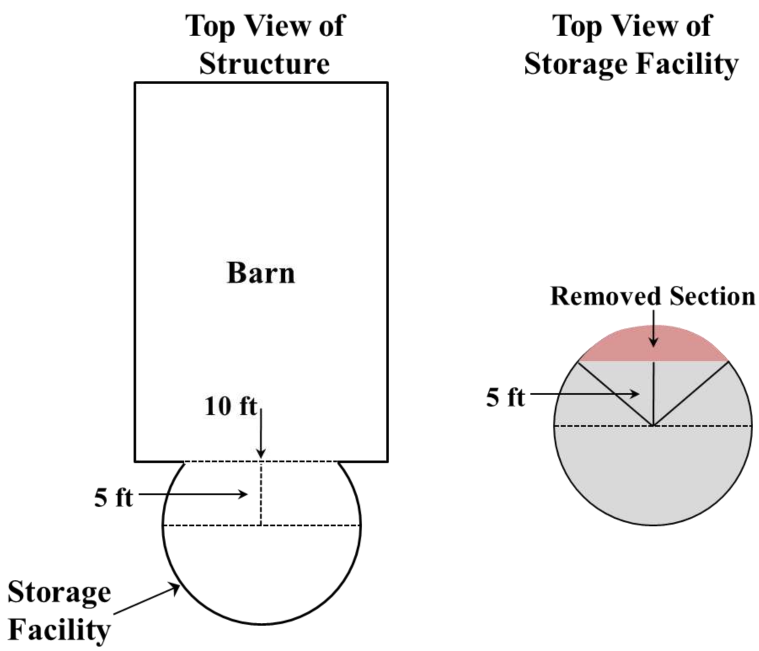
B. 10π square meters

C. 24π square meters

D. 40π square meters

26B

John stores wheat in a storage facility located on his farm. The cylinder-shaped storage facility has one flat, rectangular face that rests against the side of his barn. The barn is 5 feet from the center of the storage facility. The height of the stored grain is 30 feet, when full to capacity, and the face resting against the barn is 10 feet wide. This storage facility holds the exact amount of wheat he uses per year.



a. Determine the capacity of John's storage facility in cubic feet. ________________

b. If one bushel is 1.25 cubic feet, how many bushels can be stored? _______________

Expressing Geometric Properties with Equations (G.GPE)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Translate between the geometric description and the equation for a conic section.

9-11.G.GPE.1 – Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

DCAS-Like

27A

What are the center and radius of the circle described by the equation
$2x^2 + 2y^2 + 12x + 20y + 36 = 0$?

A. Center (3, 5); radius 4

B. Center (–3, –5); radius 4

C. Center (3, 5); radius 16

D. Center (–3, –5); radius 16

Next-Generation

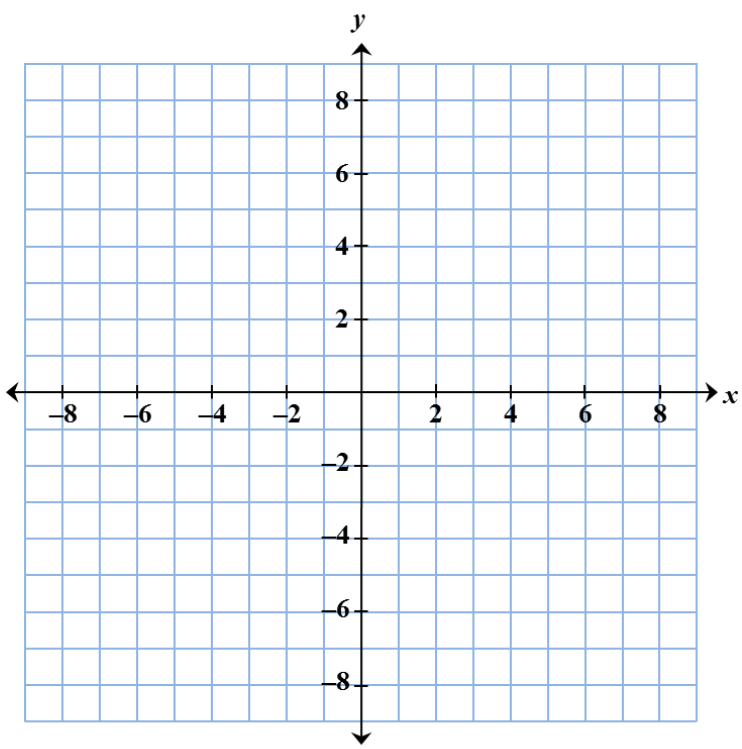
27B

A circle is described by the equation $5x^2 + 5y^2 - 50x + 20y + 100 = 0$. Find the center and radius of the circle.

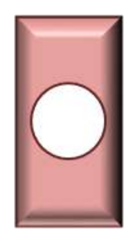
a. Center = _______________

Radius = _______________

b. Using your answer above, graph the circle.



Circle Tool

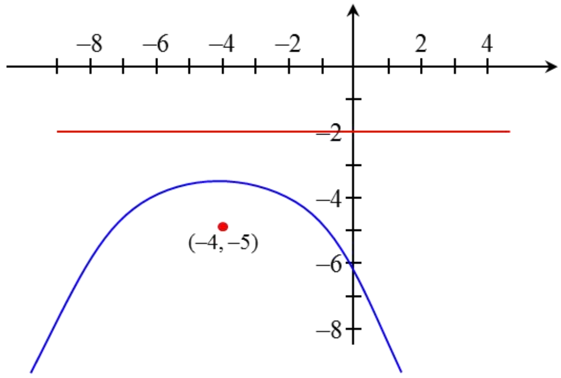


9-11.G.GPE.2 – Derive the equation of a parabola given a focus and directrix.

DCAS-Like

28A

Which equation matches this parabola? (The directrix is line $y = -2$.)



A. $y = -\frac{1}{14}x^2 - \frac{4}{7}x - \frac{37}{14}$

B. $y = \frac{1}{6}x^2 - \frac{4}{3}x + \frac{37}{6}$

C. $y = \frac{1}{14}x^2 - \frac{4}{7}x + \frac{37}{14}$

D. $y = -\frac{1}{6}x^2 - \frac{4}{3}x - \frac{37}{6}$

Next-Generation

28B

Indicate which of the following are **True** for the graph of $x^2 = -12y$.

a.	Focus is $(0, -3)$	O True	O False
b.	Directrix is $x = 3$	O True	O False
c.	Parabola is concave left	O True	O False
d.	Vertex is $(0, 0)$	O True	O False

Cluster: Use coordinates to prove simple geometric theorems algebraically.

9-11.G.GPE.4 – Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

DCAS-Like

29A

Jillian and Tammy are considering a quadrilateral $ABCD$. Their task is to prove $ABCD$ is a square.

Jillian says, "We just need to show that the slope of $\overline{AB}$ equals the slope of $\overline{CD}$, and the slope of $\overline{BC}$ equals the slope of $\overline{AD}$."

Tammy says, "We should show that $AC = BD$ and that (slope of $\overline{AC}$) × (slope of $\overline{BD}$) $= -1$."

Whose method of proof is valid?

A. Only Jillian's is valid.

B. Only Tammy's is valid.

C. Both are valid.

D. Neither is valid.

Next-Generation

29B

The vertices of quadrilateral $PQRS$ are: $P(4, 7)$, $Q(8, 4)$, $R(-1, -8)$, and $S(-5, -5)$. Complete the steps below to prove whether quadrilateral $PQRS$ is a rectangle.

a. What are the lengths of each side of quadrilateral $PQRS$?

$PQ =$ __________ $QR =$ __________

$RS =$ __________ $PS =$ __________

b. What are the slopes of $\overline{PQ}$ and $\overline{QR}$?

$\overline{PQ} =$ __________

$\overline{QR} =$ __________

c. What is the measure of angle PQR? __________

Explain how you found the measure of angle PQR.

d. Prove whether quadrilateral $PQRS$ is a rectangle. Use your answers to items a, b, and c, along with any other additional information, to support your answer.

9-11.G.GPE.5 – Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

DCAS-Like

30A

Which equation describes a line passing through $(-3, 1)$ and is parallel to $y = 4x + 1$?

A. $y = 4x + 13$

B. $y = 4x - 11$

C. $y = -\frac{1}{4}x + \frac{1}{4}$

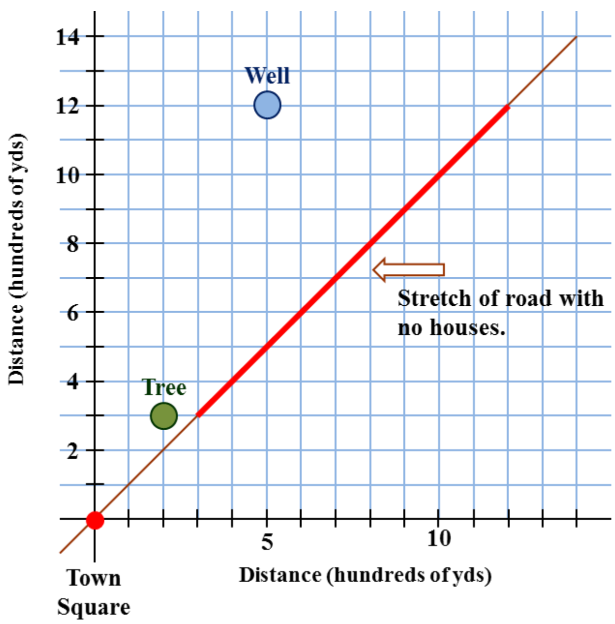
D. $y = -\frac{1}{4}x + \frac{7}{4}$

Next-Generation

30B

Euler's Village Task

You would like to build a house close to the village of Euler. There is a beautiful town square in the village, and the road you would like to build your house on begins right at the town square. The road follows an approximately northeast direction as you leave town and continues for 3,000 feet. It passes right by a large tree located approximately 200 yards east and 300 yards north of the town square. There is a stretch of the road, between 300 and 1,200 yards to the east of town, which currently has no houses. This stretch of road is where you would like to locate your house. Building restrictions require all houses sit parallel to the road. All water supplies are linked to town wells, and the closest well to this part of the road is 500 yards east and 1,200 yards north of the town square.



1. How far from the well would it be if the house was located on the road at the following locations. (For the sake of calculations, do not consider how far the house is from the road, just use the road to make calculations.)
   a. 300 yards east of town? _________________
   b. 500 yards east of town? _________________
   c. 1,000 yards east of town? _________________
   d. 1,200 yards east of town? _________________

2. The cost of piping leading from the well to the house is a major concern. Where should you locate your house in order to have the shortest distance to the well?

   Location on graph: (_____, _____)

3. If the cost of laying pipes is $22.5 per linear yard, how much will it cost to connect your house to the well? (Use shortest distance.)
   a. Distance from well: _________________
   b. Cost: _________________

4. Write a formula that the builder could use to find the cost of laying pipes to any house along this road. How would you have to change your formula for another road?

   Cost: _________________

9-11.G.GPE.6 – Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

DCAS-Like

31A

The point P divides $\overline{AB}$ in a ratio of 4:1, where $AP > BP$. If $A(-9,-5)$ and $B(11,-2)$, where is P?

A. $\left(7,-2\frac{3}{5}\right)$

B. $\left(6,-\frac{1}{4}\right)$

C. $\left(-4,-3\frac{1}{4}\right)$

D. $\left(-5,-3\frac{3}{5}\right)$

Next-Generation

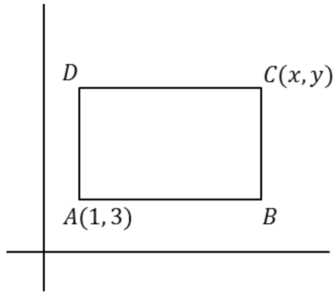
31B

Three collinear points on the coordinate plane are $A(x,y)$, $B(x+4h,y+4k)$, and $P(x+3h,y+3k)$. What is $\frac{AP}{BP}$?

9-11.G.GPE.7 – Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

DCAS-Like

32A



$ABCD$ is a rectangle where the slope of $\overline{AB}$ is 0. What is the area of the rectangle?

A. xy

B. $xy - 3$

C. $(x-1)(y-3)$

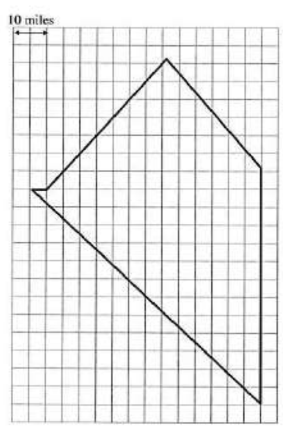
D. $2(x-1) + 2(y-3)$

Next-Generation

32B

The map at the right shows the counties in the State of Nevada. The shaded area is Esmeralda County.

The diagram below shows Esmeralda County superimposed on a grid.



a. Approximate the area of Esmeralda County. Area = _______________

b. Esmeralda County is the least populated county in Nevada with only 775 people. What is the population density of Esmeralda County? ____________________

Geometric Measurement and Dimension (G.GMD)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

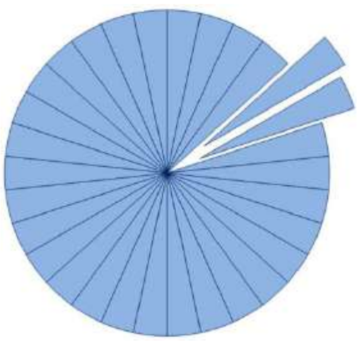
Cluster: Explain volume formulas and use them to solve problems.

9-11.G.GMD.1 – Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

DCAS-Like

33A

To estimate the area of a circle, Irene divided the circle into 30 congruent sectors. Then she combined pairs of sectors into shapes as shown below. As the shapes resemble rectangles, she treats the shapes as rectangles with the height r (radius) and the base equal to the length of the curved side of one sector. What is the area of each shape?



A. $r\dfrac{\pi r}{15}$

B. $r\dfrac{\pi r}{30}$

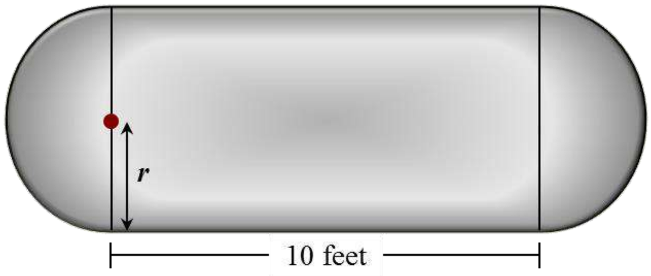
C. $r\dfrac{\pi r}{60}$

D. $r\dfrac{\pi r}{120}$

33B

People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters.

The tanks are made in the shape of a cylinder with hemispheres on the ends.



The Insane Propane Tank Company makes tanks with this shape, in different sizes.

The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, r, will be different depending on the size of the tank.

The company wants to double the capacity of their standard tank, which is 6 feet in diameter.

What should the radius of the new tank be? ____________________

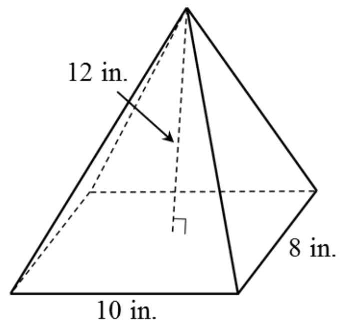
Explain your thinking and show your calculations.

9-11.G.GMD.3 – Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

DCAS-Like

34A

What is the volume of the rectangular pyramid?



A. 72 cubic inches

B. 200 cubic inches

C. 320 cubic inches

D. 960 cubic inches

Next-Generation

34B

Mike needs to buy a water tank for his business. The tank must fit inside a storage box that is shaped like a cube with side lengths of 30 feet. Water tanks are available in cylinders, cones, square pyramids, and spheres. Mike wants to buy the tank that has the largest capacity.

a. What shape tank should Mike buy? ______________________

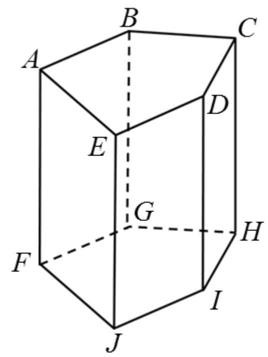
b. Explain why your choice is the best option for Mike.

Cluster: Visualize relationships between two-dimensional and three-dimensional objects.

9-11.G.GMD.4 – Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

DCAS-Like

35A



In the figure on the right, points A, E, and H are on a plane that intersects a right prism. What is the intersection of the plane with the right prism?

A. A line

B. A triangle

C. A quadrilateral

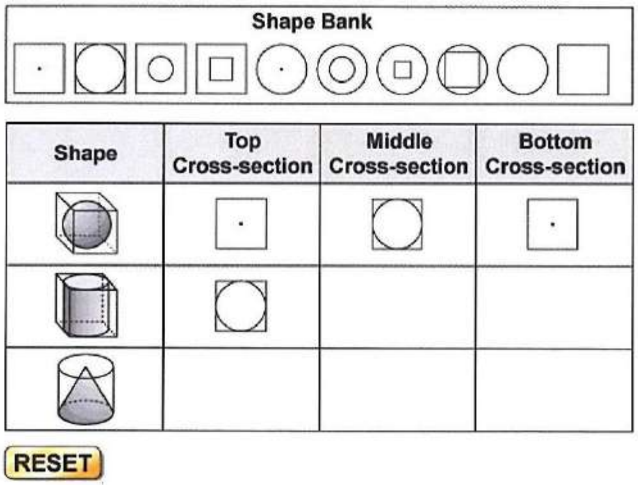
D. A pentagon

Next-Generation

35B

a. Complete the table to show the top, middle, and bottom horizontal cross-sections for each three-dimensional shape. The first row is completed.

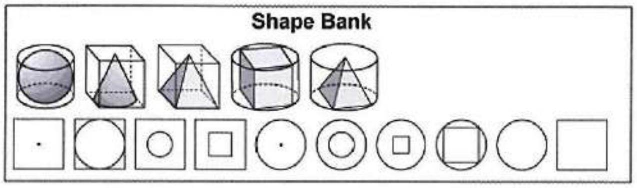
Click and drag pictures from the Shape Bank into the table. The cross-section shapes in the Shape Bank may be used more than once.

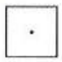
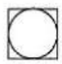


b. Complete the first row in the table to show the three-dimensional shape that matches the given horizontal cross-sections.

Complete the last row in the table by selecting one of the unused three-dimensional shapes and then providing the top, middle, and bottom horizontal cross-sections for that shape.

Click and drag pictures from the Shape Bank into the table. The cross-section shapes on the second row of the Shape Bank may be used more than once.



Shape	Top Cross-section	Middle Cross-section	Bottom Cross-section
			

RESET

Modeling with Geometry (G.MG)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

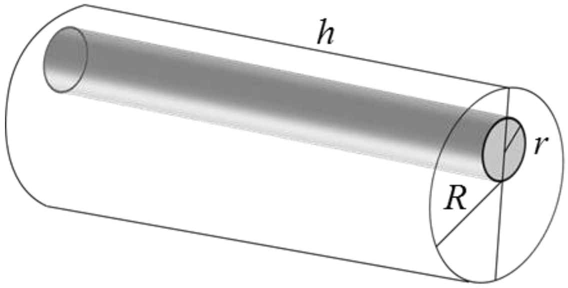
Cluster: Apply geometric concepts in modeling situations.

9-11.G.MG.1 – Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

DCAS-Like

36A

An object consists of a larger cylinder with a smaller cylinder drilled out of it as shown.



What is the volume of the object?

A. $\pi(R^2 - r^2)h$

B. $(\pi R^2 - r^2)h$

C. $(R^2 - \pi r^2)h$

D. $\pi(R - r)^2 h$

Next-Generation

36B

Stephanie has an aquarium that is in the shape of a right rectangular prism, 50 centimeters long, 30 centimeters wide, and 30 centimeters tall.

For decoration, Stephanie wants a layer of marbles in the bottom of the tank about 5 centimeters deep. The marbles have a diameter of 1 centimeter and come in bags of 500.

a. How many bags of marbles will Stephanie need? _______________

Stephanie pours <u>all</u> of the marbles into the tank. She now adds water until its level is 3 centimeters below the top of the tank.

b. How much water is in the tank? Express your answer in liters (1 liter = 1000 cubic centimeters). _______________

9-11.G.MG.2 – Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*

DCAS-Like

37A

A swimming pool is in the shape of a rectangular prism with a <u>horizontal</u> cross-section 10 feet by 20 feet. The pool is 5 feet deep and filled to capacity.

Water has a density of approximately 60 pounds per cubic foot, or 8 pounds per gallon.

What is the approximate weight of water in the pool?

A. 8,000 lb

B. 16,700 lb

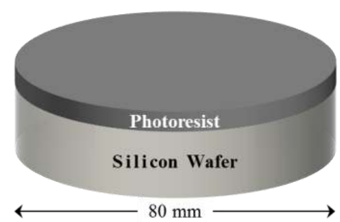
C. 60,000 lb

D. 80,000 lb

Next-Generation

37B

A silicon wafer is a circular disc 80 millimeters in diameter. One side of the wafer is coated with 0.06 milligrams of a substance called photoresist to a uniform thickness. Photoresist has a density of 1.2 milligrams per cubic millimeter.



a. What is the volume of photoresist used on the wafer?

____

b. What is the thickness of photoresist on the wafer?

____

9-11.G.MG.3 – Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

DCAS-Like

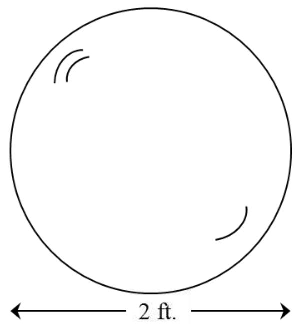
38A

Morgan is going to form a clay model of the moon. The model will have a diameter of 2 feet, and the clay she will use comes in containers as described below. What is the least number of containers Stephanie will need in order to complete the model?

Containers

Containers are $\frac{1}{2}$-ft. tall and cylinder-shaped, with 1-ft. diameters.

Model of the Moon



A. 3

B. 11

C. 16

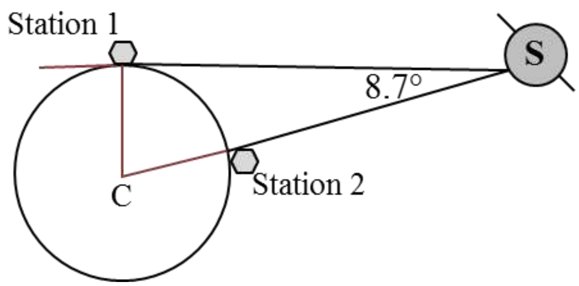
D. 22

38B

A satellite orbiting the earth uses radar to communicate with two control stations on the earth's surface. The satellite is in a geostationary orbit. That means that the satellite is always on the line through the center of the earth and Control Station 2.

From the perspective of Station 1, the satellite is on the horizon and from the perspective of Station 2, the satellite is always directly overhead as in the following diagram. The angle between the lines from the satellite to the stations is 8.7°.

Assuming the Earth is a sphere with radius 3963 miles, answer the following questions. Round all answers to the nearest whole number.

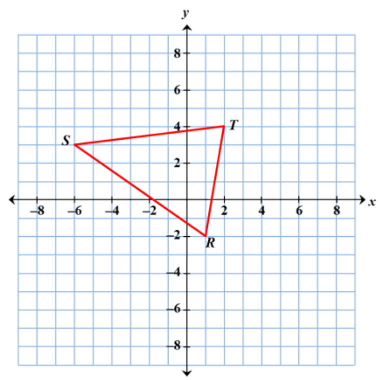


1. How many miles will a signal sent from Station 1 to the satellite and then to Station 2 have to travel? Explain your answer.
2. A satellite technician is flying from one station to the other in a direct path along the earth's surface. What is the distance she will have to fly? If she travels an average of 300 mph, how long with the trip take? Explain your answer.
3. If a signal could travel through the Earth's surface from one station to the other, what is the shortest distance the signal could travel to get from Station 1 to Station 2? Explain your answer.

Answer Key and Item Rubrics

Congruence (G.CO)

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>1A: D
(9-11.G.CO.1)</td><td>1B:
<table>
<tr><td>a.</td><td>1. Rhombus</td></tr>
<tr><td>e.</td><td>2. Isosceles Triangle</td></tr>
<tr><td>f.</td><td>3. Hexagon</td></tr>
<tr><td>l.</td><td>4. Segment</td></tr>
<tr><td>p.</td><td>5. Ray</td></tr>
<tr><td>o.</td><td>6. Angle</td></tr>
<tr><td>i.</td><td>7. Plane</td></tr>
<tr><td>j.</td><td>8. Endpoint</td></tr>
<tr><td>n.</td><td>9. Line</td></tr>
<tr><td>m.</td><td>10. Vertex</td></tr>
<tr><td>k.</td><td>11. Point</td></tr>
<tr><td>q.</td><td>12. Parallel Lines</td></tr>
<tr><td>s.</td><td>13. Perpendicular Lines</td></tr>
<tr><td>r.</td><td>14. Skew Lines</td></tr>
<tr><td>t.</td><td>15. Chord</td></tr>
</table></td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
2A: D (9-11.G.CO.2)	**2B:** a. (.5 point) The coordinates of the vertices of $\Delta A'B'C'$ are $A'(5,-1), B'(-2,-6)$, and $C'(6,-7)$ b. (.5 point) The rule could be described as $(x,-y)$. c. (.5 point)  d. (.5 point) The rule could be described as $(x-4, y-3)$. Other appropriate strategies are acceptable.
3A: D (9-11.G.CO.3)	**3B:** a. 60° b. 120° and 180° *Points Assigned* a. 1 point for 60°—“degrees” is not required b. 1 point for 2 other correct rotations—all possible rotations: 120, 180, 240, and 300 *Scoring Rubric* **2 points** **1 point:** Demonstrates minimal understanding of applying rotations **0 points:** Student’s response is incorrect, irrelevant, too brief to evaluate, or blank.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>4A: D
(9-11.G.CO.4)</td><td>4B:
a. translation
b. reflection
c. reflection
d. rotation</td></tr>
<tr><td>5A: A
(9-11.G.CO.5)</td><td>5B:
There are multiple sequences of transformations that work for the 4th and 5th pre-image/image pairs. For example, to get from image 1 to image 5 you might reflect image 1 about some vertical line, then translate the reflected image until it coincides with image 5. Or, you might translate image 1 until you can identify a mirror line in which to reflect the image to get it to coincide with image 5.</td></tr>
<tr><td>6A: A
(9-11.G.CO.6)</td><td>6B:
<table>
<tr><th>Pre-Image</th><th>Final Image</th><th>Rigid-Motion Transformation Description</th></tr>
<tr><td>$ABCD$</td><td>$A'B'C'D'$</td><td>Translate 2 units up</td></tr>
<tr><td>$A'B'C'D'$</td><td>$A"B"C"D"$</td><td>Reflex across vertical line (y-axis if drawn)</td></tr>
<tr><td>$A"B"C"D"$</td><td>$QRST$</td><td>Rotate 90° clockwise</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
</table>
</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>7A: B
(9-11.G.CO.7)</td><td>7B:
Answers will vary. Solutions may be paragraph, 2-column, flow, or other type of proof. However, statements must be justified. Either transformation-based or Euclidean-based proofs are acceptable.
Example of a transformational geometry based proof:
<table>
<tr><th>Statement</th><th>Justification</th></tr>
<tr><td>$r_m(D) = E$</td><td>given</td></tr>
<tr><td>m is the perpendicular bisector of $\overline{AB}$</td><td>given</td></tr>
<tr><td>$r_m(A) = B$</td><td>definition of a reflection</td></tr>
<tr><td>$r_m(C) = C$</td><td>definition of a reflection</td></tr>
<tr><td>$r_m(\Delta ADC) = \Delta BEC$</td><td>reflections are isometries</td></tr>
<tr><td>$\Delta ADC \cong \Delta BEC$</td><td>figures are congruent if one is the image of the other under an isometry</td></tr>
</table>
</td></tr>
<tr><td>8A: D
(9-11.G.CO.8)</td><td>8B:
a. False
b. True
c. False</td></tr>
<tr><td>9A: A
(9-11.G.CO.9)</td><td>9B:
Key: D
Rationale for choosing correct option: The steps in the construction make a copy of the angle formed between the transversal and the given line, so these angles are congruent. The construction steps have produced two lines intersected by a transversal with a pair of corresponding angles congruent. Option D states a theorem that can be used to conclude that the corresponding angles must be congruent.
Rational for choosing incorrect options: These are all principles that can be used to prove that two lines are parallel, though only option D applies to this construction.</td></tr>
</table>

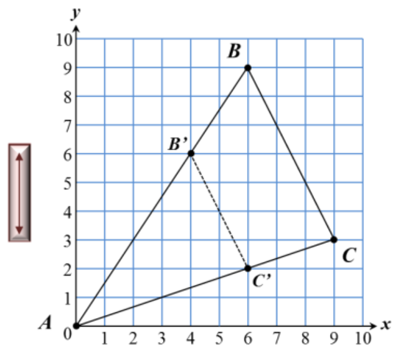
<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>10A: A
(9-11.G.CO.10)</td><td>10B:
Each item is scored independently and will receive 1 point.
a. $\angle ABD \cong \angle ABC$
b. $\overline{AC} \cong \overline{CE}$</td></tr>
<tr><td>11A: A
(9-11.G.CO.10)</td><td>11B:
a. Midpoint of $\overline{AB} = (3, 5)$
Midpoint of $\overline{BC} = (6, 4)$
b. Length of $\overline{AC} = sqrt(40)$
c. Slope of $\overline{AC} = -\frac{1}{3}$
d. Darlene is correct. The length of the segment connecting the midpoints is $sqrt(10)$ units, which is half of $sqrt(40)$. The slope of the segment connecting the midpoints is $-\frac{1}{3}$, which is equal to the slop of segment AC. Equal slopes means the segments are parallel.
Note: Other answers are possible.

Points Assigned
• Part a. – 1 point for the correct coordinates of BOTH midpoints.
• Part b. – 1 point for correct length.
• Part c. – 1 point for correct slope.
• Part d. – 1 point for stating that Darlene is correct with complete justification.

Scoring Rubric
4 points
3 points
2 points
1 point: Demonstrates minimal understanding of using coordinates to prove theorems about triangles.
0 points: Student’s response is incorrect, irrelevant, too brief to evaluate, or blank.</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
12A: B (9-11.G.CO.11)	**12B:** a. False b. False c. True
13A: D (9-11.G.CO.12)	**13B:** a. Yes b. Yes c. No d. Yes
14A: A (9-11.G.CO.13)	**14B:** All 3 angles in triangle ABC measure 60°, $AB = BC = AC = 6$, and $AO = BO = CO =$ radius of circle. So triangles ABO, BCO, and ACO are isosceles and congruent. Their base angles measure 30°, and their altitudes are perpendicular to the base and bisect the base. So we can draw DO, the altitude of ABO. DO is part of DC, the altitude of ABC, which separates congruent right triangles ACD and BCD. $AB = BC = AC = 6$, $AD = BD = 3$, $AO = BO = CO =$ radius of circle Triangles ACD and BCD are congruent 30-60-90 triangles. $$6^2 = 3^2 + CD^2$$ $$CD = 3\sqrt{3}$$ DO splits triangle ABO (isosceles triangle with 30° base angles) into two congruent 30-60-90 right triangles: AOD and BOD. They are similar to triangles ACD and BCD. In all those 30-60-90 triangles, the hypotenuse is twice as long as the short leg. So $CO = CD - DO = CD - \left(\frac{1}{3}\right) \bullet CD = 2\sqrt{3}$

Similarity, Right Triangles, and Trigonometry (G.SRT)

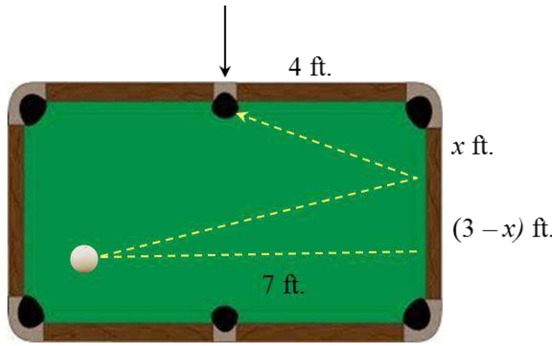
DCAS-Like Answer	Next-Generation Solution
15A: D (9-11.G.SRT.1)	**15B:** Solution 
16A: A (9-11.G.SRT.2)	**16B:** a. No – We are given that $2AX = 3XD$. This is not enough information to prove similarity. To see that in a simple way, draw an arbitrary triangle ΔAXB. Extend AX and choose a point D on the extended line so that $2AX = 3XD$. Extend BX and choose a point C on the extended line so that $2BX = XC$. Now triangles AXB and CXD satisfy the given conditions but are not similar. (If you are extremely unlucky, AXB and CXD might be similar by a different correspondence of sides. If this happens, rotate the line BC a little bit. The lengths of AX, XD, BX, XC remain the same, but the triangles are no longer similar.

DCAS-Like Answer	Next-Generation Solution
	b. Yes – We are given that $\frac{AX}{BX} = \frac{DX}{CX}$. Rearranging this proportion gives $\frac{AX}{DX} = \frac{BX}{CX}$. Let $k = \frac{AX}{XD}$. Suppose we rotate the triangle DXC 180 degrees about point X. Since AD is a straight line, DX and AX align upon rotation of 180 degrees, as do CX and BX, and so angles DXC and AXB coincide after this rotation. (Alternatively, one could observe that DXC and AXB are vertical angles, and hence congruent, giving a second argument the angles line up precisely.) Then, dilate the triangle DXC by a factor of k about the center X. This dilation moves the point D to A, since $k(DX) = AX$, and moves C to B, since $k(CX) = BX$. Then, since the dilation fixes X, and dilations take line segments to line segments, we see that the triangle DXC is mapped to triangle AXB. So the original triangle DXC is similar to AXB. (Note that we state the similarity so that the vertices of each triangle are written in corresponding order.) c. Yes – Again rotate triangle DXC so that angle DXC coincides with angle AXB. Then the image of the side CD under this rotation is parallel to the original side CD, so the new side CD is parallel to side AB. Now, apply a dilation about point X that moves the vertex C to point B. This dilation moves the line CD to a line through B parallel to the previous line CD. We already know that line AB is parallel to CD, so the dilation must move the line CD onto the line AB. Since the dilation moves D to a point on the ray XA and on the line AB, D must move to A. Therefore the rotation and dilation map the triangle DXC to the triangle AXB. Thus DXC is similar to AXB. d. Yes – Suppose we draw the bisector of angle AXC, and reflect the triangle CXD across this angle bisector. This maps the segment XC onto the segment XA; and since reflections preserve angles, it also maps segment XD onto segment XB. Since angle XCD is congruent to angle XAB, we also know that the image of side CD is parallel to side AB. Therefore, if we apply a dilation about the point X that takes the new point C to A, then the new line CD will be mapped onto the line AB, by the same reasoning used in Item c. Therefore, the new point D is mapped to B, and thus the triangle XCD is mapped to triangle XAB. So triangle XCD is similar to triangle XAB. (Note that this is not the same correspondence we had in item b. and c.)

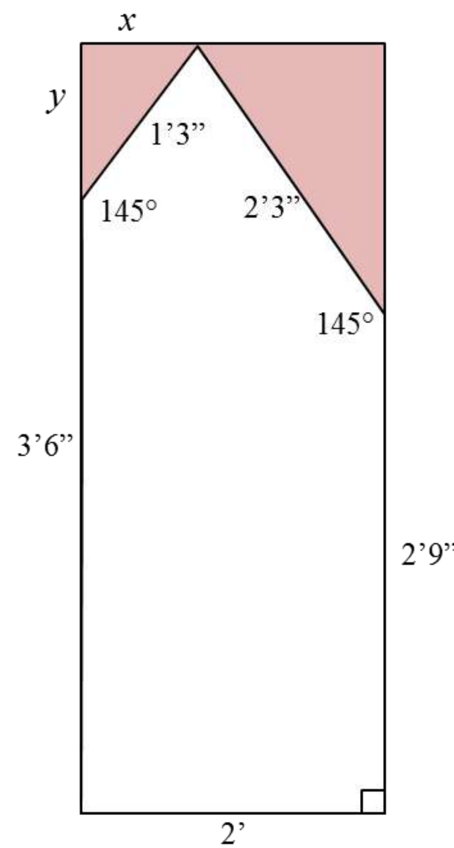
<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>17A: A
(9-11.G.SRT.3)</td><td>17B:

	Statements	Reasons
1	$AD = 6; DC = 3$ $BE = 4; EC = 2$	Given
2	$CA = CD + DA$ $CB = CE + EB$	Addition postulate
3	$\frac{CA}{CD} = \frac{9}{3} = 3; \frac{CB}{CE} = \frac{6}{2} = 3$	Substitution
4	$\frac{CA}{CD} = \frac{CB}{CE}$	Transitive property
5	$\measuredangle C \cong \measuredangle C$	Reflexive property – A quantity is congruent to itself.
6	$\Delta CDE \sim \Delta CAB$	SAS Similarity Theorem – If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

</td></tr>
<tr><td>18A: C
(9-11.G.SRT.4)</td><td>18B:
This question assesses the student’s ability to do geometric proofs.
Given $\Delta ABC \sim \Delta CBD$, then $\frac{e}{a} = \frac{a}{c}$, so $a^2 = ce$.
Given $\Delta ABC \sim \Delta ACD$, then $\frac{d}{b} = \frac{b}{c}$, so $b = cd$.
Thus $a^2 + b^2 = ce + cd = c(d + e) = c(c) = c^2$.</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
19A: C (9-11.G.SRT.5)	**19B:**  Let x be the distance from the northeast corner of the table at which Pablo wants the cue ball to hit the east wall. We start by drawing two right triangles as shown in the diagram above: one whose hypotenuse is the segment from the desired point of contact with the east wall to the north pocket, and one whose hypotenuse is the segment from the cue ball to the point of contact with the east wall. The former right triangle has legs of length 4 ft. and x ft. The latter has legs of length 7 ft., because the cue ball begins one foot away from the west wall, and $(3-x)$ ft., because it begins one foot away from the south wall. Because the angle at which the cue ball hits the wall is equal to the angle at which it rebounds off of it, and because the two right triangles already have one right angle each, we know the two triangles are similar by AA similarity, and therefore $$\frac{7}{(3-x)} = \frac{4}{x}$$ Multiplying each side by $x(3-x)$ yields $7x = 4(3-x)$ and we obtain $x = \frac{12}{11}$ ft. Therefore, Pablo wants the cue ball to contact the east wall $\frac{12}{11}$ ft. away from the northeast corner of the table.

DCAS-Like Answer	Next-Generation Solution
20A: C (9-11.G.SRT.6)	**20B:** a. $\frac{3}{x} = \frac{9}{40+x}$ $3(x+40) = 9x$ $3x + 120 = 9x$ $120 = 6x$ $20 = x$ Using Pythagorean Theorem, $AC = 60.7$ b. $\sin A = \frac{9}{60.7}$ c. $\cos A = \frac{60}{60.7}$ d. $\tan A = \frac{9}{60} \approx \frac{3}{20}$
21A: A (9-11.G.SRT.7)	**21B:** a. True b. False c. True

DCAS-Like Answer	Next-Generation Solution
22A: B (9-11.G.SRT.8)	**22B:** a. The area of the closet is equal to the area of a containing rectangle minus the areas of the two shaded right triangles. Convert all measurements to either feet or inches to avoid errors. This solution is in feet. Using right triangle trigonometry, the smaller right triangle has legs of $1.25 \sin 35° \approx 0.72$ feet (unknown measurement for x) and $1.25 \cos 35° \approx 1.02$ feet (unknown measurement for y). The larger right triangle has legs of $3.5 + 1.02 - 2.75 \approx 1.77$ feet and $2 - 0.72 \approx 1.28$ feet. The length of the rectangle is $3.5 + 1.02 \approx 4.52$ feet. The area of the closet: $(4.52 \times 2) - \frac{1}{2}(0.72)(1.02) - \frac{1}{2}(1.77)(1.28) \approx 7.54$ square feet. b. The length of the closet is 4.52 feet, so Kim cannot just buy a 2-foot wide piece—it will be too short. She will have to buy a 4.52 (about 4-feet 7-inches piece to fit the length of the closet and cut off 2 feet of it so it fits the width.) 

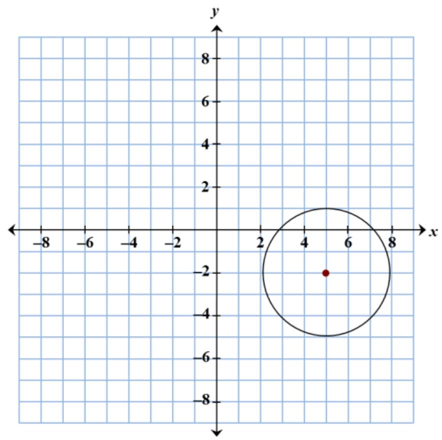
Circles (G.C)

DCAS-Like Answer	Next-Generation Solution
23A: C (9-11.G.C.1)	**23B:** To transform circle C to the larger circle D, we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and 2 units up. To enlarge circle C to the same radius as D, the enlargement ratio is the quotient of the radii: $\frac{5}{3}$.
24A: A **(9-11.G.C.2)**	**24B:** a. False b. True c. False
25A: D **(9-11.G.C.3)**	**25B:** $\angle P + \angle L + \angle M + \angle N = 360$ $4 + 18y + 10x + 10x - 6 + 16y + 6 = 360$ $20x + 34y + 4 = 360$ $20x + 34y = 356$ $10x + 17y = 178$ $\angle P + \angle M = 180$ $4 + 18y + 10x - 6 = 180$ $10x + 18y = 182$ $10x + 17y = 178$ $\underline{-1(10x + 18y = 182)}$ $10x + 17y = 178$ $-10x - 18y = -182$ $-1y = -4$ $\boldsymbol{y = 4}$ $\angle P = 76°$, $\angle L = 110°$, $\angle M = 104°$, and $\angle N = 70°$ $10x + 17y = 178$ $10x + 17(4) = 178$ $10x + 68 = 178$ $10x = 110$ $\boldsymbol{x = 11}$

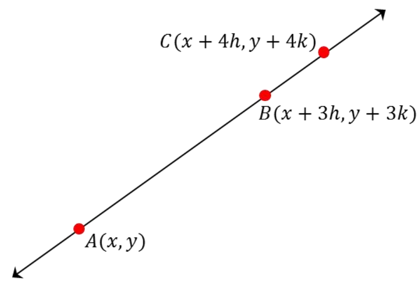
DCAS-Like Answer	Next-Generation Solution
26A: D **(9-11.G.C.5)**	**26B:** a. The intersection of the barn and the storage facility allows the construction of two right triangles, as seen from the top view of the storage facility. The distance between the side of the barn and the center of the storage facility is given as 5 ft. Using the properties of triangles and the fact that if the radius of a circle is perpendicular to a chord, then it bisects it, we can conclude that the two triangles are isosceles right triangles, with legs 5 ft. in length. We can use the Pythagorean Theorem, or the properties of special right triangles, to find the length of the hypotenuse of one of the triangles, which is also the radius of the circle, to be $5\sqrt{2}$ ft. The area of the circle, as seen from the top view of the storage facility, would be found using the formula: $A = \pi r^2$ $A = \pi(5\sqrt{2})^2 = 50\pi \text{ft}^2$, or approximately 157 ft^2 The larger triangle formed by the union of the two isosceles right triangles has a base of 10 and a height of 5, as given in the task. The area of this triangle can be found using the formula: $A = \frac{1}{2}bh$ $A = \frac{1}{2}(10)(5) = 25 \text{ ft}^2$ Since the two small isosceles right triangles "meet at the center of the circle" with their combined 45° angles, the sector of the circle that encompasses the triangles has a 90° central angle and is a quarter of the circle. The remaining sector is three-fourths of the circle. The area of the three-fourths of the circle plus the area of the triangle formed by the two isosceles right triangles equals the area of the base of the storage facility. Area of the base of the storage facility $= \frac{3}{4}(50\pi)\text{ft}^2 + 25 \text{ ft}^2 = 37.5\pi \text{ ft}^2 + 25 \text{ ft}^2 = \text{approximately } 142.8 \text{ ft}^2$ To find the capacity of the storage facility, in cubic feet, the student must then multiply the area of the base by 30 ft., the height of the grain when the storage facility is filled to capacity. $V = (142.8 \text{ ft}^2)(30 \text{ ft}) = 4284 \text{ ft}^3$. Therefore, the capacity of the storage facility is approximately 4284 ft^3.

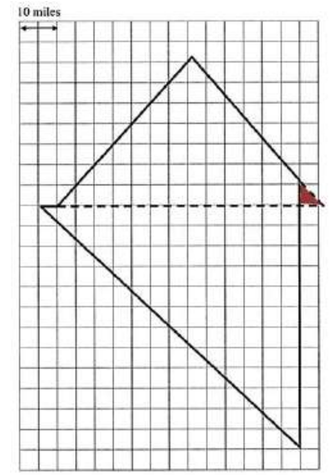
DCAS-Like Answer	Next-Generation Solution
	b. Using unit analysis to change cubic feet to bushels: $$\frac{1\text{ bushel}}{1.25\text{ ft}^3} \times 4284\text{ ft}^3 = \frac{4284}{1.25}\text{ bushels} = 3427.2\text{ bushels}$$ Note: Other strategies can be used to find the base of the storage facility, such as $\left(\frac{1}{2}\text{ circle} + \text{isosceles right triangle} + 2\text{ remaining sectors}\right)$, but this is not as efficient as the method above.

Expressing Geometric Properties with Equations (G.GPE)

DCAS-Like Answer	Next-Generation Solution
27A: B (9-11.G.GPE.1)	**27B:** 
28A: D (9-11.G.GPE.2)	**28B:** a. True b. False c. False d. True ***Key and Distractor Analysis:*** a. Key. Focus is $(0, p)$ and $p = -3$. b. Directrix is $y = -p$. Student may have confused x and y. c. Student may have confused the form of the equation. Parabola is concave down.

DCAS-Like Answer	Next-Generation Solution
	d. Key. When substituting a and b into the formula for the axis of symmetry $x = \frac{-b}{2a}$, and then substituting that value back into the given equation, one gets the vertex $(0, 0)$. Rewrite $x^2 = -12y$ as $(x-h)^2 = 4p(y-k)$ where vertex = (h, k), focus =$(h, k+p)$, and directrix $y = k - p$
29A: B (9-11.G.GPE.4)	**29B:** a. $PQ = 5$ $QR = 15$ $RS = 5$ $PS = 15$ b. $\overline{PQ} = -\frac{3}{4}$ $\overline{QR} = \frac{4}{3}$ c. Angle PQR measures 90° PQ and QR are perpendicular because the product of their slopes is -1. Perpendicular lines have 90° angles. d. $PQRS$ is a rectangle. The slope of PS is $\frac{4}{3}$, which makes segment PS perpendicular to segment PQ. This means SPQ is 90°. The slope of segment RS is $-\frac{3}{4}$, which makes segment RS perpendicular to segment PS. This means PSR is 90°. Segment QR is also perpendicular to segment RS, which means QRS is 90°. All four angles are 90°, and the opposite sides have equal lengths. This means $PQRS$ is a rectangle. Note: Other answers are possible for item d, including using the diagonals or parallel and perpendicular lines. If a different justification is used, the student must provide all slopes and side lengths as appropriate.
30A: A (9-11.G.GPE.5)	**30B:** ***Part 1:*** a. 300 yards east = 922 yards b. 500 yards east = 700 yards c. 1,000 yards east = 539 yards d. 1,200 yards east = 700 yards ***Part 2:*** Location on graph: (8.5, 8.5) Location at 850 yards east and 850 yards north of town (where the pipe is perpendicular to the road).

DCAS-Like Answer	Next-Generation Solution
	Part 3: a. Distance from well: 495 yards b. Cost: $\$22.5 \times 495 \approx \$11{,}138$ ***Part 4:*** $\text{Cost} = \$22.5 \times \sqrt{(x-5)^2 + (y-12)^2} \times 100$ OR $\text{Cost} = \$2{,}250\sqrt{(x-5)^2 + (y-12)^2}$
31A: A (9-11.G.GPE.6)	**31B:** This question assesses the student's understanding of directed line segments. $\frac{AP}{BP} = \frac{(x+3h)-x}{(x+4h)-(x+3h)}$ $= \frac{3h}{h}$ $= 3$ 

DCAS-Like Answer	Next-Generation Solution
32A: C (9-11.G.GPE.7)	**32B:**  This question assesses the student's ability to apply the area of triangles and quadrilaterals. a. Esmeralda county can be divided into a triangle (south of dotted line) and a quadrilateral. The quadrilateral north of the dotted line has an area equal to the large triangle north of the dotted line minus the small shaded triangle added to the figure. South triangle: base is about 70 miles and height is about 60 miles. Area equals: $\frac{1}{2}(60\text{ mi})(70\text{ mi}) = 2100\text{ mi}^2$ North quadrilateral is the difference between the large triangle with base 70 miles and height about 36 miles, and the shaded triangle with base and height of 6 miles. Area equals: $\frac{1}{2}(36\text{ mi})(70\text{ mi}) - \frac{1}{2}(6\text{ mi})(6\text{ mi}) = 1242\text{ mi}^2$ The area of Esmeralda County is approximately 3342 square miles. Note: It is acceptable to use 35 miles for the height of the triangle and 5×5 for the dimension of the shaded triangle. Doing so will result in a total area of 3312.5 square miles. b. The population density of Esmeralda County is: $\frac{775\text{ people}}{3342\text{ mi}^2} \approx 0.23$ people per square mile.

Geometric Measurement and Dimension (G.GMD)

DCAS-Like Answer	Next-Generation Solution
33A: A (9-11.G.GMD.1)	**33B:** ***Scoring Rubric*** Gives correct answers and shows correct reasoning such as:

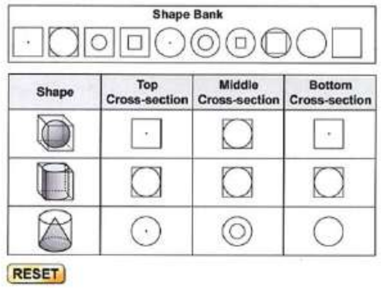
	Points
The approximate value for the radius of the new tank is 4 feet.	1
For the existing tank ▪ The volume of the cylinder is 283 or 90π ▪ The volume of the sphere is 113 or 36π ▪ The total volume is 396 or 126π	 2 2 1
For the new tank, the volume $V=\pi r^2h+\frac{4\pi r^3}{3}=10\pi r^2+\frac{4\pi r^3}{3}=2\times 126\pi$ $10r^2+\frac{4r^3}{3}=252$	2
Tries different values for r ▪ When $r=4$, $V=245.3$ ▪ When $r=5$, $V=416.6$ ▪ When $r=4.1$, $V=259.9$ OR, look at the x-intercept when graphing $y=\frac{4}{3}x^3+10x^2-252$	2
Award process points if numerical errors are made	
Total Points	***10***

DCAS-Like Answer	Next-Generation Solution
34A: C (9-11.G.GMD.3)	**34B:** ***Exemplary Response*** a. Cylinder b. The cylinder has the largest volume. Sphere $= 4500 * \pi = 14137.16$ cu. ft. Cylinder $= 6750 * \pi = 21205.75$ cu. ft. Cone $= 2250 * \pi = 7068.58$ cu. ft. Square pyramid $= 9000$ cu. ft. ***Points Assigned*** ▪ 1 point for cylinder ▪ 1 point for complete and correct justification

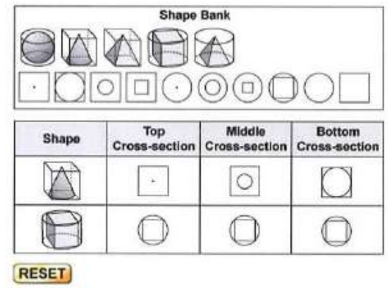
DCAS-Like Answer	Next-Generation Solution
35A: B (9-11.G.GMD.4)	**35B:**

Exemplary Response

a.



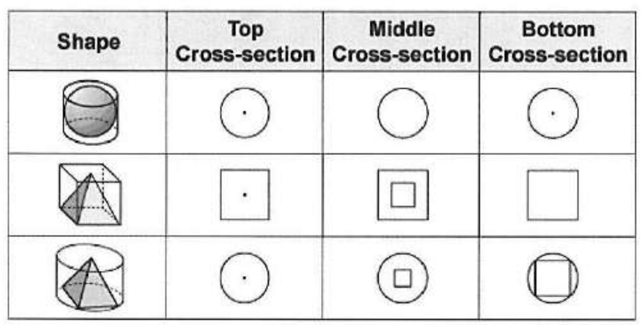
b.



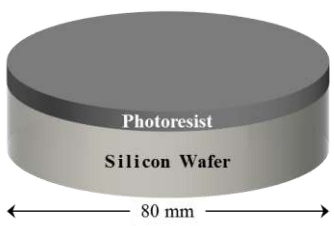
Points Assigned

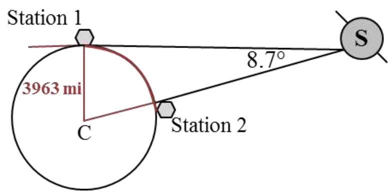
a. 3 points for ALL 5 shapes correct OR
   2 points for 3 or 4 shapes correct OR
   1 point for 1 or 2 shapes correct

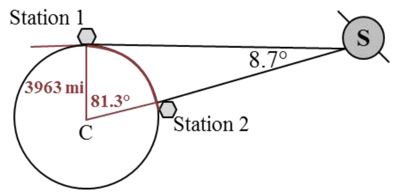
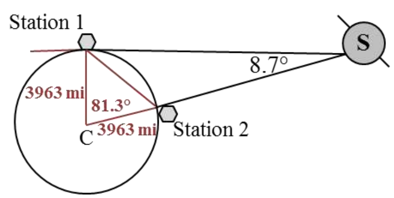
b. 2 points for correct 3-D shape in first row and ALL 3 cross-sections correct for chosen 3-D shape in second row OR
   1 point for correct 3-D shape in first row and 2 cross-sections correct for chosen 3-D shape in second row
   Note: Other possible answers for row 2 are shown below.



Modeling with Geometry (G.MG)

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>36A: A
(9-11.G.MG.1)</td><td>36B:
This question assesses the student’s ability to apply volume formulas in modeling situations.
a. The aquarium has a rectangular cross section 50 cm by 30 cm. The marbles have a diameter of 1 cm, so an array of 50×30 marbles will cover the bottom of the tank. To fill the marbles about 5 cm deep will take $50 \times 30 \times 5 = 7500$ marbles. That is $\frac{7500}{500} = 15$ bags of marbles. (Note: if a student realizes that the marbles will probably not form a 3-D array $50 \times 30 \times 5$, but have “layers” of different sizes, say 50×30 for the first, 49×29 for the second, 50×30 for the third, etc., this is acceptable. Precisely how the marbles are “packed” needs only to be reasonable.)
b. The total volume of water and marbles in the tank is $50\text{ cm} \times 30\text{ cm} \times 27 = 40{,}500\text{ cm}^3$. The volume of marbles in the tank is $7500 \times \frac{4}{3}\pi\left(\frac{1}{2}\text{ cm}\right)^3 \approx 3927\text{ cm}^3$. So the volume of water is 36,573 cm^3 or about 36.6 liters. (Treating the marbles as a solid $50\text{ cm} \times 30\text{ cm} \times 5\text{ cm}$ block of glass is incorrect; there is space between the marbles. If a student makes the case that 6 layers of marbles is closer to 5 cm tall than 5 layers, this is an acceptable solution.)</td></tr>
<tr><td>37A: C
(9-11.G.MG.2)</td><td>37B:
This question assesses the student’s ability to apply volume formulas and the concept of density in modeling situations.
a. $$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} \\ 1.2\,\frac{mg}{mm^3} &= \frac{0.06\,mg}{V} \\ V &= \frac{0.06\,mg}{1.2\frac{mg}{mm^3}} \\ V &= 0.05\,mm^3 \end{aligned}$$

</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
	b. The photoresist covers the circular wafer with a uniform thickness, essentially making it a cylinder with a height equal to the thickness: $V = \pi r^2 h$ $0.05\ mm^3 = \pi(40\ mm)^2 h$ $h = \frac{0.05\ mm^3}{\pi(40\ mm)^2}$ $h = 0.000009947 \ldots mm$ $h \approx 10^{-5} mm$
38A: B (9-11.G.MG.3)	**38B:** 1.  $\tan 8.7° = \frac{3963}{x} \approx 25{,}898$ miles The distance from Station 1 to the satellite is approximately 25,898 miles. $\sin 8.7° = \frac{3963}{y} \approx 26{,}200$ miles The distance from the satellite to the center of the Earth is approximately 26,200 miles. The distance from the satellite to Station 2 is $26{,}200 - 3963 = 22{,}237$ miles. Total distance the signal was sent is 48,135 miles.

DCAS-Like Answer	Next-Generation Solution
	2.  The distance the technician flies is now given by the length of this arc of the circle: $$3963 \bullet 81.3° \bullet \frac{\pi}{180°} \approx 5{,}620 \text{ miles}$$ To find the time taken for this flight, we divide the distance just calculated by the rate of travel, assumed in the problem to be 300 mph: $$\frac{5620}{300} \approx 19 \text{ hours}$$ If the plane is traveling at a rate of 300 miles an hour, it will take the technician approximately 19 hours to travel from Station 1 to Station 2. 3. Connecting Station 1 to Station 2 through the surface of Earth forms a triangle that is not a right triangle as shown.  The Law of Cosines is used to determine the length of a side of a triangle when the lengths of two sides and measure of the included angle are known. This is our situation. Two sides of the triangle formed are radii of the Earth (3963 miles), and the angle formed by the two radii measures 81.3°, as previously determined. ▪ Let c represent the shortest distance between Station 1 and Station 2. ▪ Let a and b represent the lengths of the radii, 3963 miles. ▪ Let 81.3° represent the measure of the included angle.

DCAS-Like Answer	Next-Generation Solution
	We solve for c using the above values in the Law of Cosines: $$c^2 = 3963^2 + 3963^2 - 2(3963)(3963)\ \cos(81.3°) \approx 26{,}659{,}524$$ Taking square roots, we find that the shortest distance the signal could travel through the Earth's surface from Station 1 to Station 2 is approximately 5,163 miles.