COMPOSITION BOOK

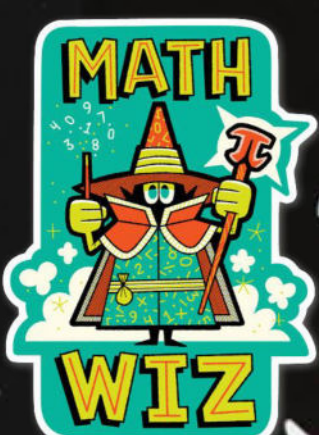
Ms. Forbes'

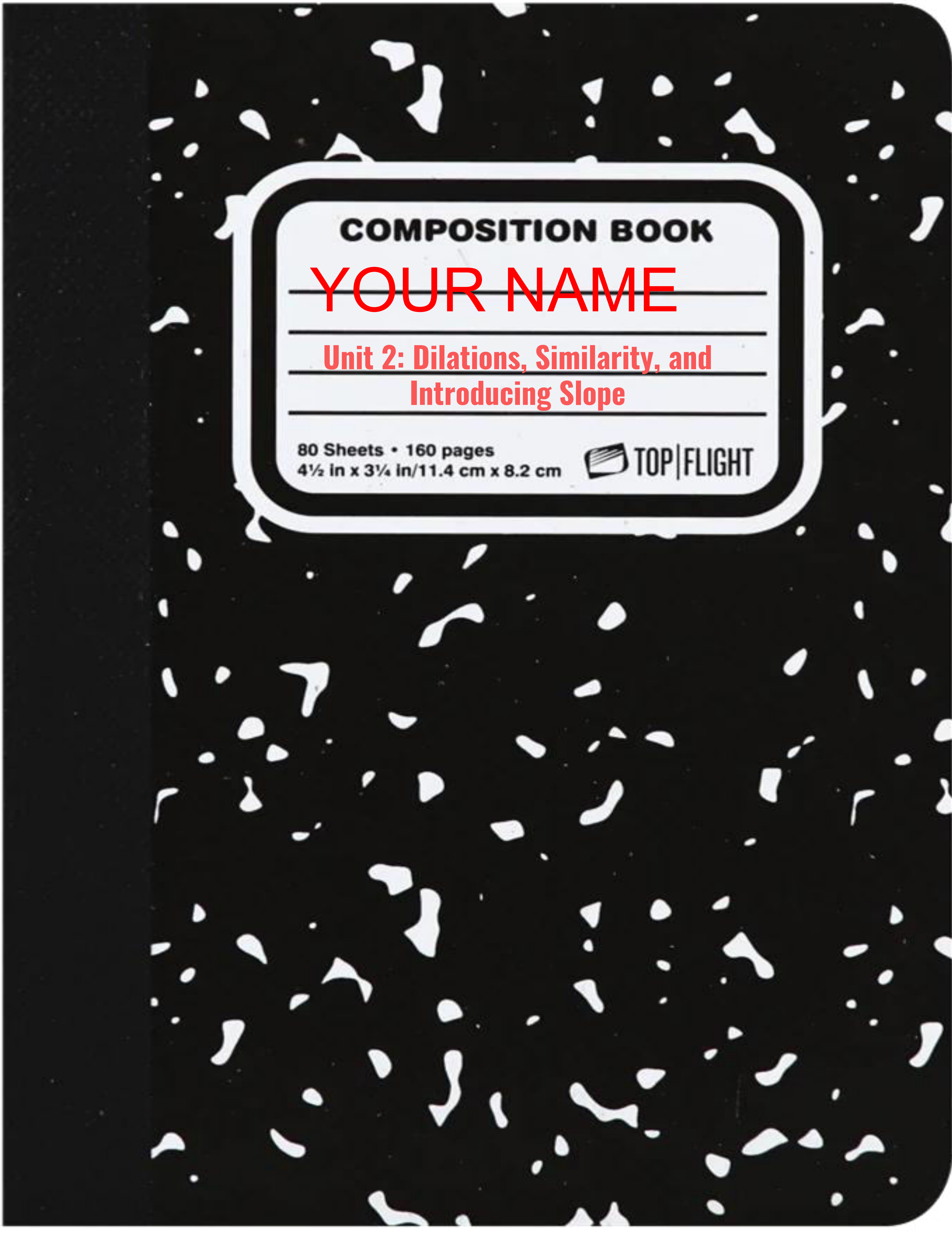
Math 8 Journal

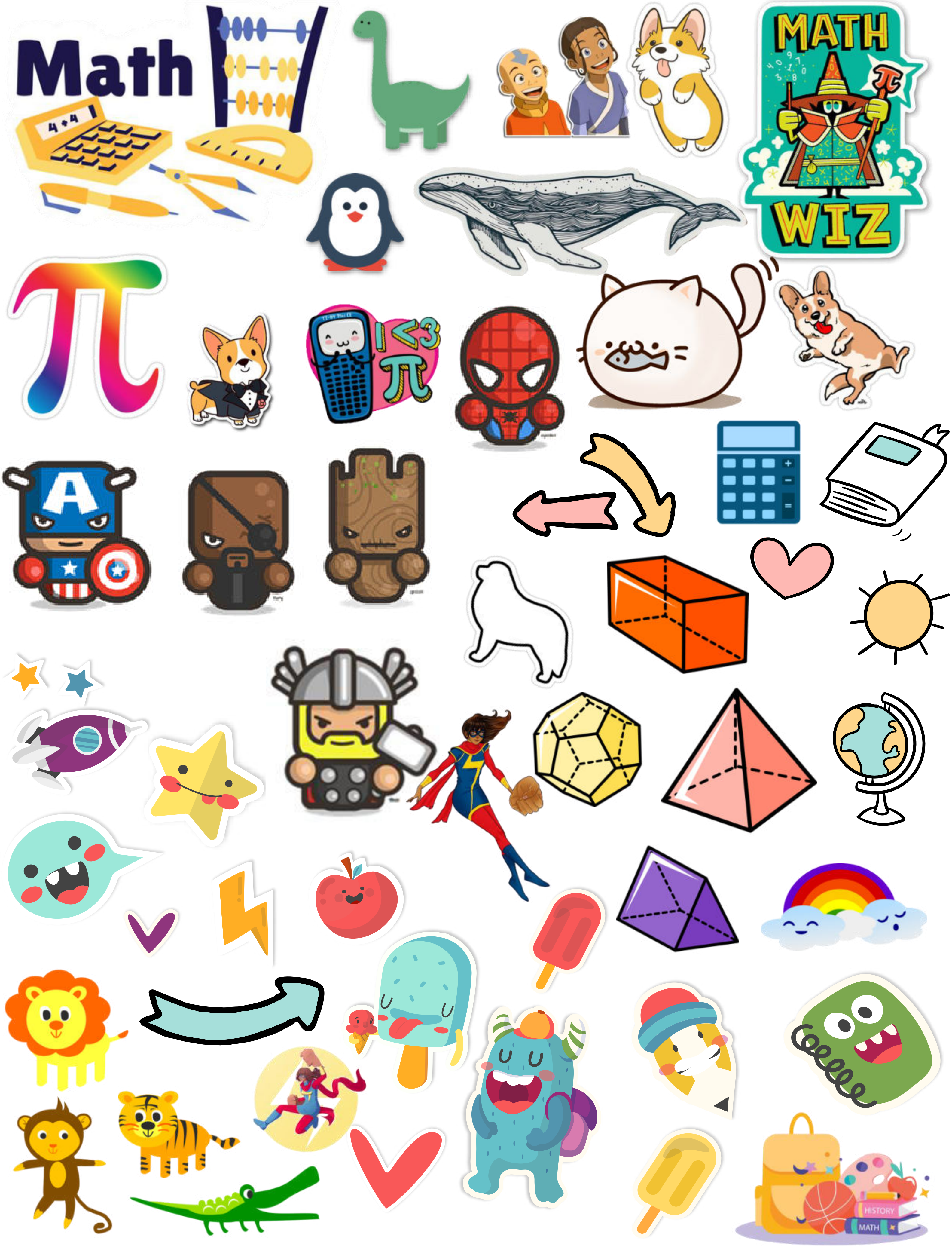
Unit 2: Dilations, Similarity, and Introducing Slope

80 Sheets • 160 pages
4½ in x 3¼ in/11.4 cm x 8.2 cm

TOP|FLIGHT




COMPOSITION BOOK
YOUR NAME
Unit 2: Dilations, Similarity, and Introducing Slope
80 Sheets • 160 pages
4½ in x 3¼ in/11.4 cm x 8.2 cm
TOP|FLIGHT


Math
4+4
MATH
WIZ
I<3 π
HISTORY
MATH


OK!
STRANGER THINGS
NOPE
COMPOSITION
OMG!
LOL!
STAR WARS

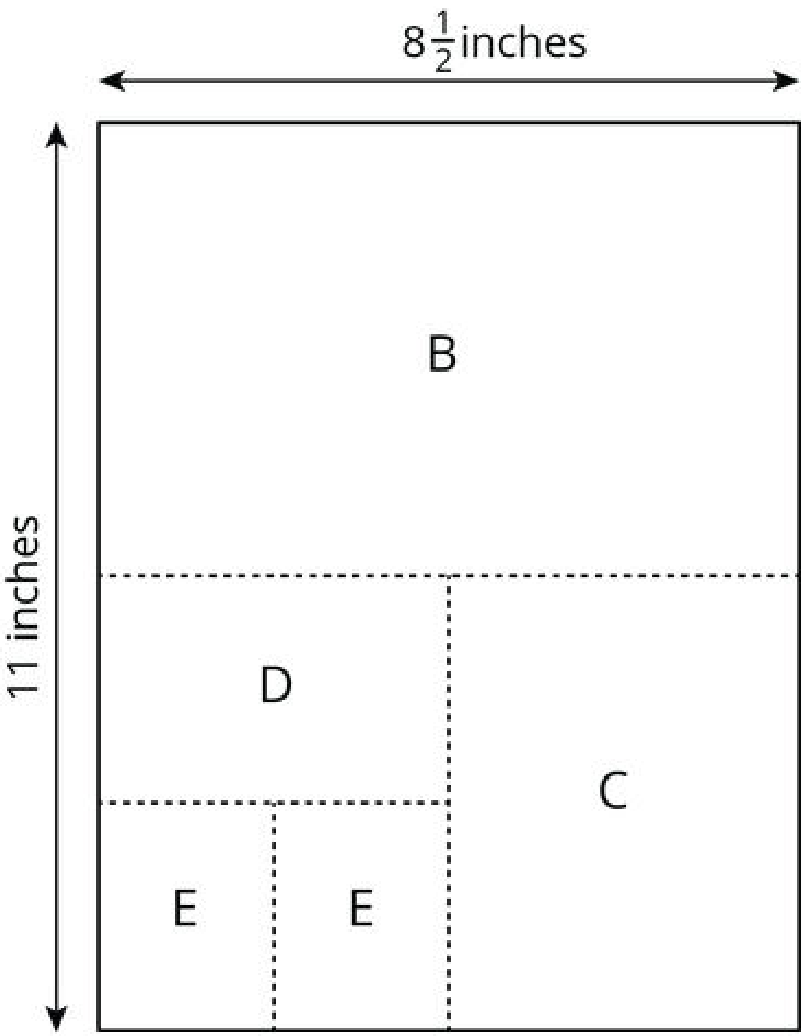
Table of Contents

Lesson 1

Lesson 1: Projecting and Scaling

Sorting Rectangles

Rectangles were made by cutting an 8 ½ - inch by 11 - inch piece of paper in half, in half again, and so on, as illustrated in the diagram. Find the lengths of each rectangle and enter them in the appropriate table.



Some of the rectangles are scaled copies of the full sheet of paper (Rectangle A). Enter the measurements of those rectangles in the table.

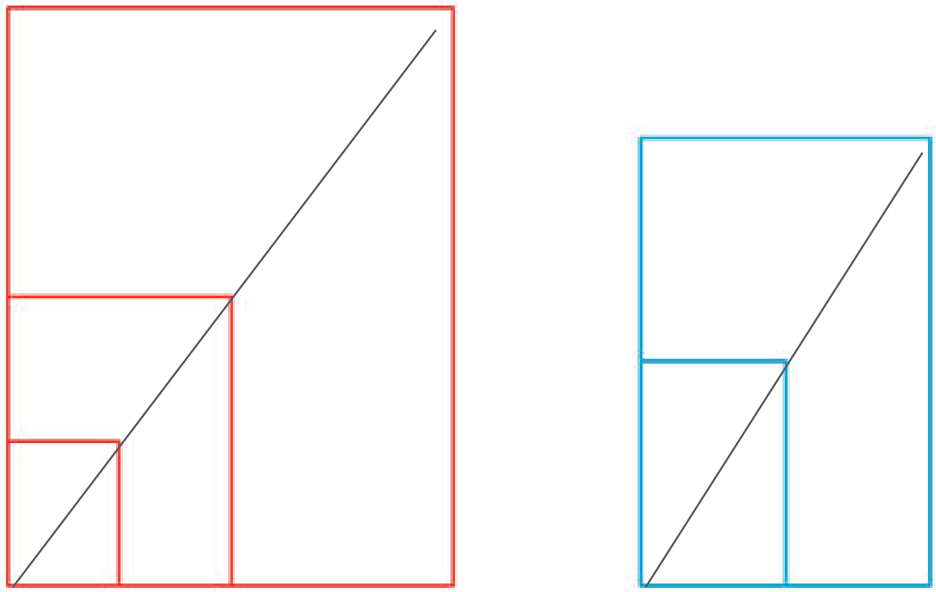
rectangle	length of short side (inches)	length of long side (inches)
A	$8\frac{1}{2}$	11

Some of the rectangles are *not* scaled copies of the full sheet of paper. Enter the measurements of those rectangles in the table.

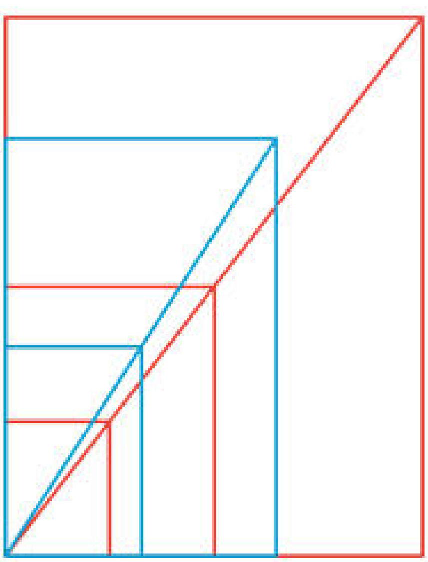
rectangle	length of short side (inches)	length of long side (inches)

Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice about the measurements of these rectangles? Look at the measurements for the rectangles that are *not* scaled copies of the full sheet. What do you notice about these measurements.

Look at the stack of rectangles that are scaled copies of the full sheet (red). Look at the stack of the other rectangles (blue). What do you notice.

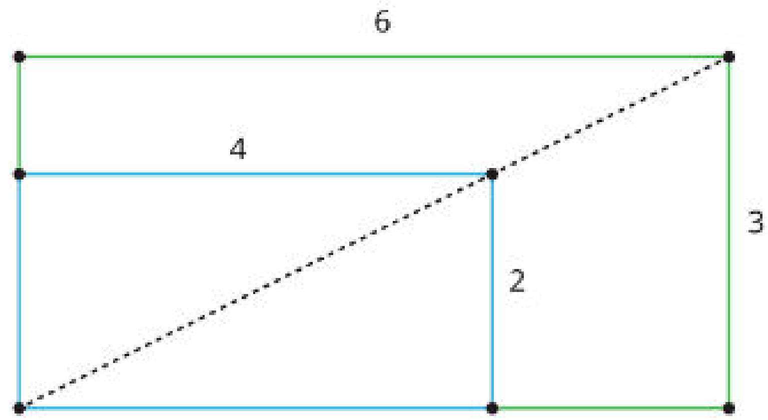


Look at the stack of *all* of the rectangles from largest to smallest. From the lines drawn, can you tell which set each rectangle came from?



Lesson 1 Summary

Scaled copies of rectangles have an interesting property. Can you see what it is?

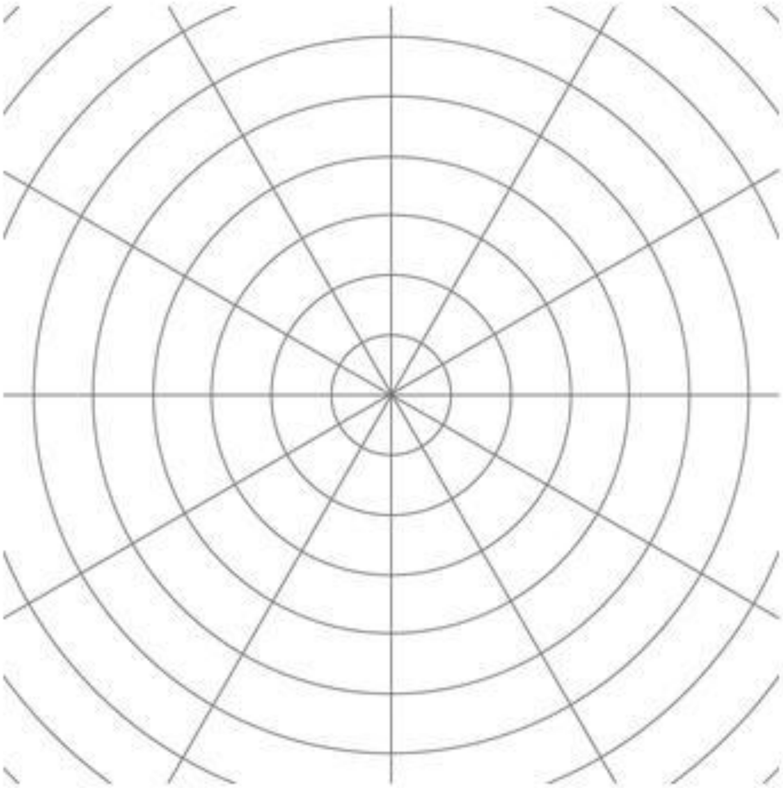


Here, the larger rectangle is a scaled copy of the smaller one (with a scale factor of $\frac{3}{2}$). Notice how the diagonal of the large rectangle contains the diagonal of the smaller rectangle. This is the case for any two scaled copies of a rectangle if we line them up as shown. If two rectangles are *not* scaled copies of one another, then the diagonals do not match up. In this unit, we will investigate how to make scaled copies of a figure.

Lesson 2

Lesson 2: Circular Grid

Concentric Circles



What do you notice? What do you wonder?

Have you ever seen a pebble dropped in a still pond? Describe what happens.

A Droplet on the Surface

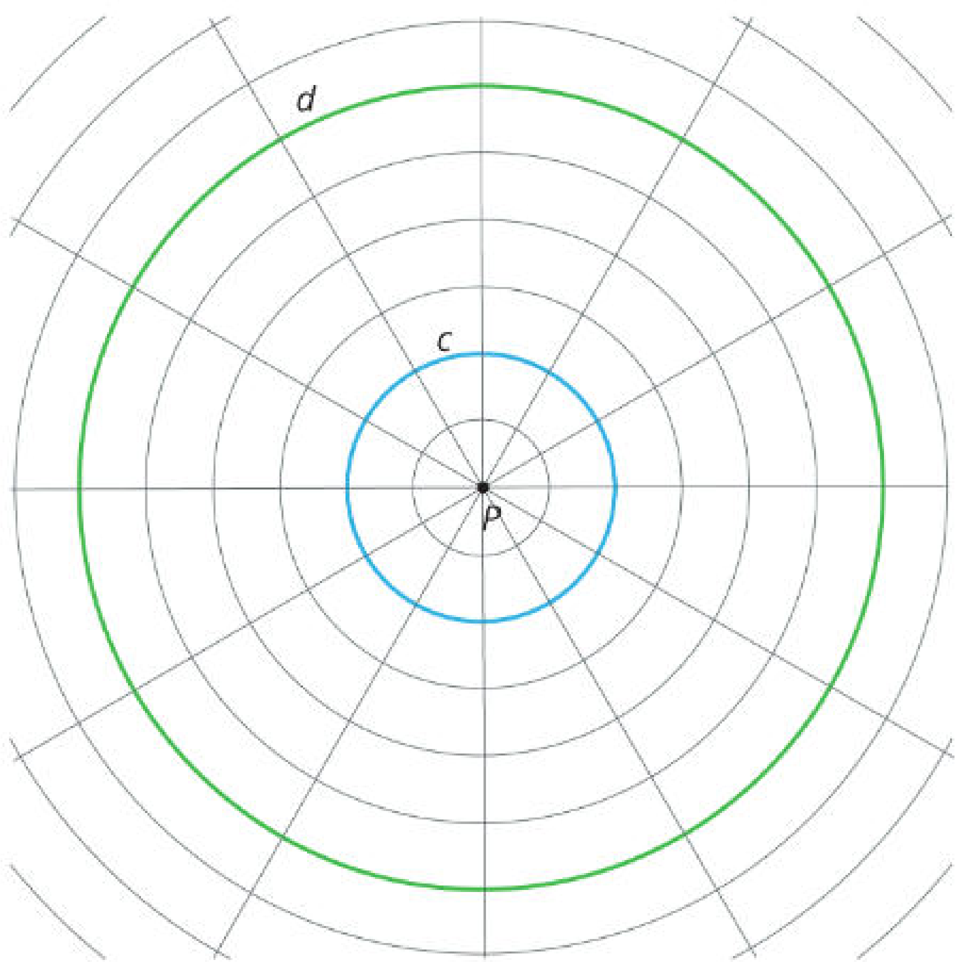
Click on the picture to use the applet to answer the following questions.

The larger Circle d is a **dilation** of the smaller Circle c. P is the **center of dilation**.

1. Draw four points *on* the smaller circle (not inside the circle!), and label them E, F, G, and H.

2. Draw the rays from P through each of those four points.

3. Label the points where the rays meet the larger circle E', F', G', and H'.



Complete the table. In the row labeled **S**, write the distance between **P** and the point on the smaller grid in units. In the row labeled L, write the distance between **P** and the corresponding point on the larger circle in grid units..

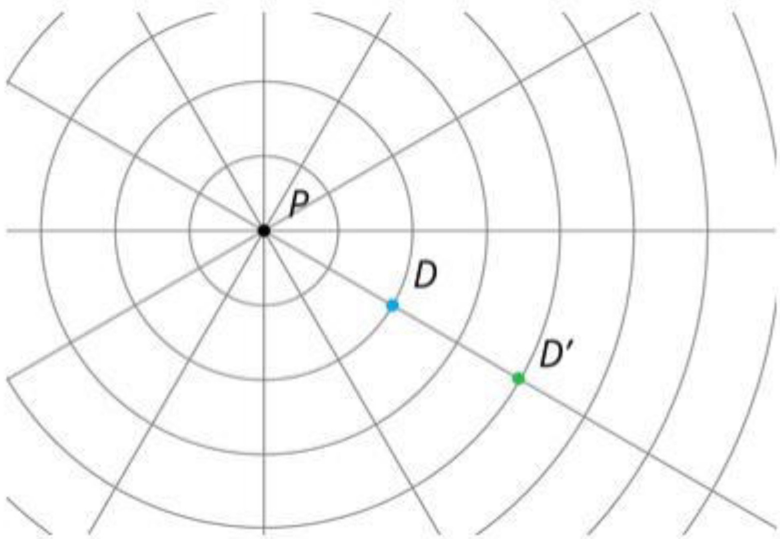
	E	F	G	H
S				
L				

The center of dilation is point ***P***. What is the *scale factor* that takes the smaller circle to the larger circle? Explain your reasoning.

Quadrilateral on a Circular Grid

You will dilate some points. In the previous activity, each point was dilated to its image using a scale factor of 3. The dilated point was three times as far from the center as the original point. When we dilate point ***D*** using ***P*** as the center of dilation and a scale factor of 2, that means we're going to take the distance from ***P*** to ***D*** and place a new point on the ray ***PD*** twice as far away from ***P***.

Click on the image to use the applet and complete the following:



Here is a polygon .

1. Dilate each vertex of polygon ***ABCD*** using ***P*** as the center of dilation and a scale factor of 2. Label the image of ***A*** as ***A',*** and label the images of the remaining three vertices as ***B'***, ***C'***, and ***D'***.

2. Draw segments between the dilated points to create polygon ***A'B'C'D'***.

3. What are some thing you notice about the new polygon?.

4. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?

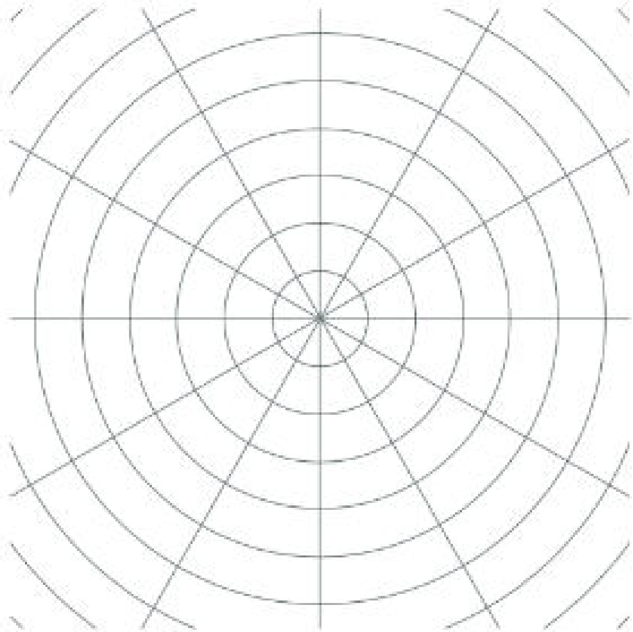
5. Dilate each vertex of polygon ***ABCD*** using ***P*** as the center of dilation and a scale factor of ½. Label the image of ***A*** as ***E***, the image of ***B*** as ***F***, the image of ***C*** as ***G*** and the image of ***D*** as ***H***.

6. What do you notice about polygon ***EFGH***?

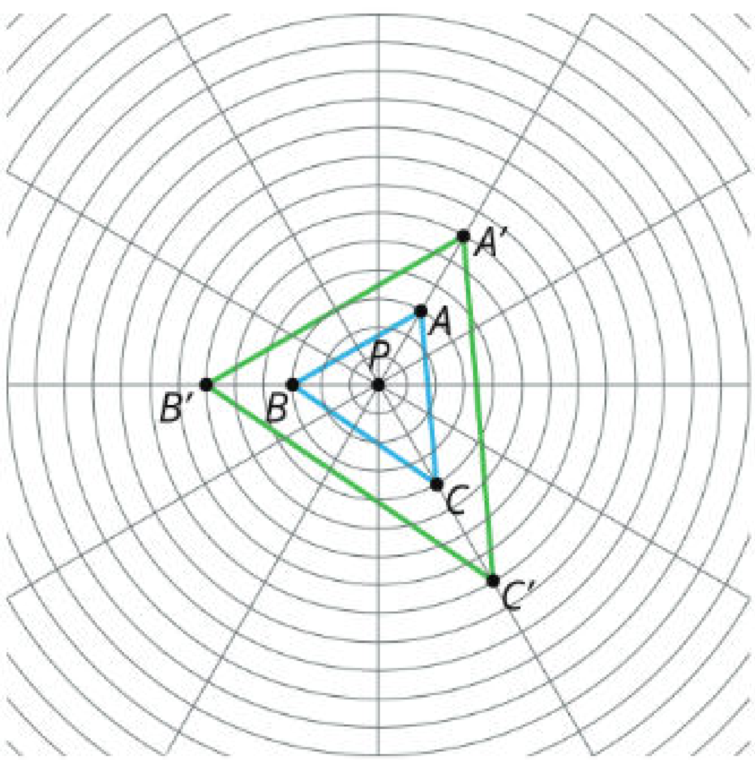
Lesson 2 Summary

A circular grid like this one can be helpful for performing **dilations**.

The radius of the smallest circle is one unit, and the radius of each successive circle is one unit more than the previous one.



To perform a dilation, we need a **center of dilation**, a scale factor, and a point to dilat
In the picture, P is the center of dilation. With a scale factor of 2, each point stays on th
same ray from P, but its distance from P doubles:



Since the circles on the grid are the same distance apart, segment PA' has twice the length of segment PA, and the same holds for the other points.

Lesson 3

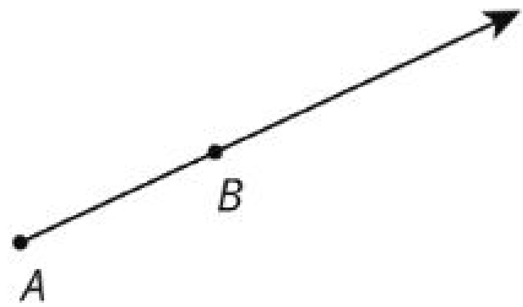
Lesson 3: Dilations with no Grid

Points on a Ray

Click on the image to use the app to complete the following

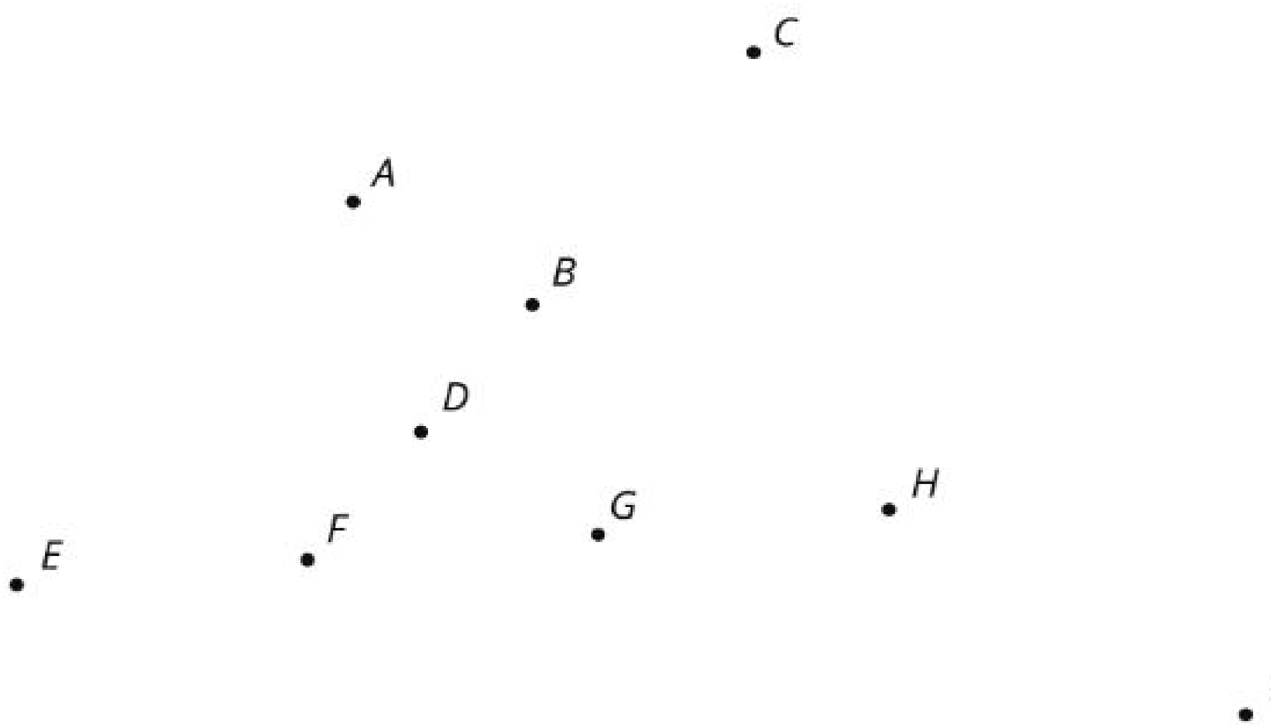
1. Find and label a point C on the ray whose distance from A is twice the distance from B to A.

2. Find and label a point D on the ray whose distance from A is half the distance from B to A.



Dilation Obstacle Course

Here is a diagram that shows nine points. Click on the image to use the applet to answer the following questions.



1. Dilate B using a scale factor of 5 and A is the center of dilation. Which point is its image?

1. Using H as the center of dilation, dilate G so that its image is E. What scale factor did you use?

1. Using H as the center of dilation, dilate E so that its image is G. What scale factor did you use?

1. To dilate F so that its image is B, what point on the diagram can you use as a center?

1. Dilate H using A as the center and a scale factor of ⅓. Which point is its image?

1. Describe a dilation that uses a labeled point as its center and that would take F to H.

Getting Perspective

Click on the picture to use the applet to perform the following directions.

1. Draw images of points ***P*** and ***Q*** using ***C*** as the center of dilation and a scale factor of 4. Label the new points ***P'*** and ***Q'***.
2. Draw the images of points ***P*** and ***Q*** using ***C*** as the center of dilation and a scale factor ½. Label the new points ***P"*** and ***Q"***.

Lesson 3 Summary

If A is the center of dilation, how can we find which point is the dilation of B with scale factor 2?

Since the scale factor is larger than 1, the point must be farther away from A than B is, which makes C the point we are looking for. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B.

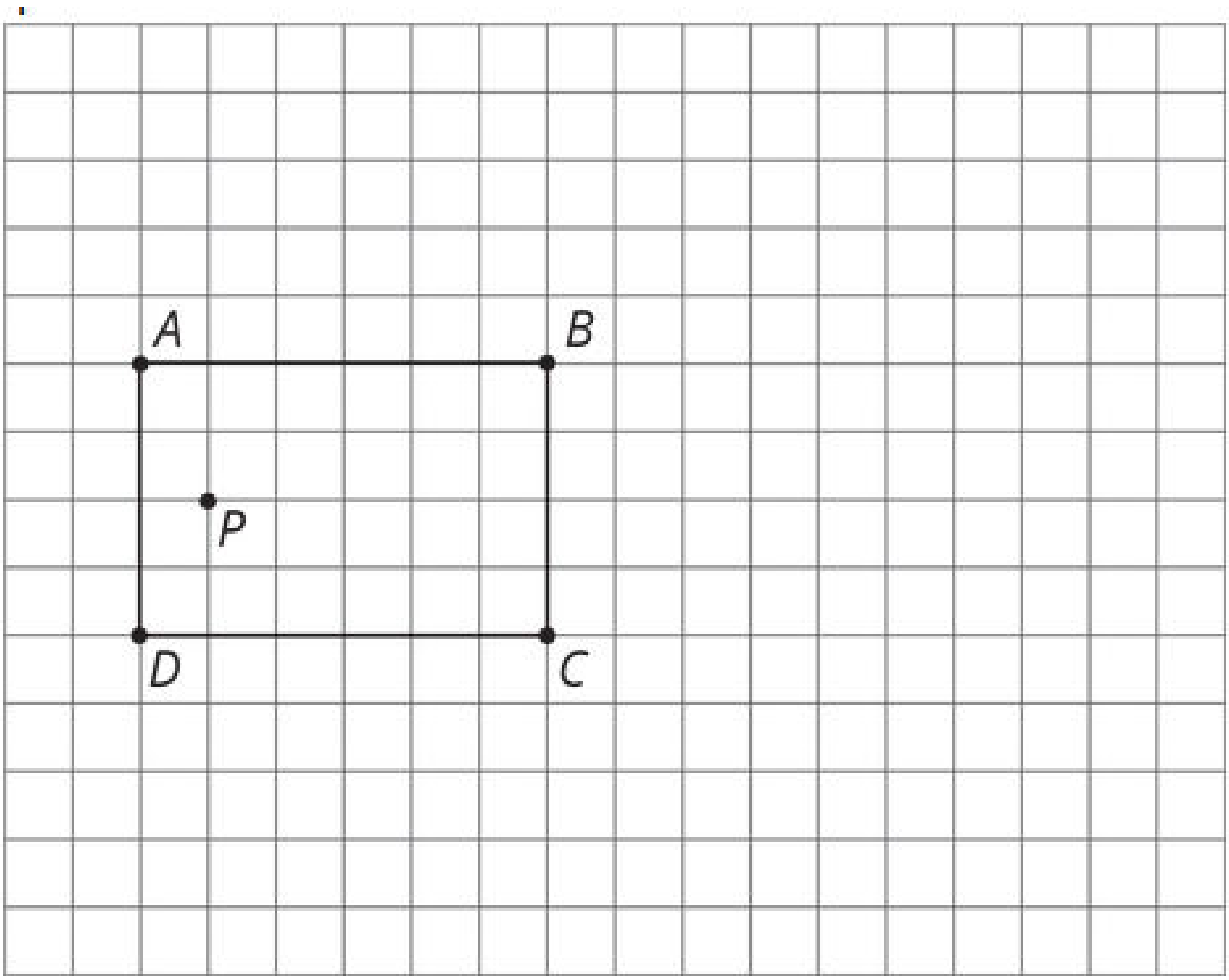
A dilation with scale factor less than 1 brings points closer. The point D is the dilation of B with center A and scale factor $\frac{1}{3}$.

Lesson 4

Lesson 4: Dilations on a Square Grid

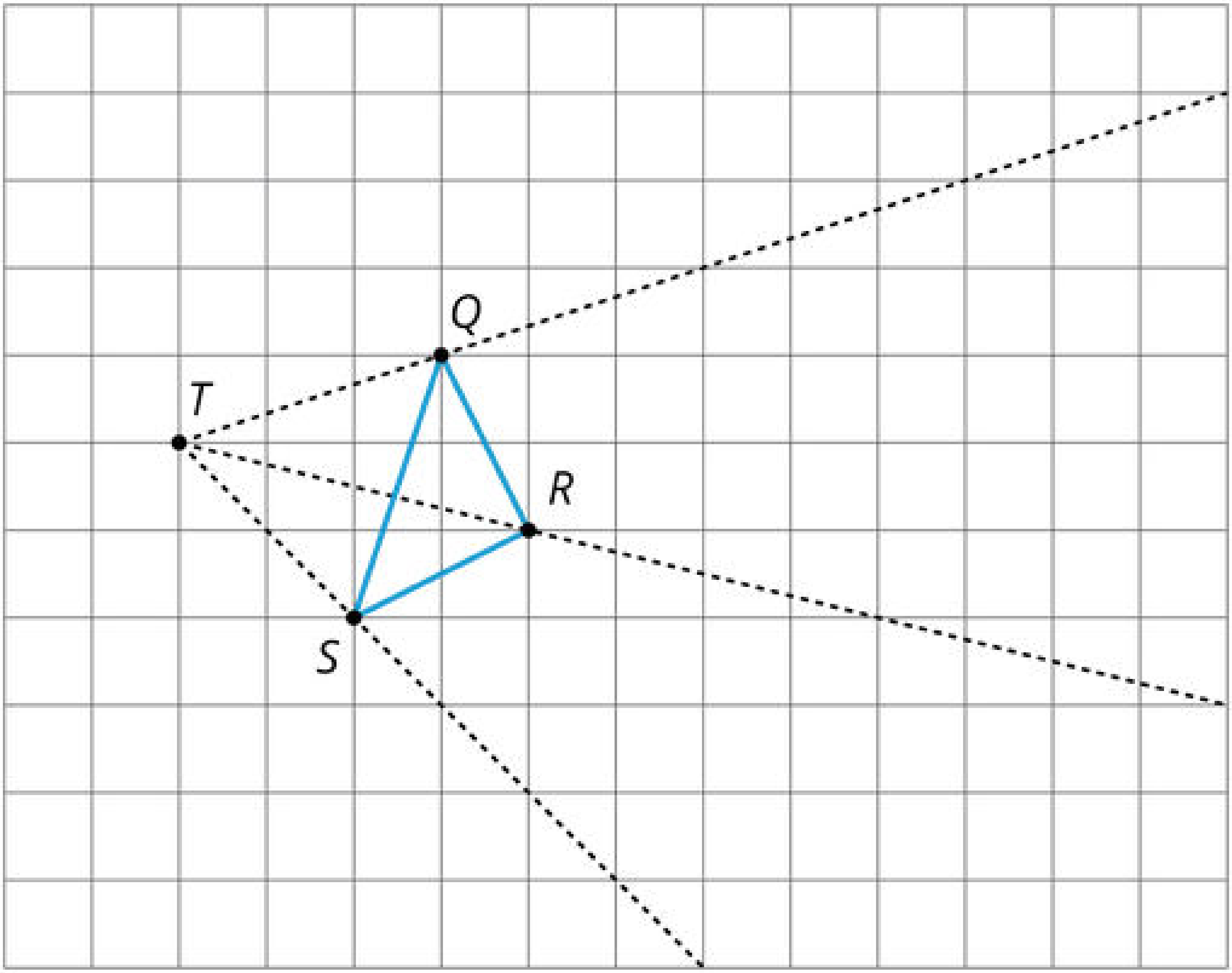
Dilations on a Grid

1. Use the line tool to find the dilation of quadrilateral ABCD with center *P* and scale factor 2.



2. Find the dilation of triangle *QRS* with center *T* and scale factor 2.

3. Find the dilation of triangle QRS with center *T* and scale factor ½



Card Sort: Matching Dilations on a Coordinate Grid

Cards 1 - 6 show a figure in the coordinate plane and describes a dilation.

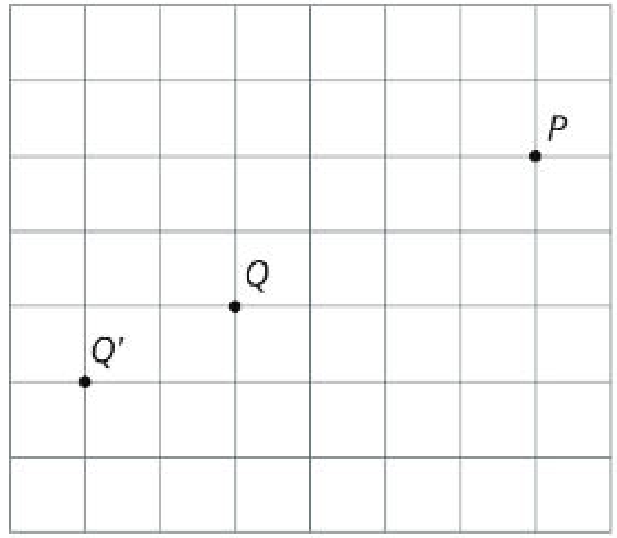
Cards A - E describe the image of the dilation for one of the numbered cards.

Match number cards with letter cards. One of the number cards will not have a match.

Lesson 4 Summary

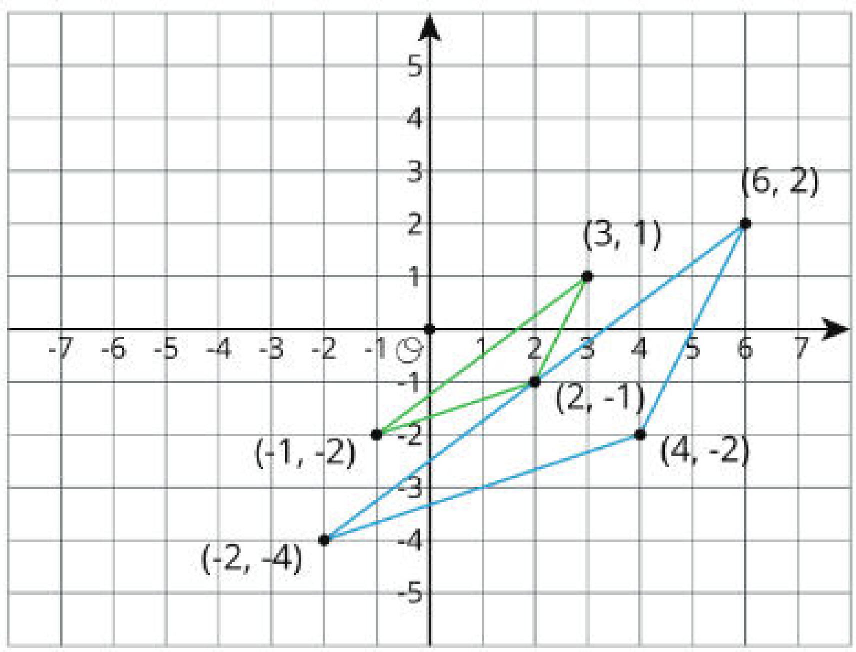
Square grids can be useful for showing dilations. The grid is helpful especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, suppose we want to dilate point Q with center of dilation P and scale factor $\frac{3}{2}$. Since Q is 4 grid squares to the left and 2 grid squares down from P, the dilation will be 6 grid squares to the left and 3 grid squares down from P (can you see why?). The dilated image is marked as Q' in the picture.



Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to *name* points, and sometimes the coordinates of the image can be found with just arithmetic.

For example, to make a dilation with center $(0, 0)$ and scale factor 2 of the triangle with coordinates $(-1, -2)$, $(3, 1)$, and $(2, -1)$, we can just double the coordinates to get $(-2, -4)$, $(6, 2)$, and $(4, -2)$.



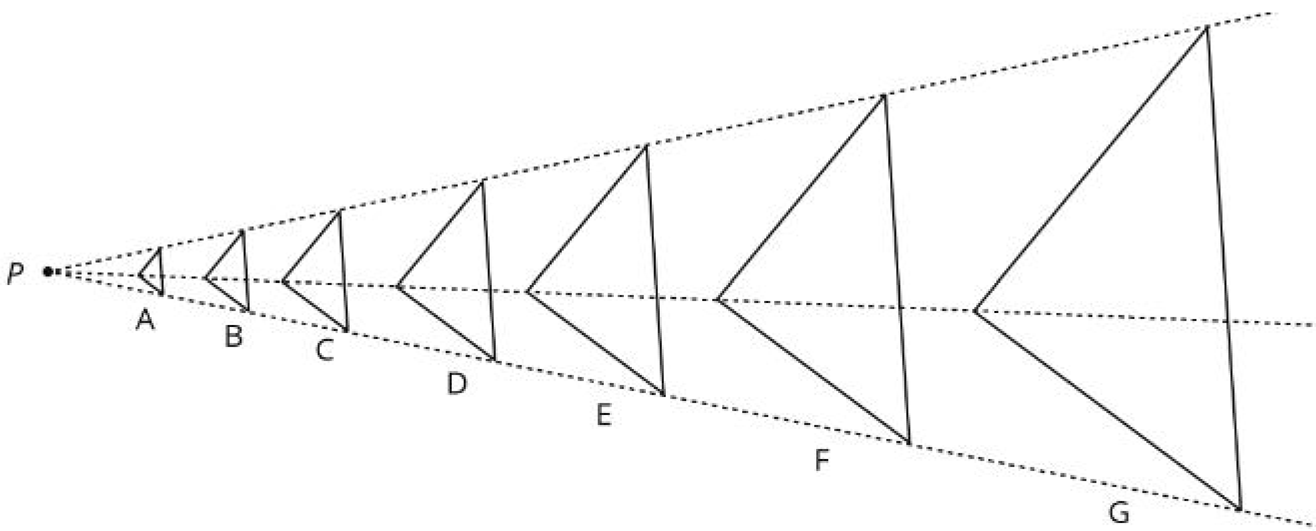
Lesson 5

Lesson 5: More Dilations

Many Dilations of a Triangle

All of these triangles are dilations of Triangle *D*. The dilations use the same center *P*, but different scale factors. What do Triangles A, B, and C have in common? What do Triangles E, F, and G have in common? What does this tell us about the different scale factors used?

Click on the image to use the applet.



Info Gap: Dilations

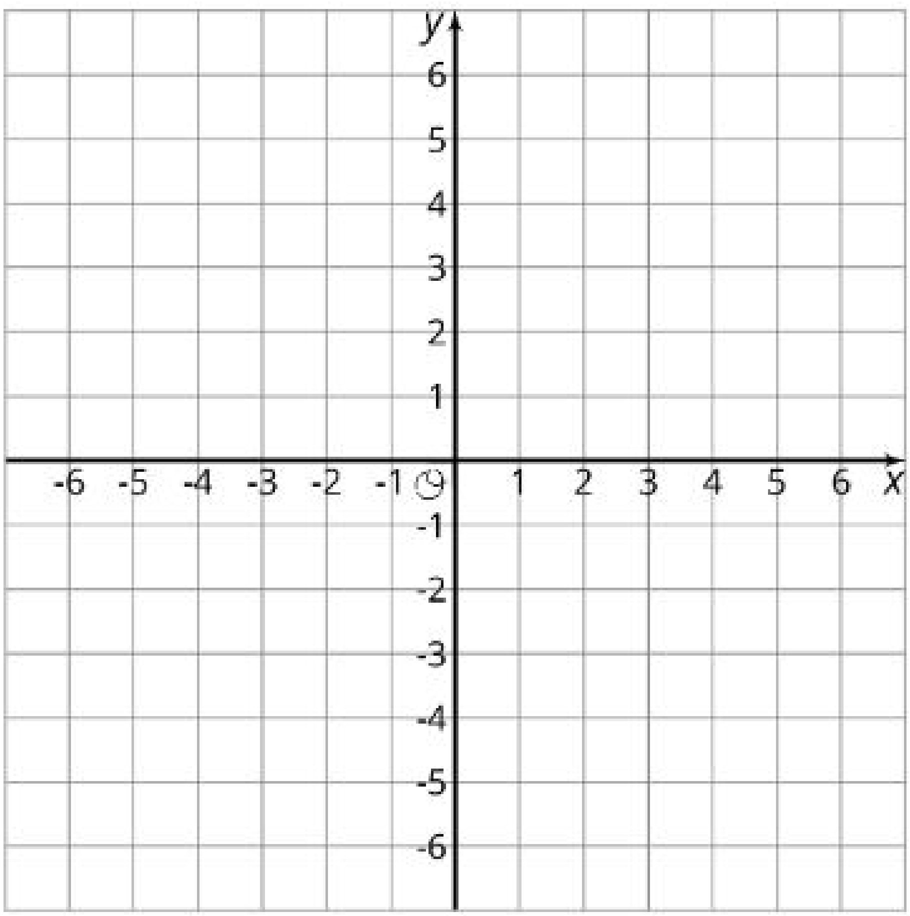
This problem was done in class. Think about what questions you would ask to solve the problem.

Info Gap: Dilations

Problem Card 1

Polygon *AFID* is dilated.

Draw the image of *AFID* under this dilation.



Lesson 5 Summary

One important use of coordinates is to communicate geometric information precisely. Let's consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires three vital pieces of information:

1. The coordinates of A, B, C, and D
2. The coordinates of the center of dilation, P
3. The scale factor of the dilation

With this information, we can dilate the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.

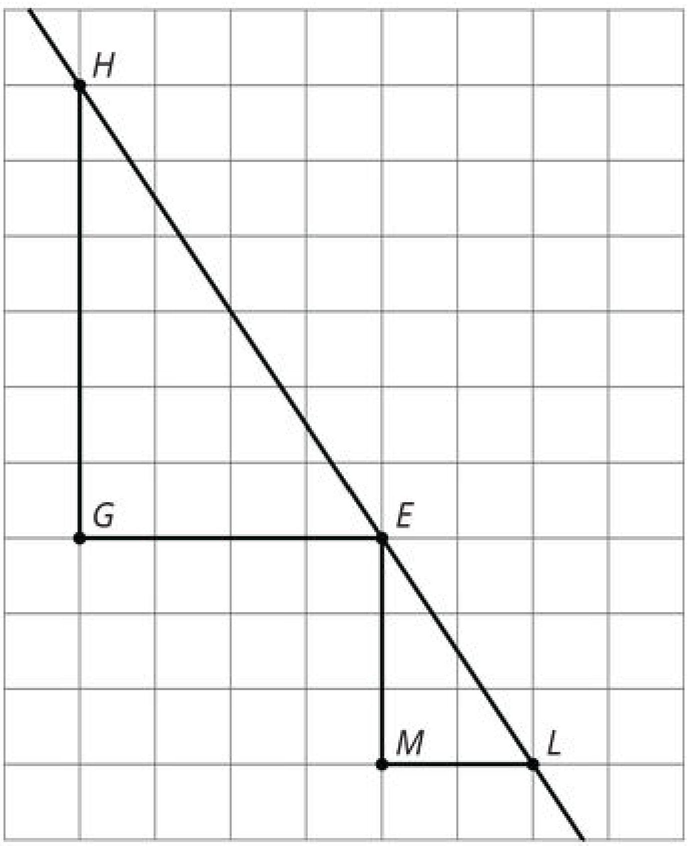
Lesson 6

Lesson 6: Similarity

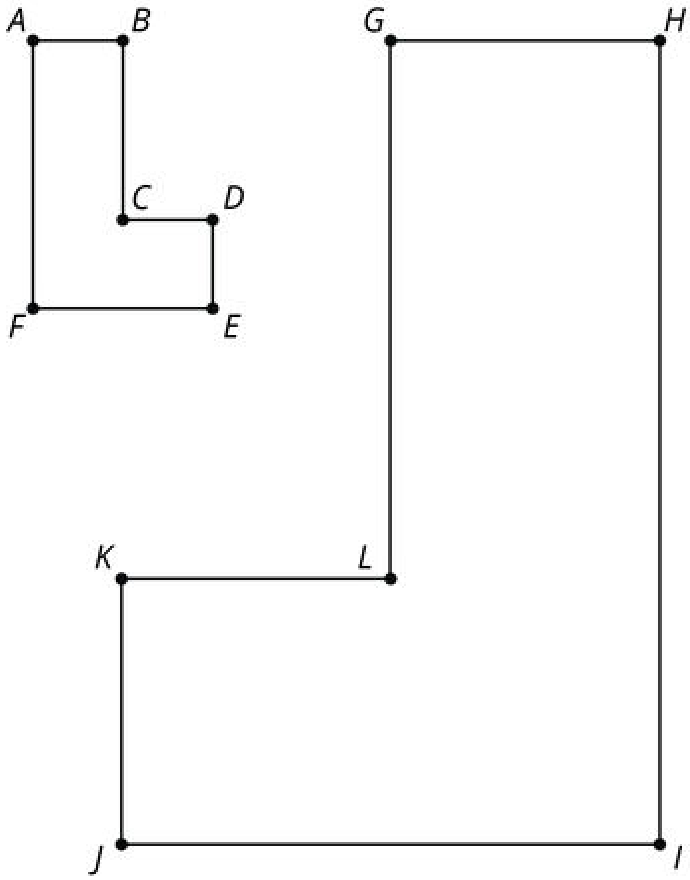
Similarity Transformations (Part 1)

What does it mean for two figures to be *similar*.

Triangle EGH and triangle LME are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.

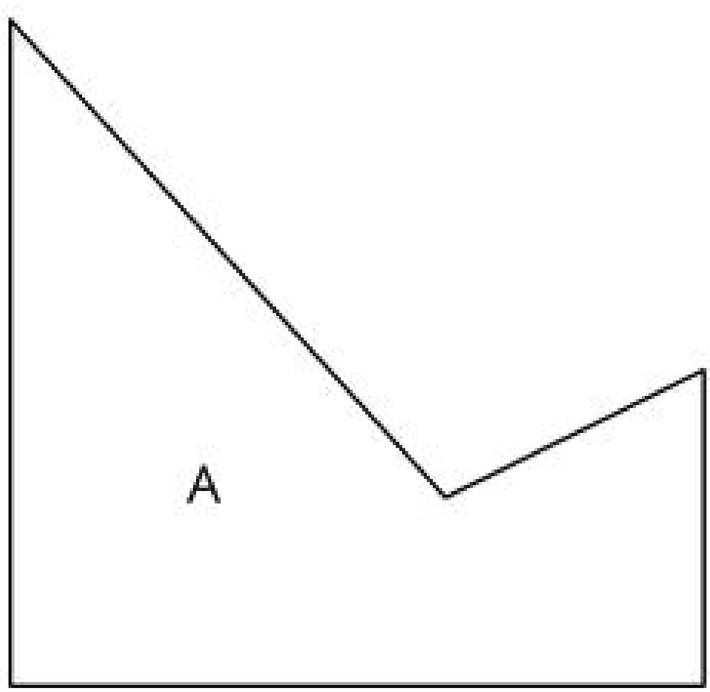


Hexagon ***ABCDEF*** and hexagon ***HGLKJI*** are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



Similarity Transformations (Part 2)

Sketch figures similar to Figure A that use only the transformations listed to show similarity.



1. A translation and a reflection. Label your sketch Figure B. Pause here so that your teacher can check your work.

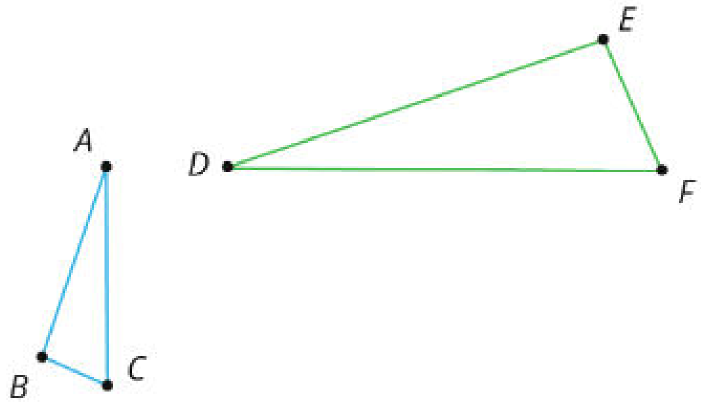
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.

3. A rotation and a reflection. Label your sketch Figure D.

4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

Lesson 6 Summary

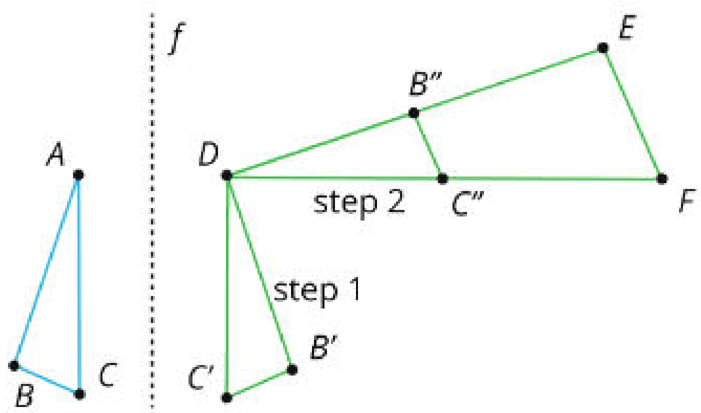
Let's show that triangle ABC is similar to triangle DEF:



Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from ABC to DEF follows these steps:

- step 1: reflect across line f
- step 2: rotate 90° counterclockwise around D
- step 3: dilate with center D and scale factor 2



Another way would be to dilate triangle ABC by a scale factor of 2 with center of dilation A, then translate A to D, then reflect over a vertical line through D, and finally rotate it so it matches up with triangle DEF. What steps would you choose to show the two triangles are similar?

Lesson 7

Lesson 7: Similar Polygons

All, Some, None: Congruence and Similarity

Choose whether each of the statements is true in *all* cases, in *some* cases, or in *no* cases.

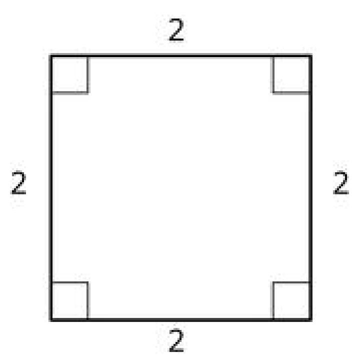
1. If two figures are congruent, then they are similar.

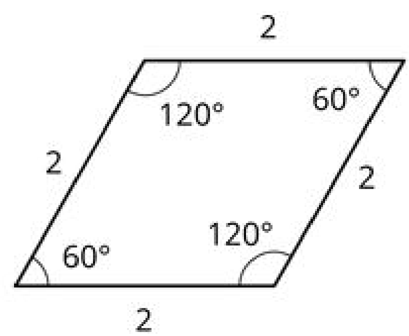
1. If two figures are similar, then they are congruent.

1. If an angle is dilated with the center of dilation at its vertex, the angle measure may change.

Are They Similar?

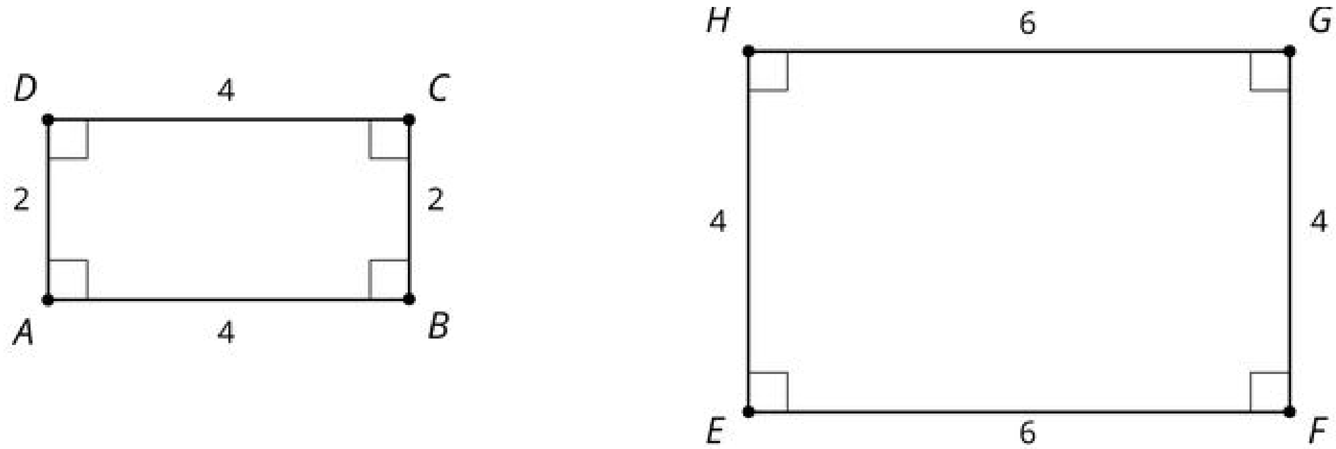
1. Let’s look at a square and a rhombus.





Priya says, “These polygons are similar because their side lengths are all the same.” Clare says, “These polygons are not similar because the angles are different.” Do you agree with either Priya or Clare? Explain your reasoning.

2. Now, let's look at rectangles *ABCD* and *EFGH.*



Jada says, "These rectangles are similar because all of the side lengths differ by 2." Lin says, "These rectangles are similar. I can dilate ***AD*** and ***BC*** using a scale factor of 2 and ***AB*** and ***CD*** using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles." Do you agree with either Jada or Lin? Explain your reasoning.

Find the Similar

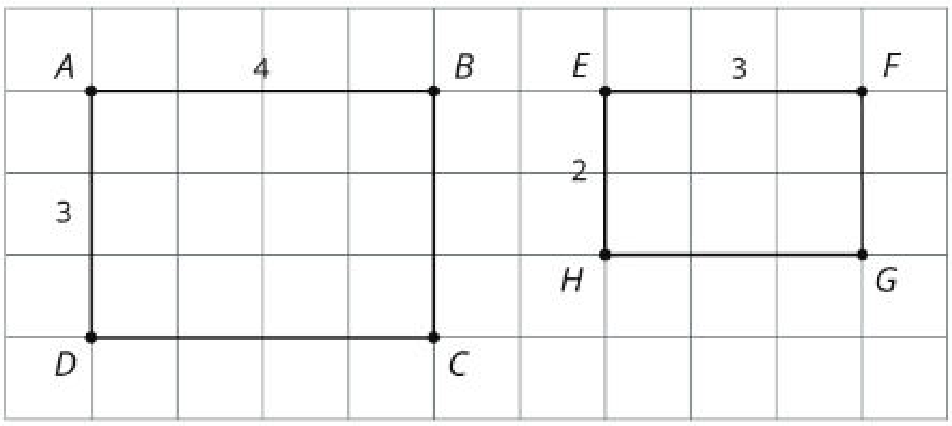
You need to find a card with a polygon that similar (but not congruent) to another card with a polygon.

Lesson 7 Summary

When two polygons are similar:

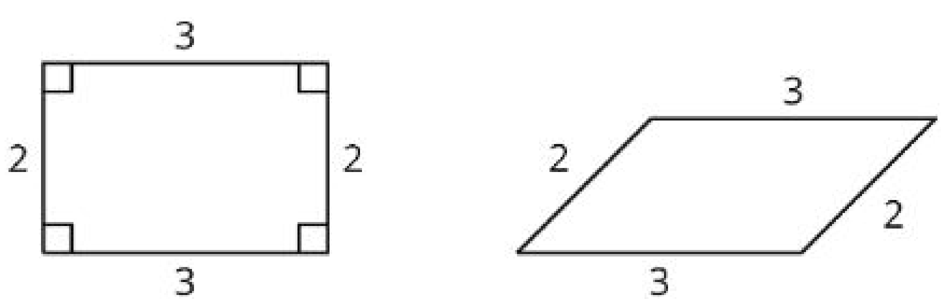
- Every angle and side in one polygon has a corresponding part in the other polygon.
- All pairs of corresponding angles have the same measure.
- Corresponding sides are related by a single scale factor. Each side length in one figure is multiplied by the scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?



It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that "looks like" isn't enough to make them similar. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match AD to EH has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond (with a scale factor of 1), but the quadrilaterals are not similar because the angles don't have the same measure:



Lesson 8

Lesson 8: Similar Triangles

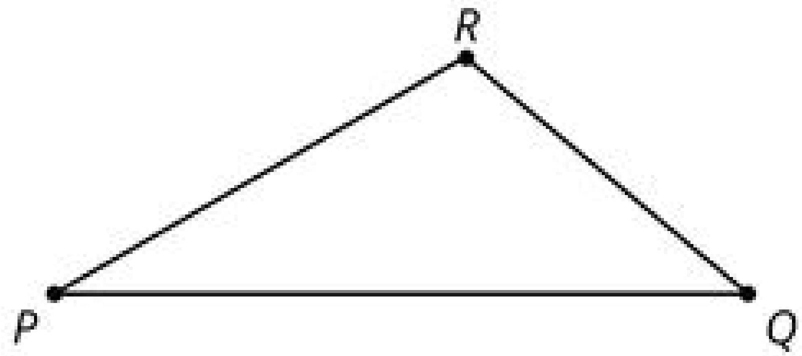
Making Pasta Angles and Triangles

You will build triangles using “pasta” and some of their given angles.
Use the [link] to complete the following activity..

1. Create a triangle using three pieces of “pasta” and angle A. Your triangle *must* include the angle you were given, but you are otherwise free to make any triangle you like.
    a. After triangle is created, measure each side length and record the length. Then measure the angles to the nearest 5 degrees and record the measurements.
    b. Find two others in the room who have the same angle A and compare your triangles. What is the same? Different? Are they congruent? Similar?
    c. How did you decide if they were or were not congruent or similar?

1. Now use more pasta and angles A, B, and C to create another triangle
    a. After triangle created, measure each side length and each angle and record the measurements.
    b. Find two others in the room who have the same angle A and compare your triangles. What is the same? Different? Are they congruent? Similar?

3. Here is triangle PQR. Break a new piece of “pasta”, different in length than segment PQ.



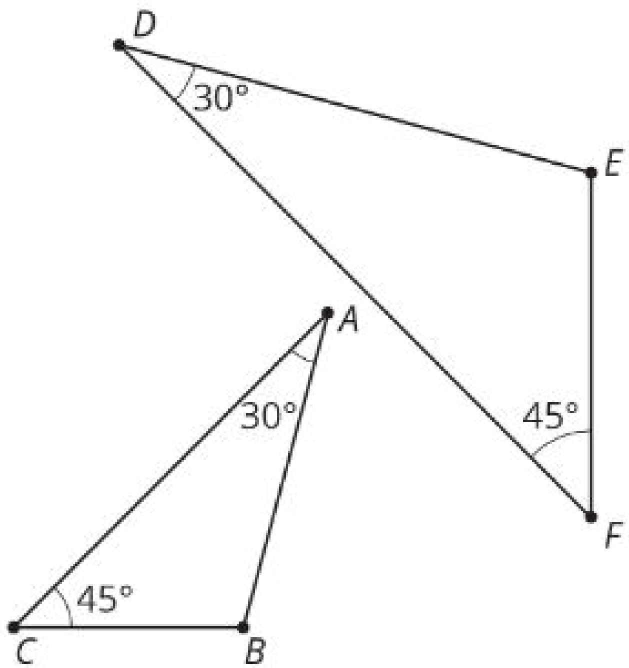
a. Is your new pasta triangle PST similar to $\triangle PQR$? Explain your reasoning.

b. If your broken piece of pasta were a different length, would the pasta triangle still be similar to $\triangle PQR$? Explain your reasoning.

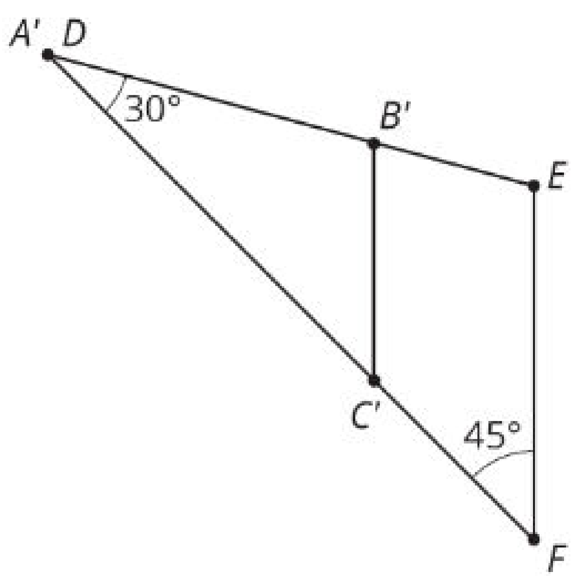
Lesson 8 Summary

We learned earlier that two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. When the polygons are triangles, we only need to check that that both triangles have two corresponding angles to show they are similar—can you tell why?

Here is an example. Triangle ABC and triangle DEF each have a 30 degree angle and a 45 degree angle.



We can translate A to D and then rotate so that the two 30 degree angles are aligned, giving this picture:



Now a dilation with center D and appropriate scale factor will move C' to F. This dilation also moves B' to E, showing that triangles ABC and DEF are similar.

Lesson 9

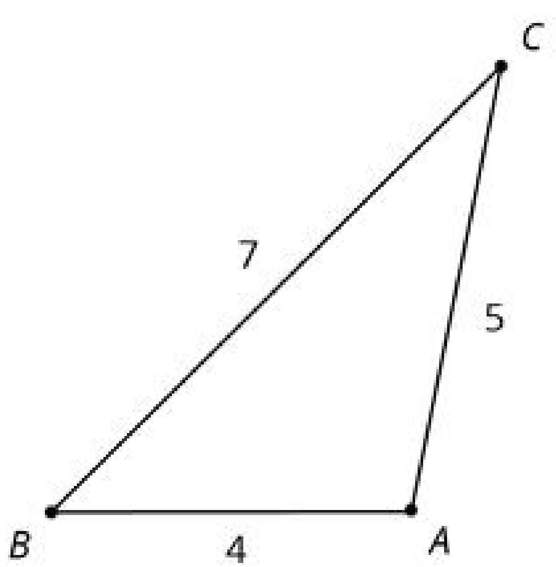
Lesson 9: Side Length Quotients in Similar Triangles

Two-three-four and Four-five-six

Triangle ***A*** has side lengths 2, 3, and 4. Triangle ***B*** has side lengths 4, 5, and 6. Is Triangle ***A*** similar to Triangle ***B***?

Quotients of Sides Within Similar Triangles

Triangle ABC is similar to triangles *DEF*, *GHI*, and *JKL*. The scale factors for the dilations that show triangle *ABC* is similar to each triangle are in the table.



triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

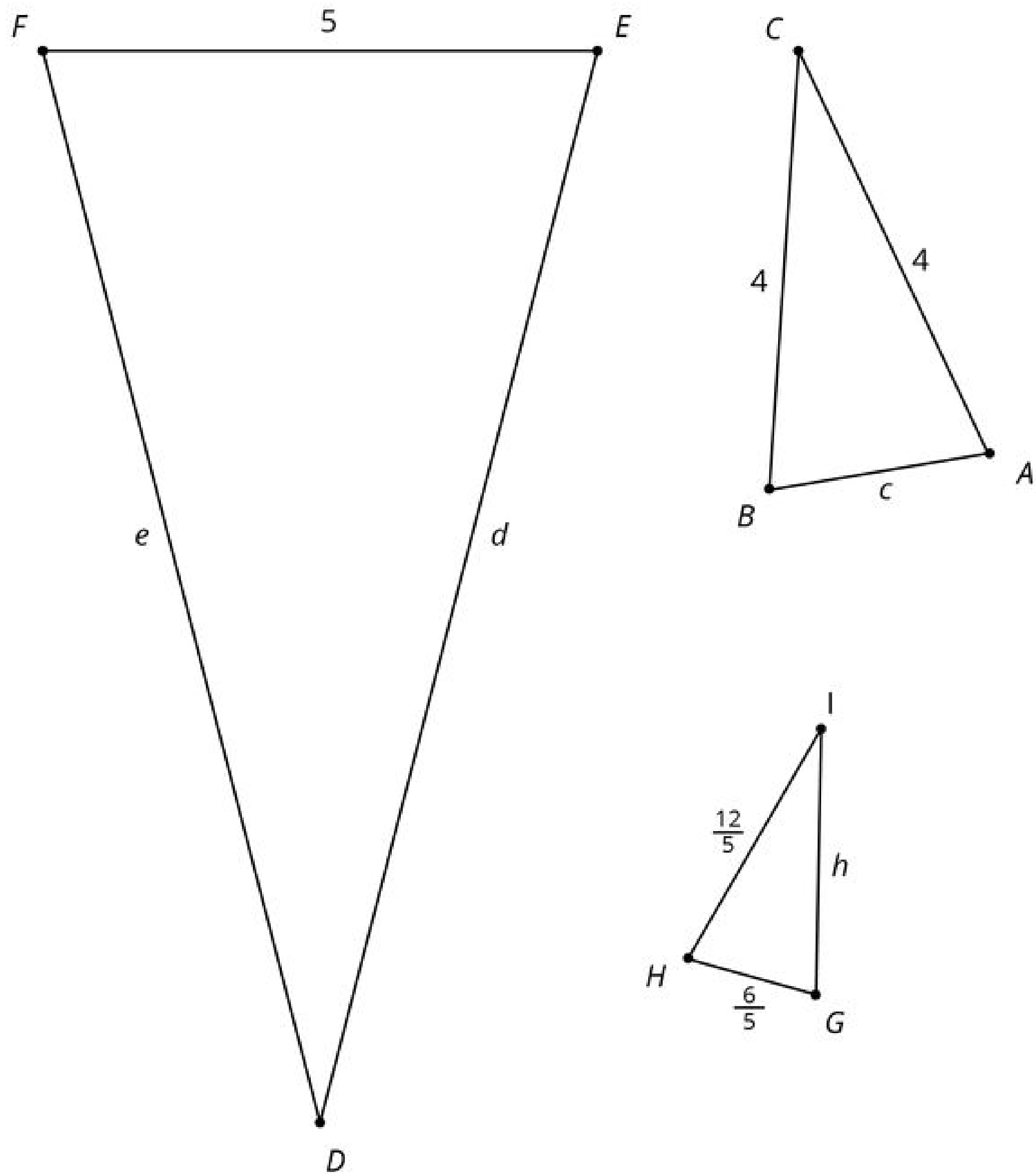
triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75		
DEF			
GHI			
JKL			

1. Find the side lengths of triangles *DEF*, *GHI*, and *JKL*. Record them in the first table.
2. For all four triangles, find the quotient of the triangle side lengths and record them in the second table. What do you notice about the quotients?

Using Side Quotiens to Find Side Lengths of Similar Triangles

There are many ways to find the values of the unknown side lengths in similar triangles. Use what you have learned so far.

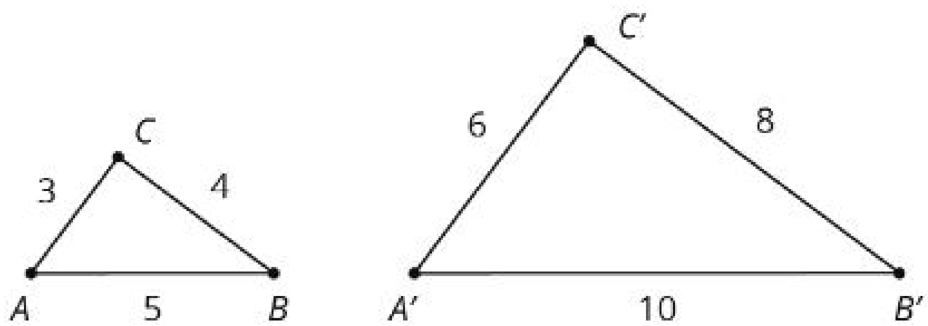
Triangles *ABC*, *EFD*, and *GHI* are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths.



c = ?
d = ?
e = ?
h = ?

Lesson 9 Summary

If two polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon. For these triangles the scale factor is 2:



Here is a table that shows relationships between the short and medium length sides of the small and large triangle.

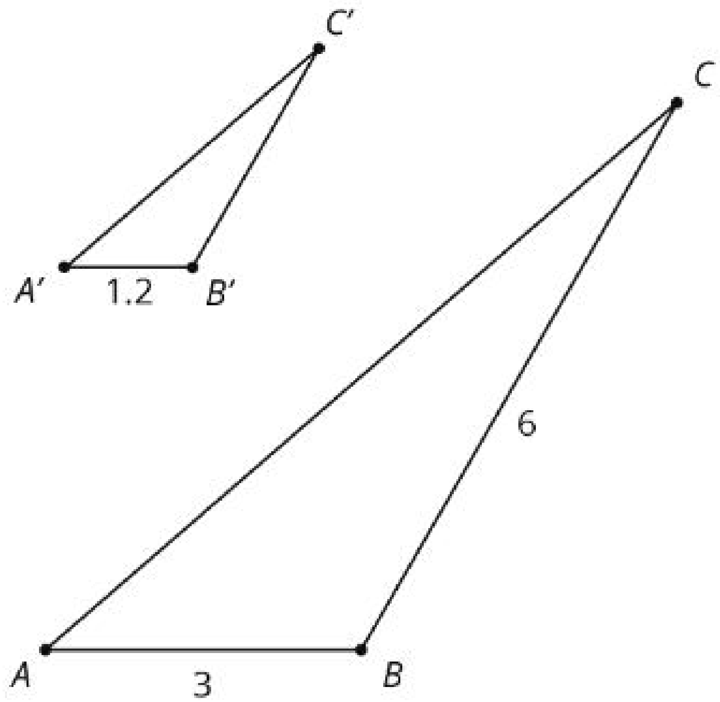
	small triangle	**large triangle**
medium side	4	8
short side	3	6
(medium side) ÷ (short side)	$\frac{4}{3}$	$\frac{8}{6} = \frac{4}{3}$

The lengths of the medium side and the short side are in a ratio of $4 : 3$. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side.

This is true for all similar polygons; the ratio between two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

Lesson 9 Summary

We can use these facts to calculate missing lengths in similar polygons. For example, triangles $A'B'C'$ and ABC shown here are similar. Let's find the length of segment $B'C'$.



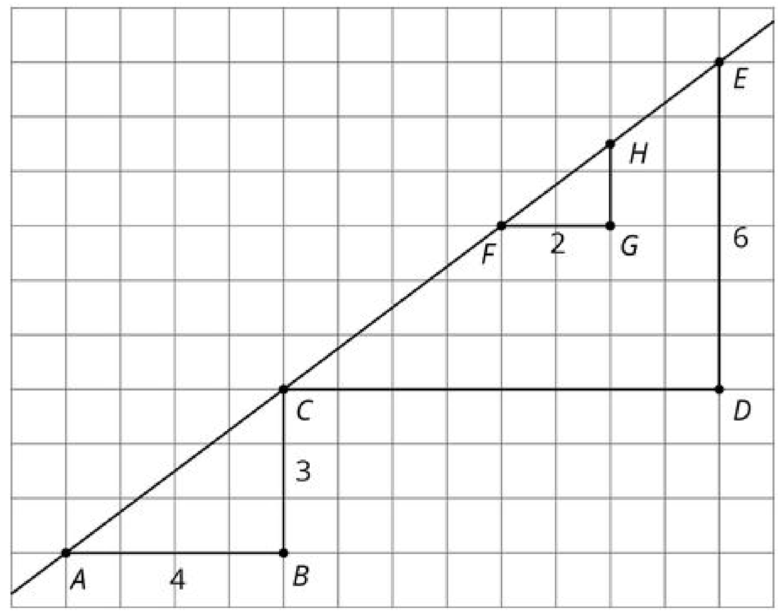
In triangle ABC, side BC is twice as long as side AB, so this must be true for any triangle that is similar to triangle ABC. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.

Lesson 10

Lesson 10: Meet Slope

Similar Triangles on the Same Line

The figure shows three right triangles, each with its longest side on the same line.



Explain why ABC and CDE are similar.

Explain why ABC and FGH are similar.

Complete the table.

triangle	vertical side	horizontal side	(vertical side) ÷ (horizontal side)
ABC			
CDE			
FGH			

Multiple Lines with the Same Slope

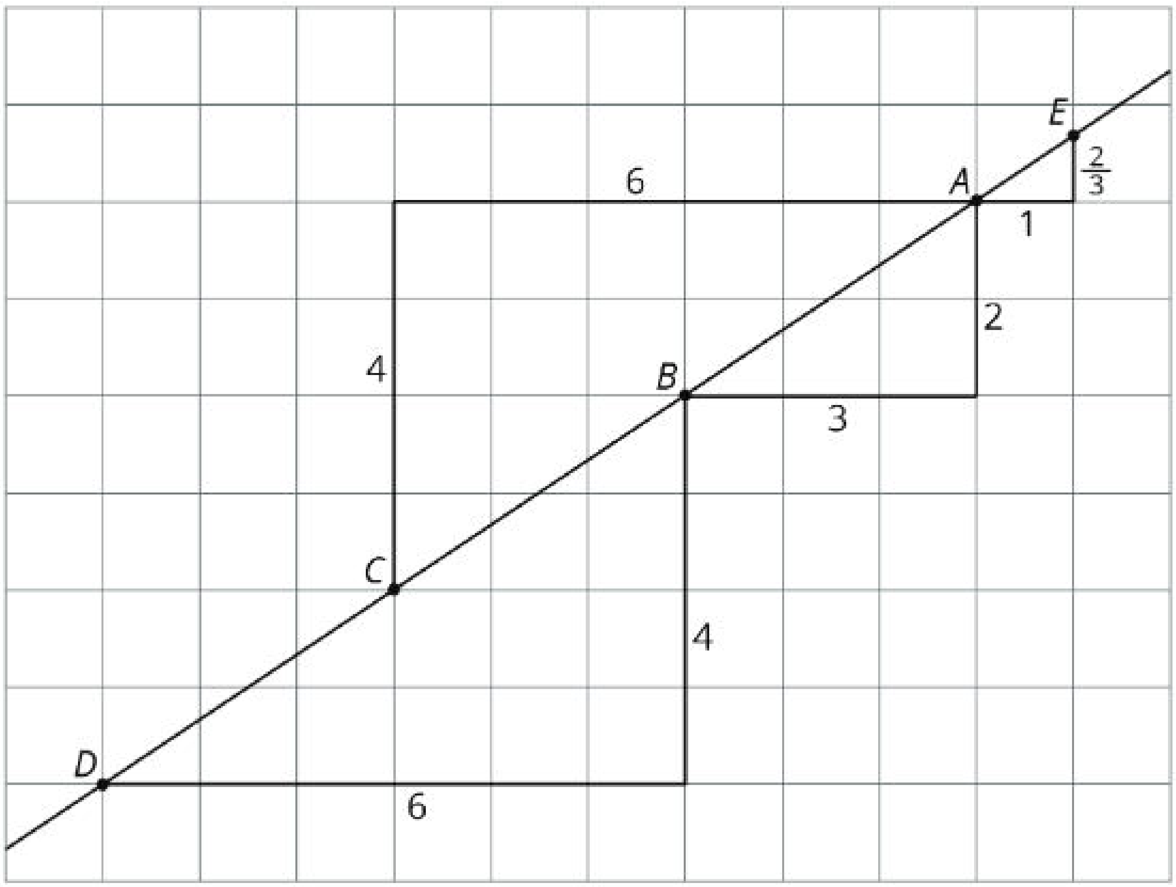
You will apply the new idea of slope. You can click the link for the digital version of this activity or do it below with the line tool.

1. Draw two line with **slope** 3. What do you notice about the two lines?

1. Draw two lines with slope ½. What do you notice about the two lines?

Lesson 10 Summary

Here is a line drawn on a grid. There are also four right triangles drawn. Do you notice anything the triangles have in common?



These four triangles are all examples of *slope triangles*. One side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The **slope** of the line is the quotient of the length of the vertical side and the length of the horizontal side of the slope triangle. This number is the same for *all* slope triangles for the same line because all slope triangles for the same line are similar.

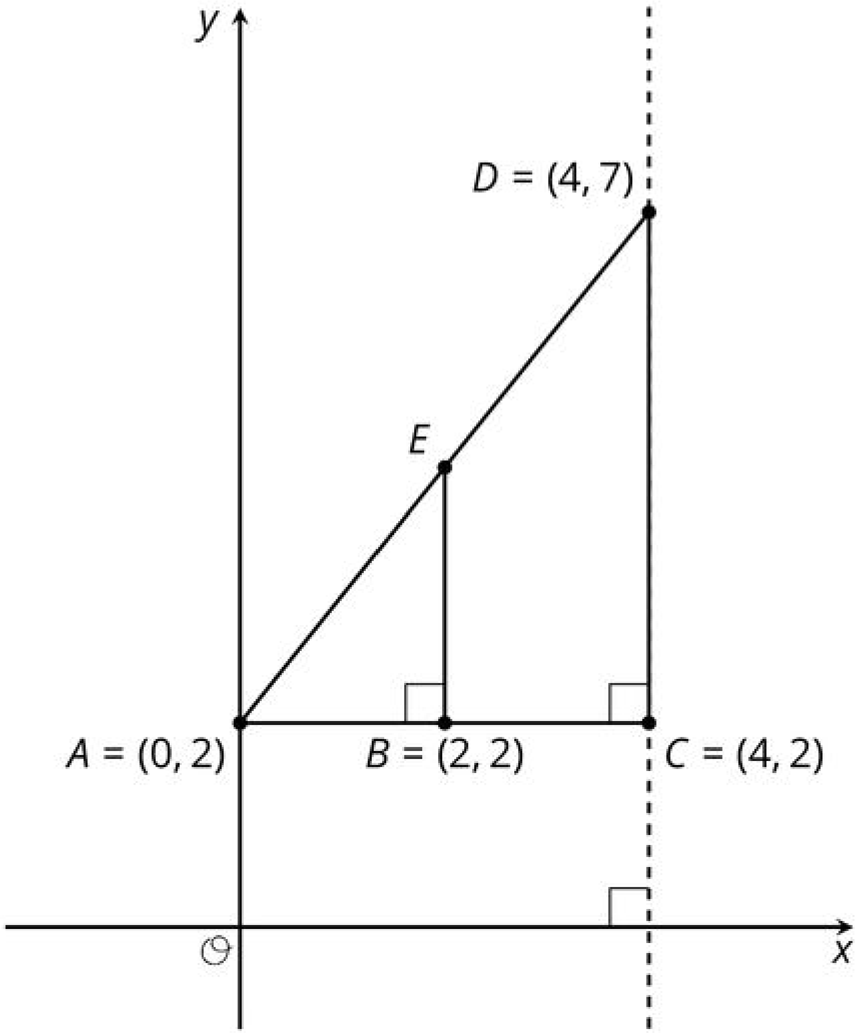
In this example, the slope of the line is $\frac{2}{3}$, which is what all four triangles have in common. Here is how the slope is calculated using the slope triangles:

- Points A and B give $2 \div 3 = \frac{2}{3}$
- Points D and B give $4 \div 6 = \frac{2}{3}$
- Points A and C give $4 \div 6 = \frac{2}{3}$
- Points A and E give $\frac{2}{3} \div 1 = \frac{2}{3}$

Lesson 11

Lesson 11: Writing Equations for Lines

Coordinates and Lengths in the Coordinate Plane

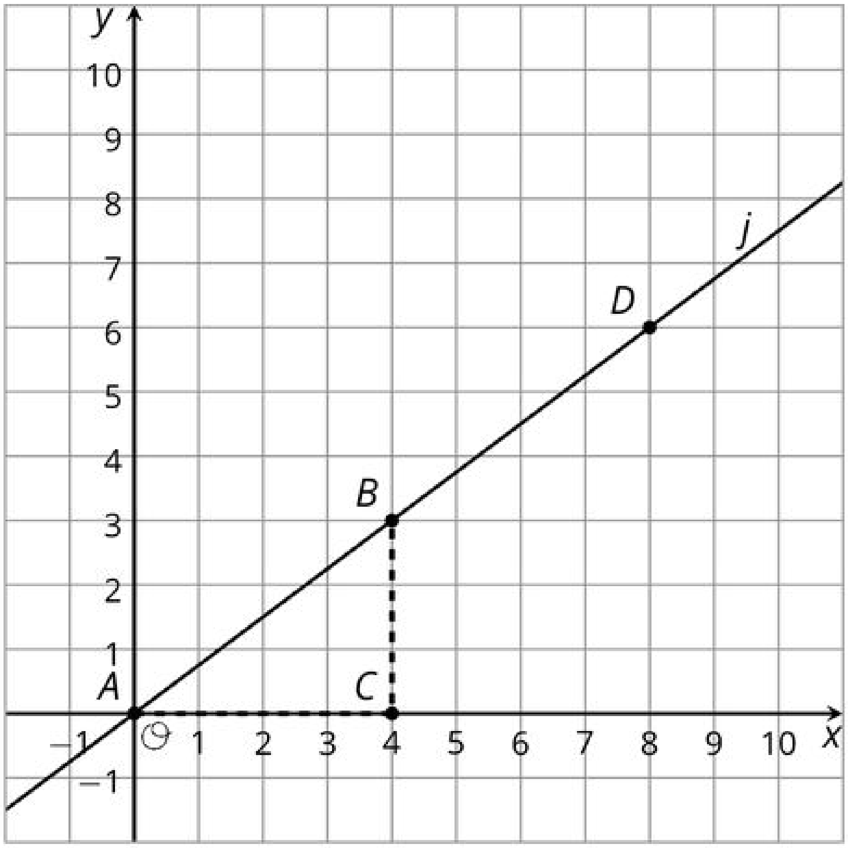


Find each of the following and explain your reasoning:

1. The length of segment BE.

1. The coordinates of E.

What We Mean by an Equation of a Line



Line j is shown in the coordinate plane.

1. What are the coordinates of B and D?

2. Is point (20, 15) on line j? Explain how you know.

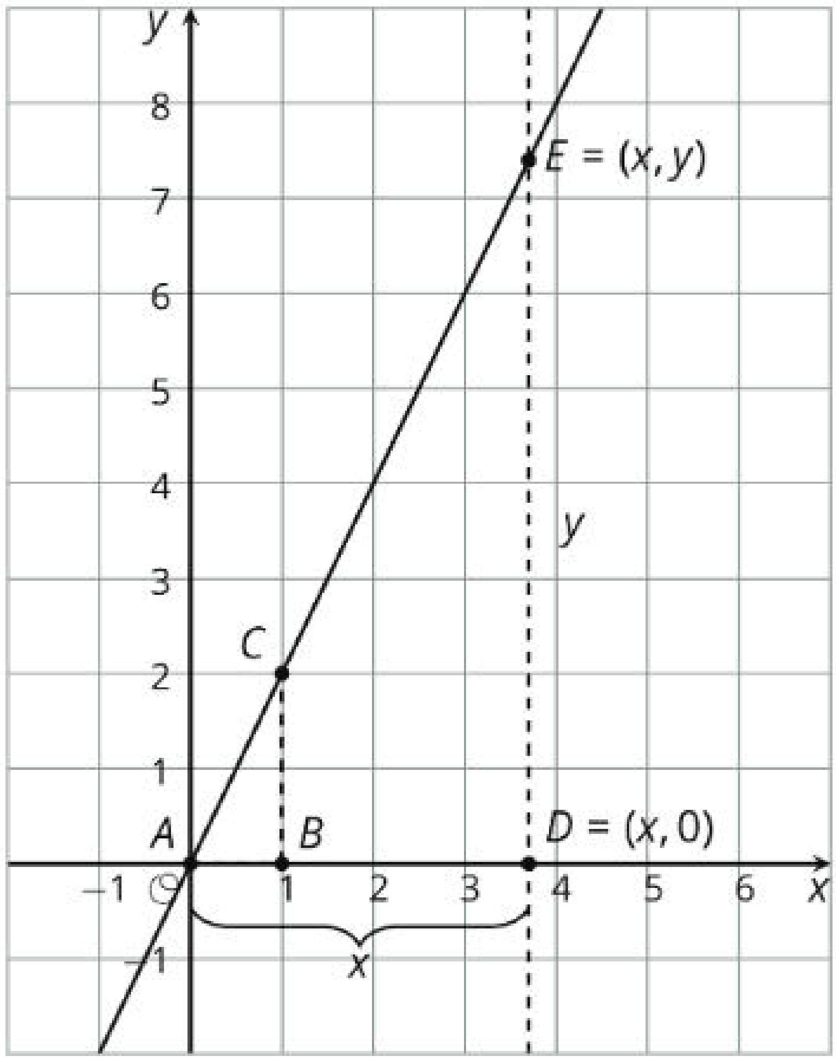
3. Is point (100, 75) on line j? Explain how you know.

4. Is point (90, 68) on line j? Explain how you know.

5. Suppose you know the x- and y-coordinates of a point. Write a rule that would allow you to test whether the point is on line j.

Lesson 11 Summary

Here are the points A, C, and E on the same line. Triangles ABC and ADE are slope triangles for the line so we know they are similar triangles. Let's use their similarity to better understand the relationship between x and y, which make up the coordinates of point E.



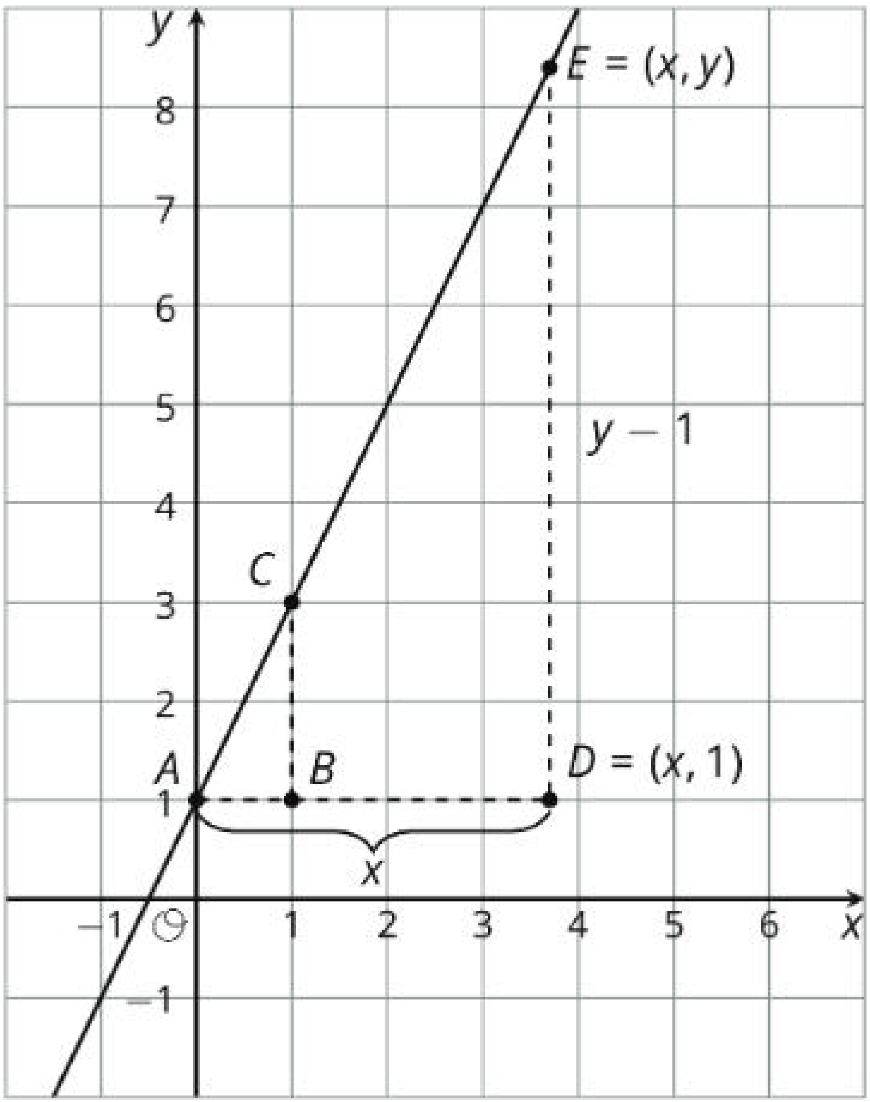
The slope for triangle ABC is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. The slope we find for triangle ADE is $\frac{y}{x}$ because the vertical side has length y and the horizontal side has length x. These two slopes must be equal since they are from slope triangles for the same line, and so:

$$\frac{2}{1} = \frac{y}{x}$$

Since $\frac{2}{1} = 2$ this means that the value of y is twice the value of x, or that $y = 2x$. This equation is true for any point (x, y) on the line!

Lesson 11 Summary

Here are two different slope triangles. We can use the same reasoning to describe the relationship between x and y for this point E.



The slope for triangle ABC is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. For triangle ADE, the horizontal side has length x. The vertical side has length $y-1$ because the distance from (x, y) to the x-axis is y but the vertical side of the triangle stops 1 unit short of the x-axis. So the slope we find for triangle ADE is $\frac{y-1}{x}$. The slopes for the two slope triangles are equal, meaning:

$$\frac{2}{1} = \frac{y-1}{x}$$

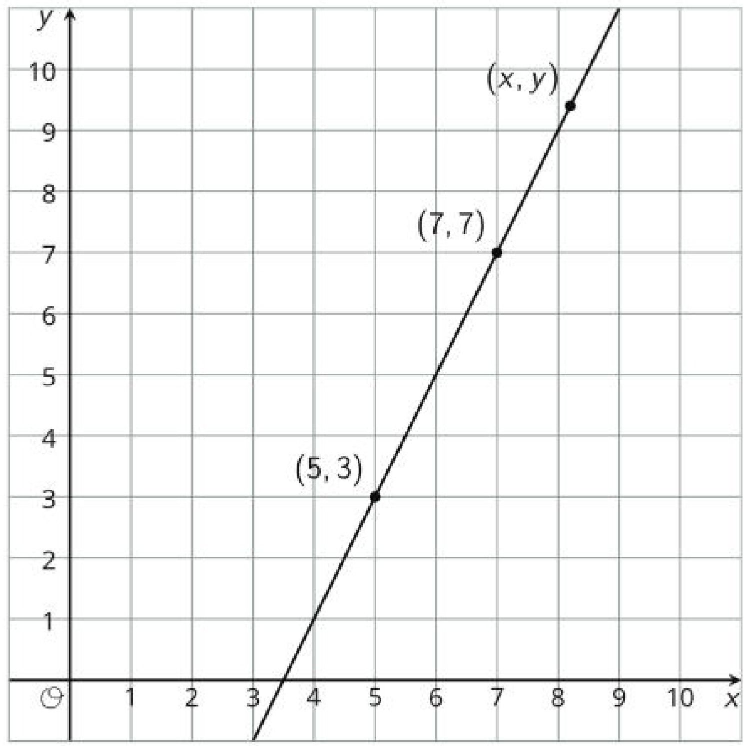
Since $y-1$ is twice x, another way to write this equation is $y-1=2x$. This equation is true for any point (x, y) on the line!

Lesson 12

Lesson 12: Using Equations for Lines

Writing Relationships from Two Points

Here is a line.



1. Using what you know about similar triangles find an equation for the line in the diagram.

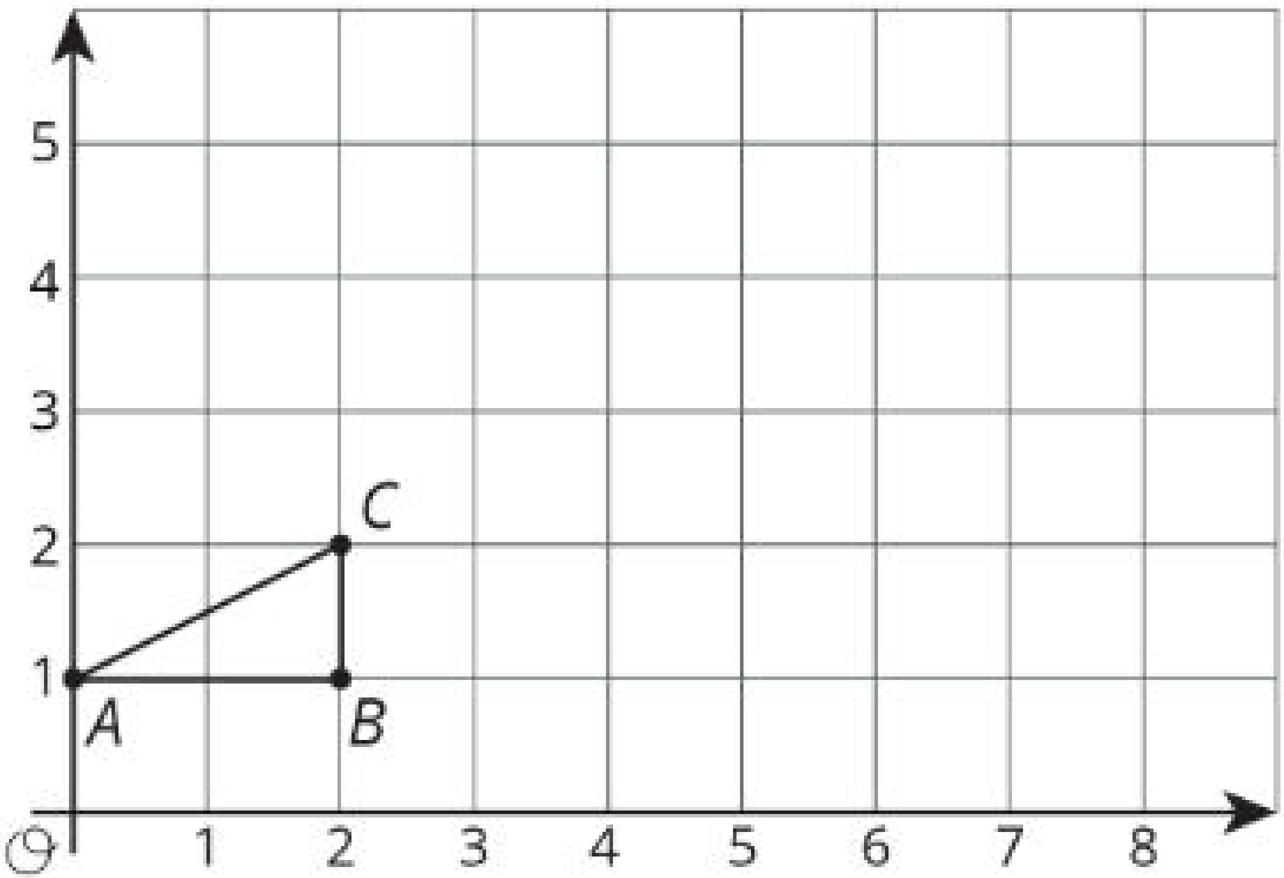
1. What is the slope of the line? Does it appear in your equation?

1. Is (9, 11) also on the line? How do you know?

1. Is (100, 193) also on the line?

Dilations and Slope Triangles

Here is triangle ABC

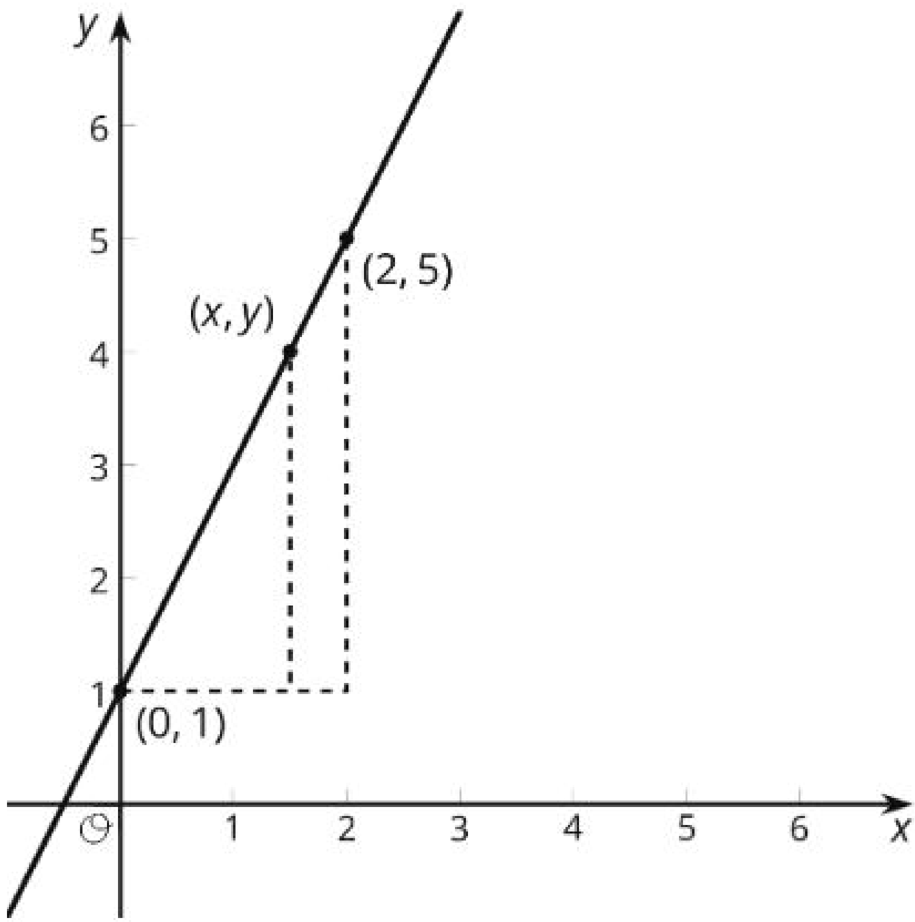


1. Draw the dilation of triangle ABC with center (0,1) and scale factor 2.
2. Draw the dilation of triangle ABC with center (0, 1) and scale factor 2.5.
3. Where is C mapped by the dilation with center (0, 1) and scale factor *s*?

1. For which scale factor does the dilation with center (0, 1) send *C* to (9, 5.5)? Explain how you know.

Lesson 12 Summary

We can use what we know about slope to decide if a point lies on a line. Here is a line with a few points labeled.



The slope triangle with vertices $(0, 1)$ and $(2, 5)$ gives a slope of $\frac{5-1}{2-0} = 2$. The slope triangle with vertices $(0, 1)$ and (x, y) gives a slope of $\frac{y-1}{x}$. Since these slopes are the same, $\frac{y-1}{x} = 2$ is an equation for the line. So, if we want to check whether or not the point $(11, 23)$ lies on this line, we can check that $\frac{23-1}{11} = 2$. Since $(11, 23)$ is a solution to the equation, it is on the line!