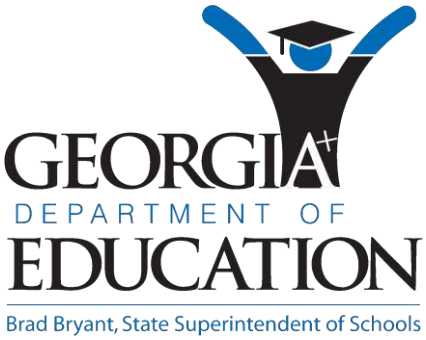

Georgia Department of Education

Secondary Math Consortium 1

November 2010

Georgia Department of Education
Secondary Math Consortium I – November 2010

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Participants Agenda
Standards Based Classroom – Instructional Framework

- Norms – "working toward solutions", introductions
- Goals – discuss article "Never Say Anything a Kid Can Say" – highlighting three gold nuggets
- Instructional Framework and the Standards-based Mathematics Classroom
- Work Math III Unit 3 task
- Share out – strategies – lesson planning
- Preview Unit 3 for Math I, II, III
- GHSGT – Make connections from Math I– Math II– Math III
- Closing – reflections
- Next Steps – questions and concerns about your role as a math coach

TED: Ideas Worth Spreading

Dan Myers – Math Class Needs a Makeover (May 2010)

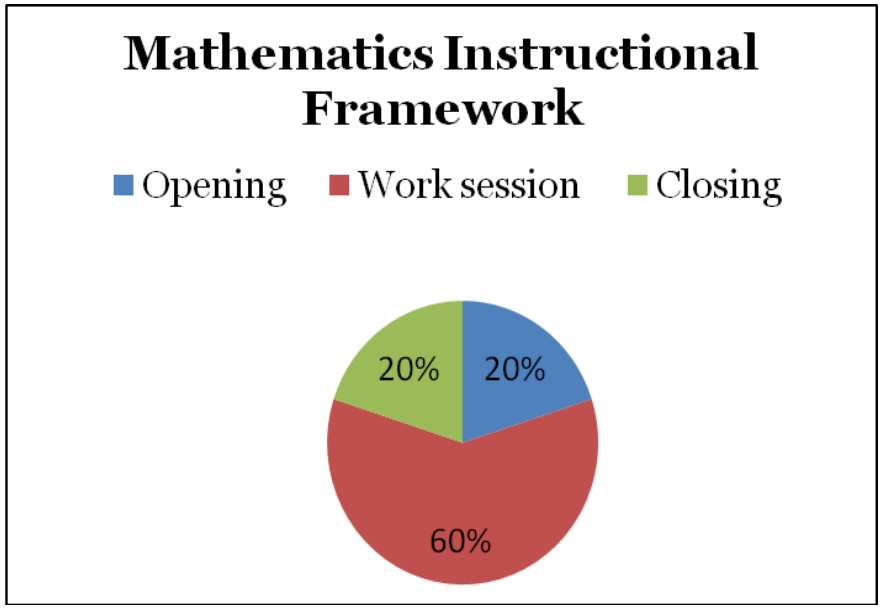
http://www.ted.com/talks/dan_meyer_math_curriculum_makeover.html

Five Problems with Teaching Math

1. Lack of initiative
2. Lack of perseverance
3. Lack of retention
4. Aversion to word problems
5. Eagerness for formula

Georgia Department of Education
Secondary Math Consortium I – November 2010

Mathematics Instructional Framework



Opening:

Discussion of standard(s), element(s), and E. Q.

Explicit instruction/activity aligned to the standards and/or elements that include a balance of:

- **Skills**
- **Conceptual understanding**
- **Problem solving**

Modeling:

- **Routines**
- **Problem-solving strategies**
- **Mathematical vocabulary**

Sets performance goals and expectations for the work session

Work Session:

Teacher:

- **Implements flexible grouping**
- **Allows student struggle time**
- **Assesses student understanding of the standards**
- **Facilitates by providing appropriate hints and asking questions**
- **Provides rigorous work**
- **Provides feedback**

Students:

- **Engage in performance tasks**
- **Collaborate with peers**
- **Communicate using LOTS**

Closing:

Teacher:

- **Selects students to share solutions**
- **Explicitly clarifies misconceptions**
- **Informally assesses student understanding**
- **Identifies future problems for adjustments in lessons and interventions**
- **Celebrates progress towards meeting standards**

Students:

- **Justify and explain approaches for solving problems**
- **Ask questions**
- **Use mathematical vocabulary, LOTS**
- **Summarize the main concepts for the day and link concepts to the standards**

Three-Part Lesson Format

Opening (10-15 minutes)	Teacher Focus	Student Focus
20% of Instructional Time - Teacher Focused		
▪ Mini review ▪ Clarify lesson ▪ Clarify questions ▪ Discuss standards and elements and E.Q. (including vocabulary) of the work using the Language of the Standards - LOTS ▪ Communicate expectations of the work ▪ Set stage for learning ▪ Provide feedback	▪ Reviews GPS standard(s) and element(s) and E. Q. using LOTS ▪ Makes clear what students will learn ▪ Clarifies and/or defines vocabulary terms ▪ Delivers mini-lesson to model strategies and/or scaffold instruction that connects to the work period ▪ Begins with a simple version of the work session task to get students engaged ▪ Makes a link to previous lessons or to work in other subject areas ▪ Demonstrates and explains ideas and methods using technology, SMARTBoard, Airliner, ActivBoard, overhead, etc. ▪ Identifies student weaknesses and misconceptions to be addressed in the work session	▪ Reads/writes standard(s) and element(s) and E. Q. ▪ Restates problem in own words ▪ Completes problems and discusses how arrived at the answer ▪ Shares thinking about problems using LOTS ▪ Responds to learning questions using LOTS ▪ Records focus of lesson in notebook ▪ Listens, takes notes on teacher demonstration or mini-lesson ▪ Completes graphic organizers to make connections for learning
Work Session (40-45 minutes)	**Teacher Focus**	**Student Focus**
60% of Instructional Time – Student Focused	Listen carefully, provide hints, observe and assess, facilitate	Communicate, read/write, defend/justify/explain answers
▪ Activity period ▪ Conferences and questions ▪ Provides feedback, written and oral using LOTS (Language of the Standards) ▪ Leads small group guidance, facilitates ▪ Connects standard/elements to work ▪ Prepares for sharing	▪ Provides instruction by using differentiated instruction and best practices strategies ▪ Allows students to share ideas before providing a suggestion ▪ Constructs questions based on student's thinking ▪ Asks questions or gives hints that do not suggest the student's answer is right or wrong in order to continue discussion/dialogue ▪ Allows for practice time ▪ Monitors/facilitates student work groups ▪ Conferences individually with students about work, goals, revisions	▪ Identifies strategies for solving problems ▪ Discusses different solutions and ways to approach solving problems ▪ Works individually, in pairs, or small groups on assigned task to practice content learned in mini-lesson ▪ Self assesses work and how work connects to standards ▪ Completes group tasks , begins preparing for sharing ▪ Participates in group presentation ▪ Reflects on good work

The Closing (10-15 minutes)	Teacher Focus	Student Focus
20% of Instructional Time – Student & Teacher Focused	Make the connection	Understand the connection
▪ Summary ▪ Share work and strategies ▪ Connect work to standards and/or elements ▪ Identify misconceptions ▪ Clarify learning goals ▪ Assess students understanding ▪ Provide feedback	▪ Connects learning to the standard(s) and/or element(s) ▪ *Can my students answer the Essential Question?* ▪ Summarizes the learning experience ▪ Gives feedback for students to successfully meet the standards ▪ Assigns follow-up work ▪ Provides time for students to record homework in agenda	▪ Reviews standards and E.Q. and understand the connections made in class to answer the questions: *Does my work meet standards? How?* ▪ Shares work with the class and receives feedback ▪ Summarizes steps of the strategy or process ▪ Writes homework in agenda

What is a Warm-Up?

A warm-up:

- Begins **immediately** when students enter the classroom
- Is an activity that takes 2-3 minutes maximum
- Is a part of the daily routine and ritual for each class
- Requires no teaching or explanation from teacher
- Connects to the lesson that is to follow
- Allows time for the teacher to take attendance
- The warm-up is part of the **OPENING**.
    - It should be the **bridge from yesterday to today**.
    - It is the preparation for what is to come.

Some examples of what a warm-up IS:

- Draw a timeline that extends from negative 10 to positive 10
- Give a practice question from the GHSGT or EOCT (*only if related to that day's standard/element*)
- Write the steps (formula) to a problem on the board
- Read paragraph/section and be ready to discuss/answer the following question: _____
- Write two things you learned yesterday and one thing you still don't understand (ticket out the door)

Some examples of what a warm-up IS NOT:

- Requires assigning the task as homework because it is too long and complicated to complete during the warm-up
- Copying standards from the board
- Looking up words and definitions in a dictionary
- Unrelated to the lesson that will follow
- Requires students to get up to get another book
- Requires directions from teacher

THE TEACHER HAS ESTABLISHED THE FOCUS FOR LEARNING

What should we see in a standards-based mathematics classroom?

- Warm-up
  -connected to the lesson
  -starts students thinking in right direction

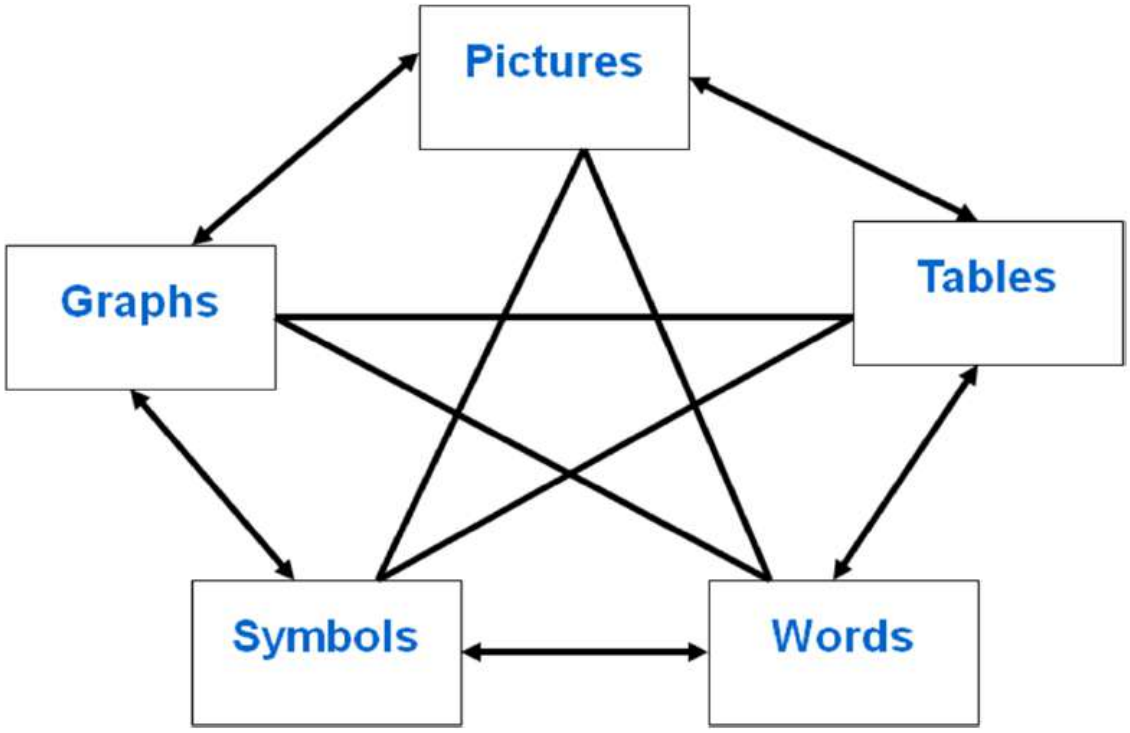
- Mini lesson, opening, setting the stage
  -checks for prior knowledge
  -reviews needed skills
  -left in view for quick access during work period

- Work period, Activity period
  -rigorous mathematics
  -use of previously learned concepts in service of new ideas
  -collaboration and verbalization
  -process skills
  -individual accountability

- Summary, Closing
  -presentation and comparison of different approaches
  -students commenting on and questioning the approaches of other students
  -teacher guiding the discussion, if necessary, to solidify concepts, skills and procedures to be learned
  -clarifying of misconceptions

Multiple Representations

Multiple Representations



Instructional programs from prekindergarten through grade 12 should enable all students to:

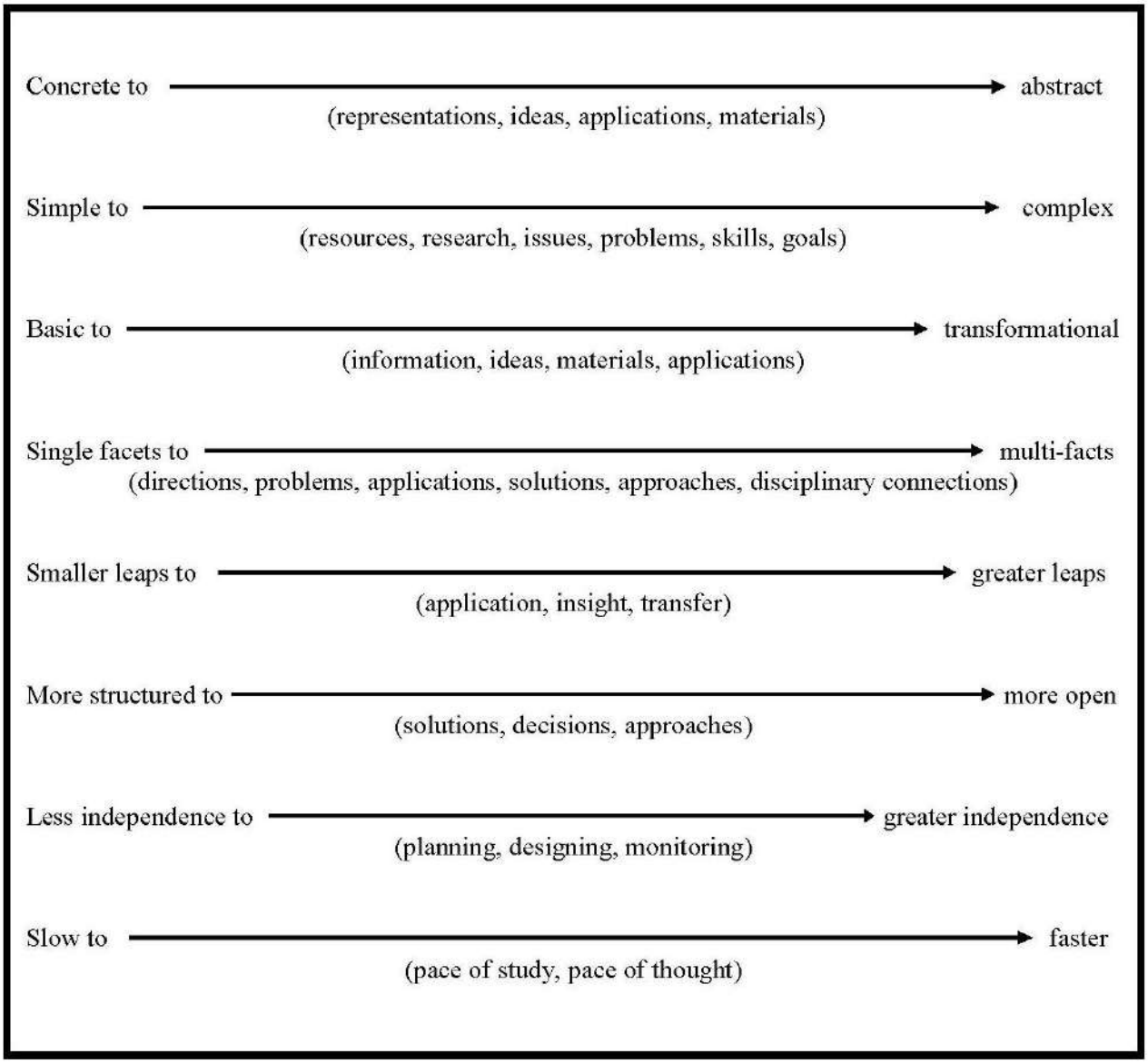
- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

If mathematics is the "science of patterns" (Steen 1988), representations are the means by which those patterns are recorded and analyzed. As students become mathematically sophisticated, they develop an increasingly large repertoire of mathematical representations and the knowledge of how to use them productively. This knowledge includes choosing specific representations in order to gain particular insights or achieve particular ends.

In grades 9–12, students' knowledge and use of representations should expand in scope and complexity. As they study new content, for example, students will encounter many new representations for mathematical concepts. They will need to be able to convert flexibly among these representations. Much of the power of mathematics comes from being able to view and operate on objects from different perspectives.

National Council of Teachers of Mathematics
Principles and Standards for School Mathematics (Representation Standards for Grades 9-12)

The Equalizer



Tomlinson

Math I Unit 3 Overview

(based on APS unit)

Unit 3 Geometry Gallery	Standards & elements MMIG3a,b,c,d,e	Math concepts/topics	Pre-requisites	Vocabulary
Formal language of reasoning and justification (communicating mathematical ideas rather than rote production of two-column proofs); Explore angles (interior & exterior sums); Triangle inequalities (including side-angle and exterior-angle inequalities); Triangle congruencies – focus is on the minimum info needed (SSS, SAS, ASA, AAS, HL); Points of concurrency (incenter, orthocenter, circumcenter, centroid); *Building on the geometry from grades 6-8--Tasks begin with the concrete to build understanding of the more abstract*				
Robotic Gallery Guards	**MM1G3a**	convex polygons, measuring angles using a protractor, measurement error, exterior angles, supplementary angles, regular polygons, conjecture and proof, Exterior Angle Sum Theorem, sums of the measures of the interior angels of a convex polygon, measure of each interior angle of a regular polygon/n-gon, measure of each exterior angle of a regular polygon/n-gon, algebraic proofs of geometric properties	using protractors to measure angles, supplementary angles, extending patterns, convex polygons, measurement error, number of degrees in triangle number,	exterior angles, interior angles
	Sum of interior and exterior angles of polygons Looking for patterns, define relationship between # sides, angles, sum of interior and exterior angles of convex polygons (formulas define relationships) Make conjectures about polygons given partial information about angles, sides			

Poor Captain Robot	**MM1G3b** Triangle inequality	triangle inequality, measures of interior angles of a triangle, measures of exterior angles of a triangle, remote interior angles, the relationship between the measures of the angles of a triangle and the lengths of the sides (side-angle inequality), the relationship between an exterior angle of a triangle and its two remote interior angles (exterior angle inequality)	using protractors to measure angles, supplementary angles, number of degrees in triangle	Triangle inequality, side-angle inequality, remote interior angles, exterior angle inequality
	Relationship between the sides and angles of any triangle; relationship between exterior and interior angles of a triangle--Triangle inequality, side-angle inequality, exterior-angle inequality			
Constructing Pennants	**MM1G3c** (SSS, ASA, SAS, AAS, HL, LL)	similarity; congruence; corresponding parts of congruent and similar figures; counterexample; investigating whether congruence of triangles can be determined given 1 or 2 pieces of information; SSS, ASA, SAS, AAS, HL, LL postulates and theorem, proof using the congruence postulates	similarity, congruence, corresponding parts of congruent and similar figures, right triangles, Pythagorean Theorem	postulate, theorem

	What is the minimum information you need to determine if triangles are congruent? Congruence postulates and theorems Introduce formal proof (algebraic & geometric) as an intro to this task. Constructing Diagonals will capitalize on "proofing."			
Constructing w Diagonals	**MM1G3d** Quadrilaterals	attributes of special quadrilaterals; use of Euclidean tools; constructions; properties of special quadrilaterals; proof, including paragraph proofs, flow proofs, and two column proofs, necessary conditions, sufficient conditions	attributes of special quadrilaterals, measuring with protractors, use of Euclidean tools, constructions	
	What minimum conditions prove the identity of a quadrilateral? Relationships between sides, angles and diagonals of quadrilaterals—organizer to summarize properties is provided Constructions, prove conjectures (two-column, flow chart, paragraph)			
Middles & Halves	**MM1G3e** Precursor to points of concurrency in triangles	classification of triangles by sides and angles; definitions of angle bisectors, perpendicular bisectors, altitudes, and medians; basic constructions, including angle bisectors, perpendicular bisector, altitudes and medians; special properties of isosceles and equilateral triangles; using geometric tools; proof	classification of triangles by sides and angles; basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisector; definitions of angle bisectors, perpendicular bisectors, and altitudes; special properties of isosceles	median (of a triangle)

			and equilateral triangles	
	Basic constructions (using a variety of tools and technology) to explore relationships between altitudes, medians, angle bisectors, perpendicular bisectors Builds for understanding of points of concurrency in Centers of Triangles task			
Centers of Triangles	**MM1G3e** Points of concurrency: incenter, orthocenter, circumcenter, and centroid	basic constructions, including angle bisectors, perpendicular bisector, medians, and altitudes; discovering points of concurrency and names, significance and applications	basic constructions, including angle bisectors, perpendicular lines, and perpendicular bisectors; definitions of angle bisectors, perpendicular bisectors, and altitudes	incenter, orthocenter, circumcenter, and centroid
	Builds on the Middle and Halves task Basic constructions (using a variety of tools and technology) to explore relationships between altitudes, medians, angle bisectors, perpendicular bisectors to discover points of concurrency—to explore relationships between the points and the type of triangles Emphasis should be placed on student conjectures on the significance of each point of concurrency and relevance in decision-making			

Math II Unit 3 Overview

(based on APS unit)

Unit 3 Circles and Spheres	**Standards & Elements MM2G3; MM2G4**	**Math concepts/topics**	**Pre-requisites**	**Vocabulary**
Properties of circles and spheres Unlike other units of the Atlanta Public Schools Curriculum Supplement, the tasks presented here vary significantly from those in the GaDOE Framework. Students are asked to prove the theorems being discovered. There is obviously not time for students to develop a formal proof, be it two column, paragraph or flowchart, for every theorem included in the unit. It is up to individual teachers to decide which theorems are to be proven and which are justified in a more informal manner. We do feel it is imperative that students prove enough theorems, particularly those in the first two tasks, to understand that properties of circles are simply applications of triangle congruence and similarity.				
Relationships among Central Angles, Arcs, and Chords	**MM2G3a,b,d**	central angle, arc, major arc, minor arc, semicircle, arc measure, arc length, congruent arcs, and chord; biconditional statements; converse statements	definitions of circle, radius, diameter, circumference; degree measure of a complete circle is 360°; biconditional statements; converse statements; triangle congruence postulates; basic constructions	central angle, arc, major arc, minor arc, semicircle, subtend, intercept, arc measure, arc length, chord
	Chords *Properties of circles are applications of triangle congruence and similarity *encourages students to keep a Circle Book			
Investigating Circle Relationships	**MM2G3a,b,d**	Inscribed angles, intercepted arcs, inscribed circles	using a protractor; exterior angle theorem; triangle congruence postulates; angle sums of a quadrilateral	Inscribed circle; inscribed polygon
	Inscribed angles			

Sunrise on the First Day of the New Year	**MM2G3a,b,d**	tangent line; secant line; relationships between the distance from the center of a circle to a line and the length of a radius of the circle; perpendicularity of a tangent to a radius of a circle at the point of tangency		tangent line, secant line
	Exploring the **measures of angles created by tangents, secants, and chords drawn to a circle** * the use of dynamic geometry software in the development and proof of properties of circles			
Investigating Lengths of Segments	**MM2G3a,b,d**	segments formed by intersecting chords, intersecting secants, intersecting tangents, and a tangent intersecting with a secant	proving triangles similar; ratios of lengths of corresponding sides of similar triangles are equal; proving triangles congruent	
	Explore the **relationships between the lengths of segments created by tangents, secants, and chords drawn to a circle** --Lengths of **segments formed by intersecting chords, intersecting secants, intersecting tangents, tangent intersecting with a secant** * use of dynamic geometry software to investigate relationships among lengths of segments formed by chords, secants, and tangents			
Finding Arc Length and the Area of a Sector	**MM2G3c**	finding the length of an arc as a portion of the circumference of a circle; finding the area of a sector as a portion of the area of a circle	Finding area and circumference of circles	arc length, sector of a circle, concentric circles

	Explore the relationship between arc length and area of sector --finding the length of an arc as a portion of the circumference of a circle; finding the area of a sector as a portion of the area of a circle			
Volume and Surface Area of a Sphere	**MM2G4a,b**	definitions of sphere, hemisphere, center of a sphere, radius of a sphere, great circle; deriving the formula for the volume and surface area of a sphere; finding the volume and surface area of a sphere given the radius; finding the radius of a sphere given the volume or surface area	definitions of center, radius, and diameter; volume and surface area of a right circular cylinder	sphere, hemisphere, center of a sphere, radius of a sphere, great circle
	derivation and application of formula for volume and surface area of a sphere; the effect of the change in the radius of a sphere on its surface area and volume			

Math III Unit 3 Overview

(based on GaDOE)

Unit 3 Exploring Exponentia	**Standards & elements** **MM3A2a,b,c,e,f,g** **[**related **MM3A1d** (analyze graphs of poly functions of higher degree)**;** **MM3A3b, d** (solve variety of equations & inequalities)**]**	**Math concepts/topics**	**Pre-requisites**	**Vocabulary**
This unit is <u>not</u> concerned with the properties of logarithms or with solving complex logarithmic equations. Rather, it focuses on building student conceptual understandings of logarithms. *This unit builds on students' understandings of inverse functions, exponential functions (including **e**), and transformations from Math 1 & Math 2. Before delving into logarithmic functions, it is important for students to understand nth roots and rational exponents, so the study of these topics is motivated in the first tasks. Review of the essential prerequisites is used as a way to motivate the study of logarithms, primarily common and natural logarithms.*				
Planet Exponentia	**MM3A2a, b** Reviews MM2A5			
	(Launch task) Reviews relationships between functions and their inverses in terms of domain and range, graphs, and equations. Begins to build the idea of rational exponents, as a new way for representing roots of expressions when determining the inverse of functions. While some of the other tasks use the same context as this launch, it is not necessary to have completed the launch task to complete other tasks in unit. This review was included because **it is extremely important for students to understand inverses to understand the way logarithmic functions are developed in the subsequent tasks**.			
How Long	**MM3A2a, b;** **MM3A2c, d, e, g** (only as related to exponential functions)			
	Students are expected to recall knowledge of writing exponential functions. Standards MM3A2c, d, e, and g will be addressed **although not fully developed**. Specifically, the need for finding the inverse of exponential expressions will be established (c), students will review characteristics of exponential functions (e), and students will explore phenomena related to exponential functions (element g).			
Population of Exponentia	**MM3A2c, e, f, g** Review MM2A2b, c, e (stressed in MM3A2e, f, g)			

	Students are expected to recall knowledge of exponential functions from Math 2. Establishes logarithmic functions as inverses of exponential functions (element c) and engages students in investigating and determining the characteristics of logarithmic functions (element e). Part 3 of the task familiarizes students with using logarithms, particularly common logarithms, to solve exponential equations. All of this is accomplished through the use of phenomena using exponential and logarithmic functions (element g). **Remember: This unit does not address the properties of logarithms. This is intentional and will be addressed in the next unit. The focus in this unit is on developing students' conceptual understanding of logarithms.**			
Modeling Natural Phenomena	**MM3A2f, g** Review ***e*** (MM2A2e; MM2A5c)			
	This task addresses transformations of graphs and exploring real phenomena related to exponential and logarithmic functions and the definition of the natural logarithm through problems concerning the natural exponential function. $f(x) = e^x$ and $f(x) = \ln x$ will be graphed. Some basic ideas of *e* will be reviewed. The task is primarily focused on graphing and applying logarithms to situations.			

TASKS - Things to Consider

Identify the standard(s) & element(s) - code and gist

What is the MATH in the task?

- What will students learn from doing the task?
- What are the knowledge and skills required to complete the task (minimal not wish list)?

What is key vocabulary?

As you experience the task, consider:

- Multiple representations
- How the task may need to be modified
- The challenges & misconceptions students may encounter
- How task could be differentiated
- Questions or concerns you may have

CCGPS Math III Unit 3, Task II – Student Edition

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 1 Student Task

In Math II you learned that an exponential function is any function in the form $f(x) = ab^x$, where a is any real number other than 0, and b is any positive number other than 1.

In this task we will revisit graphs of exponential functions and then investigate the inverses of these functions both graphically and algebraically.

1. For each of the 5 functions $f(x) = b^x$, where $b = \frac{1}{2}, \frac{3}{4}$, 2, 5, and $\frac{3}{2}$:

   a. Complete the table below by finding a y-value for each value of x.

x	y
–2	
–1	
$-\frac{1}{2}$	
0	
$\frac{1}{2}$	
1	
2	

   b. Use the points determined in *part a* to graph the function on graph paper.

2. Compare the 5 graphs in *Item 1* by discussing each of the following:

   a. domain and range

   b. intercepts

   c. asymptotes

   d. intervals of increase and decrease

3. Discuss how the value of the base b affects an exponential function.

4. Consider your graph of $f(x) = 2^x$. Describe how each of the functions listed below would transform the graph of f. Check your conjecture by graphing the functions using your graphing calculator. The points (-2, ¼) and (2, 4) are on the graph of $f(x) = 2^x$. Give the location of these points after each transformation.

   a. $g(x) = 2^{x-3}$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

   b. $j(x) = 3(2^x)$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

   c. $k(x) = 2^{-x}$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

   d. $l(x) = -2^x$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

   e. $m(x) = 2^x + 3$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

   f. $n(x) = 2^x - 1$
      transformation _____
      (- 2, ¼) → _____
      (2, 4) → _____

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 2 Student Task

5. In the next few items we will define the inverse of an exponential function. As preparation, describe the procedure for finding the inverse of a function and then list the characteristics of two functions that are inverses of each other.

6. Carefully trace the coordinate plane and your graph of the function $f(x) = 2^x$ onto a piece of patty paper.

   a. Draw the line $y = x$ on your graph.

   b. Fold the paper along the line $y = x$.

   c. Trace the curve, as you see it, on the outside of the paper.

   d. Open the paper and trace the outside curve on the inside so that you have both the graph of the exponential function and the graph of its inverse on the front of the patty paper.

   e. Using the graph on your patty paper, describe the following characteristics of the inverse of the exponential function.

   Domain: _____ Range: _____
   Asymptote: _____ Intercept(s): _____

The inverse of an exponential function is called a **logarithmic function.** In an exponential function, the input, or independent variable, is the exponent and the output, or dependent variable, is the value when the base is raised to that exponent. For example, in $f(x) = 3^x$, if $x = 4$, then $f(x) = 3^4 = 81$. Since the domains and ranges of inverse functions are interchanged, the independent variable in a logarithmic function is the value of a base raised to a given power and the dependent variable is the exponent on the base that gives that power.

To find the inverse of an exponential function, we use the same procedure we have used to find inverses of other functions- interchange x and y and then solve for y. Our first step is to start with $y = b^x$. Interchanging x and y we get the inverse $x = b^y$. In words, y is the power on b that gives x. The problem is that, to this point, you have had no notation that would allow you to express y symbolically. We will now introduce that notation. y is referred to as the "logarithim of x to the base b" and we solve $x = b^y$ for y by writing $y = \log_b x$. The formal definition is stated as follows:

$$y = \log_b x \text{ if and only if } b^y = x$$

Note that the "base" is the "base" in both expressions.

Look at a few examples:

- $10^2 = 100$ can be written as $\log_{10} 100 = 2$. Notice that the logarithm is the power on 10 that gives 100.
- $\log_4 64 = 3$ because $4^3 = 64$.
- Suppose $\log_x 144 = 2$. This is the same as saying $x^2 = 144$. Given that the base must be positive, the value of x is 12.

7. The following problems will give you some practice with logarithms. You should be able to work all parts of this item without a graphing calculator.
   a. Write each exponential equation as a logarithmic equation and each logarithmic equation as an exponential equation.
      i. $\log_{10}\left(\frac{1}{100}\right) = -2$
      ii. $5^3 = 125$
      iii. $\log_2 32 = 5$
   b. Evaluate each logarithm.
      i. $\log_{10}(0.1)$
      ii. $\log_3 81$
      iii. $\log_2\left(\frac{1}{16}\right)$
      iv. $\log_9 81$
   c. Write each logarithmic equation as an exponential equation and then solve for x.
      i. $\log_x 36 = 2$
      ii. $\log_4 4 = x$
      iii. $\log_7 1 = x$
      iv. $\log_8 x = 3$
      v. $\log_5(3x+1) = 2$
      vi. $\log_6(4x-7) = 0$

8. In this item we will examine the definition of the logarithmic function more closely. We have established that $y = b^x$ and $y = \log_b x$ are inverse functions. Remember that only one-to-one functions have inverses that are also functions and that the domains and ranges of inverse functions are interchanged.
   a. In defining the exponential function $y = b^x$, we stated that $b > 0$ and $b \neq 1$. Explain why these two facts must be true for $y = b^x$ to be an exponential function.

   b. If the base of the exponential function $y = b^x$ must be positive, what must be true about y (the range) of an exponential function. Explain.

   c. Use your responses to *parts a* and *b* to state any restrictions on the inverse of the exponential function, the function $y = \log_b x$.

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 2 Homework

Express each logarithmic equation as an exponential equation and solve.

1. $\log_2\left(\frac{1}{32}\right)=x$

2. $\log_3\left(x^2+x+3\right)=2$

3. $\log_x 5=\frac{1}{3}$

4. $\log_x 8=\frac{-3}{4}$

5. The time t, in seconds, at which the current in a circuit is I amperes can be calculated using the formula $t=-\log_2 I$. After how many seconds will the current be .25 amperes?

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 3 Student Task

For any logarithmic expression $\log_b x$, as with exponential expressions, the base must be a positive number. Logarithms which use 10 for the base are called **common logarithms** and may be denoted simply as ***log x***. (It is not necessary to write the base). The *log* button on your calculator computes common logarithms.

Understanding common logarithms can help you solve more complex exponential equations.

Suppose you were asked to solve the equation $10^x = 250$.

We know that $10^2 = 100$ and $10^3 = 1000$, so x should be between 2 and 3. Rewriting the exponential equation as a logarithmic equation with a common logarithm can help solve the problem.

$$10^x = 250 \rightarrow \log 250 = x$$

Using the calculator, we find that $x \approx 2.3979$

9. Use common logarithms to solve each of the following equations for x.
   a. $10^x = 15$

   b. $10^x = 0.3458$

   c. $3(10^x) = 2345$

   d. $-2(10^x) = -6538$

To this point you have been able to solve exponential equations *algebraically* only if you could write both side of the equation in terms of the same base. For example, you could solve $2^x = 8$ but not $2^x = 9$. Numerous times in this unit, you have solved exponential equations numerically (using tables) and graphically. You will now be able to solve those equations algebraically as well.

Consider the following problem: A bacteria culture in a Petri dish begins with a single bacterium and doubles every hour. Once the population reaches 100, the bacteria begin dying. How long does it take the population to reach 100?

From our earlier work we know the function that models this growth can be written as $f(x) = 2^x$. The equation we need to solve will then be $2^x = 100$.

We know that $2^6 = 64$ and $2^7 = 128$, so the answer must be between 6 and 7.

If we rewrite both 2 and 100 as a power with a base of 10, we have $10^a = 2$ and $10^b = 100$.

$10^a = 2 \rightarrow a = \log 2 \rightarrow a \approx .3010$
$10^b = 100 \rightarrow b = \log 100 \rightarrow b = 2$

Now, replacing 2 with $10^{0.301}$ and 100 with 10^2, we have the following:

$(10^{0.301})^x = 10^2$	
$10^{0.301x} = 10^2$	*Power of a power property*
$0.301x = 2$	*Common base property*
$x \approx 6.6439$	

Explain why you might be confident that this is the correct answer.

10. Solve each of the following equations, taken from earlier explorations, using the method described above.
    a. $2 = 1.1^x$ (Steps *i* – *iv* will help you solve this first problem.)
        i. $10^a = 2$ and $10^b = 1.1$. Find a and b.
        ii. Use substitution to write an equation in terms of powers of 10.
        iii. Use the power-to-power and common base properties.
        iv. Solve for x.
    b. $300(0.8)^x = 10$
    c. $100 = 60(2^x)$
11. Use the method described above to find the zeros of $f(x) = 2^x - 5$. Verify your work using your graphing calculator.

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 3 Homework

Evaluate each of the following *without* using your calculator.

1. log 10000

2. log 0.01

3. log 10

4. log 1

5. log 10^7

Evaluate each of the following using your calculator.

6. log 0.004

7. log ½

Use common logarithms to solve each equation.

8. $10^x = 124$

9. $-2(10^{2x}) = 1560$

Solve the following equations by writing both sides as a power of 10.

10. $3^x = 500$

11. $125(2^x) - 12 = 63$

12. The amount of drug in a patient's bloodstream peaks at 500 mg and then decreases by 30% every 5 hours.
    a. Write a function for the amount of blood remaining in the bloodstream t hours after the peak level.

    b. How long after the peak will it take for the drug level to drop to 200 mg?

CCGPS Mathematics III
Task 2: ***Exponential Functions and Their Inverses***
Day 4 Student Task

12. We want to make explicit the characteristics that are common to ALL general logarithmic functions, regardless of the base. To do this, we need to look at graphs of different logarithmic functions.

    a. Graph $y = \log_5 x$ by hand. Make a table of values to help you and don't forget to use some x-values between 0 and 1. Why might you consider $y = 5^x$ in making your table?

    b. Draw the graph of $y = \log_3 x$ by hand.

    c. Compare the graphs of $y = \log_5 x$ and $y = \log_3 x$. Which one increases faster? How can you tell?

    d. Use your graphing calculator and the graphs you drew by hand in *parts a* and *b* of this item to help you complete the following table.

	$y = \log_5 x$	$y = \log_3 x$	$y = \log_{\left(\frac{1}{3}\right)} x$	$y = \log_{\left(\frac{1}{5}\right)} x$
Domain				
Range				
Intercept				
Asymptote				
Zeros				
Increasing / Decreasing				

    e. Use the information in your table to answer the following:

        i. When do logarithmic functions increase? When do they decrease?

        ii. Give an example of a function that increases faster than any of the functions in the table.

        iii. Which of the above graphs decreased fastest?

        iv. Give an example of a function that would decrease faster than any of the ones above. Explain why you chose this function.

13. In this item we will investigate transformations of $y = \log_b x$.

a. Consider $y = \log (x - 3)$.

i. What is the parent function for this transformation?

ii. Describe the transformation of the parent function in words.

iii. Sketch the graphs of both functions.

iv. For the parent function, give:
Domain _____
Asymptote _____
Intercepts _____

v. For the transformation $y = \log (x - 3)$, give:
Domain _____
Asymptote _____
Intercepts _____

b. Consider the function $y = \log_4 (3x - 5)$.

i. To find the domain, we solve the inequality $3x - 5 > 0$. Say why.

ii. The asymptote can be determined by solving the equation $3x - 5 = 0$. Explain.

iii. Find the domain. _____ Find the asymptote. _____

iv. In general, how do we find x-intercepts of a function? Find the x-intercept for this function.

c. Find the domain, asymptote, x-intercept, and y-intercept (if applicable) of $f(x) = \log_5 (-3x + 8)$.

Domain: _____ Asymptote: _____

x-intercept: _____ y-intercept: _____

Alignment of GHSGT Math Content Descriptions to Courses/Standards/Elements

This document is adapted from the GaDOE GHSGT Math Descriptions and is intended to serve as further support to high school math teachers.

Mathematics Domains
To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics.

- **Algebra**
  Students will demonstrate the ability to
  - explore functions;
  - solve radical, simple quadratic and rational equations;
  - simplify and perform operations with radical, polynomial, and rational expressions.
  - investigate piecewise, exponential, and quadratic functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations;
  - solve equations and inequalities related to these functions;
  - explore the inverses of functions;
  - use the complex number system.

- **Geometry**
  Students will demonstrate the ability to
  - explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts;
  - apply properties of polygons; and
  - determine distances and points of concurrence.
  - understand and apply properties of right triangles and right-triangle trigonometry;
  - understand and apply properties of circles and spheres, and use them in determining related measures.

- **Data Analysis**
  Students will demonstrate the ability to
  - determine probability;
  - use both permutations and combinations to find the number of outcomes;
  - pose questions to be answered by collecting data; and
  - organize, represent, investigate, interpret, and make inferences from data.
  - demonstrate understanding of data analysis by posing questions to be answered by collecting data;
  - organize, represent, investigate, interpret, and make inferences from data;
  - compare data for two different samples and/or populations using measures of central tendency and measures of spread, including standard deviation;
  - use linear and quadratic regressions to analyze data and to make inferences.

Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need. The concepts and skills inherent in the process standards are integrated in items across the three content domains.

Overview of the Process Standards

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and with other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS

MM1P1 through MM1P5 within content from MM1A1 through MM1D3
MM2P1 through MM2P5 within content from MM2A1 through MM2D2
MM3P1 through MM3P5 within content from MM3A2 through MM3G1

Associated GPS Concepts and Skills

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Reflecting on and monitoring the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to precisely express mathematical ideas.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

Mathematics
Domain: Algebra (approximately 36% of the test)

Overview of the Domain

- Students will use graphs, tables, and simple algebraic techniques to explore and interpret the characteristics of functions.
- Students will simplify and perform operations with radical expressions, polynomials, and rational expressions.
- Students will solve radical, quadratic, and rational equations.
- Students will investigate step and piecewise functions, including greatest integer and absolute value functions.
- Students will explore exponential functions.
- Students will analyze quadratic functions in the form $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 +$
- Students will solve quadratic equations and inequalities in one variable.

Associated GPS Standards

MM1A1b,c,d,e,f,g,h,i	MM2N1a,b,c	MM3A2b,e
MM1A2a,b,c,e,f	MM2A1b,c	MM3A5a
MM1A3a,b,c,d	MM2A2a,b,c,d,f,g	
	MM2A3a,b,c	
	MM2A4b	
	MM2A5b	

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to

- **(MM1A1b)** graph and identify graphs of ba (limited to $f(x) = x^n$, where $n = 1$ t $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) =$.
  - select a graph that matches a particular function;
  - select a function that matches a given graph; and
  - understand that graphs are geometric representations of functions.

- **(MM1A1c)** graph transformations of basic functions;
  - examine and identify shifts, stretches, and shrinks of parent functions; and
  - explore and identify reflections across the *x*- and *y*-axes of parent functions.

- **(MM1A1d)** investigate and explain the characteristics of quadratic, cubic, inverse, absolute value, and square root functions (using linear functions only as a building block);
  - identify a domain (the set of inputs) and a range (the set of outputs);
  - understand set notation;
  - explore the zeros/solutions;
  - find *x*- and *y*-intercepts;
  - determine intervals of increase and decrease;

  - locate maximum and minimum values; and
  - explain end behavior.

- **(MM1A1e)** relate the characteristics of a function to a given context;
  -
  - utilize graphs, tables, and words to explain and predict the behavior of a function; and
  - understand the distinctions between discrete and continuous domains.

- **(MM1A1f)** recognize sequences as functions with domains that are whole numbers greater than zero;
  - examine sequences given in tables, algebraically, or by producing a context and identifying the corresponding function;
  - understand the difference between finite and infinite sequences; and
  - explore how and when to use a recursive definition for a given pattern or sequence.

- **(MM1A1g)** explore rates of change;
  - compare graphs of functions that have a constant rate of change (i.e., slope) versus graphs that have variable rates of change;
  - compare rates of change of linear, quadratic, square root, and other function families; and
  - explore average rates of change in regard to speed, cost, revenue, and other real-world applications.

- **(MM1A1h)** determine graphically and algebraically whether a nonlinear function has symmetry;
  - interpret if a given function has symmetry.

- **(MM1A1i)** understand that in any equation x can be interpreted as the equation $f(x) = g(x)$;
  - interpret the solutions as the x-value(s) of the intersection points of the graphs of $y1 = f(x)$ and $y2 = g(x)$;
  - use algebra to find the value of x that makes $f(x) = g(x)$ true; and
  - understand that functions are equal if they have the same domain and rule of correspondence.

- **(MM1A2a)** simplify algebraic expressions involving square roots.

- **(MM1A2b)** perform mathematical operations with square roots;
  - understand when to rationalize a denominator;
  - comprehend the equivalence of a simplified square root expression and the equivalence of a nonsimplified square root expression.

- **(MM1A2c)** add, subtract, and multiply polynomials.

- **(MM1A2e)** add, subtract, multiply, and divide rational algebraic expressions.

- **(MM1A2f)** factor expressions involving the difference/sum of two squares and trinomials in the form $ax2 + bx + c = 0$, and factor methods limited to the greatest common factor, grouping, trial and error, and special products.

- **(MM1A3a)** use either factorization or square roots to solve quadratic equations in the form $ax2 + bx + c = 0$, where $a = 1$.

- **(MM1A3b)** solve simple radical equations by isolating the variable and squaring both sides.

- **(MM1A3c)** use technology, tables, and graphs to solve equations resulting from the investigation of
  $x2 + bx + c = 0$;
  - interpret the solution of a quadratic function from a graph of the data; and
  - identify and comprehend the meaning of the x-intercepts from a table of quadratic data.

- **(MM1A3d)** solve simple rational equations that result in linear or quadratic equations.

- **(MM2N1a,b)** write square roots of negative numbers in imaginary form; write complex numbers in the form $a + bi$ in context of solving quadratic equations

- **(MM2N1c)** add, subtract, multiply, and divide complex numbers;
  - apply the associative, distributive, and commutative properties; and
  - identify and find conjugates of complex numbers.

- **(MM2A1b)** investigate and explain characteristics of a variety of piecewise-defined functions, such as absolute value and greatest integer functions; relate these characteristics to a real-life situation modeled by such a function
  - translate fluently between graphical, algebraic, and numeric representations;
  - identify the domain and range;
  - find the vertex and axis of symmetry;
  - identify the zeroes;
  - find the x- and y-intercepts;
  - identify points of discontinuity;
  - identify intervals where the value of a function is constant, increasing, or decreasing; and
  - investigate rates of change for specified intervals.

- **(MM2A1c)** solve absolute value equations and inequalities;
  - use algebraic and analytical methods; and
  - determine solutions using graphs and/or number lines,

- **(MM2A2a)** extend properties of exponents to include all integer exponents and use expressions with integer exponents to model real-world functional relationships;
  - apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to simplify and/or evaluate expressions; and understand that for any real nun $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ and apply these properties.

- **(MM2A2b)** investigate and explain characteristics of exponential functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify zeroes;
  - find *x*- and *y*-intercepts;
  - recognize and/or determine intervals where the value of a function is increasing or decreasing;
  - find maximum and minimum values;
  - investigate rates of change over intervals; and
  - recognize and explain behavior at extremes.

- **(MM2A2c)** graph exponential functions as transformations of $f(x) = ax$;
  - recognize and use transformations of $f(x) = ax$; and
  - use tables of value.

- **(MM2A2d)** solve simple exponential equations and inequalities;
  - by using algebraic and analytical methods; and
  - by reading and interpreting graphs.

- **(MM2A2f)** understand and recognize geometric sequences as exponential functions whose domains are the sequence of natural (counting) numbers;
  - interpret the constant ratio in a geometric sequence as the base of the associated exponential function; and
  - **(MM2A2g)** recognize and use concepts such as the common ratio and powers of the common ratio to solve real-world problems involving exponential growth and decay.

- **(MM2A3a)** convert between standard $y = (ax^2 + bx + c)$ and verte $y = a(x - h)^2 + k$

  forms of a quadratic function using the roots of the quadratic and the symmetry properties of the parabola; use the vertex form to locate and graph a quadratic function, e.g., when using a quadratic function to model a data relationship; translate from vertex form back to standard form to identify the parameters *a*, *b*, and *c*. 2 () y ax bx c2 () y ax hk

- **(MM2A3b)** graph quadratic functions as transformations of the function $f(x) = x2$
  - identify vertical and horizontal stretches and compressions, and vertical and horizontal translations; and
  - explore reflections across the *x*- and *y*-axes.

- **(MM2A3c)** investigate and explain characteristics of quadratic functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify the vertex and axis of symmetry;
  - find all zeroes;
  - find the *x*- and *y*-intercepts;
  - locate extrema using ordered pairs and be able to identify maximum and minimum values;
  - determine intervals of increase and decrease; and
  - investigate rates of change for specific intervals.

- **(MM2A4b)** find real and complex solutions of quadratic equations analytically; be familiar with multiple methods and recognize when a certain method is most appropriate;
  - use factoring methods and the zero-product property;
  - apply the quadratic formula;
  - explore both exact and approximate solutions; recognize when each type of solution is appropriate and why; and
  - recognize how the solutions of quadratic equations apply to a real-world situation modeled by the quadratic function, e.g., when one or both roots are meaningless in context.

- **(MM2A5b)** understand the relationship between a function and its inverse;
  - recognize and find the inverse of a function or relation using a variety of methods: interchanging the first and second coordinates of each ordered pair; using analytical (algebraic) techniques; and/or determining that two functions, *f* and *g*, are inverses by recognizing th $(f \circ g)(x) = (g \circ f)(x) = I(x) =$
  - recognize that the domain of the inverse is the range of the original relation and vice versa; and
  - understand how and why domain restrictions come into play with inverse functions and relate them to the behavior of the original function.

- **(MM3A2b)** extend properties of exponents to include rational exponents.

- **(MM3A2e)** investigate and explain characteristics of exponential and logarithmic functions including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, and rate of change.

- **(MM3A5a)** represent a system of linear equations as a matrix equation.

Mathematics
Domain: Geometry (approximately 36% of the test)
Overview of the Domain

- Students will investigate properties of geometric figures in a coordinate plane.
- Students will understand and use the language of mathematical argument and justification.
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
- Students will identify and use special right triangles.
- Students will define and apply sine, cosine, and tangent ratios to right triangles.
- Students will understand and apply the properties of circles and their associated segments and angles.
- Students will find and compare the measures of spheres.

Associated GPS

MM1G1a,c,d,e MM1G2a,b MM1G3a,b,c,d,e | MM2G1a,b MM2G2c MM2G3b,c MM2G4a,b | MM3G1a

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- **(MM1G1a)** determine the distance between two points on a coordinate grid;
  - find distances between two points on the same horizontal or vertical line; and
  - use various methods (such as the distance formula or Pythagorean theorem) to calculate the distance when given two points with coordinates ($x1$, $y1$) and ($x2$, $y2$).

- **(MM1G1c)** calculate the midpoint of a segment;
  - use various methods (such as the midpoint formula, similar triangles, averaging the endpoints, etc.) to locate the midpoint when given two points on a coordinate grid with coordinates ($x1$, $y1$) and ($x2$, $y2$); and
  - find an endpoint of a line segment when given its other endpoint and midpoint.

- **(MM1G1d)** understand the distance formula as an application of the Pythagorean theorem;
  - explore how the distance formula is derived from the Pythagorean theorem; and
  - find the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid.

- **(MM1G1e)** use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals;
  - use relationship properties of side measures, slopes, diagonals, etc., of triangles and quadrilaterals to determine unknown side lengths;
  - use side and angle theorems to prove triangles and quadrilaterals are similar and/or congruent;
  - understand the minimal information necessary to conclude that two triangles are congruent;
  - utilize properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral;

- utilize the distance formula to classify figures as triangles and quadrilaterals (e.g., squares, rectangles, trapezoids, kites, parallelograms, and rhombuses); and
- determine missing vertices of a triangle or a quadrilateral by utilizing side and angle relationships of a given figure.

- **(MM1G2a)** use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof, as appropriate, in mathematical and real-world applications;
  - utilize prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals; and
  - utilize paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication to justify conclusions regarding polygon relationships.

- **(MM1G2b)** explore and use the relationships among conditional statements;
  - determine the hypothesis and conclusion of a conditional statement, in word or in mathematical form;
  - write the converse of a conditional statement by exchanging the hypothesis and conclusion;
  - realize that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement;
  - understand that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion; and
  - utilize conditional statements to prove algebraic, geometric, and real-world concepts.

- **(MM1G3a)** determine the sum of interior and exterior angles in a polygon;
  - utilize angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon; and
  - utilize angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon.

- **(MM1G3b)** understand inequality theorems involving triangles;
  - apply the triangle inequality theorem to determine if given side lengths form a triangle;
  - utilize the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle; and
  - use the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle adding to 180 to determine the measure of an exterior angle of a triangle when given two remote interior angles.

- **(MM1G3c)** understand congruence postulates and theorems for triangles;
  - identify and use SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication; and
  - understand that SSA and AAA are not valid methods to prove triangle congruency.

- **(MM1G3d)** use and prove properties of and relationships among the following special quadrilaterals:
  - parallelograms—understand that the opposite sides are congruent, the opposite angles are congruent, the consecutive angles are supplementary, and the diagonals bisect each other;
  - rectangles—understand that the diagonals are congruent and that rectangles have all the properties of a parallelogram;
  - rhombuses—understand that the diagonals are perpendicular and bisect a pair of opposite angles and that rhombuses have all the properties of a parallelogram;
  - squares—understand that the diagonals are perpendicular and congruent and that squares have all the properties of a parallelogram;
  - isosceles trapezoids—understand that they have only one pair of parallel sides and congruent diagonals; and
  - kites—understand that the diagonals are perpendicular and that one diagonal is bisected, or the opposite sides are congruent and have congruent and perpendicular diagonals, and that kites have all the properties of a parallelogram.

- **(MM1G3e)** find and use points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles;
  - use bisectors, medians, and altitudes to find points of concurrency;
  - locate centers of circles inscribed in or circumscribed about triangles; and
  - make decisions about which center best meets a given set of conditions.

- **(MM2G1a)** determine the lengths of sides of 30°-60°-90° triangles;
  - use the fact that the length of the hypotenuse is twice the length of the shorter leg and the length of the longer leg is 'the square root of 3' times the length of the shorter leg to determine the lengths of all three sides given any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- **(MM2G1b)** determine the lengths of sides of 45°-45°-90° triangles;
  - use the fact that the length of the hypotenuse is 'the square root of 2' times the length of each leg to determine the lengths of all sides of a triangle given the length of any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- **(MM2G2c)** understand and apply the basic trigonometric ratios for right triangles.

- **(MM2G3b)** understand and use properties of and relationships among angles related to circles, such as central, inscribed, and related angles, e.g.,
  - relationship between arc measures and angle measures;
  - relationship between measures of central angles and inscribed angles; and
  - relationship of angles created by a chord with a common endpoint on the circle and the line that is tangent at that point; a secant and a tangent; two secants; and two tangents.

- **(MM2G3c)** use the properties of circles to solve problems involving the length of an arc and the area of a sector.

- **(MM2G4a)** understand, use and apply the surface area and volume of a sphere;
  - calculate surface area and volume of a sphere;
  - find the radius, diameter, and/or circumference of a sphere given the volume or a relationship from which the volume can be determined; and
  - **(MM2G4b)** determine the effect on surface area and volume when changing the radius or diameter of a sphere or vice versa.

- **(MM3G1a)** Find equations of circles.

Mathematics
Domain: Data Analysis (approximately 28% of the test)
Overview of the Domain

- Students will determine the number of outcomes related to a given event.
- Students will use the basic laws of probability.
- Students will relate samples to a population.
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).
- Students will use sample data to make informal inferences using population means and standard deviations.
- Students will determine an algebraic (limited to linear or quadratic) model to quantify the association between two quantitative variables.

Associated GPS
MM1D1a,b MM1D2a,b,c,d MM2D1a,c MM2D2b,d
MM1D3a,c

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to

- **(MM1D1a)** apply the addition and multiplication principles of counting.

- **(MM1D1b)** calculate and use simple permutations and combinations;
  - integrate the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation;
  - use diagrams to justify the classification; and
  - utilize permutation and combination formulas to determine the number of possible arrangements of real-world events.

- **(MM1D2a)** understand when an event is mutually exclusive and use diagrams, tables, and the formula *P(A or B) = P(A) + P(B)* to calculate the probability of mutually exclusive events.

- **(MM1D2b)** use diagrams, tables, and the formula *P(A and B) = P(A) times P(B after A)* to find the probabilities of dependent events and understand when an event is dependent.

- **(MM1D2c)** use diagrams, tables, and the formula $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ to calculate conditional probabilities of real-world events.

- **(MM1D2d)** use expected value to predict outcomes and make inferences.

- **(MM1D3a)** compare summary statistics from one sample data distribution to another sample data distribution;
  - interpret the mean, median, quartiles, and interquartile range of multiple data sets;
  - understand normal and binomial data distributions; and
  - describe center and variability of data distributions.

- **(MM1D3c)** understand that a random sample is used to improve the chance of selecting a representative sample;
  - determine the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample; and
  - understand that a random sample will yield unbiased results.

- **(MM2D1a)** recognize an appropriate question given a research topic and populations of interest;
  - identify potential bias created by questions.

- **(MM2D1c)** use means and standard deviations to compare data sets;
  - understand and apply various strategies for estimating means and standard deviations for comparison purposes;
  - understand various representations of data, including tables, graphs, line plots, stem-and-leaf plots, histograms, and box-and-whisker plots; know which information can be directly determined and which can only be estimated from a given representation; and
  - understand the role of *n* in comparing standard deviations of data sets, including recognizing when *n* is unknown.

- **(MM2D2b)** examine the issues of curve fitting by finding good linear fits to data using simple methods such as the median-median line by "eyeballing;"
  - decide whether a linear or quadratic model or neither is appropriate for data presented in a table or graph;
  - recognize an appropriate algebraic model given a table or graph (note: in the absence of appropriate technology, students will be expected to estimate the correct parameters for a linear or quadratic function pres $y = ax + b$ or $y = ax^2 + bx + c$); and
  - decide whether or not a particular set of data is appropriately modeled by a given function.

- **(MM2D2d)** understand issues that arise when using data to explore the relationship between two variables, including correlation, e.g., recognizing whether the fit of an algebraic model is strong, weak, or nonexistent; focus on confusion between correlation and causation.

Alignment of GHSGT Math Content Descriptions to Courses/Standards/Elements

This document is adapted from the GaDOE GHSGT Math Descriptions and is intended to serve as further support to high school math teachers.

Mathematics Domains

To provide reliable measures of student achievement and to give structure to the assessment program, the content standards contained in the GPS were grouped into content domains. Each domain was created by combining standards that share similar content characteristics. Three domains were identified for Mathematics.

- **Algebra**
  Students will demonstrate the ability to
  - explore functions;
  - solve radical, simple quadratic and rational equations;
  - simplify and perform operations with radical, polynomial, and rational expressions.
  - investigate piecewise, exponential, and quadratic functions using numerical, analytical, and graphical approaches, focusing on the use of these functions in problem-solving situations;
  - solve equations and inequalities related to these functions;
  - explore the inverses of functions;
  - use the complex number system.

- **Geometry**
  Students will demonstrate the ability to
  - explore, understand, and use the formal language of reasoning and justification in both algebraic and geometric contexts;
  - apply properties of polygons; and
  - determine distances and points of concurrence.
  - understand and apply properties of right triangles and right-triangle trigonometry;
  - understand and apply properties of circles and spheres, and use them in determining related measures.

- **Data Analysis**
  Students will demonstrate the ability to
  - determine probability;
  - use both permutations and combinations to find the number of outcomes;
  - pose questions to be answered by collecting data; and
  - organize, represent, investigate, interpret, and make inferences from data.
  - demonstrate understanding of data analysis by posing questions to be answered by collecting data;
  - organize, represent, investigate, interpret, and make inferences from data;
  - compare data for two different samples and/or populations using measures of central tendency and measures of spread, including standard deviation;
  - use linear and quadratic regressions to analyze data and to make inferences.

Process Standards

The GPS in mathematics require content to be taught in conjunction with process skills identified as the process standards. These process standards are necessary for students to master each of the mathematics content standards. Problem solving, reasoning, representation, connections, and communication are the critical dimensions of mathematical proficiency that all students need. The concepts and skills inherent in the process standards are integrated in items across the three content domains.

Overview of the Process Standards

- Students will solve problems (using appropriate technology).
- Students will reason and evaluate mathematical arguments.
- Students will communicate mathematically.
- Students will make connections among mathematical ideas and with other disciplines.
- Students will represent mathematics in multiple ways.

Associated GPS

MM1P1 through MM1P5 within content from MM1A1 through MM1D3
MM2P1 through MM2P5 within content from MM2A1 through MM2D2
MM3P1 through MM3P5 within content from MM3A2 through MM3G1

Associated GPS Concepts and Skills

- Building new mathematical knowledge through problem solving.
- Solving problems that arise in mathematics and in other contexts.
- Applying and adapting a variety of appropriate strategies to solve problems.
- Reflecting on and monitoring the process of mathematical problem solving.
- Recognizing reasoning and proof as fundamental aspects of mathematics.
- Making and investigating mathematical conjectures.
- Developing and evaluating mathematical arguments and proofs.
- Selecting and using various types of reasoning and methods of proof.
- Organizing and consolidating mathematical thinking through communication.
- Communicating mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyzing and evaluating mathematical thinking and strategies of others.
- Using the language of mathematics to precisely express mathematical ideas.
- Recognizing and using connections among mathematical ideas.
- Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognizing and applying mathematics in contexts outside of mathematics.
- Creating and using representations to organize, record, and communicate mathematical ideas.
- Selecting, applying, and translating mathematical representations to solve problems.
- Using representations to model and interpret physical, social, and mathematical phenomena.

Mathematics
Domain: Algebra (approximately 36% of the test)

Overview of the Domain

- Students will use graphs, tables, and simple algebraic techniques to explore and interpret the characteristics of functions.
- Students will simplify and perform operations with radical expressions, polynomials, and rational expressions.
- Students will solve radical, quadratic, and rational equations.
- Students will investigate step and piecewise functions, including greatest integer and absolute value functions.
- Students will explore exponential functions.
- Students will analyze quadratic functions in the form $f(x) = ax^2 + bx + c$ and $f(x) = a(x - h)^2 +$
- Students will solve quadratic equations and inequalities in one variable.

Associated GPS Standards

MM1A1b,c,d,e,f,g,h,i	MM2N1a,b,c	MM3A2b,e
MM1A2a,b,c,e,f	MM2A1b,c	MM3A5a
MM1A3a,b,c,d	MM2A2a,b,c,d,f,g	
	MM2A3a,b,c	
	MM2A4b	
	MM2A5b	

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to

- **(MM1A1b)** graph and identify graphs of ba (limited to $f(x) = x^n$, where $n = 1$ t $f(x) = |x|$, $f(x) = \sqrt{x}$ and $f(x) =$.
  - select a graph that matches a particular function;
  - select a function that matches a given graph; and
  - understand that graphs are geometric representations of functions.

- **(MM1A1c)** graph transformations of basic functions;
  - examine and identify shifts, stretches, and shrinks of parent functions; and
  - explore and identify reflections across the *x*- and *y*-axes of parent functions.

- **(MM1A1d)** investigate and explain the characteristics of quadratic, cubic, inverse, absolute value, and square root functions (using linear functions only as a building block);
  - identify a domain (the set of inputs) and a range (the set of outputs);
  - understand set notation;
  - explore the zeros/solutions;
  - find *x*- and *y*-intercepts;
  - determine intervals of increase and decrease;

  - locate maximum and minimum values; and
  - explain end behavior.

- **(MM1A1e)** relate the characteristics of a function to a given context;
  - utilize graphs, tables, and words to explain and predict the behavior of a function; and
  - understand the distinctions between discrete and continuous domains.

- **(MM1A1f)** recognize sequences as functions with domains that are whole numbers greater than zero;
  - examine sequences given in tables, algebraically, or by producing a context and identifying the corresponding function;
  - understand the difference between finite and infinite sequences; and
  - explore how and when to use a recursive definition for a given pattern or sequence.

- **(MM1A1g)** explore rates of change;
  - compare graphs of functions that have a constant rate of change (i.e., slope) versus graphs that have variable rates of change;
  - compare rates of change of linear, quadratic, square root, and other function families; and
  - explore average rates of change in regard to speed, cost, revenue, and other real-world applications.

- **(MM1A1h)** determine graphically and algebraically whether a nonlinear function has symmetry;
  - interpret if a given function has symmetry.

- **(MM1A1i)** understand that in any equation x can be interpreted as the equation $f(x) = g(x)$;
  - interpret the solutions as the x-value(s) of the intersection points of the graphs of $y1 = f(x)$ and $y2 = g(x)$;
  - use algebra to find the value of x that makes $f(x) = g(x)$ true; and
  - understand that functions are equal if they have the same domain and rule of correspondence.

- **(MM1A2a)** simplify algebraic expressions involving square roots.

- **(MM1A2b)** perform mathematical operations with square roots;
  - understand when to rationalize a denominator;
  - comprehend the equivalence of a simplified square root expression and the equivalence of a nonsimplified square root expression.

- **(MM1A2c)** add, subtract, and multiply polynomials.

- **(MM1A2e)** add, subtract, multiply, and divide rational algebraic expressions.

- **(MM1A2f)** factor expressions involving the difference/sum of two squares and trinomials in the form $ax2 + bx + c = 0$, and factor methods limited to the greatest common factor, grouping, trial and error, and special products.

- **(MM1A3a)** use either factorization or square roots to solve quadratic equations in the form $ax2 + bx + c = 0$, where $a = 1$.

- **(MM1A3b)** solve simple radical equations by isolating the variable and squaring both sides.

- **(MM1A3c)** use technology, tables, and graphs to solve equations resulting from the investigation of
  $x2 + bx + c = 0$;
  - interpret the solution of a quadratic function from a graph of the data; and
  - identify and comprehend the meaning of the *x*-intercepts from a table of quadratic data.

- **(MM1A3d)** solve simple rational equations that result in linear or quadratic equations.

- **(MM2N1a,b)** write square roots of negative numbers in imaginary form; write complex numbers in the form $a + bi$ in context of solving quadratic equations

- **(MM2N1c)** add, subtract, multiply, and divide complex numbers;
  - apply the associative, distributive, and commutative properties; and
  - identify and find conjugates of complex numbers.

- **(MM2A1b)** investigate and explain characteristics of a variety of piecewise-defined functions, such as absolute value and greatest integer functions; relate these characteristics to a real-life situation modeled by such a function
  - translate fluently between graphical, algebraic, and numeric representations;
  - identify the domain and range;
  - find the vertex and axis of symmetry;
  - identify the zeroes;
  - find the *x*- and *y*-intercepts;
  - identify points of discontinuity;
  - identify intervals where the value of a function is constant, increasing, or decreasing; and
  - investigate rates of change for specified intervals.

- **(MM2A1c)** solve absolute value equations and inequalities;
  - use algebraic and analytical methods; and
  - determine solutions using graphs and/or number lines,

- **(MM2A2a)** extend properties of exponents to include all integer exponents and use expressions with integer exponents to model real-world functional relationships;
  - apply product of powers, quotient of powers, power of a power, power of a product, and power of a quotient to simplify and/or evaluate expressions; and understand that for any real nur $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ and apply these properties.

- **(MM2A2b)** investigate and explain characteristics of exponential functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify zeroes;
  - find *x*- and *y*-intercepts;
  - recognize and/or determine intervals where the value of a function is increasing or decreasing;
  - find maximum and minimum values;
  - investigate rates of change over intervals; and
  - recognize and explain behavior at extremes.

- **(MM2A2c)** graph exponential functions as transformations of $f(x) = ax$;
  - recognize and use transformations of $f(x) = ax$; and
  - use tables of value.

- **(MM2A2d)** solve simple exponential equations and inequalities;
  - by using algebraic and analytical methods; and
  - by reading and interpreting graphs.

- **(MM2A2f)** understand and recognize geometric sequences as exponential functions whose domains are the sequence of natural (counting) numbers;
  - interpret the constant ratio in a geometric sequence as the base of the associated exponential function; and
  - **(MM2A2g)** recognize and use concepts such as the common ratio and powers of the common ratio to solve real-world problems involving exponential growth and decay.

- **(MM2A3a)** convert between standard $y = (ax^2 + bx + c)$ and verte $y = a(x - h)^2 + k$ forms of a quadratic function using the roots of the quadratic and the symmetry properties of the parabola; use the vertex form to locate and graph a quadratic function, e.g., when using a quadratic function to model a data relationship; translate from vertex form back to standard form to identify the parameters *a*, *b*, and *c*. 2 () y ax bx c2 () y ax hk

- **(MM2A3b)** graph quadratic functions as transformations of the function $f(x) = x2$
  - identify vertical and horizontal stretches and compressions, and vertical and horizontal translations; and
  - explore reflections across the *x*- and *y*-axes.

- **(MM2A3c)** investigate and explain characteristics of quadratic functions; use these characteristics to model and solve real-world problems;
  - identify domain and range;
  - identify the vertex and axis of symmetry;
  - find all zeroes;
  - find the *x*- and *y*-intercepts;
  - locate extrema using ordered pairs and be able to identify maximum and minimum values;
  - determine intervals of increase and decrease; and
  - investigate rates of change for specific intervals.

- **(MM2A4b)** find real and complex solutions of quadratic equations analytically; be familiar with multiple methods and recognize when a certain method is most appropriate;
  - use factoring methods and the zero-product property;
  - apply the quadratic formula;
  - explore both exact and approximate solutions; recognize when each type of solution is appropriate and why; and
  - recognize how the solutions of quadratic equations apply to a real-world situation modeled by the quadratic function, e.g., when one or both roots are meaningless in context.

- **(MM2A5b)** understand the relationship between a function and its inverse;
  - recognize and find the inverse of a function or relation using a variety of methods: interchanging the first and second coordinates of each ordered pair; using analytical (algebraic) techniques; and/or determining that two functions, *f* and *g*, are inverses by recognizing th $(f \circ g)(x) = (g \circ f)(x) = I(x) =$
  - recognize that the domain of the inverse is the range of the original relation and vice versa; and
  - understand how and why domain restrictions come into play with inverse functions and relate them to the behavior of the original function.

- **(MM3A2b)** extend properties of exponents to include rational exponents.

- **(MM3A2e)** investigate and explain characteristics of exponential and logarithmic functions including domain and range, asymptotes, zeros, intercepts, intervals of increase and decrease, and rate of change.

- **(MM3A5a)** represent a system of linear equations as a matrix equation.

Mathematics
Domain: Geometry (approximately 36% of the test)
Overview of the Domain

- Students will investigate properties of geometric figures in a coordinate plane.
- Students will understand and use the language of mathematical argument and justification.
- Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
- Students will identify and use special right triangles.
- Students will define and apply sine, cosine, and tangent ratios to right triangles.
- Students will understand and apply the properties of circles and their associated segments and angles.
- Students will find and compare the measures of spheres.

Associated GPS

MM1G1a,c,d,e MM1G2a,b MM1G3a,b,c,d,e MM2G1a,b MM2G2c MM2G3b,c MM2G4a,b MM3G1a

Associated GPS Concepts and Skills

Assessment of this domain will focus on student ability to

- **(MM1G1a)** determine the distance between two points on a coordinate grid;
  - find distances between two points on the same horizontal or vertical line; and
  - use various methods (such as the distance formula or Pythagorean theorem) to calculate the distance when given two points with coordinates ($x1$, $y1$) and ($x2$, $y2$).

- **(MM1G1c)** calculate the midpoint of a segment;
  - use various methods (such as the midpoint formula, similar triangles, averaging the endpoints, etc.) to locate the midpoint when given two points on a coordinate grid with coordinates ($x1$, $y1$) and ($x2$, $y2$); and
  - find an endpoint of a line segment when given its other endpoint and midpoint.

- **(MM1G1d)** understand the distance formula as an application of the Pythagorean theorem;
  - explore how the distance formula is derived from the Pythagorean theorem; and
  - find the length of a hypotenuse or a leg of a triangle plotted on a coordinate grid.

- **(MM1G1e)** use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals;
  - use relationship properties of side measures, slopes, diagonals, etc., of triangles and quadrilaterals to determine unknown side lengths;
  - use side and angle theorems to prove triangles and quadrilaterals are similar and/or congruent;
  - understand the minimal information necessary to conclude that two triangles are congruent;
  - utilize properties of parallel and perpendicular lines and angle bisectors to construct or draw the missing measure of a polygon, given a known relationship to another triangle or quadrilateral;

- utilize the distance formula to classify figures as triangles and quadrilaterals (e.g., squares, rectangles, trapezoids, kites, parallelograms, and rhombuses); and
- determine missing vertices of a triangle or a quadrilateral by utilizing side and angle relationships of a given figure.

- **(MM1G2a)** use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof, as appropriate, in mathematical and real-world applications;
  - utilize prior knowledge of quadrilateral relationships to prove or disprove classification of quadrilaterals; and
  - utilize paragraph proofs, flow proofs, two-column proofs, or any other method that relays clear communication to justify conclusions regarding polygon relationships.

- **(MM1G2b)** explore and use the relationships among conditional statements;
  - determine the hypothesis and conclusion of a conditional statement, in word or in mathematical form;
  - write the converse of a conditional statement by exchanging the hypothesis and conclusion;
  - realize that the inverse of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement;
  - understand that the contrapositive of a conditional statement is the negation of the hypothesis and conclusion of the conditional statement and then the interchange of the hypothesis and conclusion; and
  - utilize conditional statements to prove algebraic, geometric, and real-world concepts.

- **(MM1G3a)** determine the sum of interior and exterior angles in a polygon;
  - utilize angle relationships of a polygon to find a missing measure or the total interior angles measures of a specific polygon; and
  - utilize angle relationships, such as linear pairs and the exterior angle sum theorem, to determine an exterior angle of a polygon.

- **(MM1G3b)** understand inequality theorems involving triangles;
  - apply the triangle inequality theorem to determine if given side lengths form a triangle;
  - utilize the side-angle inequality theorem to determine the largest and smallest angle or side in a triangle; and
  - use the exterior-angle inequality theorem, linear pairs, or the sum of the angles of a triangle adding to 180 to determine the measure of an exterior angle of a triangle when given two remote interior angles.

- **(MM1G3c)** understand congruence postulates and theorems for triangles;
  - identify and use SSS, SAS, ASA, AAS, HL to prove/justify that given triangles are congruent through proofs including two-column, paragraph, and flow chart, or any other valid form of communication; and
  - understand that SSA and AAA are not valid methods to prove triangle congruency.

- **(MM1G3d)** use and prove properties of and relationships among the following special quadrilaterals:
  - parallelograms—understand that the opposite sides are congruent, the opposite angles are congruent, the consecutive angles are supplementary, and the diagonals bisect each other;
  - rectangles—understand that the diagonals are congruent and that rectangles have all the properties of a parallelogram;
  - rhombuses—understand that the diagonals are perpendicular and bisect a pair of opposite angles and that rhombuses have all the properties of a parallelogram;
  - squares—understand that the diagonals are perpendicular and congruent and that squares have all the properties of a parallelogram;
  - isosceles trapezoids—understand that they have only one pair of parallel sides and congruent diagonals; and
  - kites—understand that the diagonals are perpendicular and that one diagonal is bisected, or the opposite sides are congruent and have congruent and perpendicular diagonals, and that kites have all the properties of a parallelogram.

- **(MM1G3e)** find and use points of concurrency, such as incenter, orthocenter, circumcenter, and centroid, in triangles;
  - use bisectors, medians, and altitudes to find points of concurrency;
  - locate centers of circles inscribed in or circumscribed about triangles; and
  - make decisions about which center best meets a given set of conditions.

- **(MM2G1a)** determine the lengths of sides of 30°-60°-90° triangles;
  - use the fact that the length of the hypotenuse is twice the length of the shorter leg and the length of the longer leg is 'the square root of 3' times the length of the shorter leg to determine the lengths of all three sides given any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- **(MM2G1b)** determine the lengths of sides of 45°-45°-90° triangles;
  - use the fact that the length of the hypotenuse is 'the square root of 2' times the length of each leg to determine the lengths of all sides of a triangle given the length of any one of the three sides; and
  - solve problems that involve application of these side length relationships.

- **(MM2G2c)** understand and apply the basic trigonometric ratios for right triangles.

- **(MM2G3b)** understand and use properties of and relationships among angles related to circles, such as central, inscribed, and related angles, e.g.,
  - relationship between arc measures and angle measures;
  - relationship between measures of central angles and inscribed angles; and
  - relationship of angles created by a chord with a common endpoint on the circle and the line that is tangent at that point; a secant and a tangent; two secants; and two tangents.

- **(MM2G3c)** use the properties of circles to solve problems involving the length of an arc and the area of a sector.

- **(MM2G4a)** understand, use and apply the surface area and volume of a sphere;
  - calculate surface area and volume of a sphere;
  - find the radius, diameter, and/or circumference of a sphere given the volume or a relationship from which the volume can be determined; and
  - **(MM2G4b)** determine the effect on surface area and volume when changing the radius or diameter of a sphere or vice versa.

- **(MM3G1a)** Find equations of circles.

Mathematics
Domain: Data Analysis (approximately 28% of the test)
Overview of the Domain

- Students will determine the number of outcomes related to a given event.
- Students will use the basic laws of probability.
- Students will relate samples to a population.
- Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).
- Students will use sample data to make informal inferences using population means and standard deviations.
- Students will determine an algebraic (limited to linear or quadratic) model to quantify the association between two quantitative variables.

Associated GPS
MM1D1a,b MM1D2a,b,c,d MM2D1a,c MM2D2b,d MM1D3a,c

Associated GPS Concepts and Skills
Assessment of this domain will focus on student ability to

- **(MM1D1a)** apply the addition and multiplication principles of counting.

- **(MM1D1b)** calculate and use simple permutations and combinations;
  - integrate the multiplication principle to clarify the difference between permutations and combinations and when each is appropriate to use for a situation;
  - use diagrams to justify the classification; and
  - utilize permutation and combination formulas to determine the number of possible arrangements of real-world events.

- **(MM1D2a)** understand when an event is mutually exclusive and use diagrams, tables, and the formula *P(A or B) = P(A) + P(B)* to calculate the probability of mutually exclusive events.

- **(MM1D2b)** use diagrams, tables, and the formula *P(A and B) = P(A) times P(B after A)* to find the probabilities of dependent events and understand when an event is dependent.

- **(MM1D2c)** use diagrams, tables, and the formula $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$ to calculate conditional probabilities of real-world events.

- **(MM1D2d)** use expected value to predict outcomes and make inferences.

- **(MM1D3a)** compare summary statistics from one sample data distribution to another sample data distribution;
  - interpret the mean, median, quartiles, and interquartile range of multiple data sets;
  - understand normal and binomial data distributions; and
  - describe center and variability of data distributions.

- **(MM1D3c)** understand that a random sample is used to improve the chance of selecting a representative sample;
  - determine the type of sampling to be used, given a scenario, so that a survey yields results from a random population sample; and
  - understand that a random sample will yield unbiased results.

- **(MM2D1a)** recognize an appropriate question given a research topic and populations of interest;
  - identify potential bias created by questions.

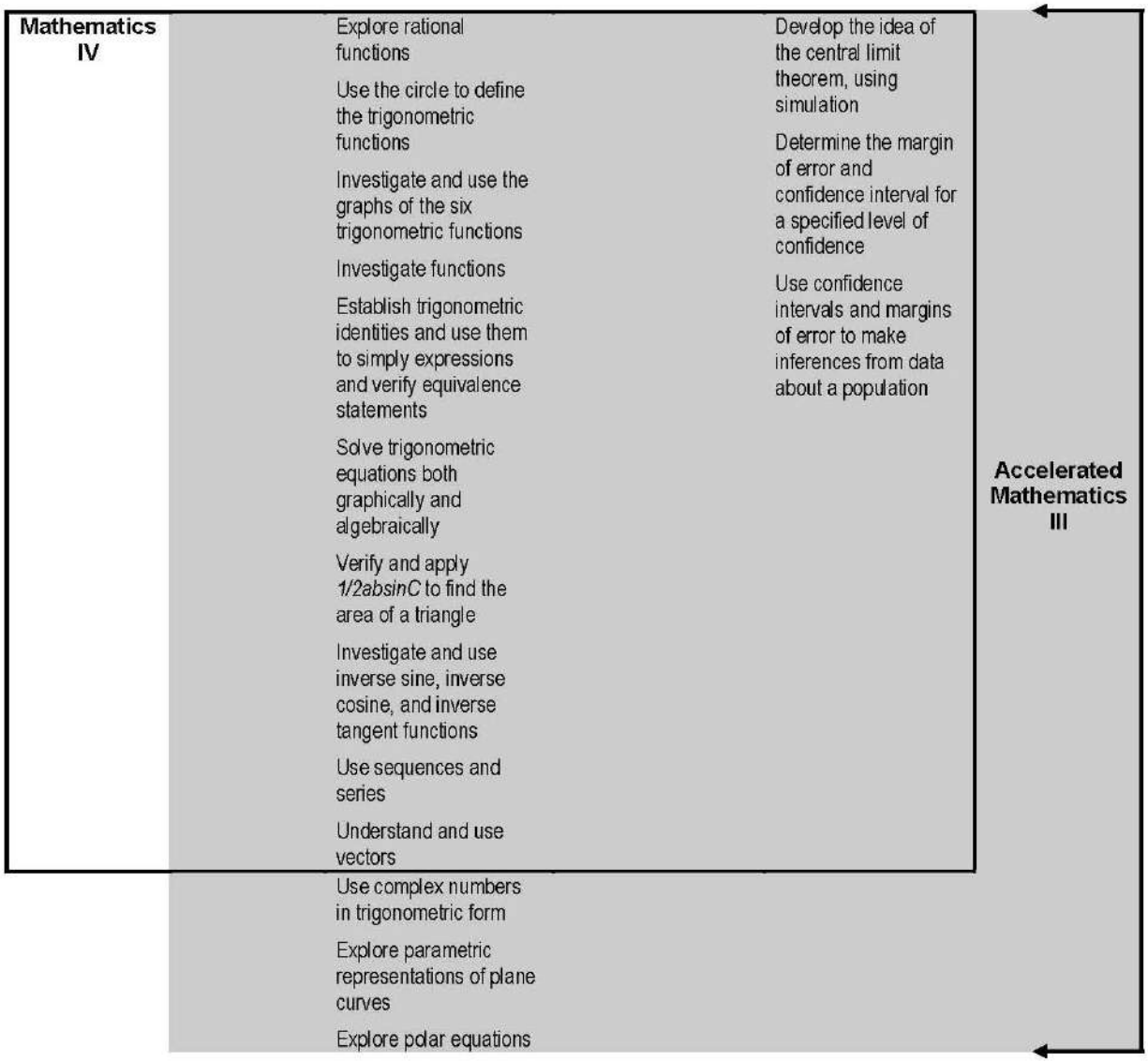
- **(MM2D1c)** use means and standard deviations to compare data sets;
  - understand and apply various strategies for estimating means and standard deviations for comparison purposes;
  - understand various representations of data, including tables, graphs, line plots, stem-and-leaf plots, histograms, and box-and-whisker plots; know which information can be directly determined and which can only be estimated from a given representation; and
  - understand the role of *n* in comparing standard deviations of data sets, including recognizing when *n* is unknown.

- **(MM2D2b)** examine the issues of curve fitting by finding good linear fits to data using simple methods such as the median-median line by "eyeballing;"
  - decide whether a linear or quadratic model or neither is appropriate for data presented in a table or graph;
  - recognize an appropriate algebraic model given a table or graph (note: in the absence of appropriate technology, students will be expected to estimate the correct parameters for a linear or quadratic function pres $y = ax + b$ or $y = ax^2 + bx + c$); and
  - decide whether or not a particular set of data is appropriately modeled by a given function.

- **(MM2D2d)** understand issues that arise when using data to explore the relationship between two variables, including correlation, e.g., recognizing whether the fit of an algebraic model is strong, weak, or nonexistent; focus on confusion between correlation and causation.

High School GPS Mathematics Course Content
(James Pratt, Cobb County)

	Number and Operations	Algebra	Geometry	Data Analysis and Probability	
Mathematics I		Explore and interpret functions Operate with radical expressions, polynomials, and rational expressions Solve simple equations	Investigate properties of geometric figures in the coordinate plane Understand and use the language of mathematical argument and justification Discover, prove, and apply properties of triangles, quadrilaterals, and other polygons	Determine the number of outcomes related to a given event Use the basic laws of probability Relate samples to a population Explore variability of data by determining the mean absolute deviation.	
Mathematics II	Represent and operate with complex numbers	Investigate step and piecewise functions, including greatest integer and absolute value functions Analyze quadratic functions Solve quadratic equations and inequalities in one variable	Understand the properties of circles Find and compare the measures of spheres	Determine an algebraic model to quantify the association between two quantitative variables	**Accelerated Mathematics I**
		Explore exponential functions Explore inverses of functions	Identify and use special right triangles Define and apply sine, cosine, and tangent ratios to right triangles	Make informal inferences about population means and standard deviations, using sample data	
Mathematics III		Analyze graphs of polynomial functions of higher degree Explore logarithmic functions as inverses of exponential functions Solve a variety of equations and inequalities Perform basic operations with matrices Use matrices to formulate and solve problems Solve linear programming problems in two variables Understand and apply matrix representations of vertex-edge graphs	Investigate the relationships between lines and circles Recognize, analyze, and graph the equations of the conic sections Investigate planes and spheres	Create probability histograms of discrete random variables Solve problems involving probabilities by interpreting a normal distribution as a probability histogram for a continuous random variable Understand the differences between experimental and observational studies by posing questions and collecting, analyzing, and interpreting data	**Accelerated Mathematics II**


Mathematics IV
Explore rational functions
Use the circle to define the trigonometric functions
Investigate and use the graphs of the six trigonometric functions
Investigate functions
Establish trigonometric identities and use them to simply expressions and verify equivalence statements
Solve trigonometric equations both graphically and algebraically
Verify and apply 1/2absinC to find the area of a triangle
Investigate and use inverse sine, inverse cosine, and inverse tangent functions
Use sequences and series
Understand and use vectors
Use complex numbers in trigonometric form
Explore parametric representations of plane curves
Explore polar equations
Develop the idea of the central limit theorem, using simulation
Determine the margin of error and confidence interval for a specified level of confidence
Use confidence intervals and margins of error to make inferences from data about a population
Accelerated Mathematics III

Never Say Anything a Kid Can Say! Article

Never Say Anything a Kid Can Say!

STEVEN C. REINHART

AFTER EXTENSIVE PLANNING, I PRESENTED what should have been a masterpiece lesson. I worked several examples on the overhead projector, answered every student's question in great detail, and explained the concept so clearly that surely my students understood. The next day, however, it became obvious that the students were totally confused. In my early years of teaching, this situation happened all too often. Even though observations by my principal clearly pointed out that I was very good at explaining mathematics to my students, knew my subject matter well, and really seemed to be a dedicated and caring teacher, something was wrong. My students were capable of learning much more than they displayed.

Implementing Change over time

THE LOW LEVELS OF ACHIEVEMENT – of many students caused me to question – how I was teaching, and my search for a – better approach began. Making a commitment to change 10 percent of my if teaching each year, I began to collect and use materials and ideas gathered from supplements, workshops, professional journals, and university classes. Each year, my goal was simply to teach a single topic in a better way than I had the year before.

STEVE REINHART, *steveJeinhart@wetn.pbs.org, teaches mathematics at Chippewa Falls Middle School, ChiPpewa Falls, WI* **54729.** *He is interested in the teaching of algebraic thinking at the middle school level and in the professional development of*

Before long, I noticed that the familiar teacher-centered, direct-instruction model often did not fit well with the more in-depth problems and tasks that I was using. The information that I had gathered also suggested teaching in nontraditional ways. It was not enough to teach better mathematics; I also had to teach mathematics better. Making changes in instruction proved difficult because I had to learn to teach in ways that I had never observed or experienced, challenging many of the old teaching paradigms. As I moved from traditional methods of instruction to a more student-centered, problem-based approach, many of my students enjoyed my classes more. They really seemed to like working together, discussing and sharing their ideas and solutions to the interesting, often contextual, problems that I posed. The small changes that I implemented each year began to show results. In five years, I had almost completely changed both *what* and *how* I was teaching.

The Fundamental Flaw

AT SOME POINT DURING THIS METAMORPHOSIS, I concluded that a fundamental flaw existed in my teaching methods. When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not! Eventually, I concluded that if my students were to ever really learn mathematics, *they* would have to do the explaining, and *I,* the listening. My definition of a good teacher has since changed from "one who explains things so well that students understand" to "one who gets students to explain things so well that they can be understood."

Getting middle school students to explain their thinking and become actively involved in classroom discussions can be a challenge. By nature, these

students are self-conscious and insecure. This insecurity and the effects of negative peer pressure tend to discourage involvement. To get beyond these and other roadblocks, I have learned to ask the best possible questions and to apply strategies that require all students to participate. Adopting the goals and implementing the strategies and questioning techniques that follow have helped me develop and improve my questioning skills. At the same time, these goals and strategies help me create a classroom atmosphere in which students are actively engaged in learning mathematics and feel comfortable in sharing and discussing ideas, asking questions, and taking risks.

Questioning Strategies That Work for Me

ALTHOUGH GOOD TEACHERS PLAN DETAILED lessons that focus on the mathematical content, few take the time to plan to use specific questioning techniques on a regular basis. Improving questioning skills is difficult and takes time, practice, and planning. Strategies that work once will work again and again. Making a list of good ideas and strategies that work, revisiting the list regularly, and planning to practice selected techniques in daily lessons will make a difference.

Create a plan.
The following is a list of reminders that I have accumulated from the many outstanding teachers with whom I have worked over several years. I revisit this list often. None of these ideas is new, and I can claim none, except the first one, as my own. Although implementing any single suggestion from this list may not result in major change, used together, these suggestions can help transform a classroom. Attempting to change too much too fast may result in frustration and failure. Changing a little at a time by selecting, practicing, and refining one or two strategies or skills before moving on to others can result in continual, incremental growth. Implementing one or two techniques at a time also makes it easier for students to accept and adjust to the new expectations and standards being established.

1. Never say anything a kid can say! This one goal keeps me focused. Although I do not think that I have ever met this goal completely in anyone day or even in a given class period, it has forced me to develop and improve my questioning skills. It also sends a message to students that their participation is essential. Every time I am tempted to tell students something, I try to ask a question instead.

2. Ask good questions. Good questions require more than recalling a fact or reproducing a skill. By asking good questions, I encourage students to think about, and reflect on, the mathematics they are learning. A student should be able to learn from answering my question, and I should be able to learn something about what the student knows or does not know from her or his response. Quite simply, I ask good questions to get students to think and to inform me about what they know. The best questions are open ended, those for which more than one way to solve the problem or more than one acceptable response may be possible.

3. Use more process questions than product questions. Product questions-those that require short answers or a yes or no response or those that rely almost completely on memory-provide little information about what a student knows. To find out what a student understands, I ask process questions that require the student to reflect, analyze, and explain his or her thinking and reasoning. Process questions require students to think at much higher levels.

4. Replace lectures with sets of questions. When tempted to present information in the form of a lecture, I remind myself of this definition of a lecture: 'The transfer of information from the notes of the lecturer to the notes of the student without passing through the minds of either." If I am still tempted, I ask myself the humbling question "What percent of my students will actually be listening to me?"

5. Be patient. Wait time is very important. Although some students always seem to have their hands raised immediately, most need more time to process their thoughts. If I always call on one of the first students who volunteers, I am cheating those who need more time to think about, and process a response to, my question. Even very capable students can begin to doubt their abilities, and many eventually stop thinking about my questions altogether. Increasing wait time to five seconds or longer can result in more and better responses.

Good discussions take time; at first, I was uncomfortable in taking so much time to discuss a single question or problem. The urge to simply tell my students and move on for the sake of expedience was considerable. Eventually, I began to see the value in what I now refer to as a "less is more" philosophy. I now believe that all students learn more when I pose a high-quality problem and give them the necessary time to investigate, process their thoughts, and reflect on and defend their findings.

Share with students reasons for asking questions. Students should understand that all their statements are valuable to me, even if they are incorrect or show misconceptions. I explain that I ask them questions because I am continuously evaluating what the class knows or does not know. Their comments help me make decisions and. plan the next activities.

Teach for success. If students are to value my questions and be involved in discussions, I cannot use questions to embarrass or punish. Such questions accomplish little and can make it more difficult to create an atmosphere in which students feel comfortable sharing ideas and taking risks. If a student is struggling to respond, I move on to another student quickly. As I listen to student conversations and observe their work, I also identify those who have good ideas or comments to share. Asking a shy, quiet student a question when I know that he or she has a good response is a great strategy for building confidence and self-esteem. Frequently, I alert the student ahead of time: 'That's a great idea. I'd really like you to share that with the class in a few minutes."

Be nonjudgmental about a response or comment. This goal is indispensable in encouraging discourse. Imagine being in a classroom where the teacher makes this comment: "WOW! Brittni, that was a terrific, insightful response! Who's next?" Not many middle school students have the confidence to follow a response that has been praised so highly by a teacher. If a student's response reveals a misconception and the teacher replies in a negative way, the student may be discouraged from volunteering again. Instead, encourage more discussion and move on to the next comment. Often, students disagree with one another, discover their own errors, and correct their thinking. Allowing students to listen to fellow classmates is a far more positive way to deal with misconceptions than announcing to the class that an answer is incorrect. If several students remain confused, I might say, "I'm hearing that we do not agree on this issue. Your comments and ideas have given me an idea for an activity that will help you clarify your thinking." I then plan to revisit the concept with another activity as soon as possible.

Try not to repeat students' answers. If students are to listen to one another and value one another's input, I cannot repeat or try to improve on what they say. If students realize that I will repeat or clarify what another student says, they no longer have a reason to listen. I must be patient and let students clarify their own thinking and encourage them to speak to their classmates, not just to me.

All students can speak louder - I have heard them in the halls! Yet I must be careful not to embarrass someone with a quiet voice. Because students know that I never accept just one response, they think nothing of my asking another student to paraphrase the soft-spoken comments of a classmate.

Is this the right answer?" Students frequently ask this question. My usual response to this question might be that "I'm not sure. Can you explain your thinking to me?" As soon as I tell a student that the answer is correct, thinking stops. If students explain their thinking clearly, I ask a "What if?" question to encourage them to extend their thinking.

Participation is not optional! I remind my students of this expectation regularly. Whether working in small groups or discussing a problem with the whole class, each student is expected to contribute his or her fair share. Because reminding students of this expectation is not enough, I also regularly apply several of the following techniques:

1. Use the think-pair-share strategy. Whole-group discussions are usually improved by using this technique. When I pose a new problem; present a new project, task, or activity; or simply ask a question, all students must think and work independently first. In the past, letting students begin working together on a task always allowed a few students to sit back while others took over. Requiring students to work alone first reduces this problem by placing the responsibility for learning on each student. This independent work time may vary from a few minutes to the entire class period, depending on the task.

After students have had adequate time to work independently, they are paired with partners or join small groups. In these groups, each student is required to report his or her findings or summarize his or her solution process. When teams have had the chance to share their thoughts in small groups, we come together as a class to share our findings. I do not call for volunteers but simply ask one student to report on a significant point discussed in the group. I might say, "Tanya, will you share with the class one important discovery your group made?" or "James, please summarize for us what Adam shared with you." Students generally feel much more confident in stating ideas when the responsibility for the response is being shared with a partner or group. Using the think-pair-share strategy helps me send the message that participation is not optional.

A modified version of this strategy also works in whole-group discussions. If I do not get the responses that I expect, either in quantity or quality, I give students a chance to discuss the question in small groups. On the basis of the difficulty of the question, they may have as little as fifteen seconds or as long as

several minutes to discuss the question with their partners. This strategy has helped improve discussions more than any others that I have adopted.

2. If students or groups cannot answer a question or contribute to the discussion in a positive way, they must ask a question of the class. I explain that it is all right to be confused, but students are responsible for asking questions that might help them understand.

3. Always require students to ask a question when they need help. When a student says, "I don't get it," he or she may really be saying, "Show me an easy way to do this so I don't have to think." Initially, getting students to ask a question is a big improvement over "I don't get it." Students soon realize that my standards require them to think about the problem in enough depth to ask a question.

4. Require several responses to the same question. Never accept only one response to a question. Always ask for other comments, additions, clarifications, solutions, or methods. This request is difficult for students at first because they have been conditioned to believe that only one answer is correct and that only one correct way is possible to solve a problem. I explain that for them to become better thinkers, they need to investigate the many possible ways of thinking about a problem. Even if two students use the same method to solve a problem, they rarely explain their thinking in exactly the same way. Multiple explanations help other students understand and clarify their thinking. One goal is to create a student-centered classroom in which students are responsible for the conversation. To accomplish this goal, I try not to comment after each response. I simply pause and wait for the next student to offer comments. If the pause alone does not generate further discussion, I may ask, "Next?" or "What do you think about _____'s idea?"

5. No one in a group is finished until everyone in the group can explain and defend the solution. This rule forces students to work together, communicate, and be responsible for the learning of everyone in the group. The learning of any one person is of little value unless it can be communicated to others, and those who would rather work on their own often need encouragement to develop valuable communication skills.

6. Use hand signals often. Using hand signals - thumbs up or thumbs down (a horizontal thumb means "I'm not sure") - accomplishes two things. First, by requiring all students to respond with hand signals, I ensure that all students are on task. Second, by observing the responses, I can find out how many students are having difficulty or do not understand. Watching students' faces as they think about how to respond is very revealing.

7. Never carry a pencil. If I carry a pencil with me or pick up a student's pencil, I am tempted to do the work for the student. Instead, I must take time to ask thought-provoking questions that will lead to understanding.

8. Avoid answering my own questions. Answering my own questions only confuses students because it requires them to guess which questions I really want them to think about, and I want them to think about all my questions. I also avoid rhetorical questions.

9. Ask questions of the whole group. As soon as I direct a question to an individual, I suggest to the rest of the students that they are no longer required to think.

10. Limit the use of group responses. Group responses lower the level of concern and allow some students to hide and not think about my questions.

11. Do not allow students to blurt out answers. A student's blurted out answer is a signal to the rest of the class to stop thinking. Students who develop this habit must realize that they are cheating other students of the right to think about the question.

Summary

LIKE MOST TEACHERS, I ENTERED THE TEACHING profession because I care about children. It is only natural for me to want them to be successful, but by merely telling them answers, doing things for them, or showing them shortcuts, I relieve students of their responsibilities and cheat them of the opportunity to make sense of the mathematics that they are learning. To help students engage in real learning, I must ask good questions, allow students to struggle, and place the responsibility for learning directly on their shoulders. I am convinced that children learn in more ways than I know how to teach. By listening to them, I not only give them the opportunity to develop deep understanding but also am able to develop true insights into what they know and how they think.

Making extensive changes in curriculum and instruction is a challenging process. Much can be learned about how children think and learn, from recent publications about learning styles, multiple intelligences, and brain research. Also, several reform curriculum projects funded by the National Science Foundation are now available from publishers. The Connected Mathematics Project, Mathematics in Context, and Math Scape, to name a few, artfully address issues of content and pedagogy.

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