COMMON CORE ASSESSMENT COMPARISON FOR MATHEMATICS

GRADES 9–11
ALGEBRA

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Prepared by:

Delaware Department of Education
Accountability Resources Workgroup
401 Federal Street, Suite 2
Dover, DE 19901

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INTRODUCTION

The purpose of this document is to illustrate the differences between the Delaware Comprehensive Assessment System (DCAS) and the expectations of the next-generation Common Core State Standard (CCSS) assessment in Mathematics. A side-by-side comparison of the current design of an operational assessment item and the expectations for the content and rigor of a next-generation Common Core mathematical item are provided for each CCSS. The samples provided are designed to help Delaware's educators better understand the instructional shifts needed to meet the rigorous demands of the CCSS. This document does not represent the test specifications or blueprints for each grade level, for DCAS, or the next-generation assessment.

For mathematics, next-generation assessment items were selected for CCSS that represent the shift in content at the new grade level. Sites used to select the next-generation assessment items include:

- Smarter Balanced Assessment Consortium
- Partnership of Assessment of Readiness for College and Career
- Illustrative Mathematics
- Mathematics Assessment Project

Using released items from other states, a DCAS-like item, aligned to the same CCSS, was chosen. These examples emphasize the contrast in rigor between the previous Delaware standards, known as Grade-Level Expectations, and the Common Core State Standards.

Section 1, DCAS-Like and Next-Generation Assessment Comparison, includes content that is in the CCSS at a different "rigor" level. The examples are organized by the CCSS. For some standards, more than one example may be given to illustrate the different components of the standard. Additionally, each example identifies the standard and is separated into two parts. Part A is an example of a DCAS-like item, and Part B is an example of a next-generation item based on CCSS.

Section 2 includes at least one Performance Task that addresses multiple aspects of the CCSS (content and mathematical practices).

How to Use Various Aspects of This Document

- Analyze the way mathematics standards are conceptualized in each item or task.
- Identify the instructional shifts that need to occur to prepare students to address these more rigorous demands. Develop a plan to implement the necessary instructional changes.
- Notice how numbers (e.g., fractions instead of whole numbers) are used in the sample items.
- Recognize that the sample items and tasks are only one way of assessing the standard.
- Understand that the sample items and tasks do not represent a mini-version of the next-generation assessment.
- Instruction should address "focus," coherence," and "rigor" of mathematics concepts.
- Instruction should embed mathematical practices when teaching mathematical content.

- For grades K–5, calculators should not be used as the concepts of number sense and operations are fundamental to learning new mathematics content in grades 6–12.
- The next-generation assessment will be online and the scoring will be done electronically. It is important to note that students may not be asked to show their work and therefore will not be given partial credit. It is suggested when using items within this document in the classroom for formative assessments, it is good practice to have students demonstrate their methodology by showing or explaining their work.

Your feedback is welcome. Please do not hesitate to contact Katia Foret at katia.foret@doe.k12.de.us or Rita Fry at rita.fry@doe.k12.de.us with suggestions, questions, and/or concerns.

* The Smarter Balanced Assessment Consortium has a 30-item practice test available for each grade level (3-8 and 11) for mathematics and ELA (including reading, writing, listening, and research). These practice tests allow students to experience items that look and function like those being developed for the Smarter Balanced assessments. The practice test also includes performance tasks and is constructed to follow a test blueprint similar to the blueprint intended for the operational test. The Smarter Balanced site is located at: http://www.smarterbalanced.org/.

Priorities in Mathematics

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Common Core State Standards for Mathematical Practices

	Mathematical Practices	Student Dispositions:	Teacher Actions to Engage Students in Practices:
Essential Processes for a Productive Math Thinker	**1. Make sense of problems and persevere in solving them**	▪ Have an understanding of the situation ▪ Use patience and persistence to solve problem ▪ Be able to use different strategies ▪ Use self-evaluation and redirections ▪ Communicate both verbally and written ▪ Be able to deduce what is a reasonable solution	▪ Provide open-ended and rich problems ▪ Ask probing questions ▪ Model multiple problem-solving strategies through Think-Aloud ▪ Promote and value discourse ▪ Integrate cross-curricular materials ▪ Promote collaboration ▪ Probe student responses (correct or incorrect) for understanding and multiple approaches ▪ Provide scaffolding when appropriate ▪ Provide a safe environment for learning from mistakes
	6. Attend to precision	▪ Communicate with precision—orally and written ▪ Use mathematics concepts and vocabulary appropriately ▪ State meaning of symbols and use them appropriately ▪ Attend to units/labeling/tools accurately ▪ Carefully formulate explanations and defend answers ▪ Calculate accurately and efficiently ▪ Formulate and make use of definitions with others ▪ Ensure reasonableness of answers ▪ Persevere through multiple-step problems	▪ Encourage students to think aloud ▪ Develop explicit instruction/teacher models of thinking aloud ▪ Include guided inquiry as teacher gives problem, students work together to solve problems, and debrief time for sharing and comparing strategies ▪ Use probing questions that target content of study ▪ Promote mathematical language ▪ Encourage students to identify errors when answers are wrong
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	▪ Create multiple representations ▪ Interpret problems in contexts ▪ Estimate first/answer reasonable ▪ Make connections ▪ Represent symbolically ▪ Talk about problems, real-life situations ▪ Attend to units ▪ Use context to think about a problem	▪ Develop opportunities for problem-solving strategies ▪ Give time for processing and discussing ▪ Tie content areas together to help make connections ▪ Give real-world situations ▪ Demonstrate thinking aloud for students' benefit ▪ Value invented strategies and representations ▪ More emphasis on the process instead of on the answer
	3. Construct viable arguments and critique the reasoning of others	▪ Ask questions ▪ Use examples and counter examples ▪ Reason inductively and make plausible arguments ▪ Use objects, drawings, diagrams, and actions ▪ Develop ideas about mathematics and support their reasoning ▪ Analyze others arguments ▪ Encourage the use of mathematics vocabulary	▪ Create a safe environment for risk-taking and critiquing with respect ▪ Provide complex, rigorous tasks that foster deep thinking ▪ Provide time for student discourse ▪ Plan effective questions and student grouping ▪ Probe students

	Mathematical Practices	Students:	Teacher(s) promote(s) by:
Modeling and Using Tools	**4. Model with mathematics**	▪ Realize that mathematics (numbers and symbols) is used to solve/work out real-life situations ▪ Analyze relationships to draw conclusions ▪ Interpret mathematical results in context ▪ Show evidence that they can use their mathematical results to think about a problem and determine if the results are reasonable—if not, go back and look for more information ▪ Make sense of the mathematics	▪ Allowing time for the process to take place (model, make graphs, etc.) ▪ Modeling desired behaviors (think alouds) and thought processes (questioning, revision, reflection/written) ▪ Making appropriate tools available ▪ Creating an emotionally safe environment where risk-taking is valued ▪ Providing meaningful, real-world, authentic, performance-based tasks (non-traditional work problems) ▪ Promoting discourse and investigations
	5. Use appropriate tools strategically	▪ Choose the appropriate tool to solve a given problem and deepen their conceptual understanding (paper/pencil, ruler, base ten blocks, compass, protractor) ▪ Choose the appropriate technological tool to solve a given problem and deepen their conceptual understanding (e.g., spreadsheet, geometry software, calculator, web 2.0 tools) ▪ Compare the efficiency of different tools ▪ Recognize the usefulness and limitations of different tools	▪ Maintaining knowledge of appropriate tools ▪ Modeling effectively the tools available, their benefits, and limitations ▪ Modeling a situation where the decision needs to be made as to which tool should be used ▪ Comparing/contrasting effectiveness of tools ▪ Making available and encouraging use of a variety of tools
Seeing Structure and Generalizing	**7. Look for and make use of structure**	▪ Look for, interpret, and identify patterns and structures ▪ Make connections to skills and strategies previously learned to solve new problems/tasks independently and with peers ▪ Reflect and recognize various structures in mathematics ▪ Breakdown complex problems into simpler, more manageable chunks ▪ "Step back" or shift perspective ▪ Value multiple perspectives	▪ Being quiet and structuring opportunities for students to think aloud ▪ Facilitating learning by using open-ended questions to assist students in exploration ▪ Selecting tasks that allow students to discern structures or patterns to make connections ▪ Allowing time for student discussion and processing in place of fixed rules or definitions ▪ Fostering persistence/stamina in problem solving ▪ Allowing time for students to practice
	8. Look for and express regularity in repeated reasoning	▪ Identify patterns and make generalizations ▪ Continually evaluate reasonableness of intermediate results ▪ Maintain oversight of the process ▪ Search for and identify and use shortcuts	▪ Providing rich and varied tasks that allow students to generalize relationships and methods and build on prior mathematical knowledge ▪ Providing adequate time for exploration ▪ Providing time for dialogue, reflection, and peer collaboration ▪ Asking deliberate questions that enable students to reflect on their own thinking ▪ Creating strategic and intentional check-in points during student work time

For classroom posters depicting the Mathematical Practices, please see: http://seancarberry.cmswiki.wikispaces.net/file/detail/12-20math.docx

Seeing Structure in Expressions (A.SSE)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Interpret the structure of expressions.

9-11.A.SSE.1 – Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

DCAS-Like

1A

Given the height h and volume V of a certain cylinder, Jull uses the following formula to compute its radius to be 20 meters. If a second cylinder has the same volume as the first, but is 100 times taller, what is its radius?

$$r = \sqrt{\frac{V}{\pi h}}$$

A. 2 meters

B. 200 meters

C. 0.2 meters

D. 2000 meters

Next-Generation

1B

Mark threw a baseball in from the outfield. The table below gives the time since the ball was thrown (in seconds) and the height of the ball (in feet).

Time (in seconds)	0	0.5	1.0	1.5	2.0	2.5
Height (in feet)	5	23.5	34.0	36.5	31.0	17.5

a. Charlie is trying to figure out a function rule that will match this data. He wants the rule to be of the form $h = -16t^2 + bt + c$. Find the values for b and c that Bobby should use in his rule. Explain your reasoning or show your work.

$b =$ ____________________ $c =$ ____________________

b. Explain what the values of b and c that you found in Part a. tell you about Mark's throw.

c. If nobody caught the ball, did it hit the ground before or after 3 seconds passed? Explain your reasoning.

9-11.A.SSE.2 – Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

DCAS-Like

2A

Leanne correctly solved the equation $x^2 + 4x = 6$ by completing the square. Which equation is part of her solution?

A. $(x+2)^2 = 8$

B. $(x+2)^2 = 10$

C. $(x+4)^2 = 10$

D. $(x+4)^2 = 22$

Next-Generation

2B

For numbers a. through e. below, determine whether each relation is equivalent to $(a+b)(x+y)$.

a. $(a+b)x + (a+b)y$	○ Yes	○ No
b. $a(x+y) + b(x+y)$	○ Yes	○ No
c. $(b+a)(y+x)$	○ Yes	○ No
d. $ax + by$	○ Yes	○ No
e. $ax + bx + ay + by$	○ Yes	○ No

Cluster: Write expressions in equivalent forms to solve problems.

9-11.A.SSE.3 – Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

DCAS-Like

3A

$(3y - 1)^4 =$

A. $81y^4 - 108y^3 + 54y^2 - 12y + 1$

B. $81y^4 - 108y^3 - 54y^2 - 12y + 1$

C. $81y^4 - 54y^3 + 108y^2 - 12y + 1$

D. $81y^4 + 54y^3 - 108y^2 - 12y + 1$

Next-Generation

3B

It is given that:

$$24 + 10x - x^2 = p - (x - 5)^2$$

Find the value of p.

$p =$ _________

9-11.A.SSE.3 – Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

a. Factor a quadratic expression to reveal the zeros of the function it defines.

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DCAS-Like

4A

What are the solutions to the equation $3x^2 + 3 = 7x$?

A. $x = \frac{7+\sqrt{85}}{6}$ of $x = \frac{7-\sqrt{85}}{6}$

B. $x = \frac{-7+\sqrt{85}}{6}$ of $x = \frac{-7-\sqrt{85}}{6}$

C. $x = \frac{7+\sqrt{13}}{6}$ of $x = \frac{7-\sqrt{13}}{6}$

D. $x = \frac{-7+\sqrt{13}}{6}$ of $x = \frac{-7-\sqrt{13}}{6}$

Next-Generation

4B

For items a. through e. below, select the two equations with equivalent zeros.

a. $y = x^2 + 14$

b. $y = x^2 + 9x + 14$

c. $y = \left(x - \frac{9}{2}\right)^2 - \frac{25}{4}$

d. $y = (x + 7)(x + 2)$

e. $y = \left(\frac{1}{2}x + 7\right)(2x + 2)$

9-11.A.SSE.4 – Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

DCAS-Like

5A

What is the nth term in the arithmetic series below? Note: $n = 1$ is first term.

$3 + 7 + 11 + 15 + 19 \ldots$

A. $4n$

B. $3 + 4n$

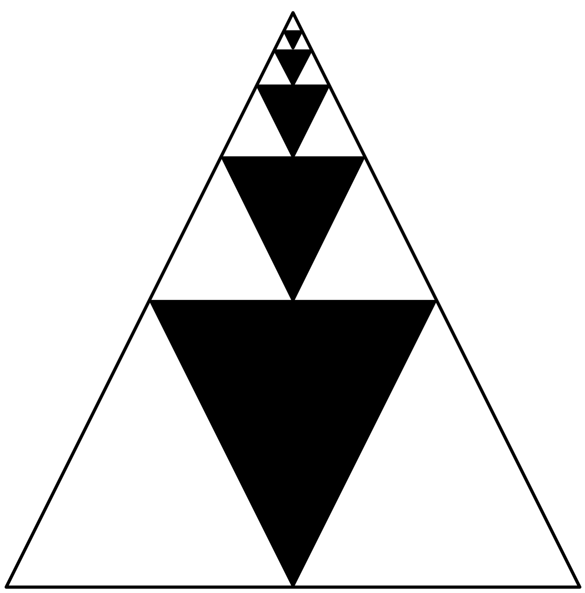
C. $2n + 1$

D. $4n - 1$

Next-Generation

5B

Consider the picture below consisting of a nested sequence of five equilateral triangles, colored in black. Each of the black triangles is made by connecting the three midpoints of the sides of the immediately larger white triangle.



Find and evaluate a sum to compute the total area of the black region (that is, the sum of the areas of the five black triangles) given that the largest triangle in the diagram has area 1.

Arithmetic with Polynomials and Rational Expressions (A.APR)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Perform arithmetic operations on polynomials.

9-11.A.APR.1 – Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

DCAS-Like

6A

$(6x^2 + 3x + 10) - 4(x^2 - 4x + 6) =$

A. $2x^2 - 13x + 34$

B. $2x^2 + 19x - 14$

C. $2x^2 + 13x - 14$

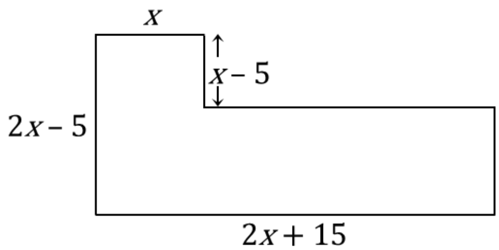
D. $2x^2 + 19x + 34$

Next-Generation

6B

Part A

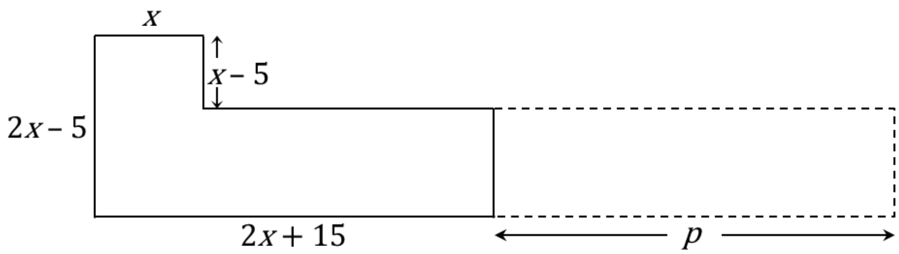
A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



Write an expression for the area, in yards, of this proposed parking lot. Explain the reasoning you used to find the expression.

Part B

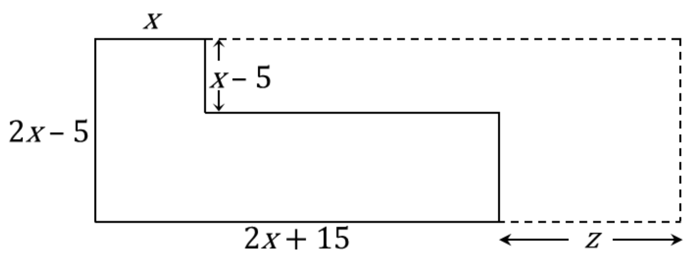
The town council has plans to double the area of the parking lot in a few years. They create two plans to do this. The first plan increases the length of the base of the parking lot by p yards, as shown in the diagram below.



Write an expression in terms of x to represent the value of p, in yards. Explain the reasoning you used to find the value of p.

Part C

The town council's second plan to double the area changes the shape of the parking lot to a rectangle, as shown in the diagram below.



Can the value of z be represented as a polynomial with integer coefficients? Justify your reasoning.

Cluster: Understand the relationship between zeros and factors of polynomial.

9-11.A.APR.2 – Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x – a* is *p(a)*, so *p(a) = 0* if and only if *(x – a)* is a factor of *p(x)*.

DCAS-Like

7A

If dividing the polynomial $f(x)$ by $(x+4)$ yields a remainder of -11, which of the following is true?

A. $f(-11) = -4$

B. $f(-11) = 4$

C. $f(-4) = -11$

D. $f(4) = -11$

Next-Generation

7B

Suppose $p(x)$ is a polynomial of degree $d > 0$.

a. If $p(0) = 0$, show that $p(x)$ is evenly divisible by x.

b. If $p(1) = 0$, shwo that $p(x)$ is evenly divisible by $x - 1$.

c. If r is a real number such that $p(r) = 0$, show that $p(x)$ is evenly divisible by $x - r$.

d. Using item c., show that p can have at most d distinct roots, that is, there can be at most d numbers $r_1, \ldots, r_d$ with $p(r_1) = \cdots = p(r_d) = 0$.

9-11.A.APR.3 – Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

DCAS-Like

8A

The graph of a polynomial function has the following x-intercepts: $0, 1, 3, 6$. Which of these expressions represents such a function?

A. $(x-1)(x-3)(x-6)$

B. $x(x+1)(x+3)(x+6)$

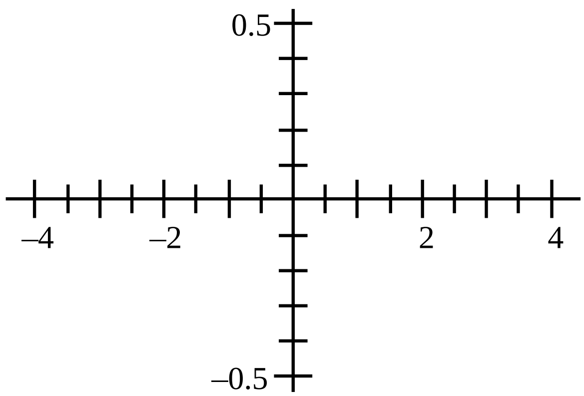
C. $(x+1)(3x+1)(6x+1)$

D. $2x(x-1)(x-3)(x-6)$

Next-Generation

8B

Mark the x-intercepts of the polynomial $f(x) = (x^2 - 5x + 6)(x-1)(x+3)$ on the graph.



Cluster: Use polynomial identities to solve problems.

9-11.A.APR.4 – Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

DCAS-Like

9A

Which values of m and n give the Pythagorean triple (17, 144, 145)?

A. $m = 6$ and $n = 7$

B. $m = 8$ and $n = 9$

C. $m = 6$ and $n = 10$

D. $m = 7$ and $n = 11$

Next-Generation

9B

Alice was having a conversation with her friend Trina, who had a discovery to share:

> *Pick any two integers. Look at the sum of their squares, the difference of their squares, and twice the product of the two integers you chose. Those three numbers are the sides of a right triangle.*

Trina had tried this several times and found that it worked for every pair of integers she tried. However, she admitted that she was not sure whether this "trick" always works or if there might be cases in which the trick does not work.

a. Using the table below, investigate Trina's conjecture for the given pairs of integers. Does her trick appear to work in all cases or only in some cases?

m	n	$m^2 + n^2$	$m^2 - n^2$	$2mn$
2	1			
2	2			
3	1			
3	2			
4	1			

b. If Trina's conjecture is true, then give a precise statement of the conjecture, using variables to represent the two chosen integers, and prove it. If the conjecture is not true, modify it so that it is a true statement and prove the new statement.

c. Use Trina's trick to find an example of a right triangle in which all of the sides have integer length, all three sides are longer than 100 units, and the three side lengths do not have any common factors.

9-11.A.APR.6 – Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

DCAS-Like

10A

Which of the following expressions is equivalent to the one shown below?

$$\frac{x^2 - 3x}{x^3 + x^2 - 12x}$$

A. $\frac{1}{x+4}$, where $x \neq 0, 3, -4$

B. $\frac{1}{x+4}$, where $x \neq 3, -4$

C. $\frac{x}{x+4}$, where $x \neq 3, -4$

D. $\frac{x}{x+4}$, where $x \neq 3, -4$

Next-Generation

10B

The US Department of Energy keeps track of fuel efficiency for all vehicles sold in the United States. Each car has two fuel economy numbers, one measuring efficient for city driving and one for highway driving. For example, a 2012 Volkswagen Jetta gets 29.0 miles per gallon (mpg) in the city and 39.0 mpg on the highway.

Many banks have "green car loans" where the interest rate is lowered for loans on cars with high *combined* fuel economy. This number is *not* the average of the city and highway economy values. Rather, the combined fuel economy (as defined by the federal Corporate Average Fuel Economy standard) for x mpg in the city and y mpg on the highway is computed as:

$$\text{combined fuel economy} = \frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)} = \frac{2xy}{x+y}$$

a. What is the combined fuel economy for the 2012 Volkswagen Jetta? Give your answer to three significant digits.

b. For most conventional cars, the highway fuel economy is 10 mpg higher than the city fuel economy. If we set the city fuel economy to be x mpg for such a car, what is the combined fuel economy in terms of x? Write your answer as a single rational function, $a(x)/b(x)$.

c. Rewrite your answer from item b. in the form of $q(x) + \frac{r(x)}{b(x)}$ where $q(x)$, $r(x)$, and $b(x)$ are polynomials and the degree of $r(x)$ is less than the degree of $b(x)$.

d. Use your answer in item c. to conclude that if the city fuel economy, x, is large, then the combined fuel economy is approximately $x + 5$.

Creating Equations (A.CED)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Create equations that describe numbers or relationships.

9-11.A.CED.1 – Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**

DCAS-Like

11A

Melinda wants to spend less than $45 to buy two T-shirts and a pair of shoes. She selects a pair of shoes that costs $24. If the cost of each T-shirt she selects is x, which inequality represents the amount that she can spend on each T-shirt?

A. $x > 21$

B. $x < 21$

C. $x > 10.5$

D. $x < 10.5$

Next-Generation

11B

A government buys x fighter planes at z dollars each, and y tons of wheat at w dollars each. It spends a total of B dollars, where $B = xz + yw$.

a. Find the number of tons of wheat the government can afford to buy if it spends a total of $100 million, wheat costs $300 per ton, and it must buy 5 fighter planes at $15 million each.

Enter your equation in the space below:

b. Find the price of fighter planes if the government bought 3 of them, in addition to 10,000 tons of wheat at $500 a ton, for a total of $50 million.

Enter your equation in the space below:

c. Find the price of a ton of wheat, given that a fighter plane costs 100,000 times as much as a ton of wheat, and that the government bought 20 fighter planes and 15,000 tons of wheat for a total cost of $90 million.

Enter your equation in the space below:

9-11.A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

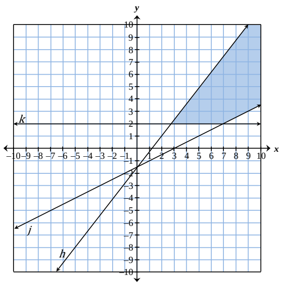
DCAS-Like

12A

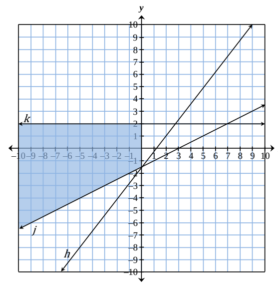
Which graph best represents the solution to the system of linear equations shown below?

- Line h: $5x - 4y \geq 6$
- Line j: $x - 2y \geq 3$
- Line k: $y \leq 2$

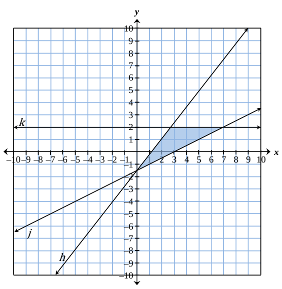
A.



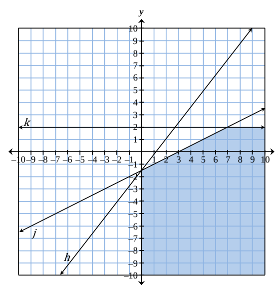
B.



C.



D.



Next-Generation

12B

David compares the sizes and costs of photo books offered at an online store. The table below shows the cost for each size photo book.

Book Size	Base Price	Cost for Each Additional Page
7-in. by 9-in.	$20	$1.00
8-in. by 11-in.	$25	$1.00
12-in. by 12-in.	$45	$1.50

The base price reflects the cost of a photo book with any number of pages up to 20. The cost for each additional page is given in the table.

1. Write an equation to represent the relationship between the cost, y, in dollars, and the number of pages, x, for each book size. Be sure to place each equation next to the appropriate book size. Assume that x is at least 20 pages.

7-in. by 9-in.
8-in. by 11-in.
12-in. by 12-in.

2. What is the cost of a 12-in. by 12-in. book with 28 pages?

3. How many pages are in an 8-in. by 11-in. book that costs $49?

9-11.A.CED.2 – Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*

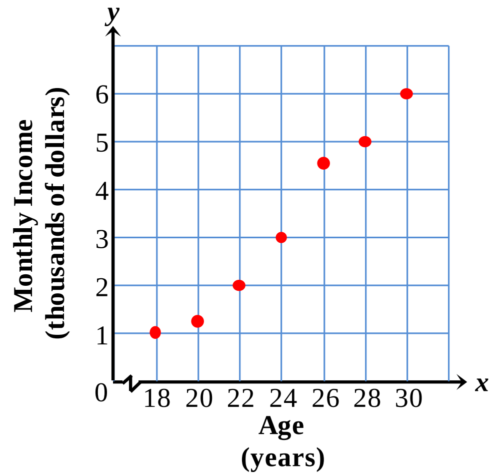
DCAS-Like

13A

The graph below shows how Daniel's monthly income increases as he grows older.

Daniel's Monthly Income

Age (years)	Monthly Income (thousands of dollars)
18	1
20	1.2
22	2
24	3
26	4.6
28	5
30	6



If the trend continues, what is the best prediction of Daniel's monthly income at age 34?

A. $6,800

B. $7,000

C. $7,800

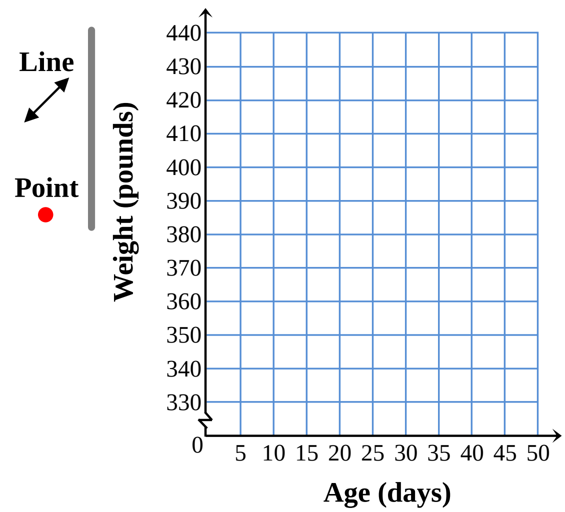
D. $8,000

13B

Andy found data on the Internet showing the weight of a baby elephant in a zoo at various ages.

Age (days)	2	7	26	35	68	500	697	910	1,164
Weight (pounds)	340	356	407	431	504	1,550	1,910	2,288	2,891

Help Andy develop a model that can be used to predict the elephant's weight for other ages where there is no data. Complete the five steps below.



1. Use the point tool to plot the first four data points on the graph. Then use the line tool to draw a line of best fit.
2. Determine the equation of your line of best fit as an initial model for the elephant's growth.
   - Use x to represent the elephant's age.
   - Use y to represent the elephant's weight.
   - Start your equation with $y =$.

Enter your equation in the space below.
$y =$

3. Use your equation to predict the weight of the elephant at 68 days.

Predicted weight of elephant at 68 days:	

4. Compare your prediction with the actual weight of the elephant at 68 days. Describe how your model should be changed to generate more accurate predictions when the elephant is older.

5. Describe, in terms of the elephant's growth, why your initial model did not work for all the data.

9-11.A.CED.3 – Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

DCAS-Like

14A

The only coins that Alexis has are dimes and quarters.

- Her coins have a total value of \$5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, d, and the number of quarters, q, Alexis has?

A. $\begin{cases} d + q = 5.80 \\ 40d + 40q = 5.80 \end{cases}$

B. $\begin{cases} d + q = 40 \\ 0.25d + 0.10q = 5.80 \end{cases}$

C. $\begin{cases} d + q = 5.80 \\ 0.10d + 0.25q = 40 \end{cases}$

D. $\begin{cases} d + q = 40 \\ 0.10d + 0.25q = 5.80 \end{cases}$

Next-Generation

14B

The coffee variety Arabica yields about 750 kg of coffee beans per hectare, while Robusta yields about 1200 kg per hectare. Suppose that a plantation has a hectares of Arabica and r hectares of Robusta.

a. Write an equation relating a and r if the plantation yields 1,000,000 kg of coffee.

Enter your equation in the space below.
$y =$

b. On August 14, 2003, the world market price of coffee was \$1.42 per kg of Arabica and \$0.73 per kg of Robusta. Write an equation relating a and r if the plantation produces coffee worth \$1,000,000.

Enter your equation in the space below.
$y =$

9-11.A.CED.4 – Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.**

DCAS-Like

15A

The distance between two points can be described by the following formula, where $d =$ distance, $(x_1, y_1) =$ starting points, and $(x_2, y_2) =$ terminal point.

$$d = \sqrt{(x_2 - x_1)^2 + (y - y_1)^2}$$

Find the equivalent equation solved for x_2, when $x_2 \geq x_1$?

A. $x_2 = x_1 + d - (y_2 - y_1)$

B. $x_2 = x_1 + \sqrt{d^2 - (y_2 - y_1)^2}$

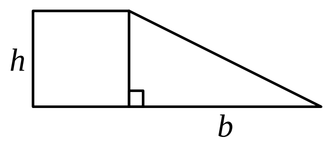
C. $x_2 = y_1 + \sqrt{d^2 - (x_2 - x_1)^2}$

D. $x_2 = x_1 + \sqrt{(y_2 - y_1)^2 - d^2}$

Next-Generation

15B

The figure below is made up of a square with height, h units, and a right triangle with height, h units, and base length, b units.



The area of this figure is 80 square units.

Write an equation that solves for the height, h, in terms of b. Show all work necessary to justify your answer.

$h =$ ________________________

Reasoning with Equations and Inequalities (A.REI)

Specific modeling standards appear throughout the high school mathematical standards and are indicated by an asterisk (*).

Cluster: Understand solving equations as a process of reasoning and explain the reasoning.

9-11.A.REI.1 – Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

DCAS-Like

16A

Solve: $3(x + 5) = 2x + 35$

- Step 1: $3x + 15 = 2x + 35$
- Step 2: $5x + 15 = 35$
- Step 3: $5x = 20$
- Step 4: $x = 4$

Which is the first *incorrect* step in the solution shown above?

A. Step 1

B. Step 2

C. Step 3

D. Step 4

Next-Generation

16B

Use the following equation to answer the question: $0.25(y - 2x) + 4 = 12$

Solve the equation for y one step at a time. Show each step in the table, describe the process, and explain the purpose of that step.

The first step is already done. You may not need all the rows, but you must show at least two more steps.

Solution Steps	Process	Purpose
$0.25(y - 2x) + 4 = 12$	Given	
$0.25(y - 2x) = 8$	Subtract 4 from each side	Combine like terms

9-11.A.REI.2 – Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

DCAS-Like

17A

The formula $P = 2\pi\left(\sqrt{\frac{L}{32}}\right)$ can be used to approximate the period of a pendulum, where L is the pendulum's length in feet and P is the pendulum's period in seconds. If a pendulum's period is 1.6 seconds, which of the following is closest to the length of the pendulum?

A. 1.4 ft

B. 4.2 ft

C. 2.1 ft

D. 3.2 ft

Next-Generation

17B

Solve the following two equations for x. Enter the solution below. If there is no solution, enter NS on the line.

a. $\sqrt{2x+1} - 5 = -2$ $x =$ ______________

b. $\sqrt{2x+1} + 5 = -2$ $x =$ ______________

Determine whether the following equations have extraneous solutions.

c. $\sqrt{x} = 5$ ○ Yes ○ No

d. $\sqrt{x} = -5$ ○ Yes ○ No

e. $\sqrt[3]{x} = 5$ ○ Yes ○ No

f. $\sqrt[3]{x} = -5$ ○ Yes ○ No

Cluster: Solve equations and inequalities in one variable.

9-11.A.REI.3 – Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

DCAS-Like

18A

Which of the following inequalities is equivalent to $5n + 4 < 19$?

A. $n < 3$

B. $n < 5$

C. $n > 3$

D. $n < 10$

Next-Generation

18B

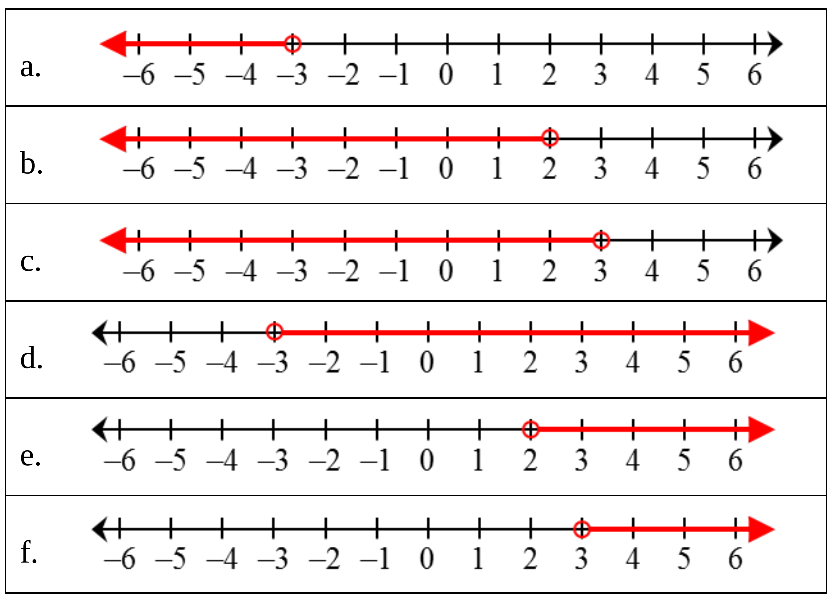
Match each inequality in items 1 through 3 with the number line in items a. to f. that represents the solution to the inequality.

To connect an inequality to its number line, first click the inequality. Then, click the number line it goes with. A line will automatically be drawn between them.

1. $-4x < -12$

2. $2(x + 2) < 8$

3. $5 - 2x < 2 - x$



9-11.A.REI.4 – Solve quadratic equations in one variable.

a. Use the method of completing the square to transforms any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.

DCAS-Like

19A

What is the solution set for $(x-2)^2 = 64$?

A. $\{-6\}$

B. $\{10\}$

C. $\{-6, 10\}$

D. $\{6, -10\}$

Next-Generation

19B

Solve the following equation:

$$(3x-2)^2 = 6x - 4$$

When you are finished, enter the solution(s) below.

Solution 1: $\frac{\square}{\square}$

Click + to enter another solution or click done

Cluster: Solve systems of equations.

9-11.A.REI.5 – Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

DCAS-Like

20A

Members of a senior class held a car wash to raise funds for their senior prom. They charged $3 to wash a car and $5 to wash a pick-up truck or a sport utility vehicle. If they earned a total of $275 by washing a total of 75 vehicles, how many cars did they wash?

A. 25

B. 34

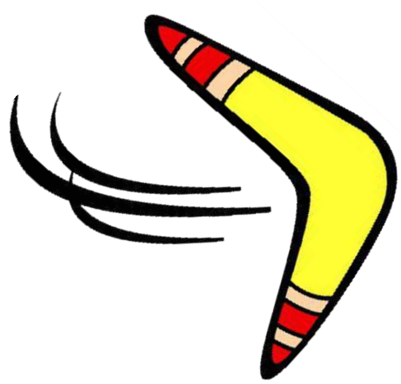
C. 45

D. 50

Next-Generation

20B

Phil and Cath make and sell boomerangs for a school event. They money they raise will go to charity.



They plan to make them in two sizes: small and large.

Phil will carve them from wood. The small boomerang takes 2 hours to carve and the large one takes 3 hours to carve. Phil has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 boomerangs of either size.

The small boomerang will make $8 for charity. The large boomerang will make $10 for charity. They want to make as much money for charity as they can.

How many small and large boomerangs should they make?

How much money will they then make?

9-11.A.REI.6 – Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

DCAS-Like

21A

What is the solution to this system of equations?

$$\begin{cases} y = -3x - 2 \\ 6x + 2y = -4 \end{cases}$$

A. $(6, 2)$

B. $(1, -5)$

C. No solution

D. Infinitely many solutions

Next-Generation

21B

A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special.

- On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials.
- On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials.

What is the cost if each lunch special?

Vegetarian: ________________
Chicken: ________________

9-11.A.REI.7 – Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

DCAS-Like

22A

Which ordered pair, (x, y), represents the solution to this system of equations?

$$y = (x+3)^2$$
$$y = x^2 + 9$$

A. $(0, 3)$

B. $(0, 9)$

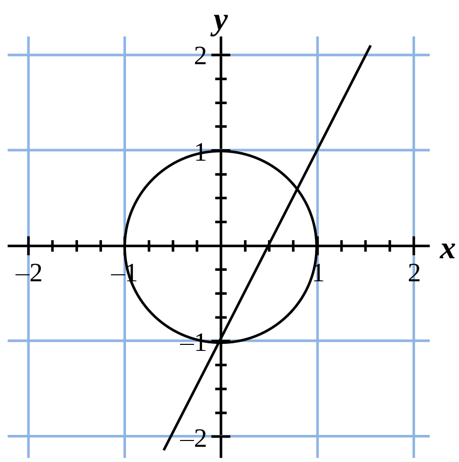
C. $(9, 0)$

D. $(3, 0)$

Next-Generation

22B

The equations $x^2 + y^2 = 1$ and $y = 2x - 1$ are graphed below.



Find all the solutions to this pair of equations.

When you are finished, enter the solution(s) below.

Solution 1: $\frac{\square}{\square}$

Click + to enter another solution or click **done**

Cluster: Represent and solve equations and inequalities graphically.

9-11.A.REI.10 – Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

DCAS-Like

23A

Which point lies on the line represented by the equation below?

$$5x + 4y = 22$$

A. $\left(-2, \frac{11}{4}\right)$

B. $\left(-1, \frac{17}{4}\right)$

C. $(2, 3)$

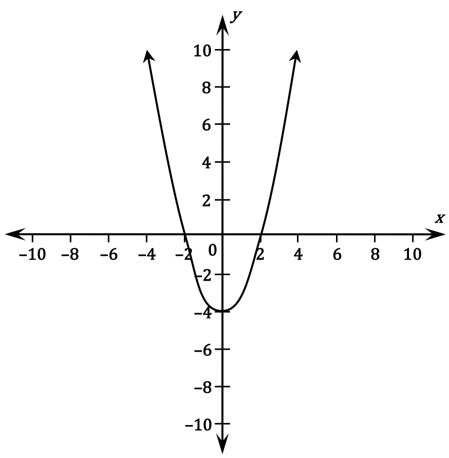
D. $(6, 2)$

Next-Generation

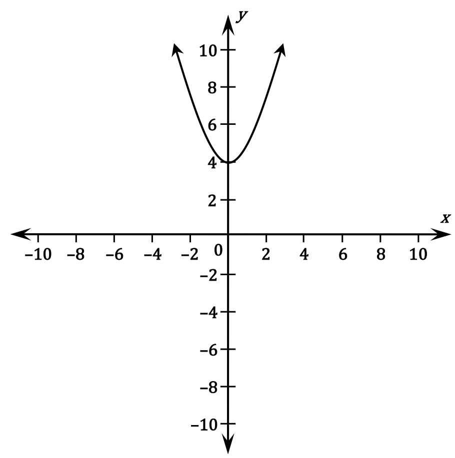
23B

Which graph could represent the solution set of $y = \sqrt{x - 4}$?

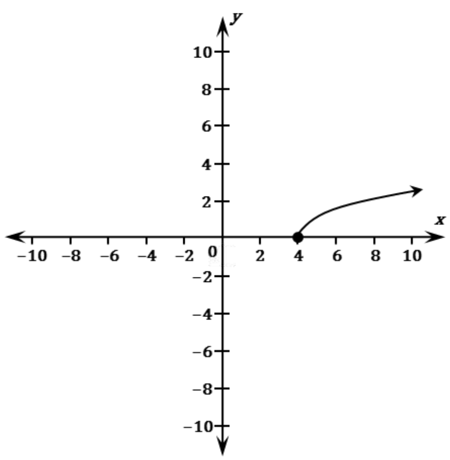
a.



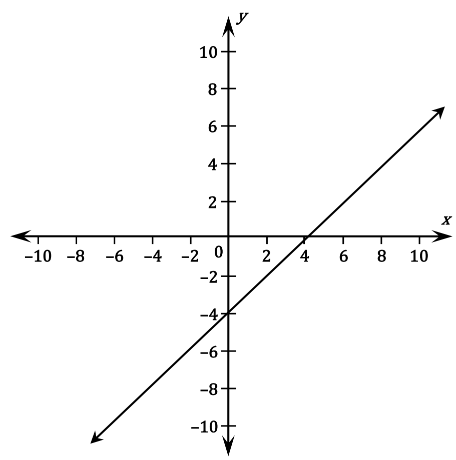
c.



b.



d.



Cluster: Represent and solve equations and inequalities graphically.

9-11.A.REI.11 – Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

DCAS-Like

24A

Sam has a total of 58 DVDs and CDs. If the number of CDs is two more than three times the number of DVDs, how many CDs does he have?

A. 42

B. 14

C. 44

D. 12

Next-Generation

24B

One automobile starts out from a town at 8 a.m. and travels at an average speed rate of 35 mph. Three hours later, a second automobile starts out to overtake the first. If the second automobile travels at an average rate of 55 mph, how long before it overtakes the first?

9-11.A.REI.12 – Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

DCAS-Like

25A

Which point lies in the solution set for the system $\begin{cases} 2y - x \geq -6 \\ 2y - 3x < 6 \end{cases}$?

A. $(-4, -1)$

B. $(3, 1)$

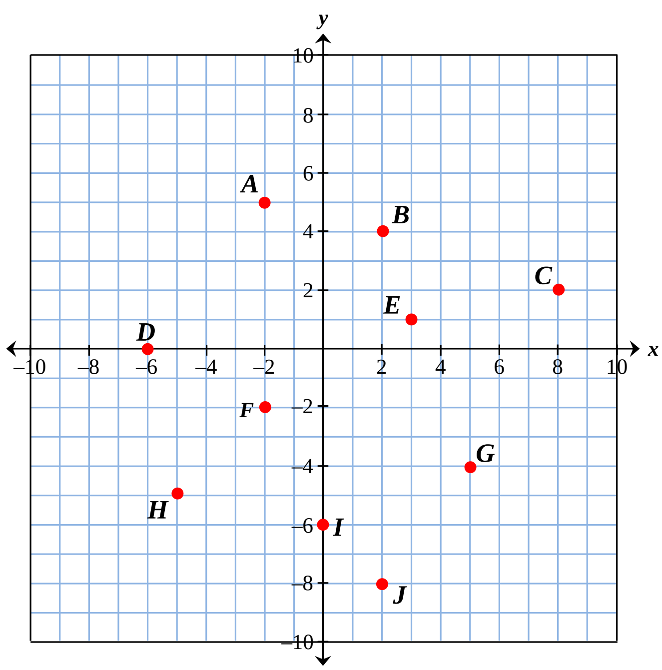
C. $(0, -3)$

D. $(4, 3)$

Next-Generation

25B

The coordinate grid below shows points A through J.



Given the system of inequalities shown below, select all the points that are solutions to this system of inequalities.

$$\begin{cases} x + y < 3 \\ 2x - y > 6 \end{cases}$$

O A O B O C O D O E

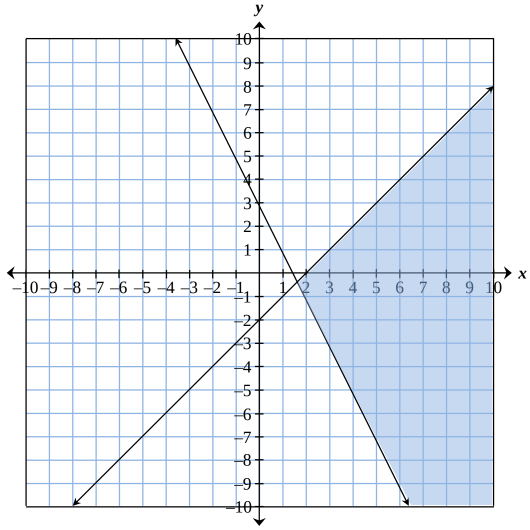
O F O G O H O I O J

9-11.A.REI.12 – Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

DCAS-Like

26A

Which system of linear inequalities is represented by this graph?



a. $\begin{cases} y \geq \frac{1}{2}x + 3 \\ y \geq x - 2 \end{cases}$

b. $\begin{cases} y \geq 2x + 3 \\ y \leq x - 2 \end{cases}$

c. $\begin{cases} 2x - y \geq 3 \\ x + y \leq 2 \end{cases}$

d. $\begin{cases} 2x + y \geq 3 \\ x - y \geq 2 \end{cases}$

Next-Generation

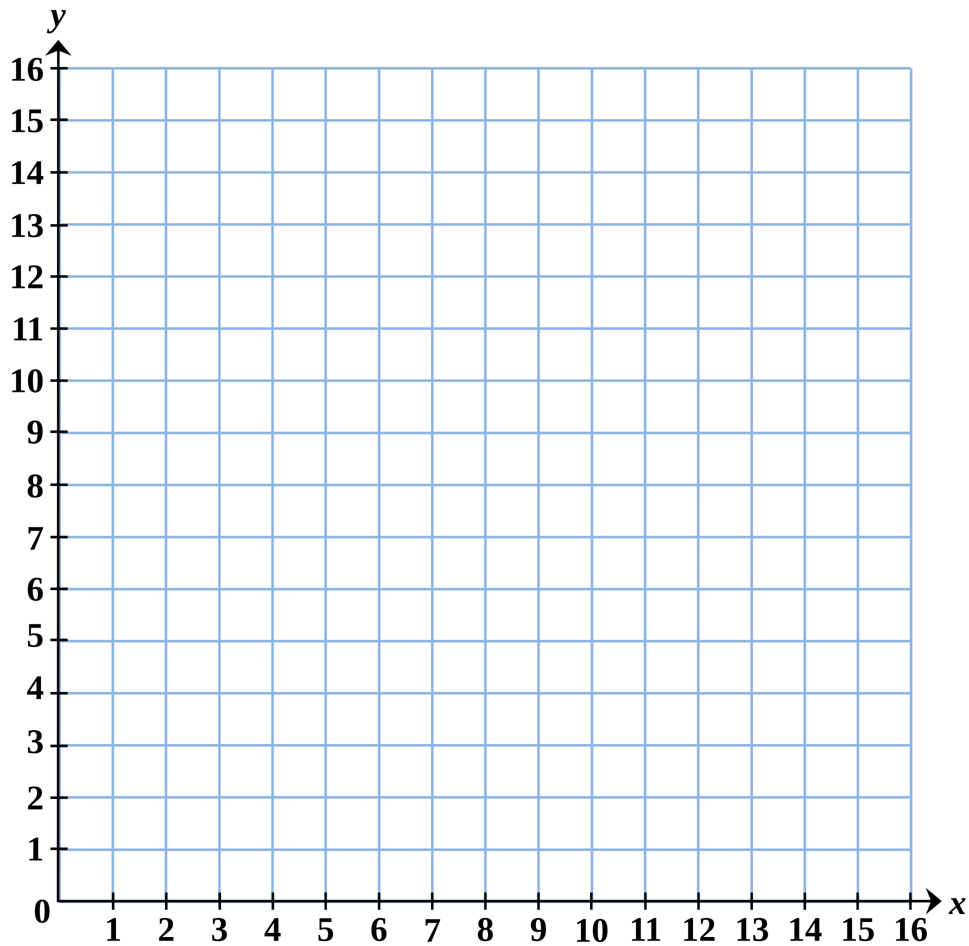
26B

Graph this system of inequalities below on the given coordinate grid.

$$\begin{cases} x + y \geq 12 \\ 20x + 30y \leq 300 \end{cases}$$

To create a line, click in the grid to create the first point on the line. To create the second point on the line, move the pointer and click. The line will be automatically drawn between the two points. Use the same process to create additional lines.

When both inequalities are graphed, select the region in your graph that represents the solution to this system of inequalities. To select a region, click anywhere in the region. To clear a selected region, click anywhere in the selected region.


y
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
x

Answer Key and Item Rubrics

Seeing Structure in Expressions (A.SSE)

DCAS-Like Answer	Next-Generation Solution
1A: A (9-11.A.SSE.1)	**1B:** a. $b = 45$ and $c = 5$ b. The value of b tells you that the initial upward velocity of the ball was 45 feet per second. The value of c tells you that the ball was thrown from a height of 5 feet. c. According to the rule, when $t = 3$, the height is -4 feet. The, the ball hit the ground before 3 seconds had passed.
2A: B (9-11.A.SSE.2)	**2B:** a. Yes b. Yes c. Yes d. No e. Yes
3A: A (9-11.A.SSE.3)	**3B:** $p = 49$
4A: C (9-11.A.SSE.3)	**4B:** *Key and Distractor Analysis:* Both b. and d. have the same zeros, -7 and -2. a. The non-real zeros are $\pm i\sqrt{14}$. b. The zeros are -7 and -2 since the polynomial factors to be the same as in item d. c. The zeros are 7 and 2. d. The zeros are -7 and -2. e. The zeros are -14 and -1.

DCAS-Like Answer	Next-Generation Solution
5A: D (9-11.A.SSE.4)	**5B:** The geometric key to the task is to note that of the various equilateral triangle sizes in the problem, each as $\frac{1}{4}$ the area of the immediately larger triangle of which it is a part (since each smaller such triangle has half its base and half its height). In particular, the area of the largest black triangle (and each of the white triangles of the same size) is $\frac{1}{4}$ of the area of the large triangle, i.e., $\frac{1}{4}(1) = \frac{1}{4}$. Similarly, the second largest black triangle has $\frac{1}{4}$ the area of one of these white triangles, so has area $\frac{1}{4} \bullet \frac{1}{4} = \frac{1}{4^2}$. Continuing in this way, the third, fourth, and fifth black triangles have respective areas $\frac{1}{4^3}$, $\frac{1}{4^4}$, and $\frac{1}{4^5}$. The sum of the areas of the black triangles can now be evaluated by the formula for a finite geometric series (with common ratio $\frac{1}{4}$): Black Area $= \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5}$ $= \frac{1}{4}\ 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4}$ $= \frac{1}{4} \bullet \frac{1-\frac{1}{4}^5}{1-\frac{1}{4}}$ $= \frac{341}{1024}$

Arithmetic with Polynomials and Rational Expressions (A.APR)

DCAS-Like Answer	Next-Generation Solution
6A: B (9-11.A.APR.1)	**6B:** *Part A* Missing vertical dimension is $2x - 5 - (x - 5) = x$ Area $= x(x-5) + x(2x+15)$ $= x^2 - 5x + 2x^2 + 15x$ $= 3x^2 + 10x$ square yards *Part B* Doubled area $= 6x^2 + 20x$ square yards Area of top left corner $= x^2 - 5x$ square yards Area of lower portion with doubled area $= 6x^2 + 20x - (x^2 - 5x)$ $= 5x^2 + 25x$ square yards Since the width remains x yards, the longest length must be $(5x^2 + 25x) \div x = 5x + 25$ yards long. So, $y = 5x + 25 - (2x + 15) = 5x + 25 - 2x - 15 = 3x + 10$ yards *Part C* If z is a polynomial with integer coefficients, the length of the rectangle, $2x + 15 + z$, would be a factor of the doubled area. Likewise, $2x - 5$ would be a factor of the doubled area. But, $2x - 5$ is not a factor of $6x^2 + 20x$. So, $2x + 15 + z$ is not a factor either. Therefore, z cannot be represented as a polynomial with integer coefficients. *Scoring Rubric:* **3 points:** The student has a solid understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student answers Part A and Part B correctly, showing all relevant work or reasoning. The student also clearly explains assumptions made in Part C as well as showing how they lead to a refutation of the conjecture that a given polynomial has integer coefficients. **2 points:** The student understands how to add, subtract, and multiply polynomials but cannot clearly articulate

DCAS-Like Answer	Next-Generation Solution
	reasoning with viable arguments associated with these tasks. The student answers Parts A and B correctly, showing all relevant work or reasoning. However, the student has flawed or incomplete reasoning associated with assumptions made in Part C that lead to a refutation of the conjecture that a given polynomial has integer coefficients. **1 point:** The student has only a basic understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student makes one or two computational errors in Parts A and B. The student also has flawed or incomplete reasoning associated with assumptions made in Part C that lead to a refutation of the conjecture that a given polynomial has integer coefficients. **0 points:** The student demonstrates inconsistent understanding of how to articulate reasoning with viable arguments associated with adding, subtracting, and multiplying polynomials. The student makes three or more computational errors in Parts A and B. The student also has missing or flawed reasoning related to determining whether a given polynomial has integer coefficients.
7A: C (9-11.A.APR.2)	**7B:** a. If 0 is a root of the function p, this means that $p(0) = 0$. Since p is a polynomial of degree d, it is given by a formula: $p(x) = a_d x^d + a_{d-1}x^{d-1} + \cdots + a_0,$ where $a_d \neq 0$. If we divide $p(x)$ by x, using long division of polynomials, we will find $p(x) = q(x)x + s$ where $q(x)$ is a polynomial of degree $d - 1$ and the remainder s is a number. Plugging in $x = 0$, we find $0 = p(0)$ $= q(0)(1 - 1) + s$ $= s$ Thus, $s = 0$ and so $p(x) - xq(x)$ and $p(x)$ is evenly divisible by x as desired. Alternatively, we see from inspection that $p(0) = a_0$ and so a_0 is 0 when $p(0) - 0$. Since all other terms of $p(x)$ have at kleast one power of x, we can conclude that $p(x)$ is evenly divisible by x. This argument is quicker than the preceding but does not generalize as readily to the other parts of the problem.

DCAS-Like Answer	Next-Generation Solution
	b. If 1 is a root of the function p, this means that $p(1) = 0$. If we divide $p(x)$ by $x - 1$, using long division of polynomials, we will find $$p(x) = q(x)(x - 1) + s$$ where $q(x)$ is a polynomial of degree $d - 1$ and the remainder s is a number. Plugging in $x = 1$, we find $$\begin{aligned} 0 &= p(1) \\ &= q(1)(1 - 1) + s \\ &= s \end{aligned}$$ Thus, $s = 0$ and so $p(x) = (x - 1)q(x)$ and $p(x)$ is evenly divisible by $(x - 1)$ as desired. c. If r is a real number which is a root of p, we have $p(r) = 0$. Performing long division, as in part b., this time dividing $p(x)$ by $x - r$ gives $$p(x) = a(x)(x - 4) + s$$ where $a(x)$ is polynomial of degree $d - 1$ and s is a real number. Plugging in r, we find $$p(r) = a(r)(0) + s$$ Since $p(r) = 0$, we conclude that $s = 0$ and so $x - r$ divides f evenly.

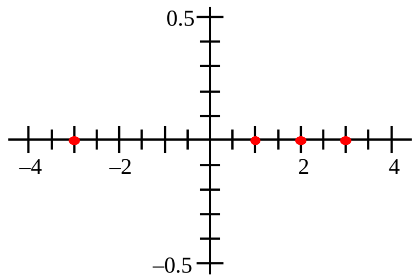
<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td></td><td>
d. If there were $d + 1$ different real numbers $r_1, \ldots, r_{d+1}$ which are all roots of p, we need to apply the argument of item c. $d + 1$ times and will find that this is not possible because it would give too many factors, of the form $x - r_i$, of p.

Concretely, applying item c. to r_1 we find
$$p(x) - p_1(x)(x - r_1)$$
where $p_1(x)$ has degree $d - 1$. Now, evaluating r_2 gives
$$0 = p(r_2) = p_1(r_2)(r_2 - r_1).$$
Since r_1 and r_2 are distinct, this means that $r_2 - r_1 \neq 0$, so we must have $p_1(r_2) = 0$.

Repeating the above argument with $p_1(x)$ in place of $p(x)$, we conclude that
$$p_1(x) = p_2(x)(x - r_2)$$
where $p_2(x)$ is a polynomial of degree $d - 2$. So we have
$$p(x) = p_2(x)(x - r_1)(x - r_2).$$
Continuing the same way we find
$$p(x) = a_d(x - r_1)(x - r_2) \ldots (x - r_d).$$
But now we see that when we plug in r_{d-1}, we get
$$p(r_{d+1}) = a_d(r_{d+1} - r_1)(r_{d+1} - r_2) \ldots (r_{d+1} - r_d)$$
and this is not zero because a_d is not zero and r_{d+1} is distinct from the other roots $r_1, \ldots, r_d$.
</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
8A: D (9-11.A.APR.3)	**8B:**

First, factor the polynomial completely to find:
$f(x) = (x-3)(x-2)(x-1)(x+3)$ and set this equal to zero: $(x-3)(x-2)(x-1)(x+3) = 0$.
Solving the equation, we find the roots are $x = 3, 2, 1, -3$. The diagram below shows these points marked on the x-axis.



DCAS-Like Answer	Next-Generation Solution
9A: B (9-11.A.APR.4)	**9B:**

a. To investigate the conjecture, we can construct a table in which we choose two integers m and n, and look at the sum of the squares, the difference of the squares, and twice the product of m and n:

m	n	m^2+n^2	m^2-n^2	$2mn$
2	1	5	3	4
2	2	8	0	8
3	1	10	8	6
3	2	13	5	12
4	1	17	15	8

In most cases, the trick seems to work; the triples (3, 4, 5), (6, 8, 10), and others given in the table are familiar Pythagorean triples. However, when $m = n = 2$, we end up with a triple containing the number zero. Since we cannot have a triangle with a side of length zero, the trick does not always work. However, we suspect that it might work as long as we make sure the three numbers generated are all positive.

DCAS-Like Answer	Next-Generation Solution
	b. In order to ensure that the three numbers generated are positive, we restrict to the case in which m and n are positive integers, and $m > n$. Also, we change the language of the conjecture slightly to reflect that the numbers are *lengths* of the sides of a right triangle, not the sides themselves: *Suppose that m and n are positive integers such that $m > n$. Then, the numbers $m^2 + n^2$, $m^2 - n^2$, and $2mn$ are the lengths of the sides of a right triangle.* To prove this, it suffices to prove that when these three numbers are squared, one square is the sum of the other two. In our table, it appeared that $m^2 + n^2$ was always the largest of the three numbers, so we conjecture that $$(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2.$$ Expanding the left side, we obtain $$(m^2 + n^2)^2 = m^4 + 2m^2n^2 + n^4.$$ Expanding the right side, we obtain $$\begin{aligned}(m^2 - n^2)^2 + (2mn)^2 &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4.\end{aligned}$$ Because these two expressions are identical, we have proven that $$(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2.$$ Therefore, of the three numbers generated, the square of one is always the sum of the squares of the other two. Furthermore, we know that all three numbers are positive. In particular, $2mn$ is positive because both m and n are positive; and $m^2 - n^2$ is positive because $m > n > 0$. Therefore, by the converse of the Pythagorean Theorem, the three numbers are the lengths of the sides of a right triangle. c. To find a triangle satisfying the given requirements, we try using Trina's trick on the integers $m = 13$ and $n = 6$. We get $m^2 + n^2 = 205$, $m^2 - n^2 = 133$, and $2mn = 156$. By the reasoning given above, we know that there is a right triangle with sides of lent 133, 156, and 205. Furthermore, the numbers 133, 156, and 205 have not common factors.

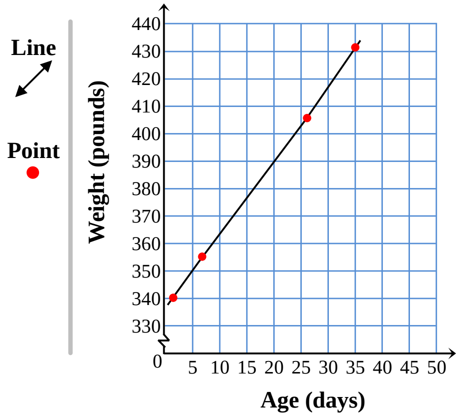
DCAS-Like Answer	Next-Generation Solution
10A: B (9-11.A.APR.6)	**10B:** a. For $x = 29.0$ mpg and $y = 39.0$ mpg, compute that combined fuel economy $= \frac{2(29.0)(39.0)}{29.0+39.0} \approx 33.265$. To three significant digits, this is 33.3 mpg. We note that this exercise is an opportunity to pay close attention to units, especially since the units of a harmonic mean of two quantities are not immediately obvious. b. For $y = x + 10$, we have combined fuel economy $= \frac{2x(x+10)}{2x+10} = \frac{2x(x+10)}{2(x+5)} = \frac{x(x+10)}{(x+5)}$. c. A student might calculate this reduction using long division, synthetic division, or grouping. For any method, we have $\frac{x(x+10)}{(x+5)} = x + 5 - \frac{25}{x+5}$ d. When x is large, $\frac{25}{(x+5)}$ is small. In particular, when $x > 20$, this term is less than 1 so the approximation of $x + 5$ is within 1 mpg of the correct value of the combined fuel economy.

Creating Equations (A.CED)

DCAS-Like Answer	**Next-Generation Solution**
11A: D (9-11.A.CED.1)	**11B:** a. We want to find the value of y. We are given $B = 100{,}000{,}000$, $w = 300$, $x = 5$, and $z = 15{,}000{,}000$. So, the equation is $100{,}000{,}000 = 5 \bullet 15{,}000{,}000 + 300y$ $100{,}000{,}000 - 75{,}000{,}000 = 300y$ $25{,}000{,}000 = 300y$ $250{,}000 = 3y$ $y = 83{,}333.3$ b. We want to find the value of z. We are given that $x = 3$, $y = 10{,}000$, $w = 500$, and $B = 50{,}000{,}000$. So, the equation is $50{,}000{,}000 = 3z + 100{,}000 \bullet 500$, OR $50{,}000{,}000 = 3z + 5{,}000{,}000$ $z = 15{,}000{,}000$ c. We want to find the value of w. We are given that $x = 20$, $y = 15{,}000$, $B = 90{,}000{,}000$, and $z = 100{,}000w$. So, the equation is $90{,}000{,}000 = 20(100{,}000w) + 15{,}000w$, which simplifies to $90{,}000{,}000 = 2{,}015{,}000w$ $w = 44.67$

DCAS-Like Answer	Next-Generation Solution
12A: D (9-11.A.CED.2)	**12B:** 1. 7-in. by 9-in. $y = x + 20$ 8-in. by 11-in. $y = x + 20$ 12-in. by 12-in. $y = 1.5x + 45$ 2. $57 3. 44 pages

DCAS-Like Answer	Next-Generation Solution
	Scoring Rubric Responses to this item will receive 0-3 points based on the following: **3 points:** The student has a solid understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model a real-life situation and uses the equations to find answers to questions within a context. The student correctly writes all three cost equations in question 1, and uses the appropriate equations from question a., or equivalent equations, to solve for the unknown cost in question 2 and the number of book pages in question 3. **2 points:** The student demonstrates some understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model the real-life situation in question 1, but does not write correct equations for all three cases. The student, however, demonstrates understanding of how to use the equations to find answers to questions within context. The answers for questions 2 and 3 represent correct calculations that may or may not use incorrect equation(s), or equivalent equations, written for question 1. **1 point:** The student has basic understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student writes equations to model a real-life situation for question 1, with one or more equations containing errors. The student demonstrates partial understanding of how to use the equations to find answers to questions within context. The answers for either question 2 or 3 represent an incorrect calculation using the equations, or equivalent equations, written for question 1. **0 points:** The student demonstrates inconsistent understanding of how to make productive use of knowledge and problem-solving strategies to solve a problem arising in everyday life. The student is unable to write any correct equation for question 1. The answers to both questions 2 and 3 are incorrect calculations using the equations, or equivalent equations, written for question 1.

DCAS-Like Answer	Next-Generation Solution
13A: A (9-11.A.CED.2)	**13B:** 1.  2. $y = 2.76x + 335$ 3. 523 pounds 4. Actual weight was 504 pounds. The model predicted too great a weight, so the slope should be decreased. 5. As the baby elephant got older, it grew a little more slowly on average than it did during the first 35 days.
14A: D (9-11.A.CED.3)	**14B:** a. We see that a hectares of Arabica will yield $750a$ kg of beans, and that r hectares of Robusta will yield $1200r$ kg of beans. So, the constraint equation is: $750a + 1200r = 1{,}000{,}000$ b. We know that a hectares of Arabica yield $750a$ kg of beans worth \$1.42/kg for a total dollar value of $1.42(750a) = 1065a$. Likewise, r hectares of Robusta yield $1200r$ kg of beans worth \$0.73/kg for a total dollar value of $0.73(1200r) = 876r$. So, the equation governing the possible values of a and r coming from the total market value of the coffee is: $1065a + 876r = 1{,}000{,}000$

DCAS-Like Answer	Next-Generation Solution
15A: B (9-11.A.CED.4)	**15B:** *Sample Top Score Response* ▪ $h^2 + \frac{1}{2}bh = 80$ ▪ $h^2 + \frac{1}{2}bh + \frac{1}{16}b^2 = 80 + \frac{1}{16}b^2$ ▪ $\left(h + \frac{1}{4}b\right)^2 = 80 + \frac{1}{16}b^2$ ▪ $h + \frac{1}{4}b = \sqrt{80 + \frac{1}{16}b^2}$ ▪ $h = \sqrt{80 + \frac{1}{16}b^2} - \frac{1}{4}b$ *Scoring Rubric* Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes productive use of knowledge and problem-solving strategies by correctly rearranging a formula to highlight a quantity of interest. **1 point:** The student demonstrates some understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes one or two minor errors in computation, such as combining a set of terms incorrectly when completing the square. **0 points:** The student demonstrates inconsistent understanding of how to solve problems by using the structure of an expression to find ways to rewrite it. The student makes little or no use of knowledge or problem-solving strategies and does not attempt to complete the square when rearranging the formula.

Reasoning with Equations and Inequalities (A.REI)

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>16A: B
(9-11.A.REI.1)</td><td>16B:
<table>
<tr><th>Solution Steps</th><th>Process</th><th>Purpose</th></tr>
<tr><td>$0.25(y-2x)+4=12$</td><td>Given</td><td></td></tr>
<tr><td>$0.25(y-2x)=8$</td><td>Subtract 4 from each side</td><td>Combine like terms</td></tr>
<tr><td>$y-2x=32$</td><td>Multiply both sides by 4</td><td>Remove the parenthesis</td></tr>
<tr><td>$y=2x+32$</td><td>Add $2x$ to both sides</td><td>Isolate y</td></tr>
</table></td></tr>
<tr><td>17A: C
(9-11.A.REI.2)</td><td>17B:
a. $\sqrt{2x+1}-5 = -2$
$\left(\sqrt{2x+1}\right)^2 = (3)^2$
$2x+1 = 9$
$2x = 8$
$x = 4$
b. $\sqrt{2x+1}+5 = -2$
$\left(\sqrt{2x+1}\right)^2 = 7$
$2x+1 = 49$
$2x = 48$
$x = 24$
c. No
d. Yes
e. No
f. No</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
18A: A (9-11.A.REI.3)	**18B:** 1. F – Students that match this inequality correctly have demonstrated an understanding of how the inequality symbol is affected when dividing by a negative number. 2. B – Students that match this inequality correctly have demonstrated an understanding of how to apply the distributive property when solving multi-step problems. 3. F – Students that match this inequality correctly have demonstrated an understanding of how to solve inequalities with variable terms on both sides.
19A: C (9-11.A.REI.4)	**19B:** *Solution to* $(3x-2)^2 = 6x - 4$ $9x^2 - 12x + 4 = 6x - 4$ $9x^2 - 18x + 8 = 0$ $(3x-2)(3x-4)$ $x = \frac{2}{3} \quad x = \frac{4}{3}$ The given equation is quadratic equation with two solutions. The task does not clue the student that the equation is quadratic or that it has two solutions. Students must recognize the nature of the equation from it structure. *Scoring* Full credit requires entering both correct solutions. Partial credit could be given for entering a single correction solution—for example, a student might divide both sides of the given equation by $3x - 2$ to obtain $3x - 2 = 2$, concluding that $x = \frac{4}{3}$, but forgetting to analyze the remaining possibility $3x - 2 = 0$.

DCAS-Like Answer	Next-Generation Solution
20A: D (9-11.A.REI.5)	**20B:** *Solutions* If one assumes that ten boomerangs are made, then the following table of possibilities may be made. The constraint on carving hours is broken when more than four large boomerangs are made.

Number of Small	Profit Made
10	80
9	82
8	84
7	86
6	88
5	90

x = small boomerangs

y = large boomerangs

Time to carve: $2x + 3y = 24$

Can only decorate ten: $x + y = 10$

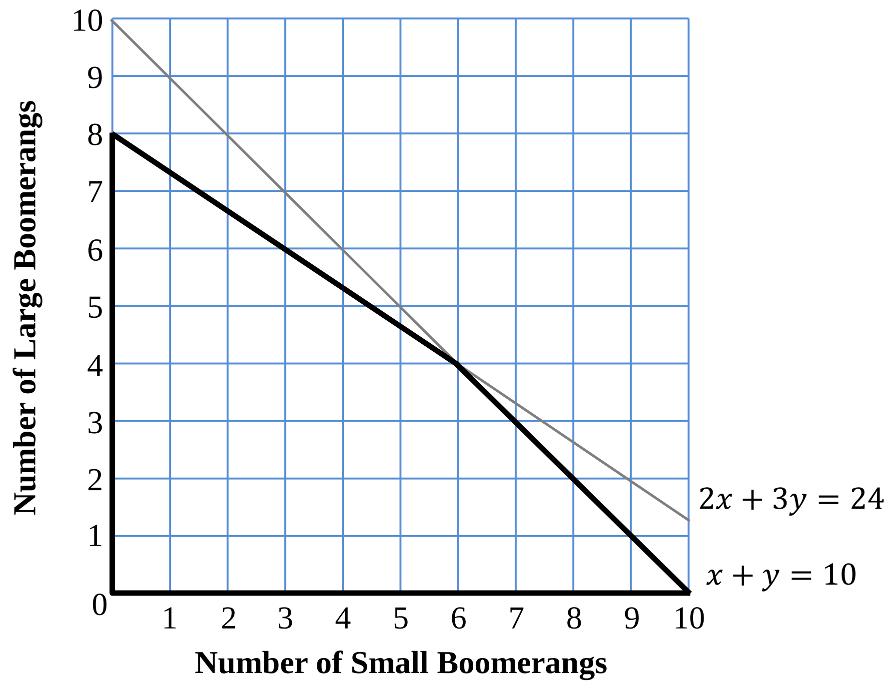
Solve system of equations

a. $x + y = 10$
$y = 10 - x$

b. $2x + 3y = 24$
$2x + 3(10 - x) = 24$
$2x + 30 - 3x = 24$
$-x = -6$
$x = 6$

If $x + y = 10$
$6 + y = 10$
$y = 4$

Phil and Cath should make 6 small and 4 large boomerangs.

DCAS-Like Answer	Next-Generation Solution
	This approach, however, does not include the possibility of making fewer than ten boomerangs. A more complete approach would be to draw a graph showing all possibilities.  The possible combinations to be checked are the integer points within the bold region on the graph. The maximum profit occurs, however, when six small and four large boomerangs are made. This profit is \$88. (This can be seen graphically by drawing lines of constant profit on the graph, e.g., $8x + 10y = 80$. This idea may emerge in discussion.)
21A: D (9-11.A.REI.6)	**21B:** Vegetarian is \$7 Chicken is \$8

DCAS-Like Answer	Next-Generation Solution
22A: B (9-11.A.REI.7)	**22B:** The solution that is clearly identifiable from the graph, the point at which the circle and line intersect, is $(0, 1)$. From the graph, we can see that there is another solution (in Quadrant I). However, it is difficult to visually determine its exact x- and y-coordinates. To find its exact location we can solve the system of equations by substitution. Let (x, y) be the intersection point. Since $y = 2x - 1$ by virtue of the point being on the line, we can substitute the quantity $(2x - 1)$ for every y appearing in the equation of the circle. $x = \frac{4}{5}$ If $x = 0$, we know $y = -1$, so we have re-discovered the first intersection point we observed. So, our second intersection point has x-coordinate equal to $\frac{4}{5}$, and we are left only having to now find its y-coordinate. We simply substitute $x = \frac{4}{5}$ in either equation and solve for y. $y = \frac{3}{5}$ Now we have that $\frac{4}{5}, \frac{3}{5}$ is also a solution.
23A: C (9-11.A.REI.10)	**23B:** *Key and Distractor Analysis* a. Confuses $y = x^2 - 4$ with $y = \sqrt{x - 4}$. b. Key c. Relates the point $(0, 4)$ on this graph to the 4 under the radicand of the given function. d. Confuses radical function with linear function.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td>24A: B
(9-11.A.REI.11)</td><td>

24B:

Solution

distance = rate × time

time/ hour	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**8.25**
Car 1 distance	0	35	70	105	140	175	210	245	280	288.75
Car 2 distance	0	0	0	0	55	110	165	220	275	288.75

Car 1: $r_1 = 35\ mph$
$t_1 = t_1$
$d_1 = r_1 t_1$
$d_1 = 35t_1$
Car 2: $r_2 = 55\ mph$
$t_2 = t_1 - 3$
$d_2 = 55(t_1 - 3)$

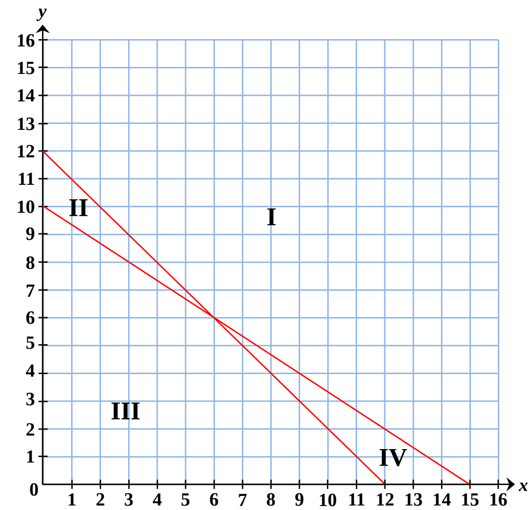
Since the distance both cars traveled is the same, $d_1 = d_2$

$d_1 = d_2$
$35t_1 = 55(t_1 - 3)$
$35t_1 = 55t_1 - 165$
$20t_1 = 165$
$t_1 = 8.25$ hours
$t_2 = t_1 - 3$
$t_2 = 8.25 - 3$
$t_2 = 5.25$ hours

It took 5.25 hours for the second car to overtake the first.
</td></tr>
</table>

DCAS-Like Answer	Next-Generation Solution
25A: B (9-11.A.REI.12)	**25B:** *Key:* G and J only *Scoring Rubric for Multi-Part Items* Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies the two correct points, G and J. The student also recognizes that points that lie in the excluded boundary or on only one of two inequalities are not solutions. **1 point:** The student has only a basic understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies the two correct points, G and J, but does not recognize that points that lie in the excluded boundary or on only one of two inequalities are not solutions and may select points A and/or I as well. **0 points:** The student demonstrates inconsistent understanding of how to determine whether a set of given points is part of the solution to a system of linear inequalities. The student identifies no correct points or only one correct point. The student also does not recognize that points that lie in the excluded boundary or on only one of two inequalities are not solutions.

DCAS-Like Answer	Next-Generation Solution
26A: D (9-11.A.REI.12)	**26B:** *Scoring Rubric* Responses to this item will receive 0-2 points based on the following: **2 points:** The student has a solid understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities and identifies the correct region that represents the solution to the system, region IV. **1 point:** The student has some understanding of how to solve a system of inequalities graphically. The student correctly graphs both inequalities but does not identify the correct region that represents the solution to the system. OR The student incorrectly graphs one or both inequalities but identifies the correct region that represents the solution to the incorrectly graphed system. **0 points:** The student demonstrates inconsistent understanding of how to solve a system of inequalities graphically. The student does not correctly graph both inequalities and/or does not identify the correct region that represents the solution to the system.

<table>
<tr><th>DCAS-Like Answer</th><th>Next-Generation Solution</th></tr>
<tr><td></td><td>


Scoring Rule Explanation

Based on the scoring rule and the scoring data for this particular item, students that create two lines representing $y = -x + 12$ and $y = \frac{-2}{3x} + 10$ and select the section of the plane represented by the intersection of $y \geq -x + 12$ and $y \leq \frac{-2}{3x} + 10$ (IV above) will receive 1 point. All other responses will receive 0 points.
</td></tr>
</table>