COMMON CORE ASSESSMENT COMPARISON FOR MATHEMATICS

GRADES 9–11
FUNCTIONS

June 2013

Prepared by:

Delaware Department of Education
Accountability Resources Workgroup
401 Federal Street, Suite 2
Dover, DE 19901

12/4/2013
Document Control No.: 2013/05/08

Table of Contents

INTRODUCTION

The purpose of this document is to illustrate the differences between the Delaware Comprehensive Assessment System (DCAS) and the expectations of the next-generation Common Core State Standard (CCSS) assessment in Mathematics. A side-by-side comparison of the current design of an operational assessment item and the expectations for the content and rigor of a next-generation Common Core mathematical item are provided for each CCSS. The samples provided are designed to help Delaware's educators better understand the instructional shifts needed to meet the rigorous demands of the CCSS. This document does not represent the test specifications or blueprints for each grade level, for DCAS, or the next-generation assessment.

For mathematics, next-generation assessment items were selected for CCSS that represent the shift in content at the new grade level. Sites used to select the next-generation assessment items include:

- Smarter Balanced Assessment Consortium
- Partnership of Assessment of Readiness for College and Career
- Illustrative Mathematics
- Mathematics Assessment Project

Using released items from other states, a DCAS-like item, aligned to the same CCSS, was chosen. These examples emphasize the contrast in rigor between the previous Delaware standards, known as Grade-Level Expectations, and the Common Core State Standards.

Section 1, DCAS-Like and Next-Generation Assessment Comparison, includes content that is in the CCSS at a different "rigor" level. The examples are organized by the CCSS. For some standards, more than one example may be given to illustrate the different components of the standard. Additionally, each example identifies the standard and is separated into two parts. Part A is an example of a DCAS-like item, and Part B is an example of a next-generation item based on CCSS.

Section 2 includes at least one Performance Task that addresses multiple aspects of the CCSS (content and mathematical practices).

How to Use Various Aspects of This Document

- Analyze the way mathematics standards are conceptualized in each item or task.
- Identify the instructional shifts that need to occur to prepare students to address these more rigorous demands. Develop a plan to implement the necessary instructional changes.
- Notice how numbers (e.g., fractions instead of whole numbers) are used in the sample items.
- Recognize that the sample items and tasks are only one way of assessing the standard.
- Understand that the sample items and tasks do not represent a mini-version of the next-generation assessment.
- Instruction should address "focus," coherence," and "rigor" of mathematics concepts.
- Instruction should embed mathematical practices when teaching mathematical content.

- For grades K–5, calculators should not be used as the concepts of number sense and operations are fundamental to learning new mathematics content in grades 6–12.
- The next-generation assessment will be online and the scoring will be done electronically. It is important to note that students may not be asked to show their work and therefore will not be given partial credit. It is suggested when using items within this document in the classroom for formative assessments, it is good practice to have students demonstrate their methodology by showing or explaining their work.

Your feedback is welcome. Please do not hesitate to contact Katia Foret at katia.foret@doe.k12.de.us or Rita Fry at rita.fry@doe.k12.de.us with suggestions, questions, and/or concerns.

* The Smarter Balanced Assessment Consortium has a 30-item practice test available for each grade level (3-8 and 11) for mathematics and ELA (including reading, writing, listening, and research). These practice tests allow students to experience items that look and function like those being developed for the Smarter Balanced assessments. The practice test also includes performance tasks and is constructed to follow a test blueprint similar to the blueprint intended for the operational test. The Smarter Balanced site is located at: http://www.smarterbalanced.org/.

Priorities in Mathematics

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Common Core State Standards for Mathematical Practices

Mathematical Practices		Student Dispositions:	Teacher Actions to Engage Students in Practices:
Essential Processes for a Productive Math Thinker	**1. Make sense of problems and persevere in solving them**	▪ Have an understanding of the situation ▪ Use patience and persistence to solve problem ▪ Be able to use different strategies ▪ Use self-evaluation and redirections ▪ Communicate both verbally and written ▪ Be able to deduce what is a reasonable solution	▪ Provide open-ended and rich problems ▪ Ask probing questions ▪ Model multiple problem-solving strategies through Think-Aloud ▪ Promote and value discourse ▪ Integrate cross-curricular materials ▪ Promote collaboration ▪ Probe student responses (correct or incorrect) for understanding and multiple approaches ▪ Provide scaffolding when appropriate ▪ Provide a safe environment for learning from mistakes
	6. Attend to precision	▪ Communicate with precision—orally and written ▪ Use mathematics concepts and vocabulary appropriately ▪ State meaning of symbols and use them appropriately ▪ Attend to units/labeling/tools accurately ▪ Carefully formulate explanations and defend answers ▪ Calculate accurately and efficiently ▪ Formulate and make use of definitions with others ▪ Ensure reasonableness of answers ▪ Persevere through multiple-step problems	▪ Encourage students to think aloud ▪ Develop explicit instruction/teacher models of thinking aloud ▪ Include guided inquiry as teacher gives problem, students work together to solve problems, and debrief time for sharing and comparing strategies ▪ Use probing questions that target content of study ▪ Promote mathematical language ▪ Encourage students to identify errors when answers are wrong
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	▪ Create multiple representations ▪ Interpret problems in contexts ▪ Estimate first/answer reasonable ▪ Make connections ▪ Represent symbolically ▪ Talk about problems, real-life situations ▪ Attend to units ▪ Use context to think about a problem	▪ Develop opportunities for problem-solving strategies ▪ Give time for processing and discussing ▪ Tie content areas together to help make connections ▪ Give real-world situations ▪ Demonstrate thinking aloud for students' benefit ▪ Value invented strategies and representations ▪ More emphasis on the process instead of on the answer
	3. Construct viable arguments and critique the reasoning of others	▪ Ask questions ▪ Use examples and counter examples ▪ Reason inductively and make plausible arguments ▪ Use objects, drawings, diagrams, and actions ▪ Develop ideas about mathematics and support their reasoning ▪ Analyze others arguments ▪ Encourage the use of mathematics vocabulary	▪ Create a safe environment for risk-taking and critiquing with respect ▪ Provide complex, rigorous tasks that foster deep thinking ▪ Provide time for student discourse ▪ Plan effective questions and student grouping ▪ Probe students

	Mathematical Practices	Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	▪ Realize that mathematics (numbers and symbols) is used to solve/work out real-life situations ▪ Analyze relationships to draw conclusions ▪ Interpret mathematical results in context ▪ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable—if not, go back and look for more information ▪ Make sense of the mathematics	▪ Allowing time for the process to take place (model, make graphs, etc.) ▪ Modeling desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ▪ Making appropriate tools available ▪ Creating an emotionally safe environment where risk-taking is valued ▪ Providing meaningful, real-world, authentic, performance-based tasks (non-traditional work problems) ▪ Promoting discourse and investigations
	5. Use appropriate tools strategically	▪ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base ten blocks, compass, protractor) ▪ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ▪ Compare the efficiency of different tools ▪ Recognize the usefulness and limitations of different tools	▪ Maintaining knowledge of appropriate tools ▪ Modeling effectively the tools available, their benefits, and limitations ▪ Modeling a situation where the decision needs to be made as to which tool should be used ▪ Comparing/contrasting effectiveness of tools ▪ Making available and encouraging use of a variety of tools
Seeing Structure and Generalizing	**7. Look for and make use of structure**	▪ Look for, interpret, and identify patterns and structures ▪ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ▪ Reflect and recognize various structures in mathematics ▪ Breakdown complex problems into simpler, more manageable chunks ▪ "Step back" or shift perspective ▪ Value multiple perspectives	▪ Being quiet and structuring opportunities for students to think aloud ▪ Facilitating learning by using open-ended questions to assist students in exploration ▪ Selecting tasks that allow students to discern structures or patterns to make connections ▪ Allowing time for student discussion and processing in place of fixed rules or definitions ▪ Fostering persistence/stamina in problem solving ▪ Allowing time for students to practice
	8. Look for and express regularity in repeated reasoning	▪ Identify patterns and make generalizations ▪ Continually evaluate reasonableness of intermediate results ▪ Maintain oversight of the process ▪ Search for and identify and use shortcuts	▪ Providing rich and varied tasks that allow students to generalize relationships and methods and build on prior mathematical knowledge ▪ Providing adequate time for exploration ▪ Providing time for dialogue, reflection, and peer collaboration ▪ Asking deliberate questions that enable students to reflect on their own thinking ▪ Creating strategic and intentional check-in points during student work time

For classroom posters depicting the Mathematical Practices, please see: http://seancarberry.cmswiki.wikispaces.net/file/detail/12-20math.docx

Interpreting Functions (F.IF)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand the concept of a function and use function notation.

9-11.F.IF.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

DCAS-Like

1A

Which relation is a function?

A.

Input	Output
1	2
2	2
3	3
4	3

B.

Input	Output
2	6
2	5
6	4
6	3

C.

Input	Output
1	2
2	4
4	6
4	8

D.

Input	Output
0	1
0	2
1	3
1	4

Next-Generation

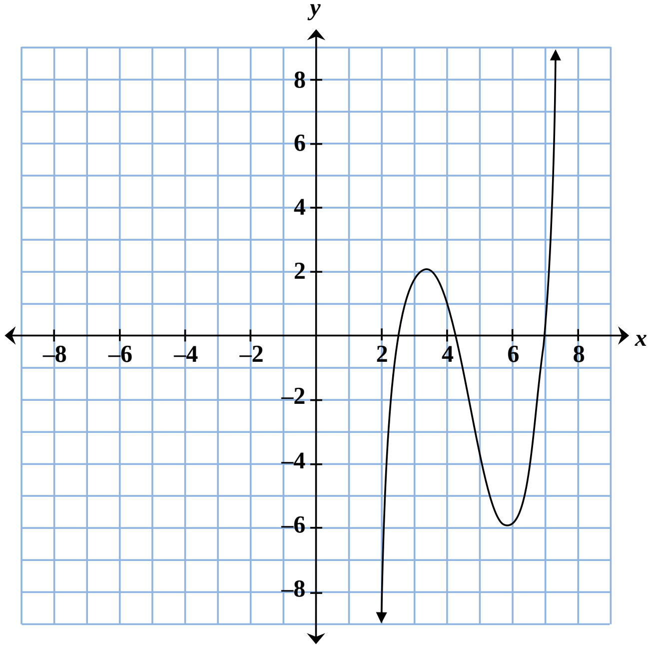
1B

For items a-d, determine whether each relation is a function.

a. $\{(0,1),(1,2),(3,1),(4,2)\}$ ○ Yes ○ No

b. $y = \pm\sqrt{4 - x^2}$ ○ Yes ○ No

c. ○ Yes ○ No



d. $\{(5,3),(2,4),(5,2)\}$ ○ Yes ○ No

9-11.F.IF.1 – Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

DCAS-Like

2A

What are the domain and range of the function $(x) = -|x - 3| + 2$?

A. Domain: all numbers less than or equal to 2. Range: all real numbers.

B. Domain: all numbers greater than or equal to 2. Range: all real numbers

C. Domain: all real numbers. Range: all numbers greater than or equal to 2.

D. Domain: all real numbers. Range: all numbers less than or equal to 2.

Next-Generation

2B

For items a. through f., determine the domain for each function. Move one or more inequalities into each box.

	Domain
a. $y = \frac{2}{x-3}$	
b. $y = \sqrt{x-5} + 1$	
c. $y = 4 - (x-3)^2$	
d. $y = \frac{7}{4-(x-3)^2}$	
e. $y = 4 - (x-3)^{\frac{1}{2}}$	
f. $y = \frac{7}{4-(x-3)^{\frac{1}{2}}}$	

$x \neq 3$	$x \neq 5$	$x > 3$	$x < 3$	$x \geq 3$	$x \leq 3$
$x > 5$	$x < 5$	$x \geq 5$	$x \leq 5$	$x = -1$	$x = 1$
$x = 19$	$x = -19$	$x =$ all real numbers			

9-11.F.IF.2 – Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

DCAS-Like

3A

If $g(x) = 3|x - 2| - x$, what is $g(0.5)$?

A. –5

B. –2

C. 1

D. 4

Next-Generation

3B

Pizza Extreme Deals!

In order to gain popularity among high school students, Pizza Extreme plans to offer a special promotion. If you show a student ID, the cost of a large pizza (in dollars) at Pizza Extreme as a function of time (measured in days since February 10th) may be described as:

$$C(t) = \begin{cases} 9, & 0 \le t \le 3 \\ 9 + t, & 3 < t \le 8 \\ 20, & 8 < t < 28 \end{cases}$$

(Assume t only takes whole number values)

a. If you want to give their pizza a try, on what date(s) should you buy a large pizza in order to get the best price?

____

b. Calculate $C(9) - C(8)$ and interpret its meaning in the context of the problem. Justify and explain your thinking.

9-11.F.IF.3 – Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

DCAS-Like

4A

Which function created the following sequence?

$3, 3, 7, 15, \ldots$

A. $f(n) = 3n^2 - 7n + 7$

B. $f(n) = n^2 - 3n + 6$

C. $f(n) = 3n^2 - 9n + 9$

D. $f(n) = 2n^2 - 6n + 7$

Next-Generation

4B

Match the sequence on the left to the correct function on the right.

Sequence	Function
a. $-12, -11.75, -11.5, -11.25 \ldots$	1. $f(n) = 256 \times 2^{(1-n)}$
b. $1, -2, 3, -4, 5, -6 \ldots$	2. $f(n) = -12.25 + 0.25n$
c. $256, 128, 64, 32, 16 \ldots$	3. $f(n) = n(-1)^{n-1}$

Cluster: Interpret functions that arise in applications in terms of the context.

9-11.F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

DCAS-Like

5A

Which function has a maximum value of 10?

A. $y = 10 + x^2$

B. $y = 10 - x^2$

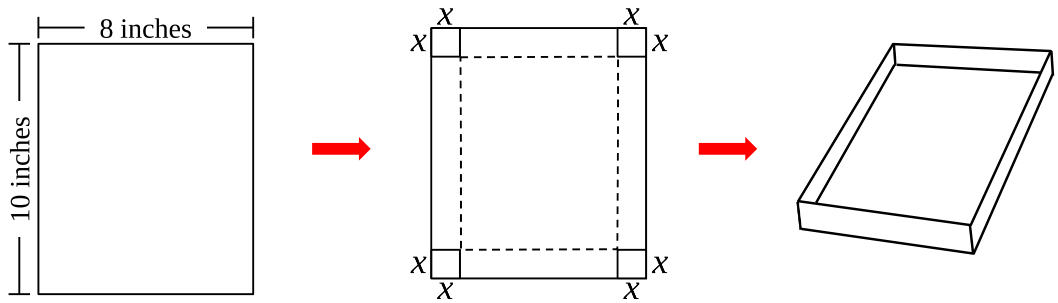
C. $y = x^2 + 10x$

D. $y = x^2 - 10x$

Next-Generation

5B

Use the diagram to answer the question.



By cutting four equal squares out of a piece of paper, folding on the dotted lines shown in the diagram, and taping the corners, an open rectangular box can be created.

Part A

Write an equation that relates the volume of the box (V) to the side length of the cut-out squares (x). Simplify your equation as much as possible by multiplying and combining like terms.

$V =$

Part B

What is the domain (x) and range (V) of your equation? Explain your reasoning or show your work in the spaced provided in the table.

Enter the domain and range as inequalities after the phrase in the answer space.

Domain:	
Range:	
Work:	

9-11.F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

DCAS-Like

6A

Which function has an x-intercept of 7?

A. $y = 7 + x^2$

B. $y = 7 - x^2$

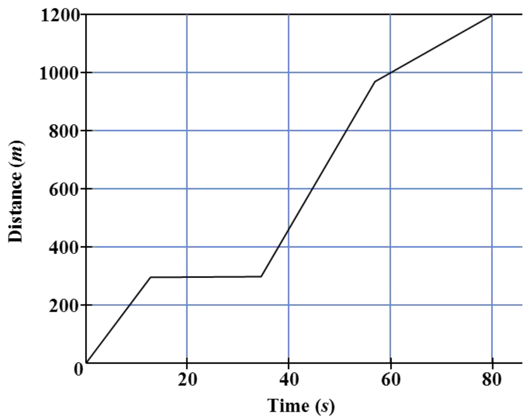
C. $y = \sqrt{7} - \sqrt{x}$

D. $y = \sqrt{7} + \sqrt{x}$

Next-Generation

6B

The graph below shows the distance a car traveled for 80 seconds.



a. Complete an input-output table to represent the function for every 10 seconds.

Input	Output
0	
10	
20	
30	
40	
50	
60	
70	
80	

b. Approximately how long was the car stationary? ______________________

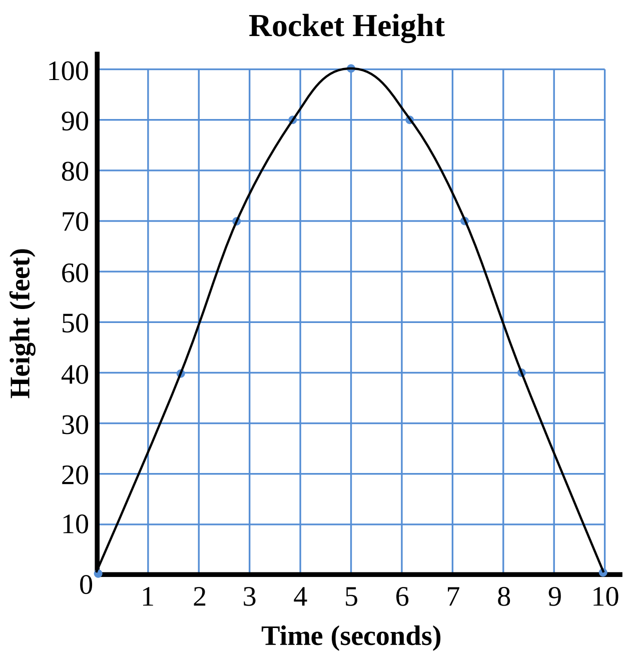
9-11.F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

DCAS-Like

7A

The graph shows the height of a rocket over 10 seconds.

Which situation is best represented by the graph?



A. Three seconds after take-off, the rocket was at a height of 70 feet. At 5 seconds, the rocket reached its greatest height of 110 feet. Ten seconds after take-off, it hit the ground.

B. Three seconds after take-off, the rocket was at a height of 75 feet. At 5 seconds, the rocket reached its greatest height of 100 feet. Ten seconds after take-off, it hit the ground.

C. Two seconds after take-off, the rocket was at a height of 75 feet. At 5 seconds, the rocket reached its greatest height of 100 feet. Five seconds after take-off, it hit the ground.

D. Once second after take-off, the rocket was at a height of 50 feet. At 5 seconds, the rocket reached its greatest height of 100 feet. Five seconds after take-off, it hit the ground.

7B

The Parks and Recreation Department is planning a tournament for club lacrosse teams in the area. Sixty-four teams have entered to play. Teams will be placed in a single elimination bracket at random. Four fields are available for tournament use. Games will only be played on Saturdays from 8 a.m. until 4 p.m. Games are 40 minutes in length. Ten minutes is allotted for half time and 10 minutes for teams to warm up. A team can plan only one round each Saturday.

Your job is to report how many fields are being used each Saturday during the tournament. The parks department also needs to know how much available field space they have each weekend for other activities.

a. The parks and recreation department has been asked to find field space for a soccer tournament coming to town. They will also only play on Saturday at the same time as the lacrosse tournament. If 24 soccer teams are entering the tournament, which weekend can they begin (assuming all games will be played on the same day)?

____

b. How many weekends is the field used for the lacrosse tournament?

____

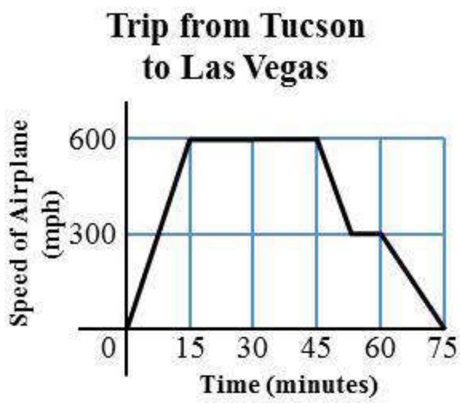
9-11.F.IF.4 – For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

DCAS-Like

8A

The graph shows the speed of an airplane during a trip from Tucson to Las Vegas.

Which situation is best represented by the graph?



A. An airplane took off and increased speed steadily for 15 minutes. For 30 minutes, the plane flew at 600 mph. The airplane then steadily decreased speed until it landed 20 minutes later.

B. An airplane took off and increased speed steadily for 15 minutes. For 30 minutes, the plane flew at 600 mph. The airplane began to slow down steadily until its speed was 300 mph. The airplane circled the airport for about 7 minutes, and then slowed down steadily until it landed 15 minutes later.

C. An airplane took off and increased speed steadily for 15 minutes. For 30 minutes, the plane flew at 600 mph. The airplane began to slow down steadily until the speed was 100 mph. The airplane circled the airport for about 10 minutes, and then slowed down steadily until it landed 15 minutes later.

D. An airplane took off and increased speed steadily for 15 minutes. For 45 minutes, the plane flew at 600 mph. The airplane began to slow down steadily until the speed was 300 mph. The airplane circled the airport for about 7 minutes, and then slowed down steadily until it landed 75 minutes later.

8B

Isabella owes a balance of $300 on her credit card. She has stopped making purchases with the card, and she plans to make a $40 payment each month until her debt is paid and her credit card balance is $0. The monthly rate is 1.5%, and interest is added each moth to the balance that remains.

Part A

Consider the spreadsheet. In a spreadsheet, each entry (cell) is referred to by its column letter and row number. For example, 260.00 is the entry in cell D2 of this spreadsheet.

	A	B	C	D	E
1	Month	Amount Owed ($)	Monthly Payment ($)	Remaining Amount Owed After Payment ($)	Amount Owed After 1.5% interest Charge ($)
2	1	300.00	40.00	260.00	263.90
3	2	263.90	40.00		

A3	B3	C3	D3	E3

0.015	1.015	×	÷	+	–

a. Drag the tiles to write a formula to find the value of cell D3.

$D3 =$ __________ __________ __________

b. Drag the tiles to write a formula to find the value of cell E3.

$E3 =$ __________ __________ __________

Part B

Fill in the blanks with values to correctly complete the spreadsheet. Use dollar amounts written as decimals rounded to the nearest cent.

	A	B	C	D	E
1	Month	Amount Owed ($)	Monthly Payment ($)	Remaining Amount Owed After Payment ($)	Amount Owed After 1.5% Interest Charge ($)
2	1	300.00	40.00	260.00	263.90
3	2	263.90	40.00		
4	3		40.00		

Part C

Fill in the blanks based on your calculations. Use dollar amounts written as decimals rounded to the nearest cent.

a. At the end of the sixth month, how much will Isabella still owe on the credit card? $__________

b. Isabella will finish paying off her credit card debt in __________ months.

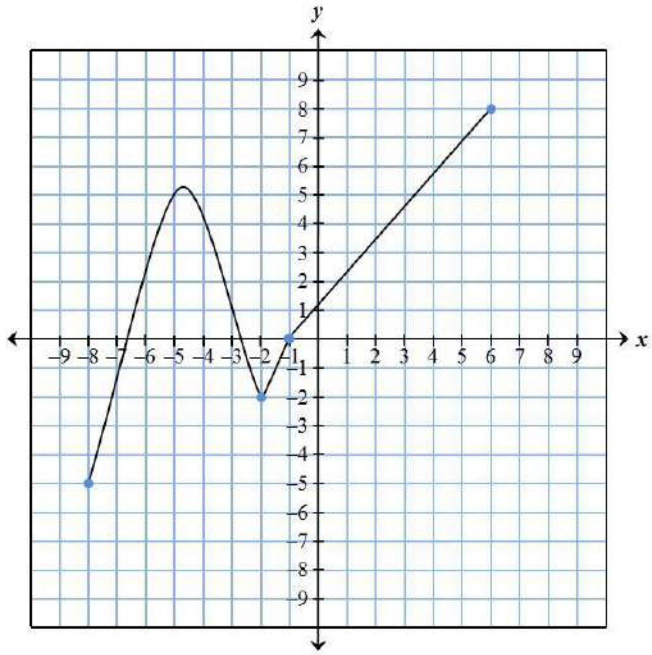
c. What is the amount of Isabella's last payment? $__________

9-11.F.IF.5 – Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

DCAS-Like

9A

Which expression represents the range of the function shown in the graph below?



A. $-8 \le x \le 6$

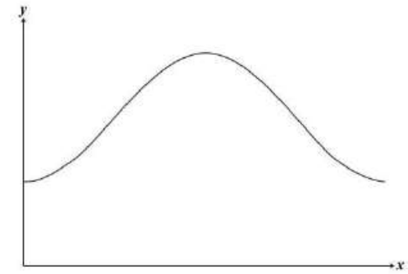
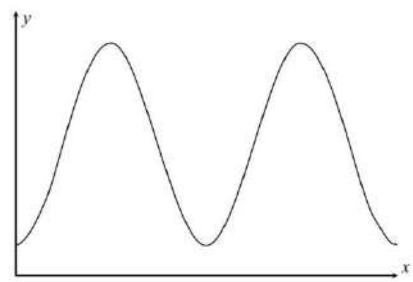
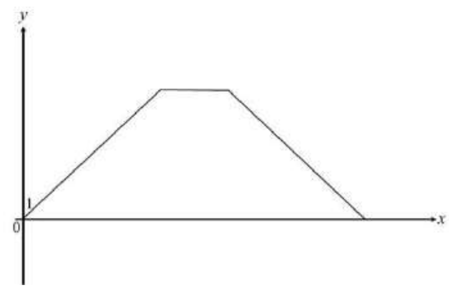
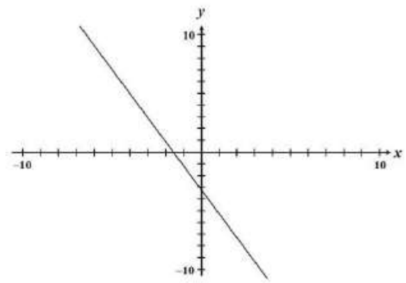
B. $-5 \le x \le 8$

C. $-8 \le y \le 6$

D. $-5 \le y \le 8$

9B

Match each graph to one feature of the function and one context description.

<table>
<tr><th>Context or Feature</th><th>Table/Context/Equation</th><th>Graph</th></tr>
<tr><td>a. This function has a maximum value of 80 and a minimum value of 10.</td><td>i. A continuous function, including the following points:
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>-5</td><td>-7</td><td>-9</td><td>-11</td></tr></table></td><td>I.
</td></tr>
<tr><td>b. This function increases, then remains constant, then decreases. The y-intercept is $(0, 0)$ and the range is from $[0, 10]$.</td><td>ii. The temperature in San Diego in one day.</td><td>II.
</td></tr>
<tr><td>c. The domain of this function is all reals. The slope, or rate of change, for this function is –2. This function has y-intercept at $(0, -3)$.</td><td>iii. This function represents the height off the ground of a rider on a Ferris wheel as they make two complete rotations on the ride.</td><td>III.
</td></tr>
<tr><td>d. The domain of this function is from $[0, 24]$. This function increases to its maximum value, then decreases to the same value as the y-intercept.</td><td>iv. This function represents distance versus time: Rashid walked to the store at a constant rate, bought groceries, and then walked home at the same constant rate.</td><td>IV.
</td></tr>
</table>

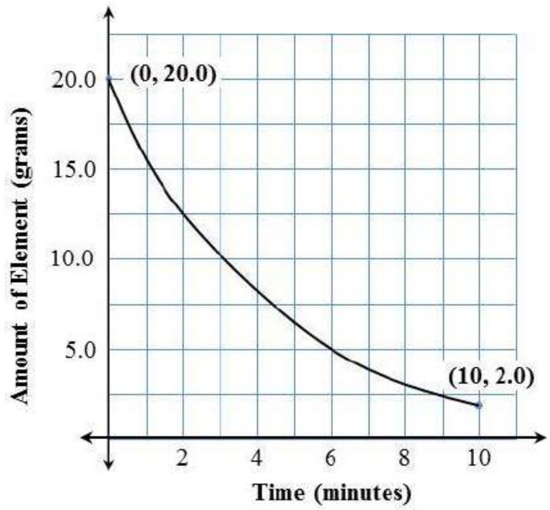
Context or Feature	Table/Context/Equation	Graph
a.		
b.		
c.		
d.		

9-11.F.IF.6 – Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

DCAS-Like

10A

The graph models the amount of a radioactive element present over the course of a 10-minute experiment.



What is the average rate of change of the amount of the element over the 10-minute experiment?

A. –0.2 g/min

B. –-1.8 g/min

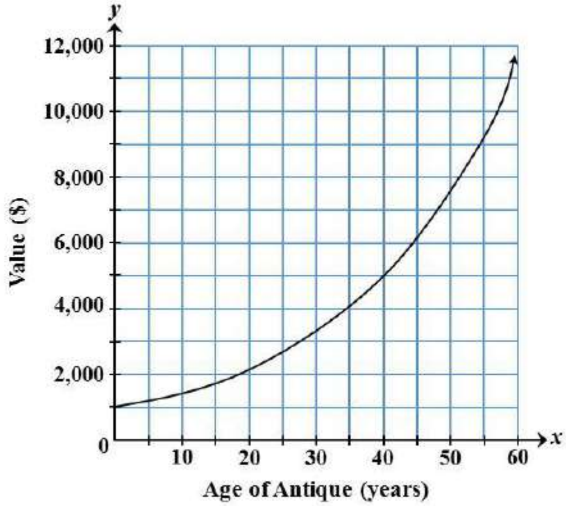
C. –2.0 g/min

D. –5.0 g/min

Next-Generation

10B

The value of an antique has increased exponentially, as shown in this graph.



Based on the graph, estimate to the nearest $50 the average rate of change in value of the antique for the following time intervals:

a. From 0 to 20 years: $________________

b. From 20 to 40 years: $________________

Cluster: Analyze functions using different representations.

9-11.F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

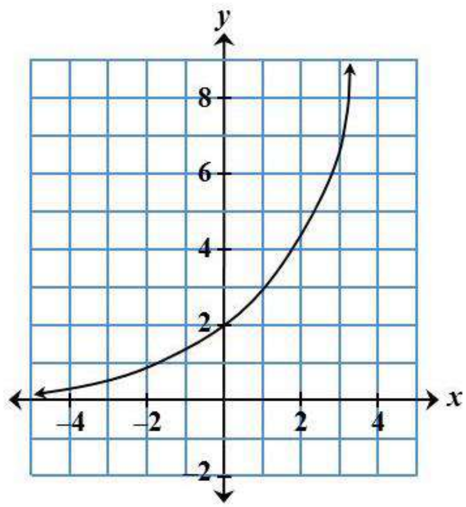
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

DCAS-Like

11A

The graph shows an exponential function.



What is the equation of the function?

A. $y = \left(\frac{2}{3}\right)^x$

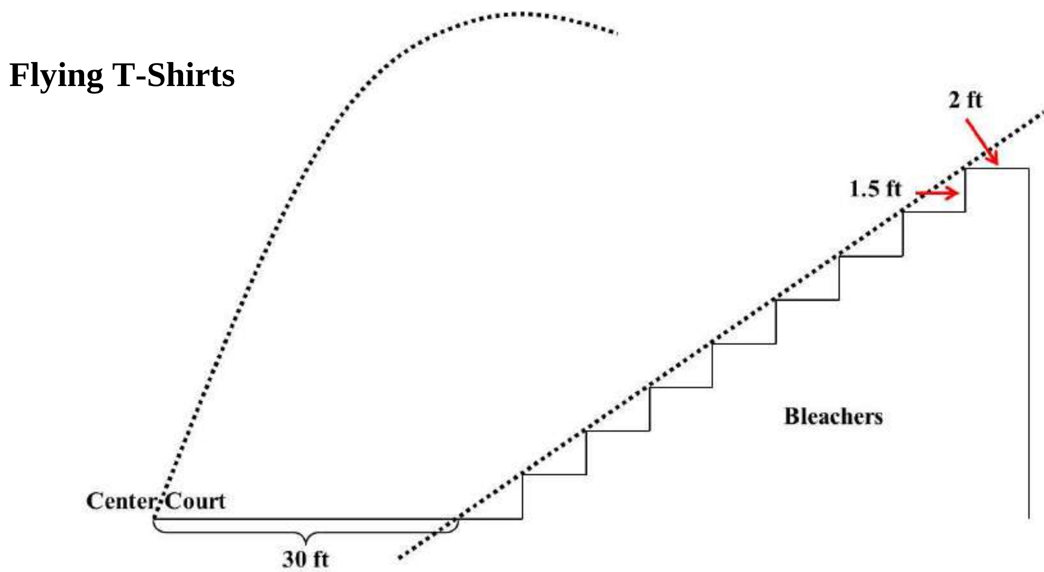
B. $y = 2(3)^x$

C. $y = 2\left(\frac{3}{2}\right)^x$

D. $y = 3\left(\frac{2}{3}\right)^x$

11B

At a varsity basketball game, free t-shirts are being thrown into the audience from a floor launcher. Eric wants to make sure his girlfriend catches the t-shirt, so they need to figure out which bleacher row she should sit in. Eric launches from center court. The t-shirt's travel path, according to the manufacturer's manual, is represented by the equation $y(x) = -0.05x^2 + 2.5x$, where y represents the vertical height in feet in terms of the horizontal distance traveled, x, in feet. The bleachers begin 32 feet from center court and each bleacher row has a height of 1.5 feet and a width of 2 feet as shown in the diagram. Use a line to represent the bleachers which touches the front edge of each row of seats and touches the floor 30 feet from center court as shown in the diagram. NOTE: the diagram is not drawn to scale.



a. Using this information, determine which row Eric's girlfriend should sit it. ______________

b. If the support structure for the ceiling of the gym is 40 feet above the floor, is there any danger that the t-shirt will strike part of the support structure?

9-11.F.IF.7 – Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

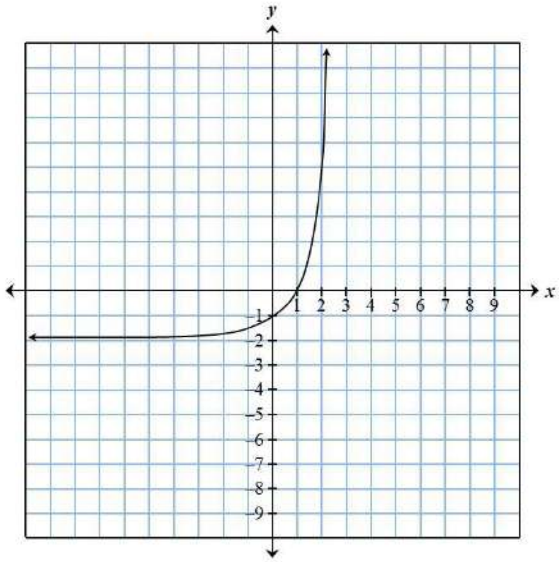
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

DCAS-Like

12A

Which function is represented by the graph below?



A. $y = e^x - 2$

B. $y = e^x + 2$

C. $y = 2 - e^x$

D. $y = -2 - e^x$

12B

Consider the following four functions:

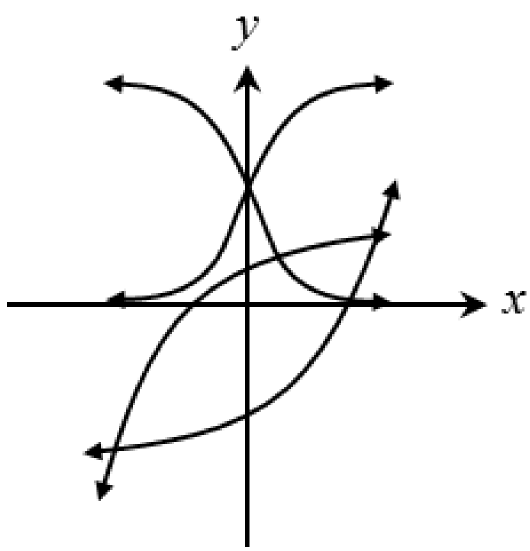
a. $f(x) = \frac{3}{1+e^{-3x}}$

b. $g(x) = 1 - \frac{e^{-x}}{2}$

c. $h(x) = -2 + \frac{e^{x}}{2}$

d. $k(x) = \frac{3}{1+e^{3x}}$

Below are four graphs of functions shown for $-2 \leq x \leq 2$. Match each function with its graph and explain your choice.



9-11.F.IF.8 – Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential.*

DCAS-Like

13A

The intercept form of a linear equation is shown below. It allows for a quick identification of the equation's intercepts. What is the intercept form of $y = 2x + 4$?

A. $-\frac{x}{4} + \frac{y}{2} = 1$

B. $-\frac{x}{2} + \frac{y}{4} = 1$

C. $\frac{x}{2} - \frac{y}{4} = 1$

D. $\frac{x}{4} - \frac{y}{2} = 1$

Next-Generation

13B

Write the function $y - 3 = \frac{2}{3}(x - 4)$ in the equivalent form **most** appropriate for identifying the slope and y-intercept of the function. What are the slope and y-intercept?

$y =$ ______________________

Slope = ______________________

y-intercept = __________________

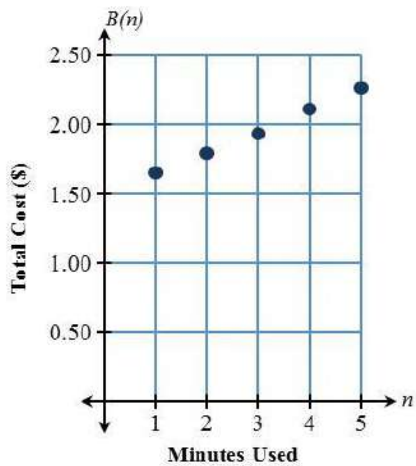
9-11.F.IF.9 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

DCAS-Like

14A

A phone call using a prepaid card consists of a fixed fee to place the call plus an additional fee for each minute of the call. The cost of an n-minute phone call with a card from Company A is $A(n) = \$0.99 + \$0.25n$, where n is a positive integer.

The cost of an n-minute phone call with a card from Company B is shown in the graph below.



Which statement below must be true?

A. The per minute fee for Company B is greater than Company A.

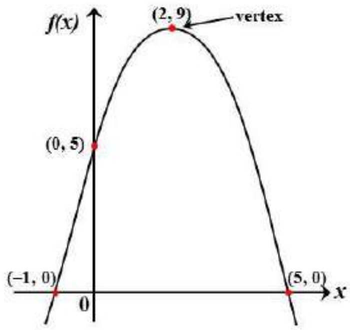
B. The fixed fee for Company B is greater than Company A.

C. A call using Company B will always cost more than the same length call using Company A.

D. A two-minute call with Company B is less than Company A.

14B

A portion of the graph of a quadratic function $f(x)$ is shown in the xy-plane. Selected values of a linear function $g(x)$ are shown in the table.



x	$g(x)$
–4	7
–1	1
2	–5
5	–11

For each comparison below, use the dropdown menu to select a symbol that correctly indicates the relationship between the first and the second quantity.

Dropdown Menu
$=$
$<$
$\leq$
$>$
$\geq$

First Quantity	Comparison	Second Quantity
The y-coordinate of the y-intercept $f(x)$		The y-coordinate of the y-intercept $g(x)$
$f(3)$		$g(3)$
Maximum value of $f(x)$ on the interval $-5 \leq x \leq 5$		Maximum value of $g(x)$ on the interval $-5 \leq x \leq 5$
$\dfrac{f(5) - f(2)}{5 - 2}$		$\dfrac{g(5) - g(2)}{5 - 2}$

Building Functions (F.BF)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Build a function that models a relationship between two quantities.

9-11.F.BF.1 – Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

DCAS-Like

15A

To fix a clogged pipe, Dripmaster Plumbing charges a one-time fee of \$75 plus \$40 per hour. NoClog Plumbers charges a one-time fee of \$50 plus \$70 per hour for the same service. Which function shows the difference in charges between the two companies for a repair taking h hours?

A. $\text{difference} = \$20 - \$35h$

B. $\text{difference} = \$25 - \$30h$

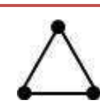
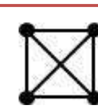
C. $\text{difference} = \$25 - \$110h$

D. $\text{difference} = \$30 - \$25h$

Next-Generation

15B

The following chart indicates the maximum number of connecting line segments, y, that can be drawn connecting x points, where no three points lie on the same line.

Number of Points (x)	1	2	3	4	5
Maximum Number of Connecting Line Segments (y)	0	1	3	6	10
Example					

a. The relationship between x and y is represented by the equation $y = kx(x - 1)$ for any positive number of points x. Use the information in the table to determine the value of the real number k.

$k =$ ____________________________

b. Use the equation from part a. to determine the maximum number of line segments that can be drawn connecting 100 points, no three of which lie on the same line.

____

9-11.F.BF.1 – Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

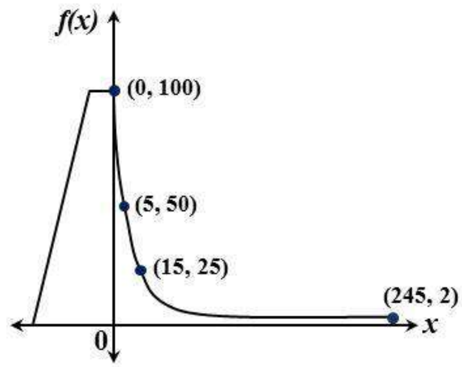
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

DCAS-Like

16A

At the top of the water slide, Jessica sits 100 feet above the ground. She begins her descent and quickly drops to a height of 50 feet while moving only 5 feet forward. She drops to a height of 25 feet upon travelling 15 feet forward, eventually coming to rest 2 feet above the ground at the end of the 245-foot-long slide.

Which function models Jessica's entire descent down the water slide?



A. $f(x) = 100 - 10x$

B. $f(x) = \frac{500}{x+5}$

C. $f(x) = \frac{2}{5}x^2 - 12x + 100$

D. $f(x) = \frac{265-x}{10}$

16B

A rabbit population can increase at a rapid rate if left unchecked. Assume that 10 rabbits are put in an enclosed wildlife ranch and the rabbit population triples each year for the next 5 years, as shown in the table.

Year	Rabbit Population
0	10
1	30
2	90
3	270
4	810
5	2430

a. Let y represent the number of rabbits after x years. Drag the tiles to the appropriate slots to build a function rule that could be used to model y as a function of x, where x is a non-negative integer.

$3x$	3^x	x^3	x	0	3	10	30	•	+

$y =$ __________ __________ __________

New Population: A group of rabbits of a different kind is placed in a second enclosed wildlife ranch. This new population of rabbits doubles each year if left unchecked.

b. Which of the following statements must be true about the model for the new rabbit population compared to the model you developed for the original rabbit population? Select all that apply.

- ☐ The base of the exponent will change from 3 to 2.
- ☐ The function rule will be quadratic.
- ☐ The coefficient will become 2.
- ☐ As the number of years increases, the graph of this model will be less steep than the graph of the original model.
- ☐ The y-intercept of the graph will be different.
- ☐ As the number of years increases, the graph of this model will be steeper than the graph of the original model.

c. Compare the two rabbit population models. How many rabbits would you need to start within the new rabbit population to have at least the same number of rabbits as in the original model after 5 years? Clearly explain how you found your answer.

9-11.F.BF.1 – Write a function that describes a relationship between two quantities.*
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

DCAS-Like

17A

Every day when commuting to and from work, Jay drives his car a total of 45 miles. His car already has 2,700 miles on it.

Which function shows the total number of miles Jay's car will have been driven after n more days?

A. $d(n) = 60$

B. $d(n) = 60n$

C. $d(n) = 45 + 2{,}700$

D. $d(n) = 2{,}700 + 45n$

Next-Generation

17B

Tom is doing an experiment adding golf balls to a glass jar containing water. The picture and the table show what happens to the height of the water as Tom adds golf balls.

Number of Golf Balls, x	Height of Water in Centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8

a. Drag tiles to complete the sentences and the equation below based on the results of Tom's experiment.

golf balls	change	glass jars	water height

1.16	1.2	1.3	9.0	12.0	13.8

The height of the water changes at an average rate of about ___________ centimeters per golf ball. If these data were graphed with the number of golf balls as the independent variable, the y-intercept for the graph would be about __________ centimeters. This means that for zero __________ , the __________ is 9 centimeters.

Tom's table and graph can be represented by the trend line with the equation

$$y = \_\_\_\_\_\_\_\_\_\_ x + \_\_\_\_\_\_\_\_\_\_ .$$

b. There are several ways that Tom could modify the conditions of his experiment.

What modifications would increase the rate of change in the height of the water level with respect to the number of golf balls? Select all that apply.

- ☐ Use larger golf balls
- ☐ Add 5 cm of water to the glass jar
- ☐ Decrease the diameter of the glass jar
- ☐ Drop the golf balls into the glass jar two at a time
- ☐ Drop the golf balls into the jar at a faster rate

c. Tom forgot to write down the initial height of the water in glass jar B, but he measured the water height at 9 centimeters after adding two golf balls.

Question 1: When Tom creates graphs of the data from both experiments, how will the y-intercepts of the graphs be different for glass jar A versus glass jar B? Explain how you know.

Question 2: How will the rate of change in the experiment using glass jar B be different than the rate of change in the experiment using glass jar A? Explain how you know.

Question 3: Suppose glass jar B has a water height of 5 centimeters with no golf balls, and the water height increases at a rate of 2 centimeters per golf ball added. Tom continues to add golf balls to each glass jar. He discovers that there are a number of golf balls at which the height of the water in each glass jar is the same. How many golf balls will be in each jar when the water in each reaches the same height?

9-11.F.BF.1 – Write a function that describes a relationship between two quantities.*

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

DCAS-Like

18A

A sequence t is defined as $t(n) = 0.57 - 0.06n$, where $n \geq 1$. Which is an equivalent recursive definition for sequence t?

A. $t(n) = 0.57;\ \ t(n+1) = t(n) - 0.06,$ for $n \geq 1$

B. $t(n) = 0.51;\ \ t(n+1) = t(n) - 0.06,$ for $n \geq 1$

C. $t(n) = 0.57;\ \ t(n+1) = t(n) - 0.51,$ for $n \geq 1$

D. $t(n) = 0.51;\ \ t(n+1) = t(n) - 0.51,$ for $n \geq 1$

Next-Generation

18B

A new social networking website was made available. The website had 10 members its first week. Beginning the second week, the creators of the website have a goal to triple the number of members every week.

For Part A and Part B below, select the appropriate expression.

0	1	3	7	10
$3n + 7$	$3n + 10$	$30(n-1)$	$10(3^{n-1})$	$3(10^{n-1})$
$f(n-1) + 2$	$f(n-1) + 30$	$3f(n-1)$	$3f(n-1) + 10$	$f(3n-1)$

Part A

Determine an explicit formula for $f(n)$, the number of members the creators have a goal of getting n weeks after the website is made available.

$f(n) =$ ____________________

Part B

Determine a recursive formula for $f(n)$.

$f(n) =$ ____________________ for $n >$ ____________________

$f(1) =$ ____________________

9-11.F.BF.1 – Write a function that describes a relationship between two quantities.*

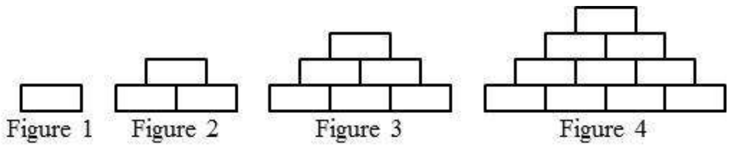
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

DCAS-Like

19A

Observe the pattern below.



Which expression represents the number of rectangles in the n^{th} figure?

A. $2n - 1$

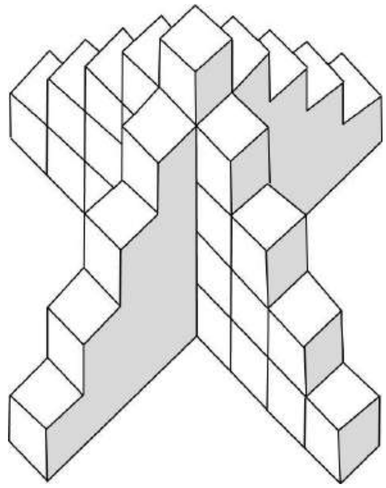
B. $2n^2 - 1$

C. $\frac{n^2+1}{2}$

D. $\frac{n(n+1)}{2}$

19B

Use this diagram to answer the following question:



The tower is 5 cubes high.

a. How many cubes are needed to build this tower? ____________________

b. How many cubes are needed to build a tower like this that is 12 cubes high?

____

c. Create a function to calculate the number of cubes needed for a tower n cubes high.

____

9-11.F.BF.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

DCAS-Like

20A

Which function will generate the n^{th} term of the sequence $-\frac{1}{2}, 1, \frac{7}{2}, 7, \ldots$

A. $f(n) = \frac{1-2n^2}{2}$

B. $f(n) = \frac{n^2-2}{2}$

C. $f(n) = \frac{n-2}{2}$

D. $f(n) = \frac{n-3}{4}$

Next-Generation

20B

The first four terms of a sequence are shown below.

$$8, 12, 18, 27, \ldots$$

Write a recursive function for this sequence.

9-11.F.BF.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

DCAS-Like

21A

To calculate her target heart rate, Shayla subtracts her age, a, from 220 and then takes 70% of the difference. Which equation should Shayla use to calculate r, her target heart rate?

A. $r = 0.7(a - 220)$

B. $r = 0.7(220 - a)$

C. $r = 0.7(220) - a$

D. $r = 220 - 0.7(a)$

Next-Generation

21B

A company purchases $24,500 of new computer equipment. For tax purposes, the company estimates that the equipment decreases in value by the same amount each year. After 3 years, the estimated value is $9,800.

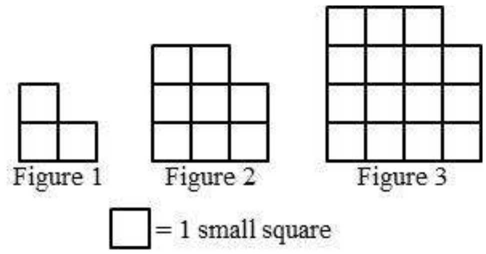
Write an explicit function that gives the estimated value of the computer equipment n years after purchase.

9-11.F.BF.2 – Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

DCAS-Like

22A

The first 3 figures in a pattern are shown.



Which function represents $f(n)$, the number of small squares in figure n?

A. $f(n) = n^2 - 1$

B. $f(n) = 2n^2 + 1$

C. $f(n) = (n + 1)^2 + 1$

D. $f(n) = (n + 1)^2 - 1$

Next-Generation

22B

Sequence I: $3, 5, 9, 17, 33, \ldots$

Sequence I is an increasing sequence. Each term in the sequence is greater than the previous term.

a. If this same pattern of differences continues for the terms in Sequence I, what are the 6th, 7th, and 20th terms?

6th term – ______________

7th term – ______________

20th term – ______________

b. Write an algebraic expression (rule) that can be used to determine the n^{th} term of Sequence II, which is the difference between the $(n + 1)^{\text{st}}$ term and the n^{th} term of Sequence I.

Cluster: Build new functions from existing functions.

9-11.F.BF.3 – Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

DCAS-Like

23A

Which of the following most accurately describes the translation of the graph $y = -2(x-6)^2 - 1$ to the graph of $y = -2(x-4)^2$?

A. Up 1 and 2 to the right

B. Up 1 and 2 to the left

C. Down 1 and 2 to the right

D. Down 1 and 2 to the left

Next-Generation

23B

Use the equation to answer the question.

$y = f(x + A) + B$

A function $f(x)$ can be useful for mathematical modeling once the type of function is identified. The function $f(x)$ can be generalized by adding parameters. The equation shown can be used for different models because the parameters can be adjusted to fit the available data.

Describe how each parameter (A and B) affects the graph of the function $f(x) = x^4$. Include specific information about how positive and negative values affect the graph for each parameter in your answer.

Parameter A

Parameter B

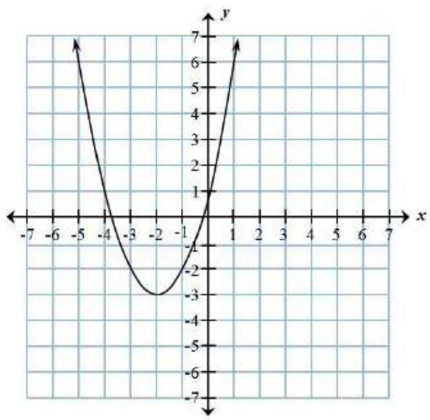
9-11.F.BF.3 – Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

DCAS-Like

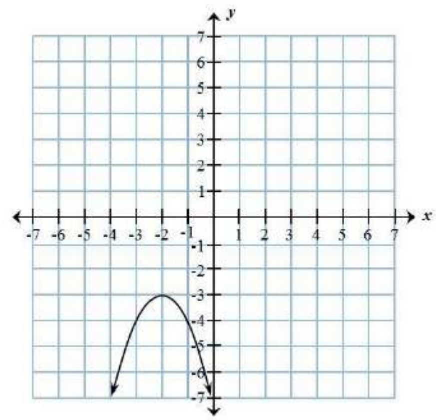
24A

Which graph best represents $y = (x + 2)^2 - 3$?

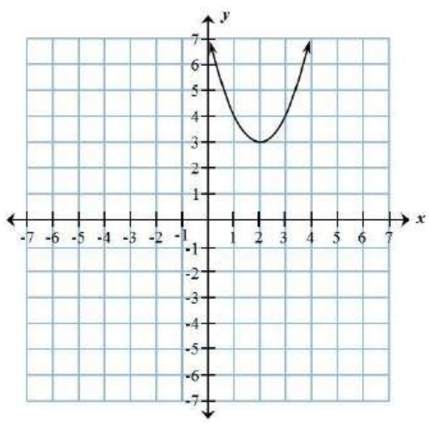
A.



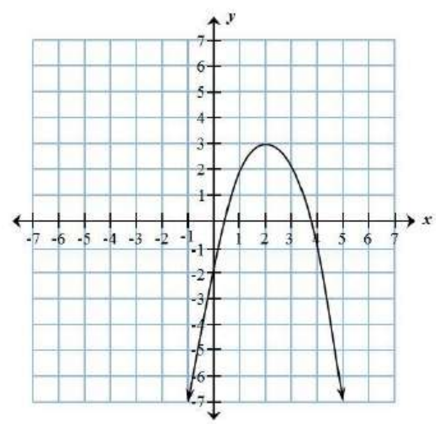
C.



B.

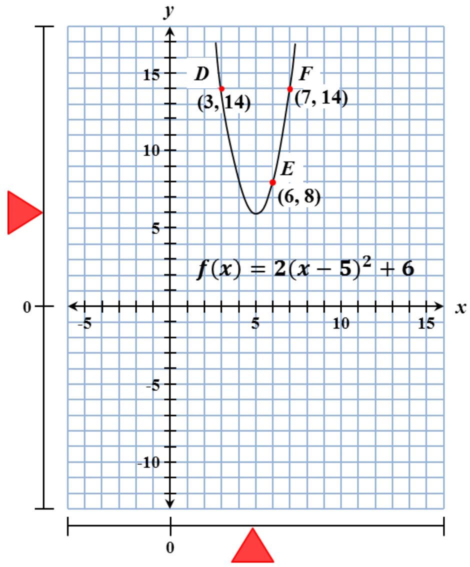


D.



24B

a. The graph of the quadratic function $f(x) = 2(x-5)^2 + 6$ is shown.



A new function, $p(x)$, is created from the existing function, such that $p(x) = -f(x)$. You may use the coordinate plane and the sliders to show the graph of the new function if you would like. The graph will not be scored.

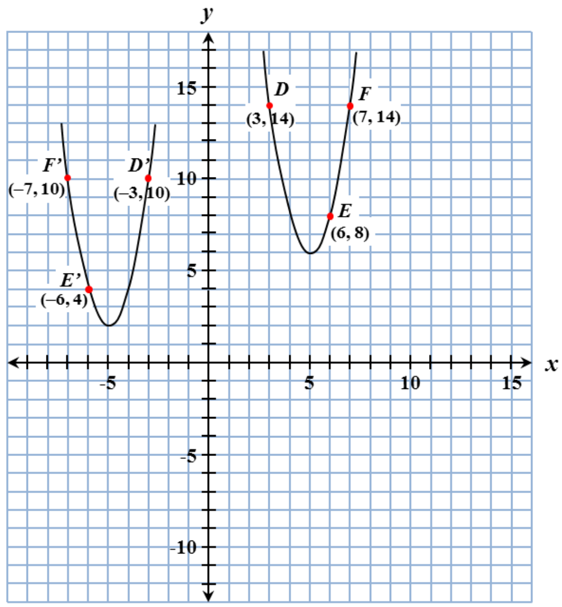
Fill in the blanks to give the coordinates of *D'*, *E'*, and *F'* that lie on the graph of the new function $p(x)$ and that are the images of *D*, *E*, and *F* that lie on graph of $f(x)$.

D' (____, ____)

E' (____, ____)

F' (____, ____)

b. The graph of the quadratic function $f(x) = 2(x-5)^2 + 6$ is shown.



The graph of a new function, $g(x)$, is obtained by applying a congruence transformation to the graph of $f(x)$, which takes the points $D, E,$ and F to the points $D', E',$ and F', respectively.

1. Describe a sequence of congruence transformations that gives the graph of the new function $g(x)$.
2. Write an equation for $g(x)$.
3. Compare your equation for $g(x)$ to the equation of the original function, $f(x)$. How do the differences in the equations reveal the transformations you described in question 1.

9-11.F.BF.4 – Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ *for* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

DCAS-Like

25A

Which function is the inverse of $f(x) = x^3 + 6$?

A. $f^{-1}(x) = x^3 + 6$

B. $f^{-1}(x) = \sqrt[3]{x} + 6$

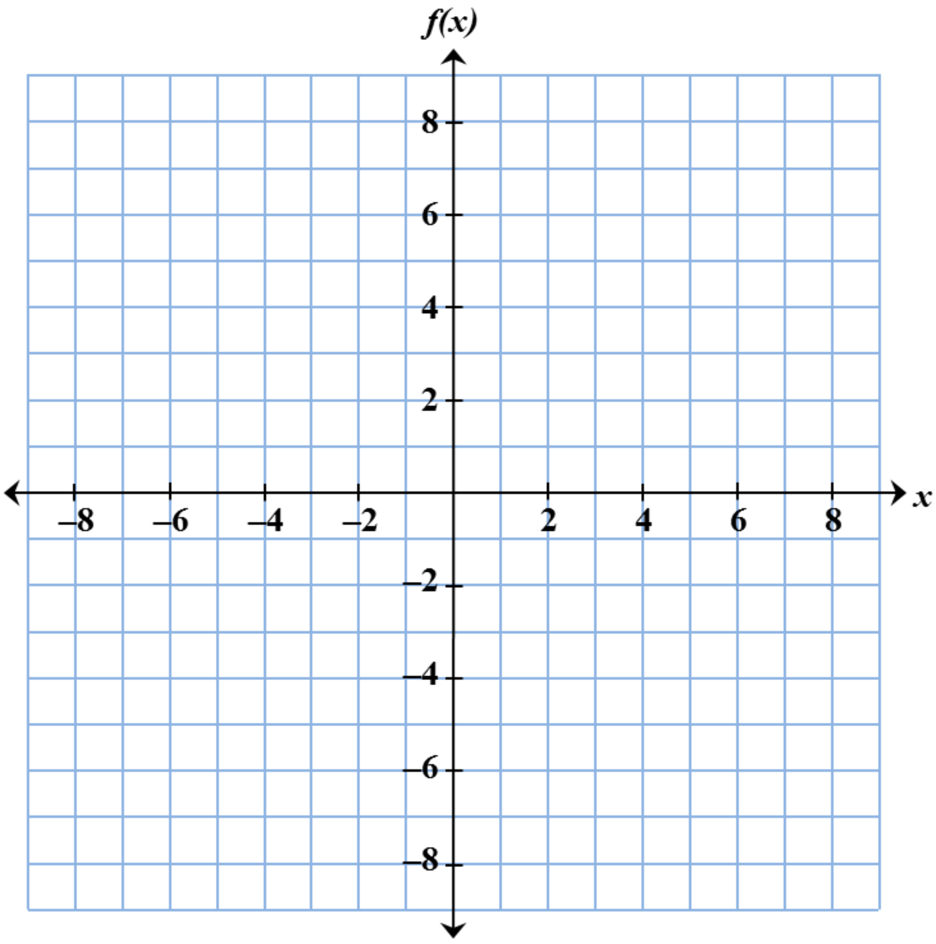
C. $f^{-1}(x) = \sqrt[3]{x} - 6$

D. $f^{-1}(x) = \sqrt[3]{x - 6}$

Next-Generation

25B

Draw the graph of the inverse of $f(x) = -\frac{3}{2}x - 3$ on the coordinate grid below.



Linear, Quadratic, and Exponential Models* (F.LE)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Construct and compare linear, quadratic, and exponential models and solve problems.

9-11.F.LE.1 – Distinguish between situations that can be modeled with linear functions and with exponential functions.*

a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.*

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.*

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.*

DCAS-Like

26A

Christy and Derron set goals for improving their recorded times for running a mile. Which statement best describes these goals?

Christy: Complete each new run in 5 fewer seconds than the previously recorded run.

Derron: Complete each new run in 5% less time than the previously recorded run.

A. Christy's goal can be modeled with an exponential function, while Derron's goal can be modeled with a linear function.

B. Christy's goal can be modeled with a linear function, while Derron's goal can be modeled with an exponential function.

C. Both goals can be modeled with exponential functions.

D. Both goals can be modeled with linear functions.

Next-Generation

26B

The following tables show the values of linear, quadratic, and exponential functions at various values of x. Indicate which function type corresponds to each table. Justify your choice.

A		B		C		D	
x	$f(x)$	$g(x)$	x	x	$h(x)$	x	$m(x)$
1	6	7	1	1	6	1	56
2	9	14	2	2	9	2	28
3	12	28	3	3	14	3	14
4	15	56	4	4	21	4	7

Table A: $f(x) =$ ____________________

Table B: $g(x) =$ ____________________

Table C: $h(x) =$ ____________________

Table D: $m(x) =$ ____________________

9-11.F.LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

DCAS-Like

27A

It takes 28 minutes for a certain bacteria population to double. If there are 5,241,763 bacteria in this population at 1:00 p.m., which of the following is closest to the number of bacteria in millions at 2:30 p.m. on the same day?

A. 80

B. 40

C. 20

D. 15

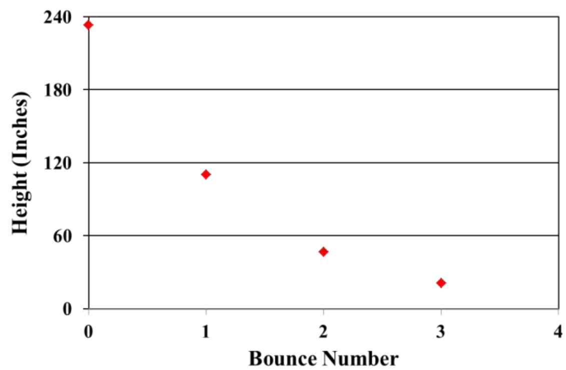
Next-Generation

27B

In science class, some students dropped a basketball and allowed it to bounce. They measured and recorded the highest point of each bounce.

The students' data is shown in the table and scatterplot. The first data point (n) represents the height of the ball the moment the students dropped it.

Bounce Number $\boldsymbol{n}$	**Measured Height (Inches)** $\boldsymbol{h(n)}$
0	233
1	110
2	46.6
3	21



In this task you will choose a function to model the data and use the model to answer some questions.

a. Compute the first three values in the last column of the table below.

Bounce Number $\boldsymbol{n}$	**Measured Height (Inches)** $\boldsymbol{h(n)}$	**Factor by Which Bounce Height Decreased** $\boldsymbol{h(n) \div h(n-1)}$
0	233	
1	110	
2	46.6	
3	21	

b. Let n be the bounce number and $h(n)$ be the height. Consider the following general forms for different kinds of models where a and b represent numbers:

Model 1	Model 2	Model 3	Model 4
$\boldsymbol{h(n) = a \bullet n + b}$	$h(n) = a \bullet n^2 + b$	$h(n) = \frac{a}{n} + b$	$h(n) = a \bullet e^{bn}$

Which of the models shown is most appropriate to use for the given data? ________________

Given the data above, what are reasonable values for a and b if we want to create a specific model to fit the data? Write the appropriate values in the equation below.

(Based on the choice students make above, they are given the appropriate template below. Here is the template for the exponential model.)

$$h(n) = \boxed{}\, e^{\boxed{}\, n}$$

c. Mika said,

> The model I came up with is $h(n) = 233 \bullet e^{-0.8n}$. I used it to predict that after 50 bounces, the height of the bounces will be less than a thousandth of an inch. It is good to have the model because it would be very difficult to measure such small heights.

What is the best way to characterize Mika's claim?

1. Mika's claim is true. The whole point of using models is to make predictions.
2. Mika's claim is true, but she should give a more precise bound for the height of the ball after 50 bounces because the heights will be much, much smaller than one thousandth of an inch.
3. Mika is correct that the model predicts that the bounces will all be less than a thousandth of an inch, but in reality the ball will be at rest before it has bounced 50 times.
4. Mika is not using the model appropriately. Models cannot be used to make predictions past the given data, only between data points.
5. Mika is not using the model appropriately. The model does not fit the data very well, so it cannot be used to make predictions that far in the future.
6. Mika's claim is not true. The model states that the ball will be at rest before it gets to 50 bounces, so the bounce heights will be zero, which is easy to measure.

9-11.F.LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

DCAS-Like

28A

Which rule applies to the table below?

x	–2	–1	0	1	2
y	81	27	9	3	1

A. $y = 9 \bullet 3^x$

B. $y = 3 \bullet 9^x$

C. $y = 3 \bullet \left(\frac{1}{9}\right)^x$

D. $y = 9 \bullet \left(\frac{1}{3}\right)^x$

Next-Generation

28B

a. In a cellular regeneration experiment, Jaydon Laboratory found that, for cells put in containers with a particular growth medium, the number of cells at the end of each week was double the number of cells at the end of the previous week.

The data for the first 6 weeks of the experiment are shown in the table. Fill in the blanks to complete the table for weeks 7–10.

Week	Number of Cells in Medium
1	15
2	30
3	60
4	120
5	240
6	480
7	
8	
9	
10	

b. Assume that as the experiment continues, the number of cells at the end of each week continues to be double the number of cells at the end of the previous week. Let w_n represent the number of cells in the growth medium in week n.

Drag the tiles to write a recursive definition for the sequence that represents the number of cells in the growth medium at the end of each week.

Week	Number of Cells in Medium
1	15
2	30
3	60
4	120
5	240
6	480

an integer	**a real number**	**2**	**15**	**30**
$+$	$-$	$\bullet$	$\div$	$\boldsymbol{w_{n-1}}$
$\boldsymbol{n \geq 0}$	$\boldsymbol{n \geq 1}$	$\boldsymbol{n \geq 2}$	$\boldsymbol{n \geq 6}$	$\boldsymbol{n \geq 15}$

$w_1 =$ ____________________

$w_n =$ ____________________, where n is ____________________, such that

____

c. Let w_n represent the number of cells in the growth medium at the end of week n. Which of these statements are true about the explicit formula for w_n?

Select all that apply.

- ☐ $w_n = 15 + 15 \bullet 2(n-1)$
- ☐ $w_n = 15 + 15 \bullet 2(n)$
- ☐ $w_n = 15 \bullet 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \bullet 15 \bullet 2^{n-1}$
- ☐ $w_n = \frac{1}{2} \bullet 15 \bullet 2^{n}$
- ☐ $n \geq 1$, where n is an integer
- ☐ $n \geq 1$, where n is a real number
- ☐ n can be any real number

d. Consider the table of data about the cellular regeneration experiment.

1. If the number of cells continues to grow according to the pattern shown in the table, at what week number will the number of cells exceed one billion?
2. Explain how the process you used to find the week number relates to either the recursive model or the explicit model you constructed in the previous questions.

9-11.F.LE.2 – Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*

DCAS-Like

29A

The cost to fill a car's tank with gas and get a car wash is a linear function of the capacity of the tank. The costs of a fill-up and a car wash for three different customers are shown in the table. Write an equation for the function in slope-intercept form. Then, find the cost of a fill-up and a car wash for a customer with a truck whose tank size is 22 gallons.

Tank Size (gal) (x)	Total Cost ($) $f(x)$
11	21.45
15	28.25
17	31.65

A. $f(x) = 1.50x + 3.00$; Cost for truck = $36.00

B. $f(x) = 1.70x + 2.75$; Cost for truck = $40.15

C. $f(x) = 0.59x + 1.62$; Cost for truck = $14.60

D. $f(x) = 1.60x + 2.25$; Cost for truck = $27.45

Next-Generation

29B

You are an engineer hired by the state parks service to analyze a trail that runs along an eroding cliff. The parks service wants to know how soon the edge of the cliff will erode to within 5 meters of the trail.

A geologist collected the data shown in the table from careful measurements of satellite photos.

Year	Distance from Trail (meters)
1985	20.50
1990	19.75
1995	19.00
2000	18.25
2005	17.50

Part A

Use the geologist's data to develop an equation that models this situation. Use D for distance and T for time.

Part B

In 2012, a careful measurement shows the cliff edge is now 16.40 meters from the trail.

Write a brief report to the parks service explaining:

a. The meaning of your equation in Part A in terms of the situation;

b. Your prediction of the year the cliff edge will reach a distance of 5 meters from the trail;

c. How you determined the date in your prediction; and

d. How reliable you think your prediction is and why.

9-11.F.LE.3 – Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*

DCAS-Like

30A

Which statement best describes the growth of the functions $f(x) = 4^x$ and $g(x) = x^4$ after $x = 4$?

A. $f(x)$ grows faster than $g(x)$.

B. $g(x)$ grows faster than $f(x)$.

C. Both $f(x)$ and $g(x)$ grow at the same rate.

D. $f(x)$ grows faster than $g(x)$ until $x = 8$, where g(x) begins to grow faster than $f(x)$.

Next-Generation

30B

Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

Choice 1: Two dollars for each bag of leaves.

Choice 2: She will be paid for the number of bags of leaves she rakes as follows: two cents for one bag, four cents for two bags, eight cents for three bags, and so on with the amount doubling for each additional bag.

a. If Celia rakes five bags of leaves, should she opt for payment method 1 or 2? ____________

b. What if she rakes 15 bags of leaves? ____________

c. How many bags of leaves does Celia have to rake before method 2 pays more than method 1? ____________

9-11.F.LE.4 – For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*

DCAS-Like

31A

Bacteria in a culture are growing exponentially with time, as shown in the table below.

Day	Bacteria
0	100
1	200
2	400

Which of the following equations expresses the number of bacteria, y, present at any time, t?

A. $y = 100 + 2^t$

B. $y = (100) \bullet (2)^t$

C. $y = 2^t$

D. $y = (200) \bullet (2)^t$

Next-Generation

31B

A biologist is culturing a new strain of bacteria, starting with just a few cells. When she first observes the bacteria colony, the population is 1,000 cells and is doubling every 20 minutes. This rate of growth continues for the rest of the day.

Part A

Write a function $P(t)$ that represents the population of the bacteria colony as a function of time in minutes.

$P(t) =$

Part B

How long will it take the population to reach 8,000?

Part C

How long will it take the population to reach 20,000?

Cluster: Interpret expressions for functions in terms of the situation they model.

9-11.F.LE.5 – Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.*

DCAS-Like

32A

The number of bacteria present in a laboratory sample after t days can be represented by $500(2^t)$. What is the initial number of bacteria present in this sample?

A. 250

B. 500

C. 750

D. 1000

Next-Generation

32B

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x) = 5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

a. Exactly how many fish did the fisherman release into the lake?

b. Find b if you know the lake contains 33 fish after eight weeks. Show step-by-step work.

c. Instead, now suppose the $P(x) = 5b^x$ and $b = 2$. What is the weekly percent growth rate in this case? What does this mean in everyday language?

9-11.F.LE.5 – Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.*

DCAS-Like

33A

Population growth of a country is modeled by the function below, where t is time in years. Based on the model, which is true about the country?

$$P = 10^7 \bullet 1.04^t$$

a. Since reaching 10 million people, the population was growing by 0.04% each year.

b. Since reaching 10 million people, the population was growing by 4% each year.

c. Since reaching 100 million people, the population was growing by 0.04% each year.

d. Since reaching 100 million people, the population was growing by 4% each year.

Next-Generation

33B

Lauren keeps records of the distances she travels in a taxi and what she pays:

Distance, d, in miles	**Fare, F, in dollars**
3	8.25
5	12.75
11	26.25

a. If you graph the ordered pairs (d, F) from the table, they lie on a line. How can you tell this without graphing them?

b. Write a function for the given data..

c. Explain what the values stand for in the context of the problem?

Trigonometric Functions (F.TF)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

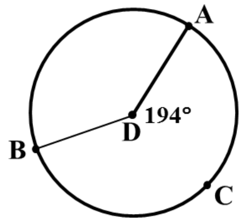
Cluster: Extend the domain of trigonometric functions using the unit circle.

9-11.F.TF.1 – Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

DCAS-Like

34A

What is the length of $\widehat{AB}$ if radius $\overline{AD} = 14$?



A. Length of $\widehat{AB}$ is 47.4 linear units.

B. Length of $\widehat{AB}$ is 40.45 linear units.

C. Length of $\widehat{AB}$ is 23.7 linear units.

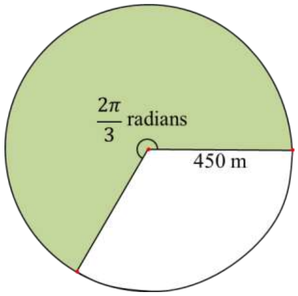
D. Length of $\widehat{AB}$ is 2716 linear units.

34B

Crops are often grown using a technique called center pivot irrigation that results in circular shaped fields.

Here is a satellite image taken over fields in Kansas that use this type of irrigation system.

If the irrigation pipe is 450 m in length, what is the area that can be irrigated after a rotation of $\frac{2\pi}{3}$ radians?



9-11.F.TF.2 – Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

DCAS-Like

35A

What value of x in the interval $0° \leq x \leq 180°$ satisfies the equation $\sqrt{3}\tan x + 1 = 0$?

A. $-30°$

B. $30°$

C. $60°$

D. $150°$

Next-Generation

35B

Match each radian measure with its corresponding unit circle coordinate.

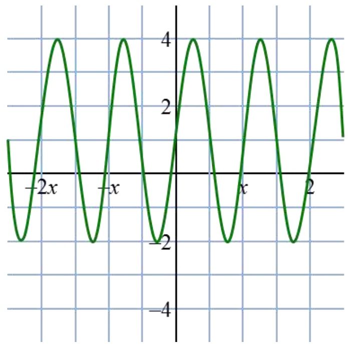
1.	$\cos\frac{4\pi}{3}$	a.	$\frac{1}{2}$
2.	$\sin\frac{5\pi}{6}$	b.	$\frac{-1}{2}$
3.	$\cos\frac{7\pi}{4}$	c.	$\frac{\sqrt{2}}{2}$
4.	$\sin\frac{8\pi}{3}$	d.	$\frac{-\sqrt{2}}{2}$
5.	$\cos\frac{-17\pi}{6}$	e.	$\frac{\sqrt{3}}{2}$
6.	$\sin\frac{-3\pi}{4}$	f.	$\frac{-\sqrt{3}}{2}$

Cluster: Model periodic phenomena with trigonometric functions.

9-11.F.TF.5 – Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

DCAS-Like

36A



Which of the following equations can be used to describe the graph shown above?

A. $y = 4\sin 2(x - 1)$

B. $y = 3\sin(2x) + 1$

C. $y = 2\sin(3x) + 1$

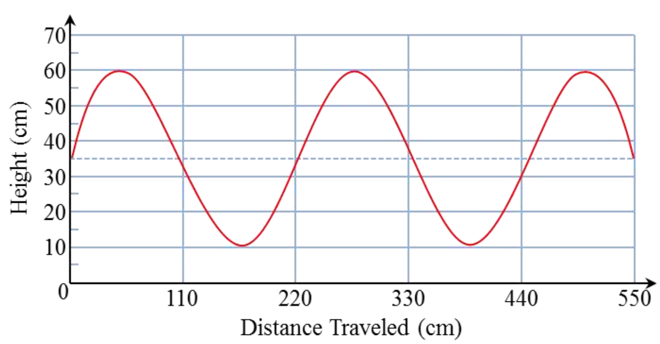
D. $y = 3\sin\left(\frac{1}{2}x\right) + 1$

Next-Generation

36B

A new type of LED bicycle light can be attached to the spokes of the front wheel at different distances from the center of the wheel. When the bicycle is moving forward at a constant speed, a person looking at the bicycle from the side would see the light following the pattern shown in the graph.


Safety Light Motion



The motion of the light can be modeled with the trigonometric function: $f(x) = E\sin(Fx) + G$

In the equation, the letters E, F, and G represent parameters based on the physical situation, and x represents the distance the bike has moved.

Part A

Determine the numerical values of the three parameters, including the units for each.

$E =$	
$F =$	
$G =$	

Part B

Explain how each of the three parameters is related to the physical situation being modeled. Use the start of the sentence given in each box.

Parameter E represents
Parameter F represents
Parameter G represents

Cluster: Prove and apply trigonometric identities.

9-11.F.TF.8 – Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.

DCAS-Like

37A

If $\cos x = \frac{-4}{7}$, find $\cos 2x$.

A. $\frac{-17}{49}$

B. $\frac{16}{49}$

C. $\frac{32}{49}$

D. $\frac{-4}{49}$

Next-Generation

37B

Match each expression on the left with the equivalent expression on the right.

1.	$\cos\frac{\pi}{2}\cos\frac{\pi}{3} + \sin\frac{\pi}{2}\sin\frac{\pi}{3}$	a.	$\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$
2.	$\cos\frac{\pi}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{2}\sin\frac{\pi}{3}$	b.	$\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$
3.	$\sin\frac{\pi}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{2}$	c.	$\cos\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$
4.	$\sin\frac{\pi}{2}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{2}$	d.	$\cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

Answer Key and Item Rubrics

Interpreting Functions (F.IF)

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>1A: A
(9-11.F.IF.1)</td><td>1B:
Key and Distractor Analysis
a. Y – All x-coordinates are unique, so it meets the definition of a function.
b. N – An input of $x = 1$ has two corresponding outputs, $y = \sqrt{3}$ and $y = -\sqrt{3}$, so it fails to meet the definition of a function.
c. Y – This is a function since for each value chosen along the x-axis, there is exactly one y-value on the graph that corresponds to it.
d. N – This is not a function since the input of 5 has two corresponding output values, 3 and 2.</td></tr>
<tr><td>2A: D
(9-11.F.IF.1)</td><td>2B:
a. In order of evaluation
◆ Operation: Subtract 3 from x. This does not restrict the domain since we can subtract 3 from any number.
◆ Operation: Divide 2 by the above result. This means that $x - 3$ cannot equal 0, so $x \neq 3$.
Domain is $x \neq 3$.
b. In order of evaluation:
◆ Operation: Subtract 5 from x. This does not restrict the domain since we can subtract 5 from any number.
◆ Operation: Take the square root of the above result. This means that $x - 5$ cannot be negative, so $x \geq 5$.
◆ Operation: Add 1 to the above result. This does not restrict the domain, since we can add 1 to any number.
Domain is $x \geq 5$.</td></tr>
</table>

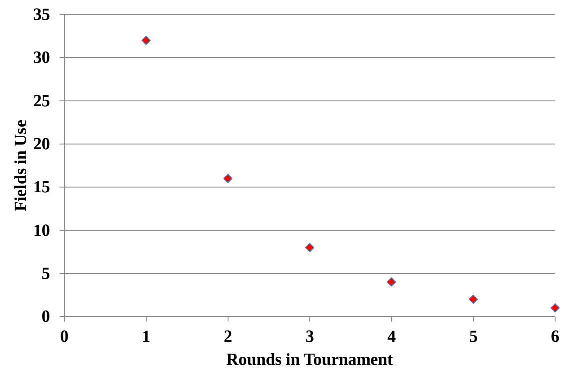
DCAS-Like Answer	Next-Generation Solution
	c. In order of evaluation: ◆ Operation: Subtract 3 from x. This does not restrict the domain since we can subtract 3 from any number. ◆ Operation: Square the above result. This does not restrict the domain since we can square any number. ◆ Operation: Subtract the above result from 4. This does not restrict the domain since we can subtract any number from 4. Domain is all real numbers. d. In order of evaluation: ◆ Operation: Subtract 3 from x. This does not restrict the domain since we can subtract 3 from any number. ◆ Operation: Square the above result. This does not restrict the domain since we can square any number. ◆ Operation: Subtract the above result from 4. This does not restrict the domain since we can subtract any number from 4. ◆ Operation: Divide 7 by the above result. This means that the result of step C cannot equal 0, so $$(x-3)^2 \neq 4.$$ Thus, $$\begin{aligned} x-3 &\neq 2 \\ x &\neq 5 \\ \text{and} \quad x-3 &\neq -2 \\ x &\neq 1 \end{aligned}$$ Domain is all x except $x = 1, 5$.

DCAS-Like Answer	Next-Generation Solution
	e. In order of evaluation: ◆ Operation: Subtract 3 from x. This does not restrict the domain since we can subtract 3 from any number. ◆ Operation: Raise the above result to the power of $\frac{1}{2}$. This means that $x - 3$ cannot be negative, so $x \geq 3$. ◆ Operation: Subtract the above result from 4. This does not restrict the domain since we can subtract any number from 4. The domain is $x \geq 3$. f. In order of evaluation: ◆ Operation: Subtract 3 from x. This does not restrict the domain since we can subtract 3 from any number. ◆ Operation: Raise the above result to the power of $\frac{1}{2}$. This means that $x - 3$ cannot be negative, so $x \geq 3$. ◆ Operation: Subtract the above result from 4. This does not restrict the domain since we can subtract any number from 4. ◆ Divide 7 by the above result. This means that the result of the previous operation cannot equal 0, so $$(x-3)^{\frac{1}{2}} - 4 \neq 0$$ $$(x-3)^{\frac{1}{2}} \neq 4$$ Thus, $$x - 3 \neq 16$$ $$x \neq 19$$ The domain is $x > 3$, $x \neq 19$—that is, all numbers greater than or equal to 3 except 19.
3A: D (9-11.F.IF.2)	**3B:** a. February 10, 11, 12, and 13 b. On the ninth day, February 19th, the cost would be \$20. On the eighth day, February 18th, the cost would be \$17, a savings of \$3 if bought on $C(8)$.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>4A: D
(9-11.F.IF.3)</td><td>4B:
<table>
<tr><th></th><th>Sequence</th><th></th><th>Function</th></tr>
<tr><td>a.</td><td>$-12, -11.75, -11.5, -11.25 \ldots$</td><td>1.</td><td>$f(n) = 256 \times 2^{1-n}$</td></tr>
<tr><td>b.</td><td>$1, -2, 3, -4, 5, -6 \ldots$</td><td>2.</td><td>$f(n) = -12.25 + 0.25n$</td></tr>
<tr><td>c.</td><td>$256, 128, 64, 32, 16 \ldots$</td><td>3.</td><td>$f(n) = n(-1)^{n-1}$</td></tr>
</table>
a – 1; b – 3; c–1</td></tr>
<tr><td>5A: B
(9-11.F.IF.4)</td><td>5B:
Part A
$V = 4x^3 - 36x^2 + 80x$
Part B
<table>
<tr><td>Domain:</td><td>All real numbers such that $0 < x < 4$</td></tr>
<tr><td>Range:</td><td>All real numbers such that $0 < V \leq 52.5$</td></tr>
<tr><td>Work:</td><td>Since the paper is 8 inches wide, x has to be less than 4 or there will not be anything to fold up.
The domain of the equation has to be positive numbers because of the context. The upper bound can be found by locating the maximum for the equation. To find the maximum for my equation, I graphed it on my calculator and then used the Trace feature to determine the y-value at that point.</td></tr>
</table>
Note: Other methods may be used for the work part, including using the vertex formula.
Points Assigned

1 point for a correct and simplified equation
1 point for correct domain
1 point for correct range
1 point for explaining method or showing work to determine both the domain and range
</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>6A: C
(9-11.F.IF.4)</td><td>6B:
a.
<table>
<tr><th>Input</th><th>Output</th></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>10</td><td>300</td></tr>
<tr><td>20</td><td>300</td></tr>
<tr><td>30</td><td>300</td></tr>
<tr><td>40</td><td>430</td></tr>
<tr><td>50</td><td>700</td></tr>
<tr><td>60</td><td>1000</td></tr>
<tr><td>70</td><td>1100</td></tr>
<tr><td>80</td><td>1200</td></tr>
</table>
b. About 20s</td></tr>
</table>

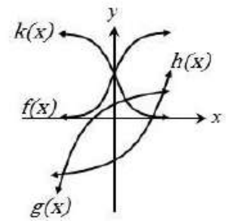
<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>7A: B
(9-11.F.IF.4)</td><td>
7B:
Students could take multiple paths to do this problem. The first part of the problem is very open-ended, so they can use a variety of strategies to report the amount of fields being used and fields that are open. Students need to first find how much field space is available (4 fields × 8 time slots = 32 fields). Students then need to calculate how many fields will be in use each round. Students can find this with or without knowledge of exponential functions. Tables or graphs will most likely be the route students will use.
<table>
<tr><th>Rounds of the Tournament</th><th>Fields Used</th><th>Fields open</th></tr>
<tr><td>1</td><td>32</td><td>0</td></tr>
<tr><td>2</td><td>16</td><td>16</td></tr>
<tr><td>3</td><td>8</td><td>24</td></tr>
<tr><td>4</td><td>4</td><td>39</td></tr>
<tr><td>5</td><td>2</td><td>30</td></tr>
<tr><td>6</td><td>1</td><td>31</td></tr>
</table>


a. If 24 teams are playing, they will need 12 fields. Students will then analyze their table, graph, or equation and find that there are that many fields available in week 2.
b. Six weeks, this can lead into a discussion about the domain of the function and its graph. Since the tournament started in week 1 and ends at week 6, the domain would be $\{1, 2, 3, 4, 5, 6\}$. Another discussion could develop about why $1 \leq x \leq 6$ would not be a correct domain. Only the integers between 1 and 6 make sense in this problem. They will not have round 4.5 of a tournament.
</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution		
8A: B (9-11.F.IF.4)	**8B:**		
	Part A	***Points***	***Total***
	a. Student drags the tiles to write a formula to find the value of cell D3: $D3 = \underline{B3 - C3}$	1	2
	b. Student drags the tiles to write a formula to find the value of cell E3: $E3 = \underline{D3 \times 1.015}$ OR $E3 = \underline{1.015 \times D3}$	1	
	Part B		2
	Student fills in the blanks in row 3 with the following correct answers: ▪ Slot A: 223.90 or 223.9 ▪ Slot B: 227.26	1	
	Student fills in the blanks in row 4 with the following correct answers: ▪ Slot C: 227.26 ▪ Slot D: 187.26 ▪ Slot E: 190.07	1	
	Part C		
	a. 75.11 or 75.12	1	3
	b. 8	1	
	c. 35.64	1	
	Total Possible	7	**7**

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>9A: D
(9-11.F.IF.5)</td><td>9B:
<table>
<tr><th>Context or Feature</th><th>Table/Context/Equation</th><th>Graph</th></tr>
<tr><td>a.</td><td>iii.</td><td>II.</td></tr>
<tr><td>b.</td><td>iv.</td><td>III.</td></tr>
<tr><td>c.</td><td>i.</td><td>IV.</td></tr>
<tr><td>d.</td><td>ii.</td><td>I</td></tr>
</table></td></tr>
<tr><td>10A: B
(9-11.F.IF.6)</td><td>10B:
Each item is scored independently and will receive 1 point.
a. From 0 to 20 years: $100
b. From 20 to 40 years: $150</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
11A: C (9-11.F.IF.7)	**11B:** a. Impose a coordinate graph on the diagram with center court at the origin. The given equation is the parabola in the diagram (drawn only partially). The equation of the line that defines the bleachers is a line with slope $\frac{1.5}{2} = \frac{3}{4}$. Using the given point $(30, 0)$ where the bleachers begin rising with that slope, the equation of the line is $y = \frac{3}{4}x - 22.5$. The intersection of the linear equation and the given parabolic equation will give both the horizontal distance (x) and vertical height (y) where the t-shirt arrives at the stands. The point of intersection can be found symbolically, but it is more likely that students will solve it by using intersection on a graphing calculator. The point of intersection is $(45, 11.25)$, which means that the t-shirt has travelled 45 feet horizontally when it strikes the line representing the bleachers at a height of 11.25 feet. This places the shirt in the 8th row by this logic: This point would be $45 - 30 = 15$ feet into the bleachers horizontally, which would place it in the 8th row. Each row is 2-feet wide, so 15 feet would be in the middle of the 8th row, which ends at 16 feet. OR The point would be 11.25-feet high, which would place it vertically in the 8th row. Each row is 1.5-feet high, so 11.25 feet would be in the middle of the 8th riser which ends at 12 feet. b. No. Using the maximum finding capabilities of a graphing calculator or a table, the maximum height indicated by the flight equation is 31.25 feet.

DCAS-Like Answer	Next-Generation Solution
12A: A (9-11.F.IF.7)	**12B:** *Solution 1: Evaluating Functions* The graphs of the four functions are identified below, followed by an explanation of how the identification can be made.  Observe that $f(x)$ and $k(x)$ always take positive values because the exponential function takes positive values. Evaluating these functions when $x = 1$ gives a value of about 2.86 for $f(1)$ and about 0.14 for $k(1)$. This information determines the graphs of f and k as indicated in the picture above. As for g and h, we may determine this by evaluating at $x = 0$. Here we find $g(0) = 1 - \frac{e^0}{2} = \frac{1}{2}$ and $h(0) = -2 + \frac{e^0}{2} = \frac{-3}{2}$. *Solution 2: Abstract Reasoning* As above, observation about the structure of the expressions shows that the graphs of f and k have to be the two which are above the x-axis. To determine which is which, note that e^x is an increasing function of x, that is as the value of x increases so does the value of e^x. Thus, $1 + e^{3x}$ is an increasing positive function of x. Thus, $\frac{1}{1+e^{3x}}$ is a decreasing positive function of x and this tells us which graph is the graph of k and which one is the graph of f. Alternatively, the same reasoning can be applied to determine that $1 + e^{-3x}$ is a decreasing function of x and so $\frac{3}{1+e^{-3x}}$ is an increasing function of x. The reasoning of the previous paragraph shows that both g and h are increasing functions of x. On the other hand, h increases exponentially fast, like e^x, while g increases much more slowly, taking values closer and closer to 1 as x increases.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>13A: B
(9-11.F.IF.8)</td><td>13B:
$y = \frac{2}{3}x + \frac{1}{3}$</td></tr>
<tr><td>14A: B
(9-11.F.IF.9)</td><td>14B:
<table>
<tr><th>First Quantity</th><th>Comparison</th><th>Second Quantity</th></tr>
<tr><td>The y-coordinate of the y-intercept $f(x)$</td><td>></td><td>The y-coordinate of the y-intercept $g(x)$</td></tr>
<tr><td>$f(3)$</td><td>></td><td>$g(3)$</td></tr>
<tr><td>Maximum value of $f(x)$ on the interval $-5 \leq x \leq 5$</td><td>=</td><td>Maximum value of $g(x)$ on the interval $-5 \leq x \leq 5$</td></tr>
<tr><td>$\frac{f(5) - f(2)}{5 - 2}$</td><td>></td><td>$\frac{g(5) - g(2)}{5 - 2}$</td></tr>
</table>
</td></tr>
</table>

Building Functions (F.BF)

DCAS-Like Answer	Next-Generation Solution
15A: B (9-11.F.BF.1)	**15B:** a. $y = kx(x - 1)$ when $x - 2 \quad y = 1$ $y = kx(x - 1)$ $1 = k \bullet 2(2 - 1)$ $1 = k \bullet 2(1)$ $1 = k \bullet 2$ $\frac{1}{2} = k$ when $x = 3 \quad y = 3$ $y = kx(x - 1)$ $3 = k \bullet 3(3 - 1)$ $3 = k \bullet 3(2)$ $3 = k \bullet 6$ $\frac{1}{2} = k$ b. $y = \frac{1}{2}x(x - 1)$ $x = 100 \quad y = ?$ $y = \frac{1}{2} \bullet 100(100 - 1)$ $y = \frac{1}{2} \bullet 100(99)$ $y = 50(99)$ $y = 4{,}950$

DCAS-Like Answer	Next-Generation Solution		
16A: B (9-11.F.BF.1)	**16B:**	*Points*	*Maximum*
	a. Student places the tiles to make an appropriate function rule that describes y as a function of x. Acceptable answers: $y = \underline{10 \bullet 3^x}$ OR $y = \underline{3^x \bullet 10}$	1	1
	b. Student selects *both* correct statements that apply to the population model: ▪ The base of the exponent will change from 3 to 2. ▪ As the number of years increases, the graph of this model will be less steep than the graph of the original model.	1	1
	c. Student gives the correct answer: 76 rabbits Student provides a complete and correct explanation that: ▪ Includes correct use of equation. ▪ Shows that he/she rounded the answer up to the nearest whole rabbit. ▪ Work shown may include Model A: After 5 years, $y - 10 \bullet 3^5 = 2430$ rabbits. Model B: $2430 = a \bullet 2^5$ $a = 75.94$ (Partial credit: Student provides a correct, but weak, explanation.)	1 2* (1)*	3
	Total Points Possible	5	5

DCAS-Like Answer	Next-Generation Solution		
17A: D (9-11.F.BF.1)	**17B:**	*Points*	*Maximum*
	a. Student drags all 6 correct tiles to their appropriate positions: The height of the water changes at an average rate of about **1.2** centimeters per golf ball. If these data were graphed with the number of golf balls as the independent variable, the y-intercept for the graph would be about **9.0** centimeters. This means that for zero **golf balls** , the **water height** is 9 centimeters. Tom's table and graph can be represented by the trend line with the equation $y = 1.2x + 9.0$	1	1
	b. Student selects all correct statements that apply: ▪ Use larger golf balls ▪ Decrease the diameter of the cylinder	1	1

DCAS-Like Answer	Next-Generation Solution		
	c. *Question 1:* The y-intercept for glass jar B will be less OR lower than the y-intercept for glass jar A. Possible explanations: ▪ The water height in jar B is lower than the water height in jar A when each jar contains the same number of golf balls. ▪ The water height in jar B for two golf balls is the same as the water height ion jar A for no golf balls. This means that the water height in jar B will be less than 9 centimeters when the 2 golf balls are removed. *Question 2:* The rate of change using glass jar B will be higher/greater/have a larger effect than the rate of change using glass jar A. Possible explanations: ▪ Because glass jar B is smaller/has a small radius or diameter than glass jar A.	 1 1 1 1	8

<table>
<tr><th>DCAS-Like Answer</th><th colspan="3">Next-Generation Solution</th></tr>
<tr><td></td><td>

Question 3: Answer: “5” or “3”

Possible explanations:

- The student gives a table, equation, ordered pairs, steps, or pattern involving both cylinders to support his or her answer. Example table is shown below:

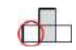
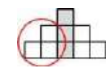
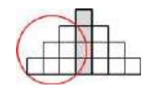
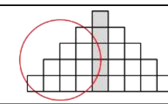
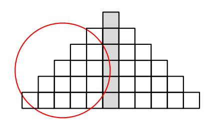
Number of Golf Balls	Height of Water in cm in New Cylinder $y = 2x + 5$	Height of Water in cm in New Cylinder $y = 1.2x + 9$
0	5	9.0
1	7	10.2
2	9	11.5
3	11	12.7
4	13	13.8
5	15	15.0

While the correct answer is 5 total golf balls, students may give an answer of 3 golf balls to indicate the number that would be added to the 2 golf balls shown to make a total of 5. Credit should be given for an answer of 3 golf balls if students clearly explain that this is what they have done.

</td><td>2
2</td><td></td></tr>
<tr><td></td><td>Total Points Possible</td><td>10</td><td>10</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
18A: B (9-11.F.BF.1)	**18B:** *Part A* $f(n) = 10(3^{n-1})$ *Part B* $f(n) = 3f(n-1)$ for $n > 1$ $f(1) = 10$ *Scoring Rubric* Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to explain and apply mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student correctly selects $f(n)$ for the explicit formula in Part A. The student also completely defines the correct recursive formula in Part B, selecting the correct $f(n)$ definition, condition for n, and initial value for $f(1)$. **1 point:** The student understands how to explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student can identify both the explicit formula in Part A and the correct $f(n)$ definition in Part B, but does not correctly identify the condition for n and/or the initial value for $f(1)$ in Part B. **0 points:** The student has an inconsistent understanding of how to explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency for writing recursive and explicit functions to describe the relationship between two quantities. The student does not correctly select the $f(n)$ definitions for both the explicit formula in Part A and the $f(n)$ definition in part B.

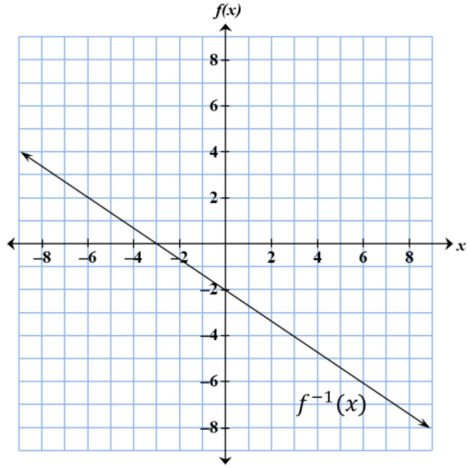
<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>19A: D
(9-11.F.BF.1)</td><td>19B:
a. 45 cubes are needed to build this tower.
b. Tower 12-cubes high:
$2(12^{2)} - 12$
$2(144) - 12 = 288 - 12$
$= 276$ cubes
c. Student can derive function using algebraic reasoning or models.
height $\quad n = 2$
$2\big(n(n-1)\big) + n$
$2(n^2 - n) + n$
$2n^2 - n$ is function used to calculate the number of cubes needed for a tower n cubes high.</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td></td><td>
<table>
<tr><td>$n = 2$</td><td>$2(2 \times 1) + 2$
$4(1) + 2 = 6$</td><td></td></tr>
<tr><td>$n = 3$</td><td>$2(3 \times 2) + 3$
$4(3) + 3 = 15$</td><td></td></tr>
<tr><td>$n = 4$</td><td>$2(4 \times 3) + 4$
$4(6) + 4 = 28$</td><td></td></tr>
<tr><td>$n = 5$</td><td>$2(5 \times 4) + 5$
$4(10) + 5 = 45$</td><td></td></tr>
<tr><td>$n = 6$</td><td>$2(6 \times 5) + 6$
$4(15) + 6 = 66$</td><td></td></tr>
</table>
Scoring Rubric
4 points: Correctly identifies the 5-high tower as requiring 45 cubes (a.) and the 12-high tower as requiring 276 cubes (b.). The function (c.) . . .
3 points: Correct answer to parts a. and b., but (function) is incomplete or is hard to follow.
2 points: Part a. is correct. May be errors in part b. and (function) is incomplete or absent.
1 point: Errors in part a. Part b. wrong or even missing.
0 points: Some work but generally without clear understanding of the goals of the calculation. For example, student might simply count visible blocks in the figure.
</td></tr>
<tr><td>20A: B
(9-11.F.BF.2)</td><td>20B:
Correct response will receive 1 point.
$f(n) = \frac{3}{2} \bullet f(n-1)$ for $n > 1$, where $f(1) = 8$</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th colspan="3">Next-Generation Solution</th></tr>
<tr><td>21A: B
(9-11.F.BF.2)</td><td colspan="3">21B:
Correct response will receive 1 point.
$f(n) = 24{,}500 - 4900n$</td></tr>
<tr><td>22A: D
(9-11.F.BF.2)</td><td colspan="3">22B:
a. 6^{th} term – 65
7^{th} term – 129
20^{th} term – 1,048,577 (not referenced in question)
b. $f(n) = 2^n + 1$</td></tr>
<tr><td>23A: B
(9-11.F.BF.3)</td><td colspan="3">23B:
Parameter A – A affects the horizontal position of the vertex of the graph. The sign of A is opposite of the direction of the shift. Negative values will shift the vertex right and positive values shift the vertex left.
Parameter B – B affects the vertical position of the vertex. Positive values of B shift the vertex up and negative values shift the vertex down.

Points Assigned
▪ Parameter A – 1 point for correctly identifying that the horizontal position is affected.
▪ Parameter A – 1 point for correctly explaining the effects of positive and negative values.
▪ Parameter B – 1 point for correctly identifying that the vertical position is affected.
▪ Parameter B – 1 point for correctly explaining the effects of positive and negative values.</td></tr>
<tr><td rowspan="3">24A: A
(9-11.F.BF.3)</td><td>24B:</td><td>Points</td><td>Maximum–8</td></tr>
<tr><td>a. Student fills in all six blanks giving the correct coordinate points.
D' (3, –14), E' (6, –8), and F' (7, –14)</td><td>1</td><td>1</td></tr>
<tr><td>b. Student correctly describes a sequence of congruence transformations: The graph of $f(x)$ is reflected across the y-axis and translated down 4 units, or the graph of $f(x)$ is translated down 4 units and then reflected across the y-axis.</td><td>1</td><td>7</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution		
	Student gives a correct equation in function notation: $g(x) = 2(x+5)^2 + 2$, or any mathematically equivalent equation.	2*	
	(Partial credit: gives correct equation not in function notation.)	(1)*	
	Student gives a clear and correct explanation including equation/graph relationship, discussing how both the reflection and the translation are seen in the equation.	4*	
	▪ The graph of $f(x)$ is reflected across the y-axis and translated down 4 units. The reflection across the y-axis is described as follows: $f(x) = f(-x)$ $= 2[(-x) - 5]^2 + 6$ $= 2(x^2 + 10x + 25) + 6$ $= 2(x+5)^2 + 6$	(2)*	
	▪ The graph is shifted down 4 units, so together $g(x) = r(x) - 4 = f(-x) - 4 = 2(x+5)^2 + 2$ OR	(2)*	
	▪ The graph is shifted down 4 units, so $t(x) = f(x) - 4 = 2(x-5)^2 + 6 - 4 = 2(x-5)^2 + 2$ ▪ The graph of $t(x)$ is reflected across the y-axis. The reflection across the y-axis is described as follows: $g(x) = t(-x)$ $= 2[(-x) - 5]^2 + 2$ $= 2(x^2 + 10x + 25) + 2$ $= 2(x+5)^2 + 2$		
	Note: If the student answer parts b. and c. correctly based on their identified transformations in part a., they will earn full credit for those parts.		
	(Partial credit: Student's explanation shows some idea about the concept that includes a reference between graph and equation.		

DCAS-Like Answer	Next-Generation Solution
25A: D (9-11.F.BF.4)	**25B:** Sample top-score response:  Correct line graphs will receive 1 point. Key: line containing *y*-intercept (0, –2) and slope of $-\frac{2}{3}$

Linear, Quadratic, and Exponential Models (F.LE)

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>26A: B
(9-11.F.LE.1)</td><td>26B:
Consecutive differences in Table A are constant ($9-6=12-9=15-12=3$), indicating a linear function.
Consecutive quotients in Table B are constant ($\frac{14}{7}=\frac{28}{14}=\frac{56}{28}=2$), indicating an exponential function.
Similarly, the constant quotient for Table D is $\frac{1}{2}$. Neither consecutive differences nor quotients are constant in Table C, and its ordered pairs are related by the equation $y=x^2+5$. So, Table A should be labeled “linear,” Table B and D “exponential,” and Table C “quadratic.”</td></tr>
<tr><td>27A: B
(9-11.F.LE.2)</td><td>27B:
a.
<table>
<tr><th>Bounce Number
$\boldsymbol{n}$</th><th>Measured Height (Inches)
$\boldsymbol{h(n)}$</th><th>Factor by Which Bounce Height Decreased
$\boldsymbol{h(n) \div h(n-1)}$</th></tr>
<tr><td>0</td><td>233</td><td></td></tr>
<tr><td>1</td><td>110</td><td>2.12</td></tr>
<tr><td>2</td><td>46.6</td><td>2.36</td></tr>
<tr><td>3</td><td>21</td><td>2.22</td></tr>
</table>
b. Model 4 is the best choice. The best choice for a is $h(n)=a \bullet e^{bn}$, b is a good choice as would any value based on two of the points given in the table or the value you would get by averaging the quotients in column 3 of the table from Item a.
c. The best characterization is statement 3.</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th colspan="3">Next-Generation Solution</th></tr>
<tr><td rowspan="3">28A: D
(9-11.F.LE.2)</td><td>28B:</td><td>Points</td><td>Maximum</td></tr>
<tr><td>a. Student fills in all four blanks with the correct answers:

<table>
<tr><th>Week</th><th>Number of Cells in Medium</th></tr>
<tr><td>1</td><td>15</td></tr>
<tr><td>2</td><td>30</td></tr>
<tr><td>3</td><td>60</td></tr>
<tr><td>4</td><td>120</td></tr>
<tr><td>5</td><td>240</td></tr>
<tr><td>6</td><td>480</td></tr>
<tr><td>7</td><td>960</td></tr>
<tr><td>8</td><td>1920</td></tr>
<tr><td>9</td><td>3840</td></tr>
<tr><td>10</td><td>7680</td></tr>
</table></td><td>1</td><td>1</td></tr>
<tr><td>b. Student drags the tiles to write the correct recursive sequence:
$w_1 = 15$
$w_n = w_{n-1} \bullet 2$, where n is an integer, such that $n \geq 2$</td><td>1</td><td>1</td></tr>
</table>

<table>
<tr><th>DCAS-Like Answer</th><th colspan="3">Next-Generation Solution</th></tr>
<tr><td></td><td>c. Student selects all three of the statements that apply:
<table>
<tr><td>☐ $w_n = 15 + 15 \bullet 2(n-1)$</td><td>☐ $w_n = 15 + 15 \bullet 2(n)$</td></tr>
<tr><td>☒ $w_n = 15 \bullet 2^{n-1}$</td><td>☐ $w_n = \frac{1}{2} \bullet 15 \bullet 2^{n-1}$</td></tr>
<tr><td>☒ $w_n = \frac{1}{2} \bullet 15 \bullet 2^{n}$</td><td>☒ $n \geq 1$, where n is an integer</td></tr>
<tr><td>☐ $n \geq 1$, where n is a real number</td><td>☐ n can be any real number</td></tr>
</table></td><td>1</td><td>1</td></tr>
<tr><td></td><td>d. Student gives the correct answer: 27
Student provides a sufficient explanation. Some examples are:
Using the explicit formula $w_n = 15 \bullet 2^{n-1}$: $15 \bullet 2^{n-1} \geq 1{,}000{,}000{,}000$, so
$2^{n-1} \geq 66{,}666{,}666.\bar{6}$
$ln(2^{n-1}) \geq ln(66{,}666{,}666.\bar{6})$
$(n-1) \bullet ln(2) \geq ln(66{,}666{,}666.\bar{6})$
$n - 1 \geq \frac{ln(66{,}666{,}666.\bar{6})}{ln(2)}$ or approximately
$n - 1 \geq 25.99$
$n \geq 26.99$
So, $n = 27$ OR
Using the recursive formula: $w_1 = 15$
$w_n = w_{n-1} \bullet 2, n \geq 1$, where n is an integer</td><td>1*
1*</td><td>2</td></tr>
<tr><td></td><td>Total Points Possible</td><td>5</td><td>5</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
29A: B (9-11.F.LE.2)	**29B:** *Part A* $D = -0.15(T - 1985) + 20.50$ *Part B* a. The mathematical model predicts that the cliff will erode an average of 15 centimeters a year based on the geologist's data. b. Plugging a distance of 5 meters into my equation, I then solved for T and get a date of 2088. c. $5 = -0(T - 1985) + 20.50$ $0.15(T - 1985) = 15.50$ $T - 1985 = 103.3$ $T \approx 2088$ d. My prediction should be accurate. When I put $T = 2012$ into my equation, the result was 16.45 meters, a difference of only 5 centimeters from the actual measurement. *Scoring Notes* ▪ Part A: Other solutions are possible such as having $T =$ years since 1985. ▪ Part B.: Student should both show work and explain the model. *Points Assigned* ▪ 1 point for writing a correct equation in Part A. ▪ 0–2 points for a holistic judgment of the student's solution and explanation in Part B. *Scoring Rubric* **3 points** **2 points** **1 point:** Demonstrates minimal understanding of creating and applying linear equations. **0 points:** Student's response is incorrect, irrelevant, too brief to evaluate, or blank.

DCAS-Like Answer	Next-Generation Solution
30A: A (9-11.F.LE.3)	**30B:** a. 1 b. 2 c. 10
31A: B (9-11.F.LE.4)	**31B:** *Part A* $P(t) = 1000(2)^{\frac{t}{20}}$ or $1000e^{\left(\frac{ln2}{20}\right)t}$ or $10^{\left(\frac{log2}{20}\right)t} + 3$ *Part B* 60 minutes or 1 hour *Part C* 86.4 minutes *Points Assigned* ▪ 1 point for writing a correct function in Part A. ▪ 1 point for the correct time in Part B. ▪ 1 point for the correct time in Part C. ▪ Note: Points for Parts B and C can be received for correct values based on an incorrect equation in Part A. *Scoring Rubric* **3 points** **2 points** **1 point:** Demonstrates minimal understanding of using logarithms to write and solve exponential equations. **0 points:** Student’s response is incorrect, irrelevant, too brief to evaluate, or blank.

DCAS-Like Answer	Next-Generation Solution
32A: B (9-11.F.LE.5)	**32B:** a. 5 fish b. 1.27 c. 100%
33A: B (9-11.F.LE.5)	**33B:** a. The fare per mile increases at a constant rate. b. $F = 2.25d + 1.5$ c. \$2.25 is the cost per mile for the taxi ride. \$1.50 is the usage fee to ride in a taxi.

Trigonometric Functions (F.TF)

DCAS-Like Answer	Next-Generation Solution
34A: A (9-11.F.TF.1)	**34B:** Using the formula: $A = \frac{1}{2} r^2 \theta$ $A = \frac{1}{2}(450)^2 \left(\frac{2\pi}{3}\right)$ $(.5 * 450^2 * \frac{2\pi}{3} = 212057.5041)$ The area is approximately 212,058 square meters.
35A: D (9-11.F.TF.2)	**35B:** 1. $\cos \frac{4\pi}{3}$ — b. $\frac{-1}{2}$ 2. $\sin \frac{5\pi}{6}$ — a. $\frac{1}{2}$ 3. $\cos \frac{7\pi}{4}$ — c. $\frac{\sqrt{2}}{2}$ 4. $\sin \frac{8\pi}{3}$ — e. $\frac{\sqrt{3}}{2}$ 5. $\cos \frac{-17\pi}{6}$ — f. $\frac{-\sqrt{3}}{2}$ 6. $\sin \frac{-3\pi}{4}$ — d. $\frac{-\sqrt{2}}{2}$

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>36A: B
(9-11.F.TF.5)</td><td>36B:

Part A

<table>
<tr><td>$E =$</td><td>25 cm</td></tr>
<tr><td>$F =$</td><td>164° per cm or $\frac{\pi}{110}$</td></tr>
<tr><td>$G =$</td><td>35 cm</td></tr>
</table>

Part B

<table>
<tr><td>Parameter E represents the distance the light is from the center of the wheel.</td></tr>
<tr><td>Parameter F represents the circumference of the wheel, which is the distance it travels horizontally during one full rotation (360°).
OR
Parameter F represents the period $= \frac{2\pi}{frequency}$</td></tr>
<tr><td>Parameter G represents the radius of the wheel.</td></tr>
</table>

Points Assigned
▪ Part A: 1 point for correct value and units for EACH parameter—up to 3 points.
▪ Part B: 1 point for correct description of EACH parameter—up to 3 points.</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
	Scoring Rubric **6 points** **5 points** **4 points** **3 points** **2 points** **1 point:** Demonstrates minimal understanding of parameters that represent amplitude, frequency, and midline. **0 points:** Student's response is incorrect, irrelevant, too brief to evaluate, or blank.
37A: A (9-11.F.TF.8)	**37B:** 1. $\cos\frac{\pi}{2}\cos\frac{\pi}{3}+\sin\frac{\pi}{2}\sin\frac{\pi}{3}$ a. $\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ 2. $\cos\frac{\pi}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{2}\sin\frac{\pi}{3}$ b. $\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$ 3. $\sin\frac{\pi}{2}\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos\frac{\pi}{2}$ c. $\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ 4. $\sin\frac{\pi}{2}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos\frac{\pi}{2}$ d. $\cos\left(\frac{\pi}{2}+\frac{\pi}{3}\right)$