COMPOSITION BOOK

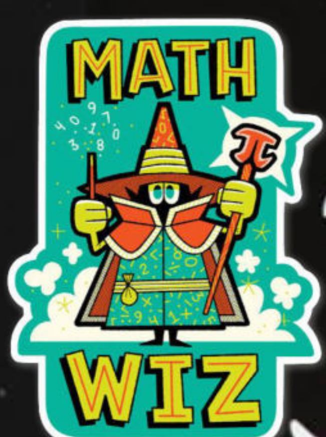
Ms. Forbes'

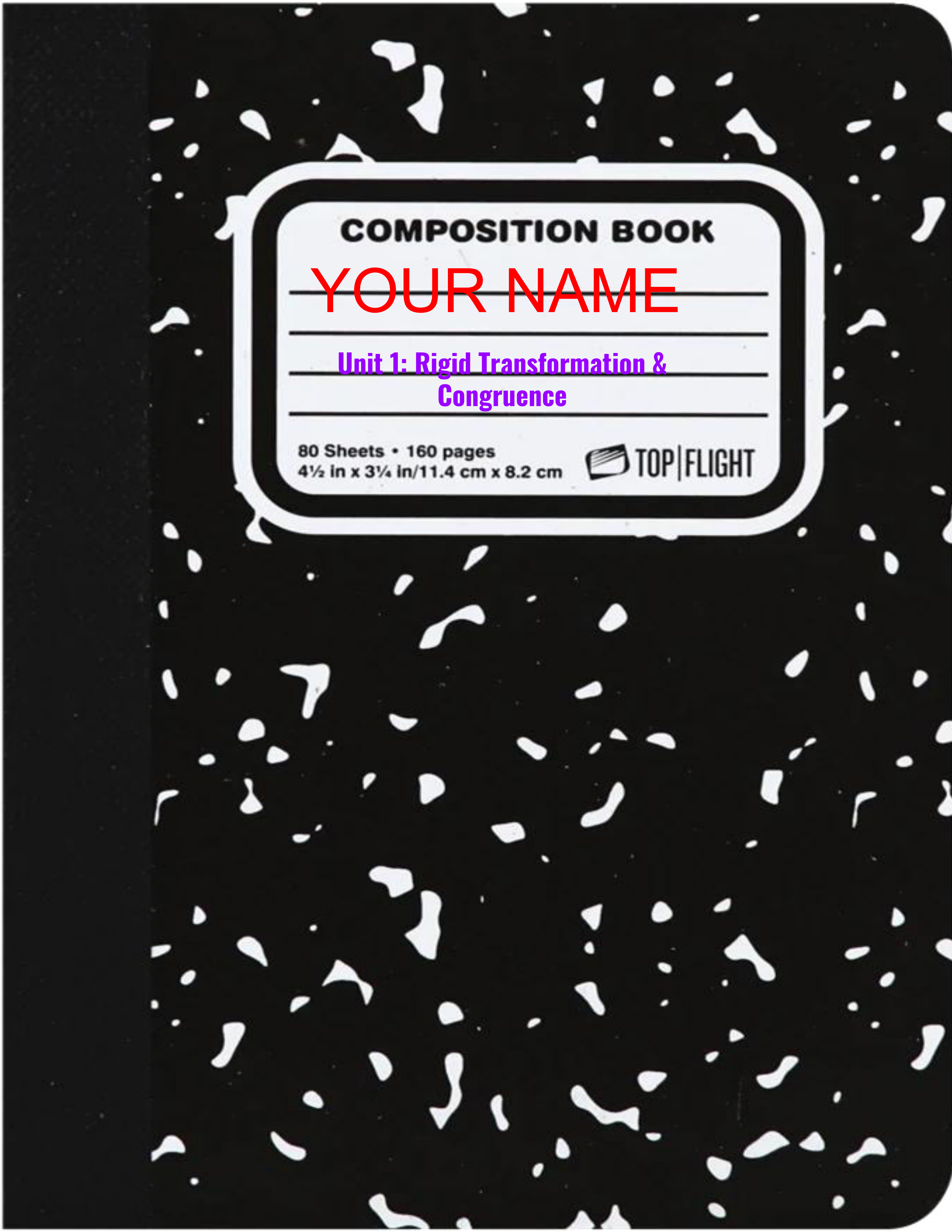
Math 8 Journal

Unit 1: Rigid Transformation & Congruence

80 Sheets • 160 pages
4½ in x 3¼ in/11.4 cm x 8.2 cm

TOP|FLIGHT

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YOUR NAME
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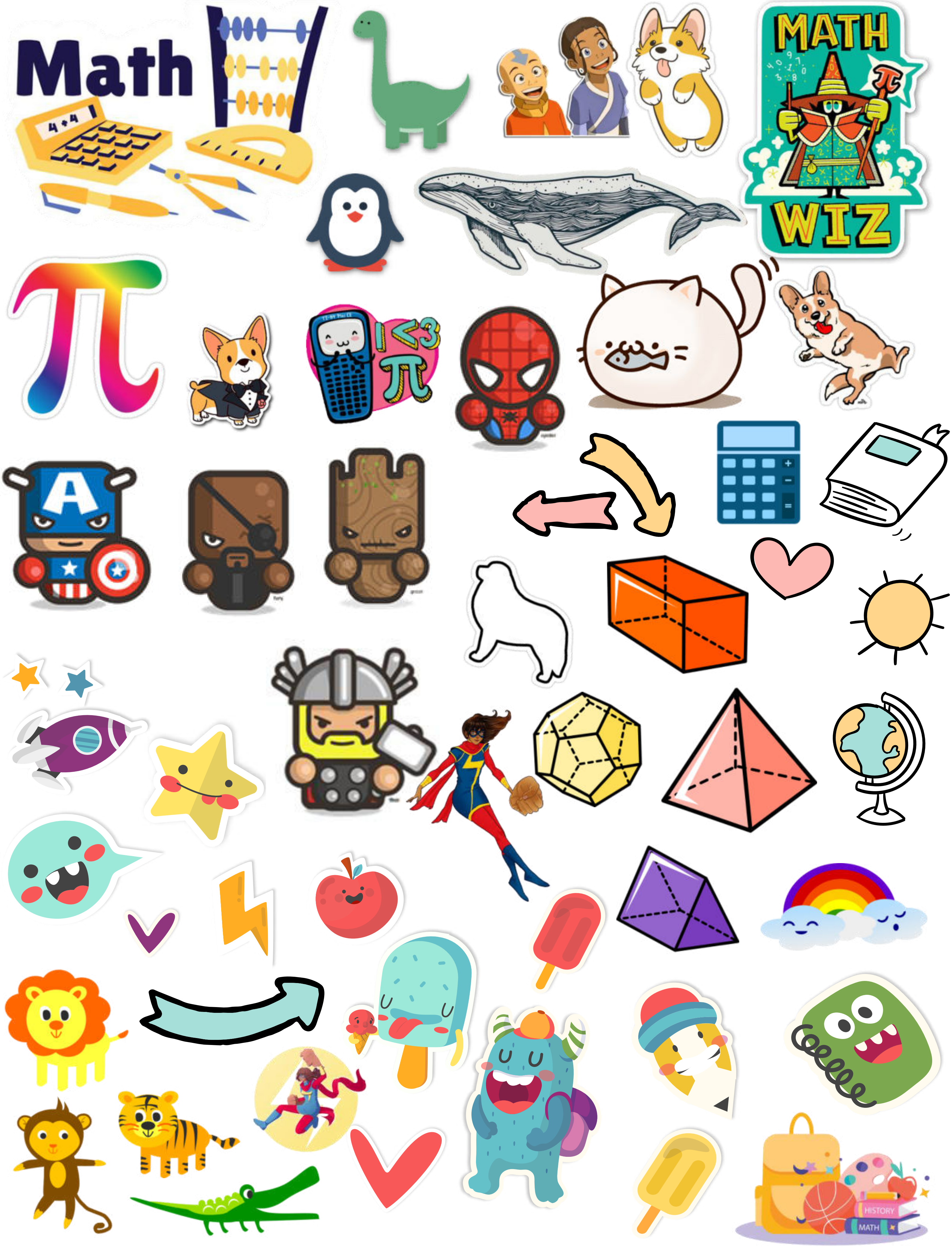
Math
4+4
MATH
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HISTORY
MATH

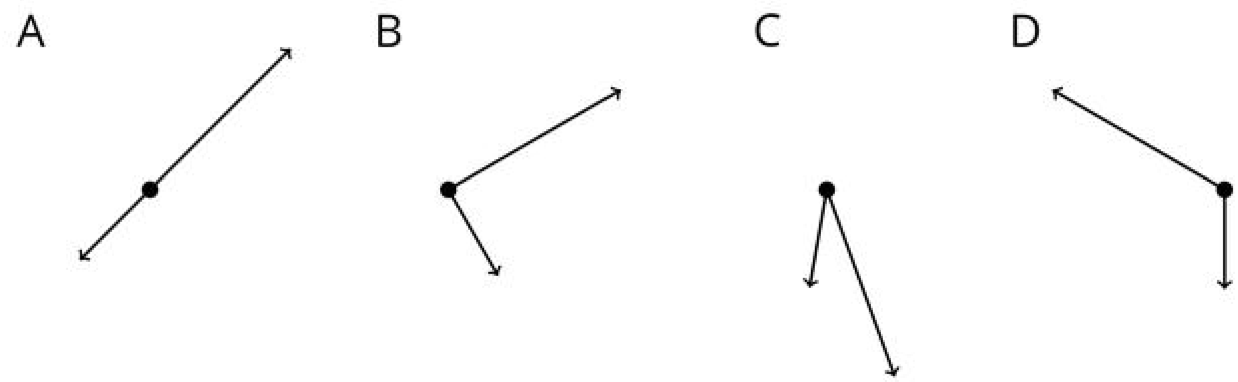

OK!
STRANGER THINGS
NOPE
COMPOSITION
OMG!
LOL!
STAR WARS

Table of Contents

Lesson 1

Lesson 1: Moving in the Plane

Part 1:
Which one doesn't belong and why?



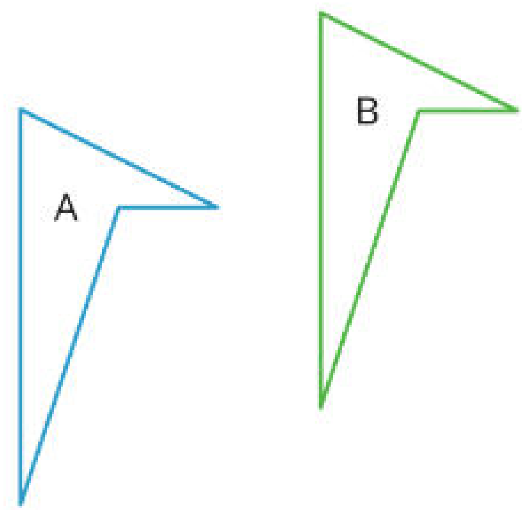
Click the link for the applet. Write a description of the moves of Dance "A".

Click the link for the applet. Write a description of the moves of Dance "B".

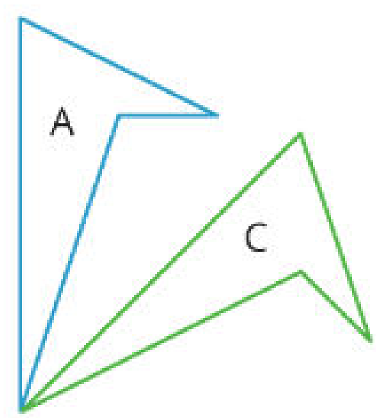
Lesson 1 Summary

Here are two ways for changing the position of a figure in a plane without changing its shape or size:

- Sliding or shifting the figure without turning it. Shifting Figure A to the right and up puts it in the position of Figure B.



- Turning or rotating the figure around a point. Figure A is rotated around the bottom vertex to create Figure C.

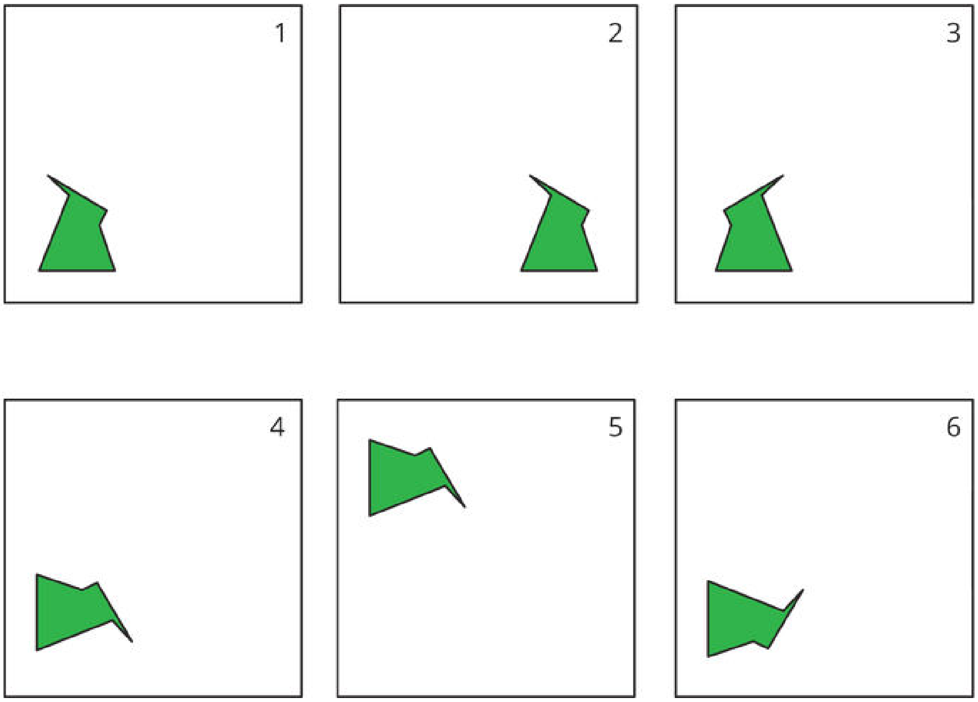


Lesson 2

Lesson 2: Naming the Moves

How Did You Make That Move?

Here is another set of dance moves.



1. Describe each move or say if it is a new move.
   a. Frame 1 to Frame 2

   b. Frame 2 to Frame 3

   c. Frame 3 to Frame 4

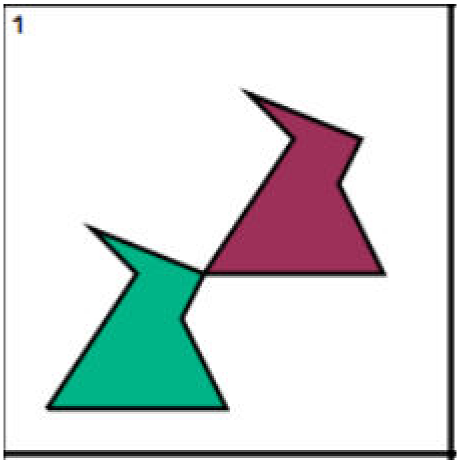
   d. Frame 4 to Frame 5

   e. Frame 5 to Frame 6

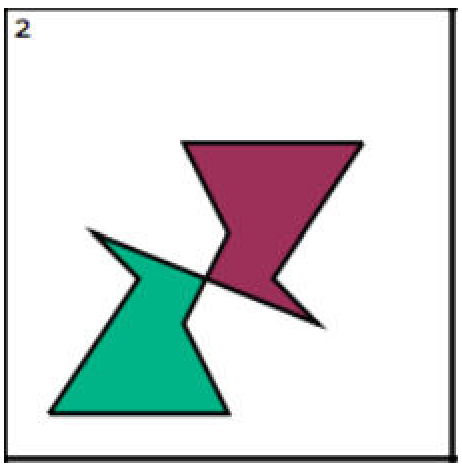
Sort the cards into categories according to the type of moves they show. Describe each category and why it is different from the others.

Click on each image to see how each image changes.

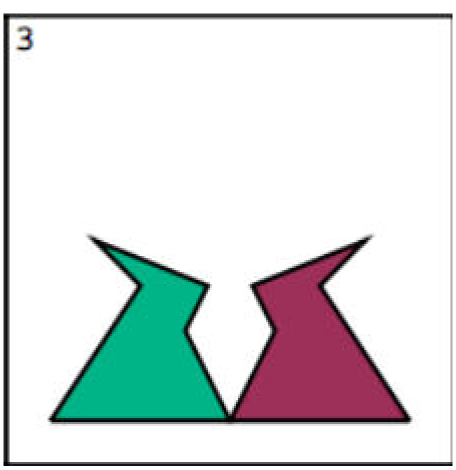
Translation



Rotation



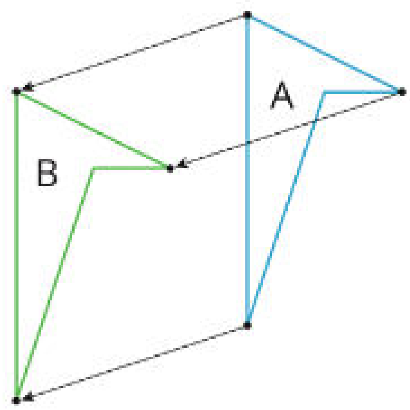
Reflection



Lesson 2 Summary

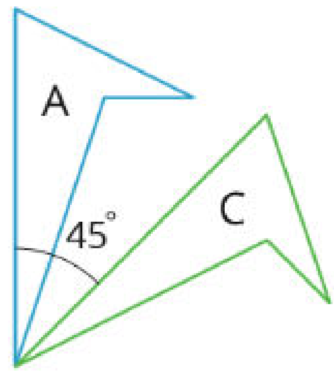
Here are the moves we have learned about so far:

- A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.

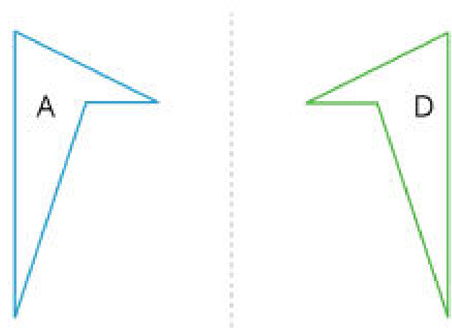


- A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.

Lesson 2 Summary



- A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



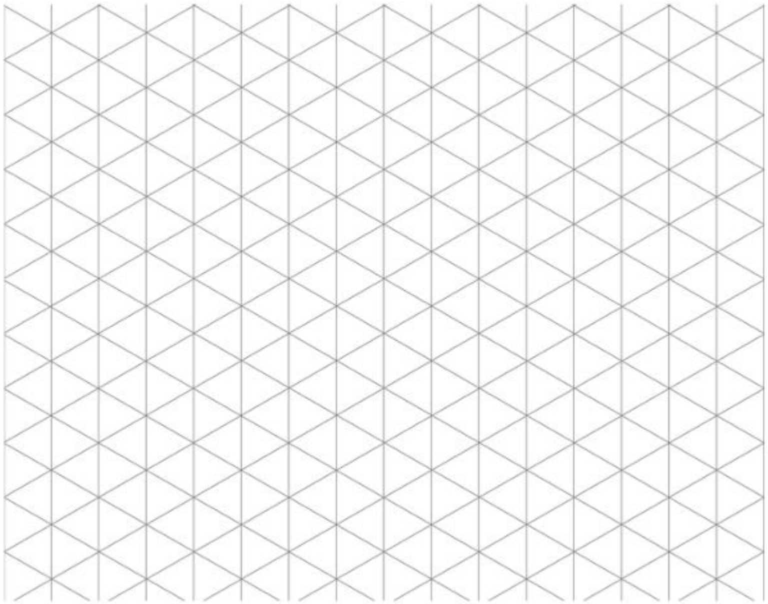
We use the word **image** to describe the new figure created by moving the original figure. If one point on the original figure moves to another point on the new figure, we call them **corresponding points.**

Lesson 3

Lesson 3: Grid Moves

The Isometric Grid

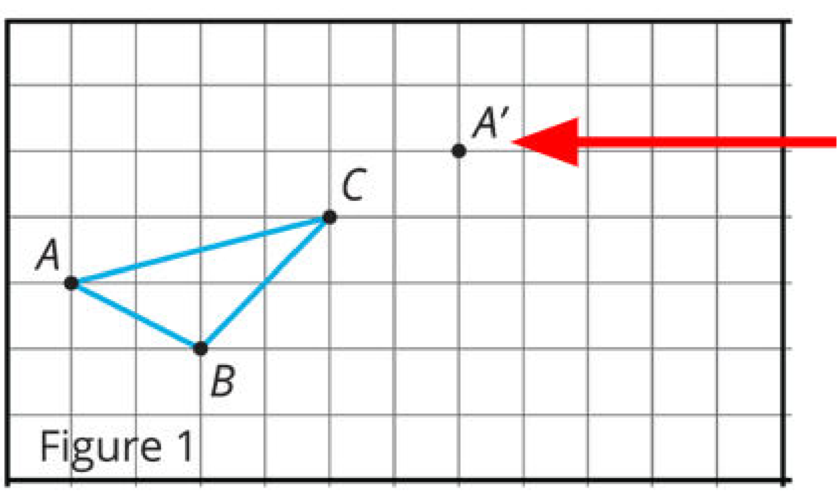
What do you notice? What do you wonder?



Transformation Information

We call point ***A'***, "**A prime**."

After a transformation, it corresponds to A in the original figure.



Follow the directions below each statement to tell GeoGebra how you want the figure to move. It is important to notice that GeoGebra uses vectors to show translations. A *vector* is a quantity that has magnitude (size) and direction. It is usually represented by an arrow.

These applets are sensitive to clicks. Be sure to make one quick click, otherwise it may count a double-click.

After each example, click the reset button, and then move the slider over for the next question.

Click here for GeoGebra.

1. Translate triangle ABC so that A goes to A'.
    a. Select the Vector tool.

    b. Click on the original point A and then the new point A'. You should see a vector.

    c. Select the Translate by Vector tool.

    d. Click on the figure to translate, and then click on the vector.

2. Translate triangle ABC so that C goes to C'.

3. Rotate triangle ABC 90° counterclockwise using center O.
    a. Select the Rotate around Point tool.

    b. Click on the figure to rotate, and then click on the center point.

    c. A dialog box will open; type the angle by which to rotate and select the direction of rotation.

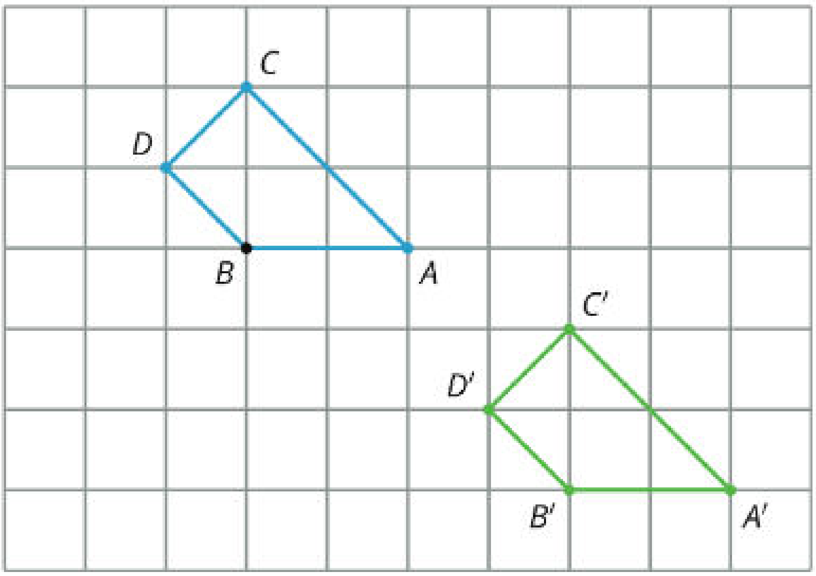
    d. Click on ok.

4. Reflect triangle ABC using line ℓ.
    a. Select the Reflect about Line tool.

    b. Click on the figure to reflect, and then click on the line of reflection.

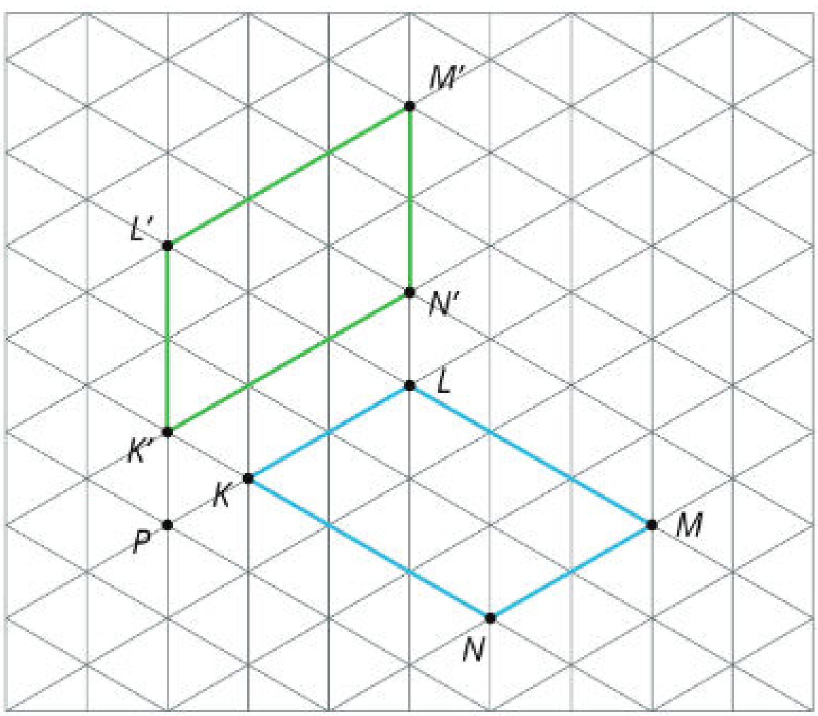
Lesson 3 Summary

When a figure is on a grid, we can use the grid to describe a transformation. For example, here is a figure and an image of the figure after a move.



Quadrilateral $ABCD$ is translated 4 units to the right and 3 units down to the position of quadrilateral $A'B'C'D'$.

A second type of grid is called an *isometric grid*. The isometric grid is made up of equilateral triangles. The angles in the triangles all measure 60 degrees, making the isometric grid convenient for showing rotations of 60 degrees.



Here is quadrilateral $KLMN$ and its image $K'L'M'N'$ after a 60-degree counterclockwise rotation around a point P.

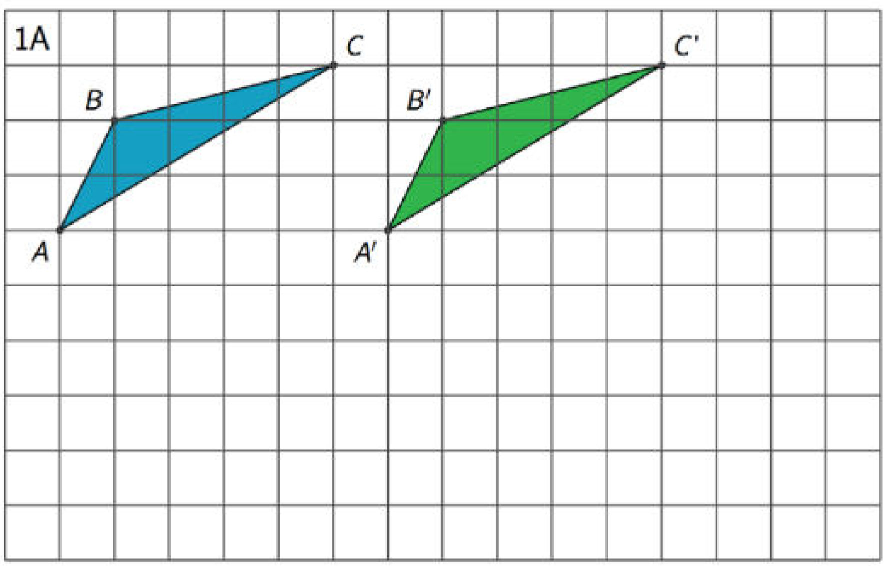
Lesson 4

Lesson 4: Making the Moves

Make That Move

Transformation - move (like a reflection) or a sequence of moves

How would you describe the transformation from Triangle ABC to Triangle A’B’C’.



Transformations encompass translations, rotations, and reflections. There are other types of transformations, but for now, we will focus on these three types.

A to B to C

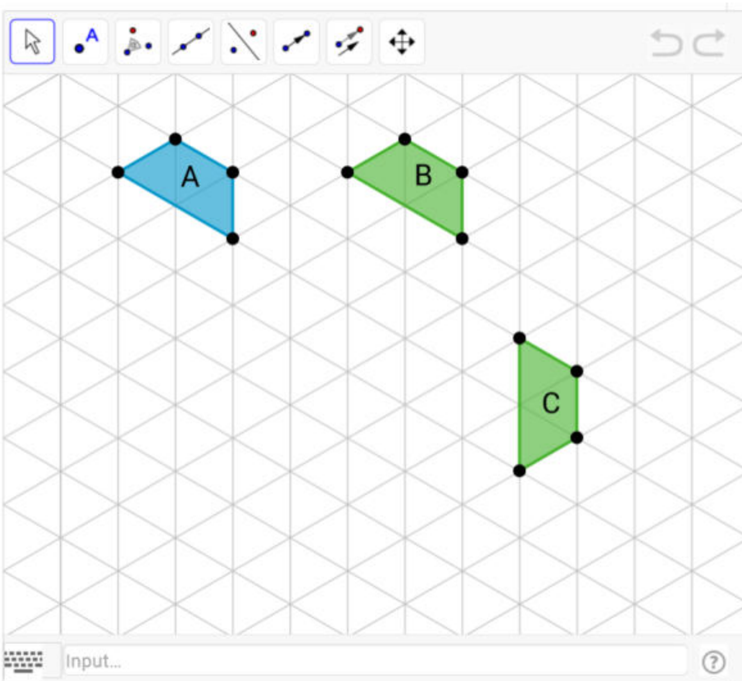
Can you imagine a single translation, rotation, or reflection that would take one bird to another?

How could we use translations, rotations, and reflections to take on bird to another?

When we do one or more transformations in a row to take one figure to another, it is called a **sequence of transformations.**

Here are some figures on an isometric grid. Explore the transformation tools in the tool bar. Directions are below the applet if you need them).

Click image for the applet.



1. Name a transformation that takes Figure A to Figure B. Name a transformation that takes Figure B to FIgure C.

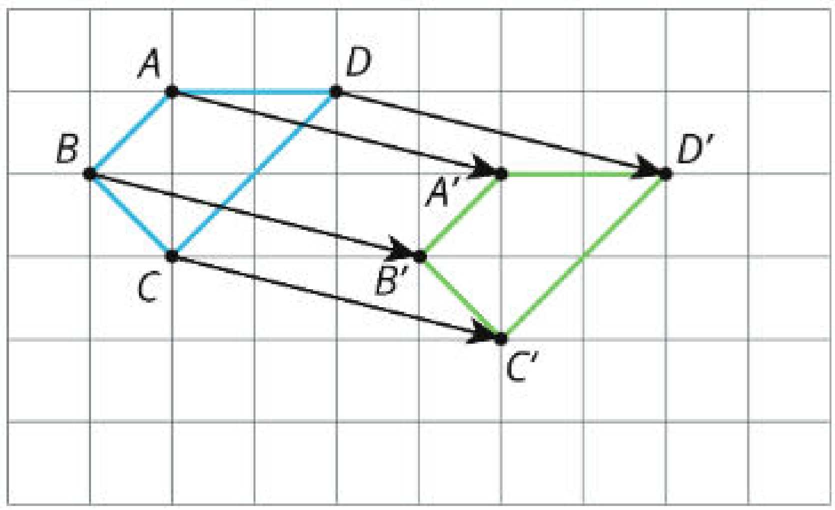
1. What is one sequence of transformations that takes Figure A to Figure C? Explain how you know.

Lesson 4 Summary

A move, or combination of moves, is called a **transformation**. When we do one or more moves in a row, we often call that a **sequence of transformations**. To distinguish the original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol $'$ attached, as in A' (pronounced "A prime").

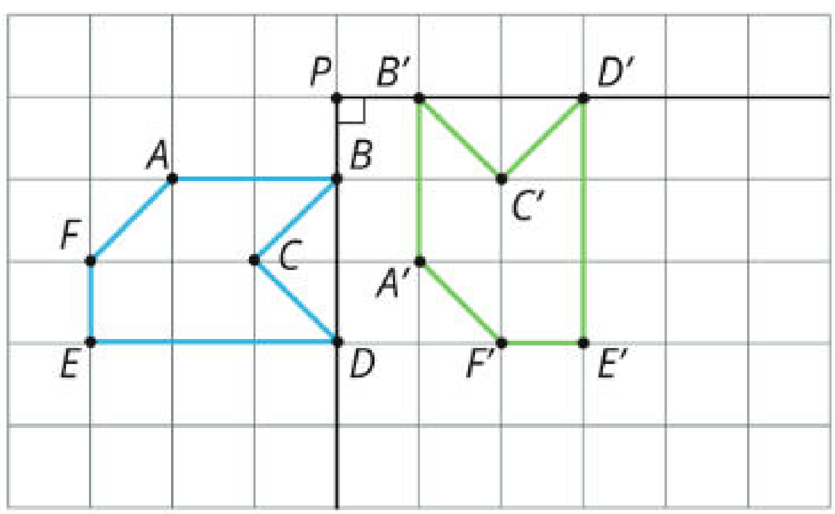
- A translation can be described by two points. If a translation moves point A to point A', it moves the entire figure the same distance and direction as the distance and direction from A to A'. The distance and direction of a translation can be shown by an arrow.

For example, here is a translation of quadrilateral $ABCD$ that moves A to A'.



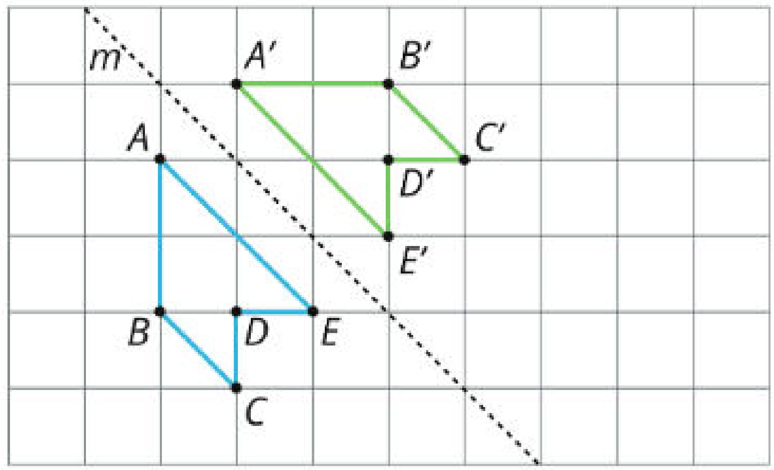
- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.

For example, hexagon $ABCDEF$ is rotated 90° counterclockwise using center P.



Lesson 4 Summary

- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.
  For example, pentagon $ABCDE$ is reflected across line m.



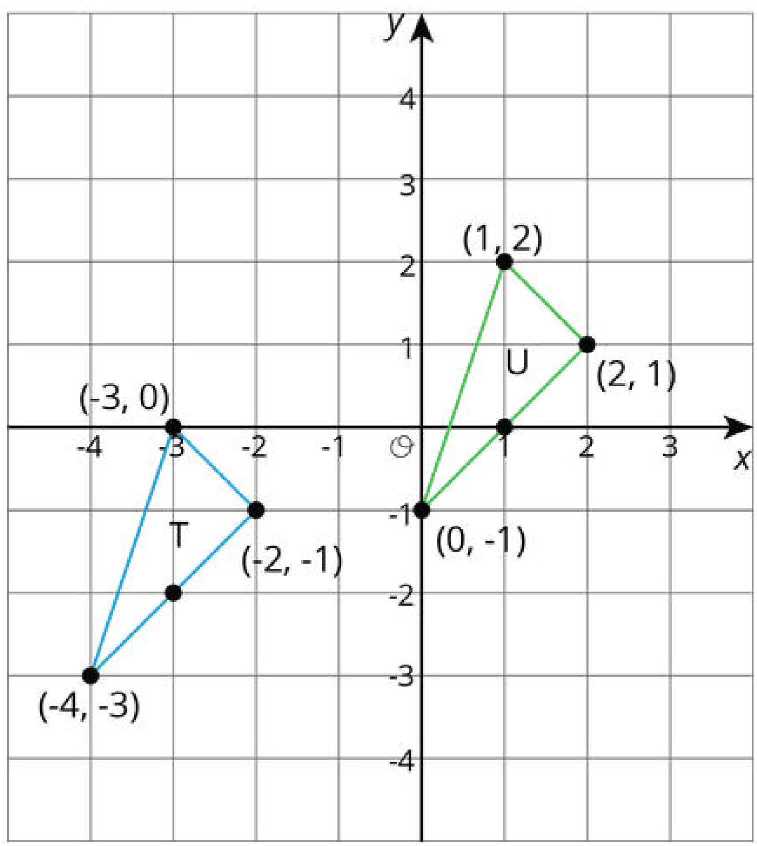
Lesson 5

Lesson 5: Coordinate Moves

Translating Coordinates

How would you describe a translation? Is there more than one way to describe the same translation?

Select all the translations that take Triangle T to Triangle U. There may be more than one correct answer.



1. Translate (-3, 0) to (1, 2).

2. Translate (2, 1) to (-2, -1).

3. Translate (-4, -3) to (0, -1).

4. Translate (1, 2) to (2, 1).

Transformation of a Segment

Click here for the applet. It has instructions for the first 3 questions built into it. Move the slider marked “question” when you are ready to answer the next one.

1. Rotate segment AB 90 degrees counterclockwise around center B by moving the slider marked 0 degrees. The image of A is named C. What are the coordinates of C?

1. Rotate segment AB 90 degrees counterclockwise around center A by moving the slider marked 0 degrees. The image B is named D. What are the coordinates of D

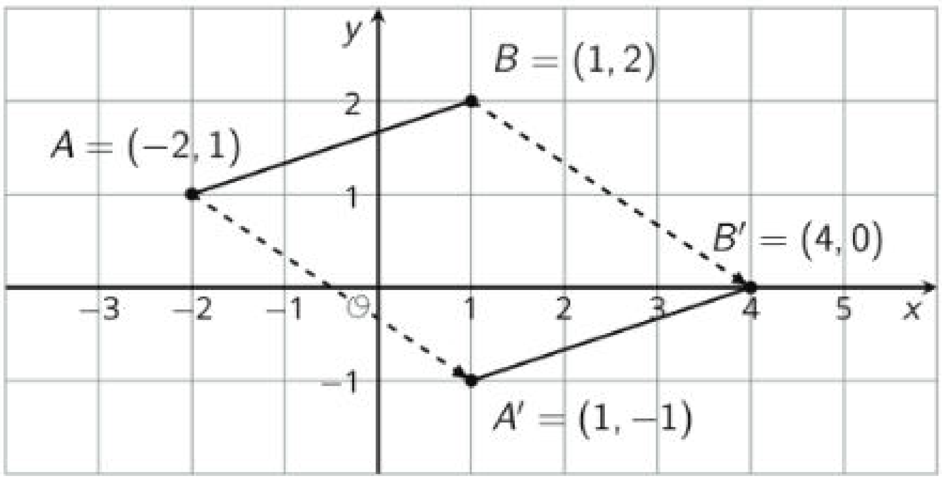
1. Rotate segment AB 90 degrees counterclockwise around (0,0) by moving the slider marked 0 degrees. The image A is named E and the image of B is named F. What are the coordinates of B and F?

1. Compare the two 90-degree counterclockwise rotations of segment AB. What is the same about the images of these rotations? What is differen?

Lesson 5 Summary

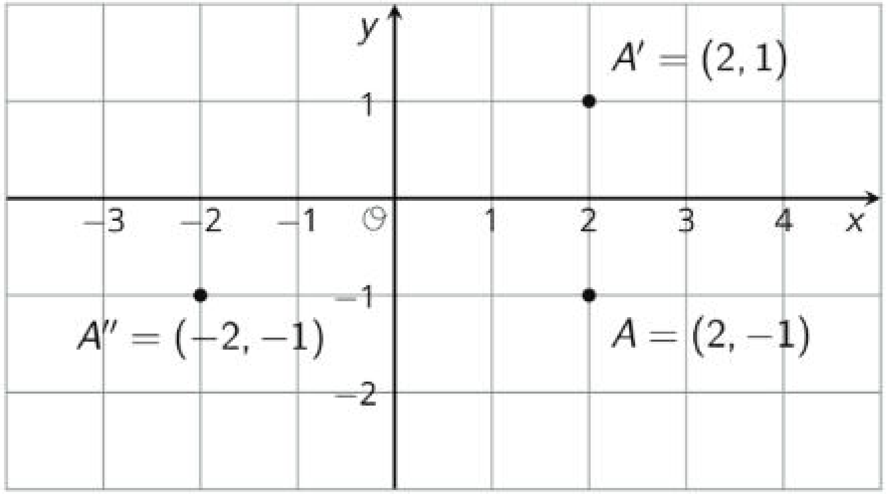
We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.



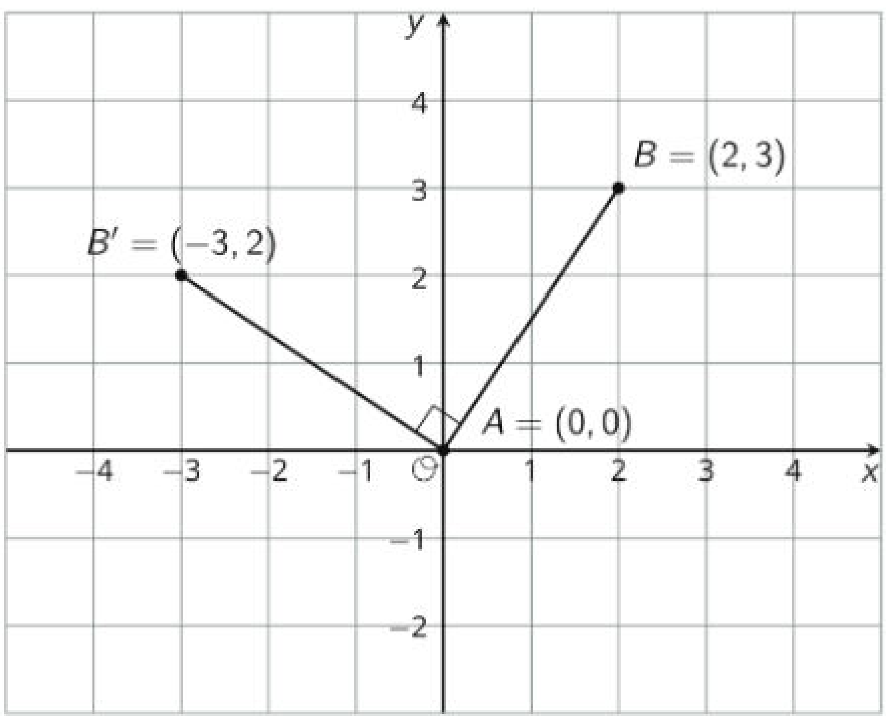
Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are $(2, -1)$ across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are $(2, 1)$. Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are $(-2, -1)$.

Lesson 5 Summary



Reflections across other lines are more complex to describe.

We don’t have the tools yet to describe rotations in terms of coordinates in general. Here is an example of a 90° rotation with center $(0,0)$ in a counterclockwise direction.



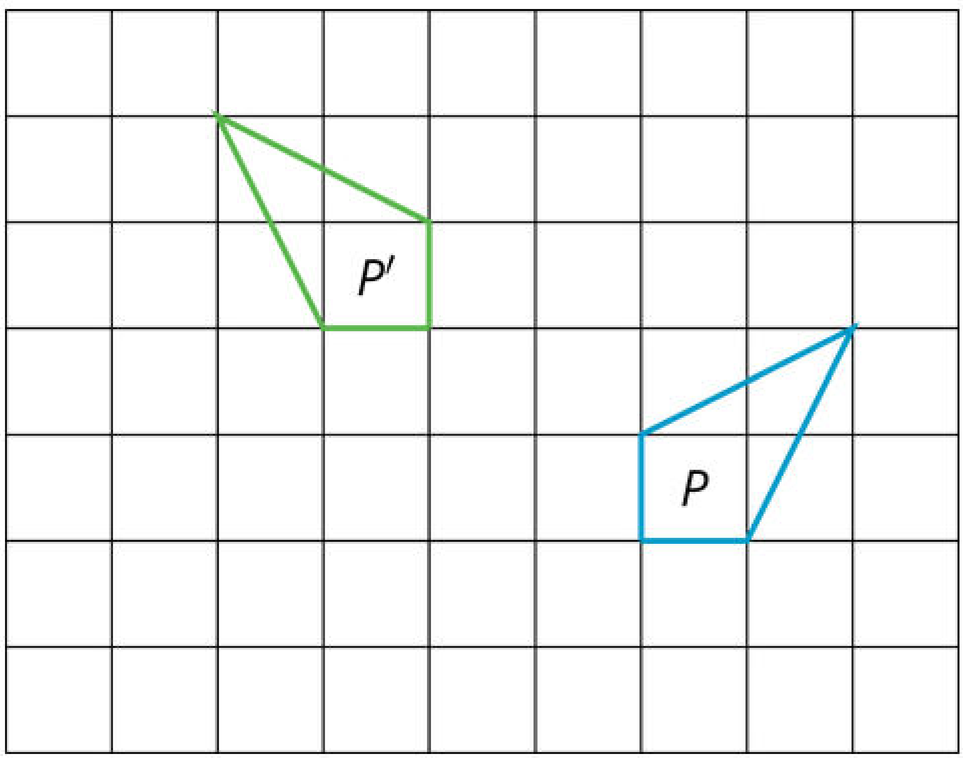
Point A has coordinates $(0,0)$. Segment AB was rotated 90° counterclockwise around A. Point B with coordinates $(2,3)$ rotates to point B' whose coordinates are $(-3,2)$.

Lesson 6

Lesson 6: Describing Transformations

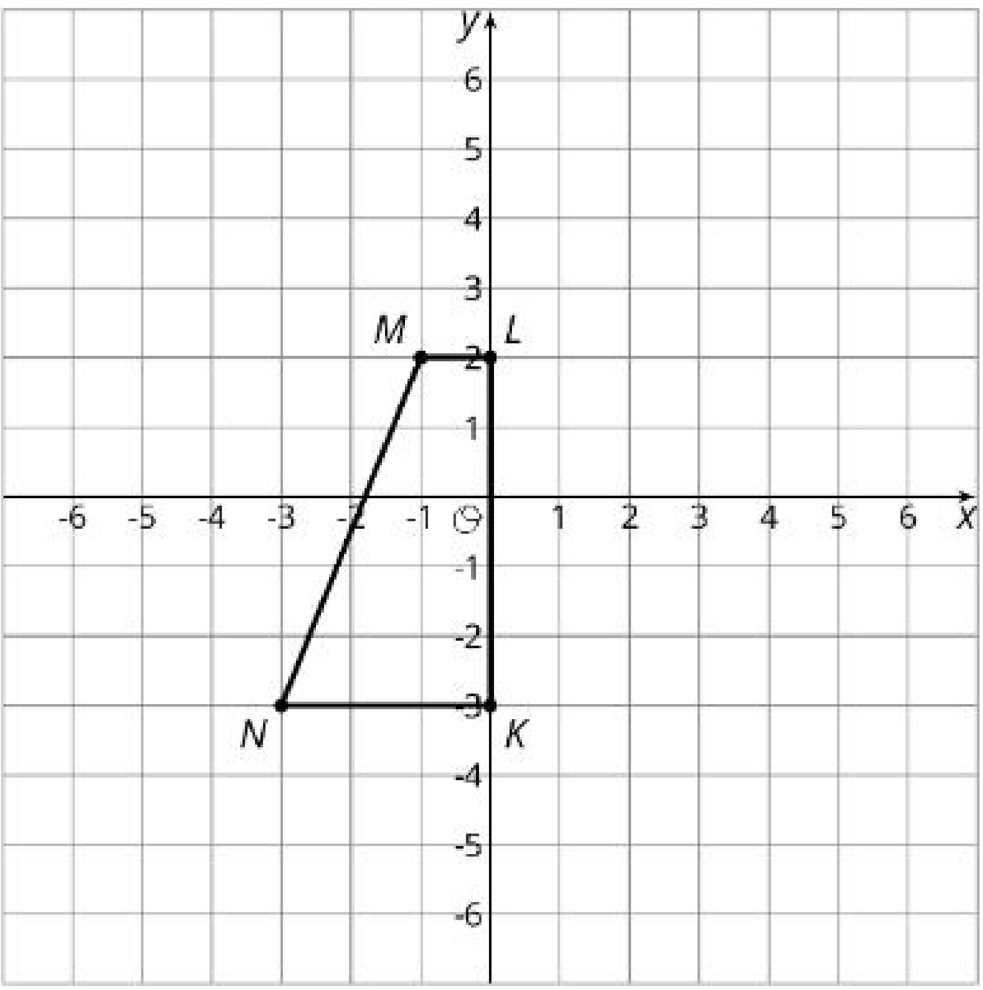
Finding a Center of Rotation

Andre performs a 90-degree counterclockwise rotation of Polygon P and gets Polygon P’, but he does not say what the center of the rotation is. Can you find the center?



This Info-Gap Activity was done during class. You have a problem card where you have to think about what you need to know to do the problem.

Ask specific questions to solve the problem.



Polygon *K'L'M'N'* is the image of *KLMN* after some transformations.

Find *K'L'M'N'*.

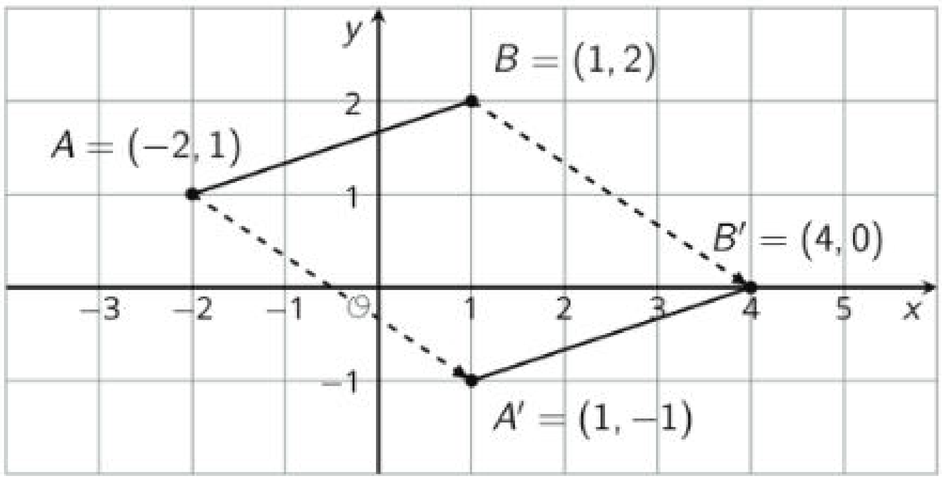
Coordinate planes allow us to communicate information about transformations precisely.

There is certain information needed for each type of transformation.
Translation - distance of vertical and horizontal components
Rotation - center of rotation, direction of rotation, and angle of rotation
Reflection - line of reflection

Lesson 6 Summary

We can use coordinates to describe points and find patterns in the coordinates of transformed points.

We can describe a translation by expressing it as a sequence of horizontal and vertical translations. For example, segment AB is translated right 3 and down 2.



Reflecting a point across an axis changes the sign of one coordinate. For example, reflecting the point A whose coordinates are $(2, \text{-}1)$ across the x-axis changes the sign of the y-coordinate, making its image the point A' whose coordinates are $(2, 1)$. Reflecting the point A across the y-axis changes the sign of the x-coordinate, making the image the point A'' whose coordinates are $(\text{-}2, \text{-}1)$.

Lesson 7

Lesson 7: No Bending or Stretching

Sides and Angles

In this activity, you will be performing transformations.

1. Translate Polygon A so point P goes to point P'. What is the length of each side in grid units. Click here for applet.

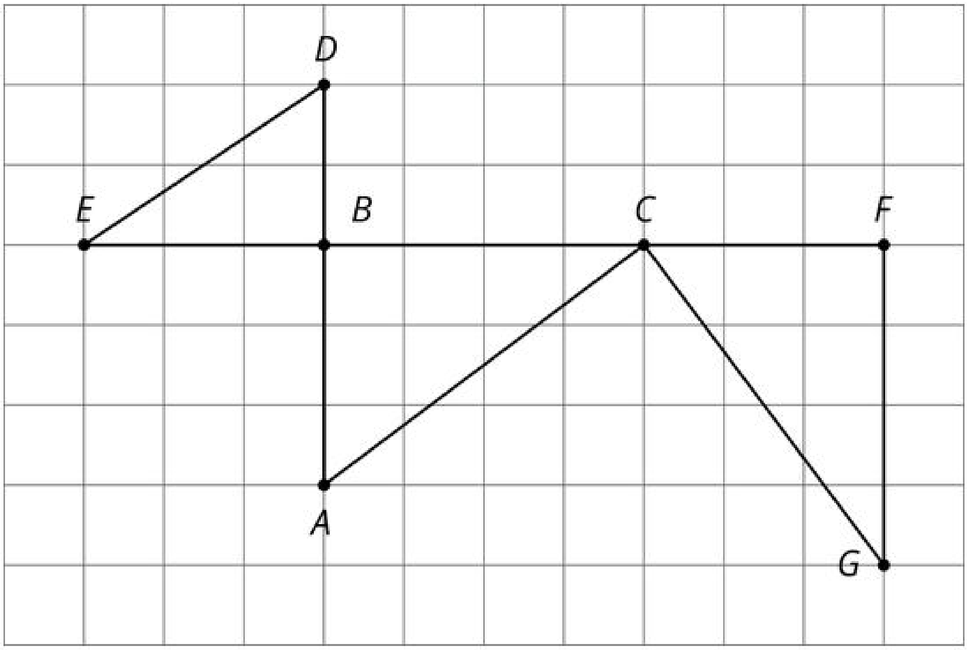
1. Reflect Pentagon C across line ℓ. What is the length of each side, in grid units and what is the measure of each interior angle. Click here for the applet.

Corresponding sides - pair of matching sides that are in the same spot on two different shapes

Corresponding angles - a pair of matching angles that are in the same spot in two different shapes

Which One?

Here is a grid showing triangle ABC and two other triangles.



You can use a rigid transformation to take triangle ABC to one of the other triangles.

Using the applet (click here) answer the following questions:

1. Which triangle? Explain how you know.

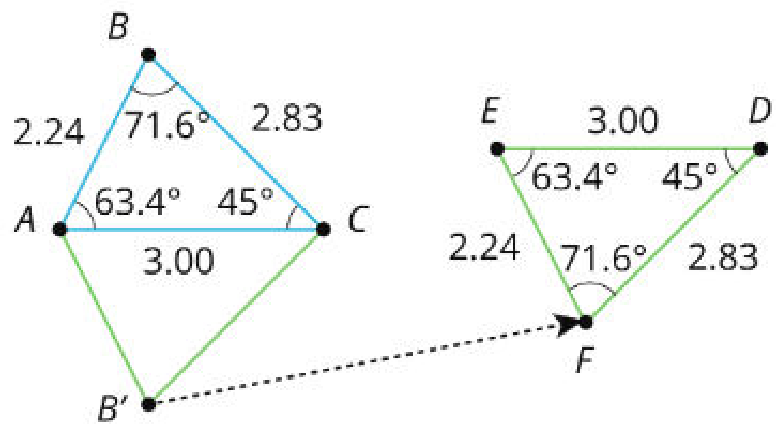
1. Describe a rigid transformation that takes ABC to the triangle you selected.

Lesson 7 Summary

The transformations we've learned about so far, translations, rotations, reflections, and sequences of these motions, are all examples of **rigid transformations**. A rigid transformation is a move that doesn't change measurements on any figure.

Earlier, we learned that a figure and its image have corresponding points. With a rigid transformation, figures like polygons also have **corresponding sides** and **corresponding angles**. These corresponding parts have the same measurements.

For example, triangle EFD was made by reflecting triangle ABC across a horizontal line, then translating. Corresponding sides have the same lengths, and corresponding angles have the same measures.



measurements in triangle ABC	corresponding measurements in image EFD
$AB = 2.24$	$EF = 2.24$
$BC = 2.83$	$FD = 2.83$
$CA = 3.00$	$DE = 3.00$
$m\angle ABC = 71.6^\circ$	$m\angle EFD = 71.6^\circ$
$m\angle BCA = 45.0^\circ$	$m\angle FDE = 45.0^\circ$
$m\angle CAB = 63.4^\circ$	$m\angle DEF = 63.4^\circ$

Lesson 8

Lesson 8: Rotation Patterns

Rotating a Segment

In the applet (click here), create a segment AB and a point C that is not on segment AB.

1. Rotate segment AB 180° around point B.
2. Rotate segment AB 180° around point C.

Construct (make) the midpoint of segment AB with the Midpoint tool

1. Rotate segment AB 180° around its midpoint. What is the image of A?

1. What happens when you rotate a segment 180°?

A Pattern of Four Triangles

Here is a diagram built with three different rigid transformations of triangle ABC.

Use the applet (click here) to answer the questions. It may be helpful to reset the image after each question.

1. Describe a rigid transformation that takes triangle ABC to triangle CDE.

1. Describe a rigid transformation that takes triangle ABC to triangle EFG.

1. Describe a rigid transformation that takes triangle ABC to triangle GHA.

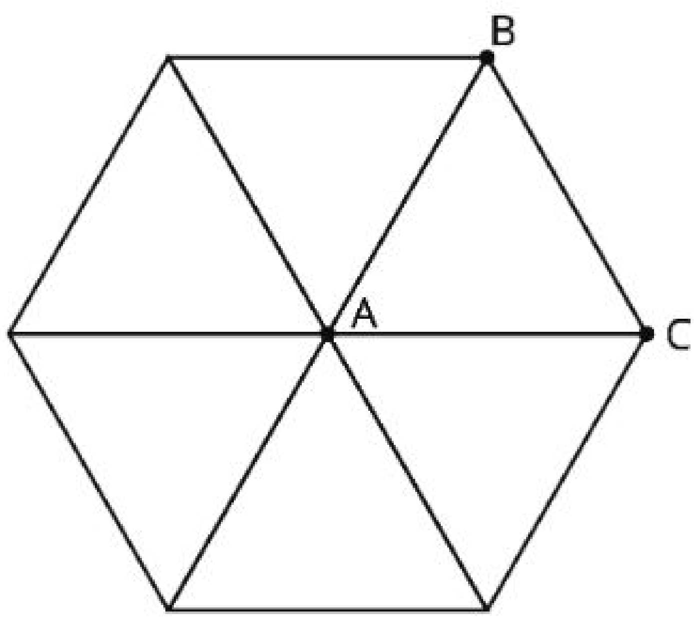
1. Do segments AC, CE, EG, and GA all have the same lengths? Explain your reasoning.

Lesson 8 Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

- The segment maps to itself (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment (if the center of rotation is *not* on the segment).

We can also build patterns by rotating a shape. For example, triangle ABC shown here has $m(\angle A) = 60$. If we rotate triangle ABC 60 degrees, 120 degrees, 180 degrees, 240 degrees, and 300 degrees clockwise, we can build a hexagon.



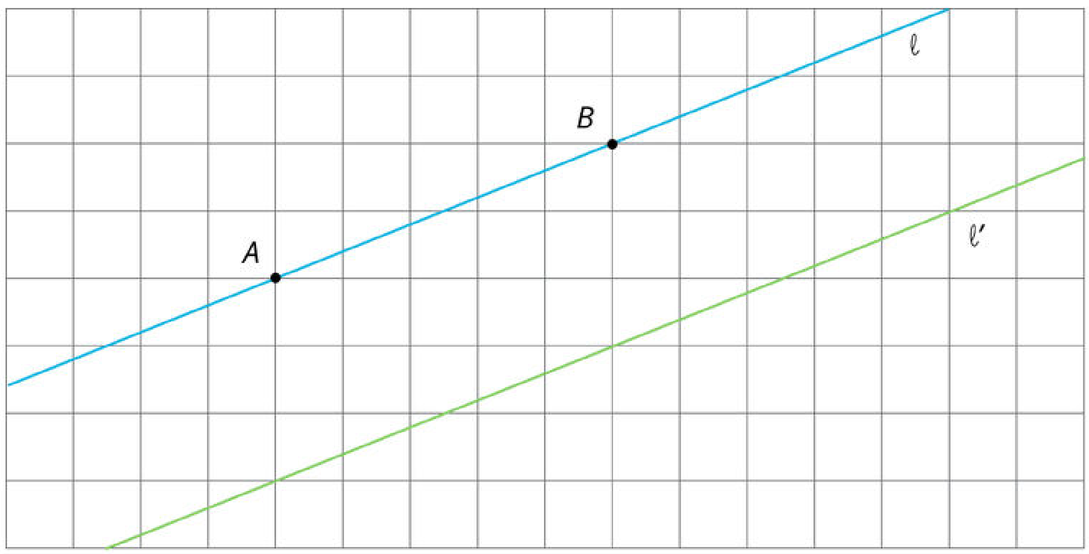
Lesson 9

Lesson 9: Moves in Parallel

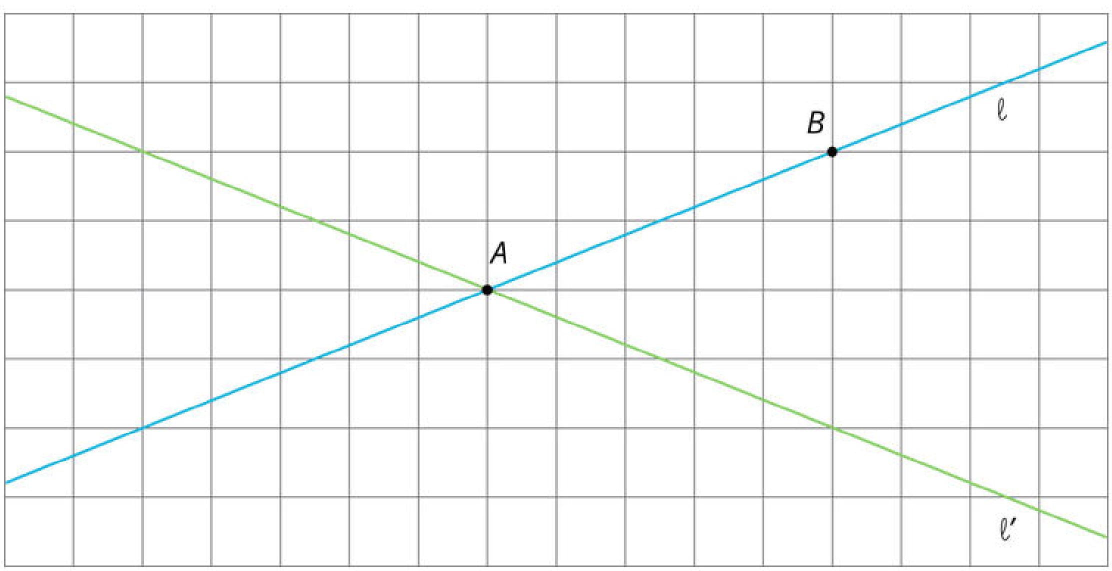
Rotating a Segment

For each diagram, describe a translation rotation, or reflection that takes line ℓ to line ℓ'. Then using the tools, plot and label A' and B', the images of A and B.

1.



2.



Parallel Lines

Using the applet (click here), perform the following:

1. Translate lines a and b 3 units up and 2 units to the right.
   What do you notice about the changes that occur to the lines after the translation? What is the same in the original and the image?

1. Rotate lines a and b counterclockwise 180 degrees using K as the center of rotation.
   What do you notice about the changes that occur to lines a and b after the rotation? What is the same in the original and in the image?

1. Reflect lines a and b across line h.
   What do you notice about the changes that occur to lines a and b after the reflection? What is the same in the original and the image?

Let's Do Some 180's

Using the applet (click here), perform the following:

1. The diagram shows a line with points labeled A, C, D, B.
   a. On the diagram draw the image of the line and points A, C, and B after the line has been rotated 180 degrees around point D.
   b. Label the images of the points A', B', and C'.
   c. What is the order of all seven points? Explain or show your reasoning. (You can upload a screenshot from the applet.)

1. The diagram shows a line with points A and C on the line and a segment AD where D is not on the line.
   a. Rotate the figure 180 degrees about point C. Label the image of A as A' and the image D as D'
   b. What do you know about the relationship between angle CAD and angle CA'D'? Explain or show your reasoning.

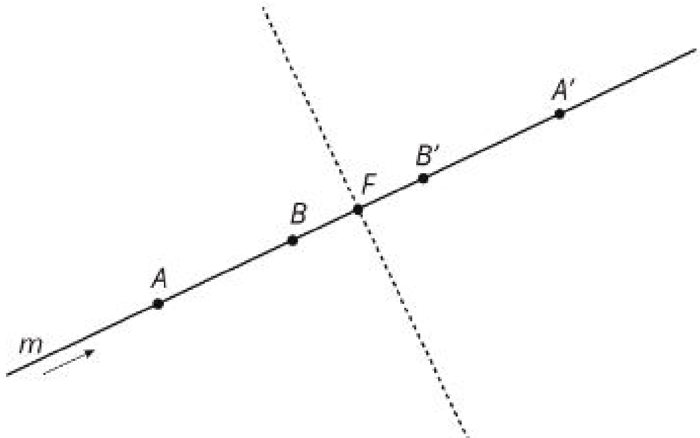
3. The diagram show two lines ℓ and m that intersect at a point O with point A on ℓ and point D on m.

a. Rotate the figure 180 degrees around O. Label the image of A as A' and the image of D as D'
b. What do you know about the relationship between the angles in the figure? Explain or show your reasoning.

Lesson 9 Summary

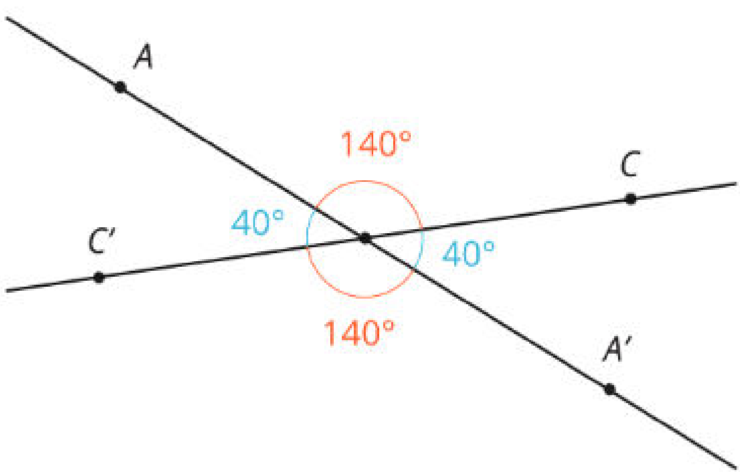
Rigid transformations have the following properties:

- A rigid transformation of a line is a line.
- A rigid transformation of two parallel lines results in two parallel lines that are the same distance apart as the original two lines.
  - Sometimes, a rigid transformation takes a line to itself. For example:



  - A translation parallel to the line. The arrow shows a translation of line m that will take m to itself.
  - A rotation by 180° around any point on the line. A 180° rotation of line m around point F will take m to itself.
  - A reflection across any line perpendicular to the line. A reflection of line m across the dashed line will take m to itself.

These facts let us make an important conclusion. If two lines intersect at a point, which we'll call O, then a 180° rotation of the lines with center O shows that **vertical angles** are congruent. Here is an example:



Rotating both lines by 180° around O sends angle AOC to angle $A'OC'$, proving that they have the same measure. The rotation also sends angle AOC' to angle $A'OC$.

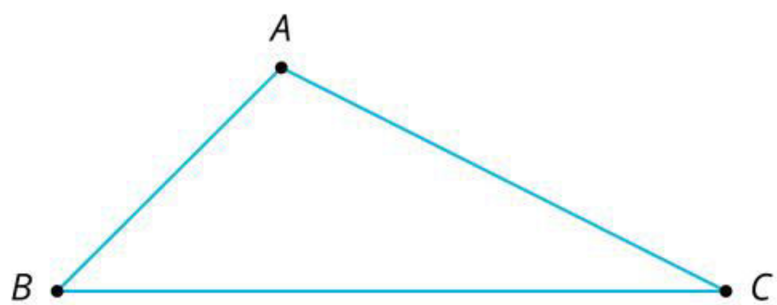
Lesson 10

Lesson 10: Composing Figures

Triangle Plus One

Using the applet, (click here) perform the following steps on triangle ABC.

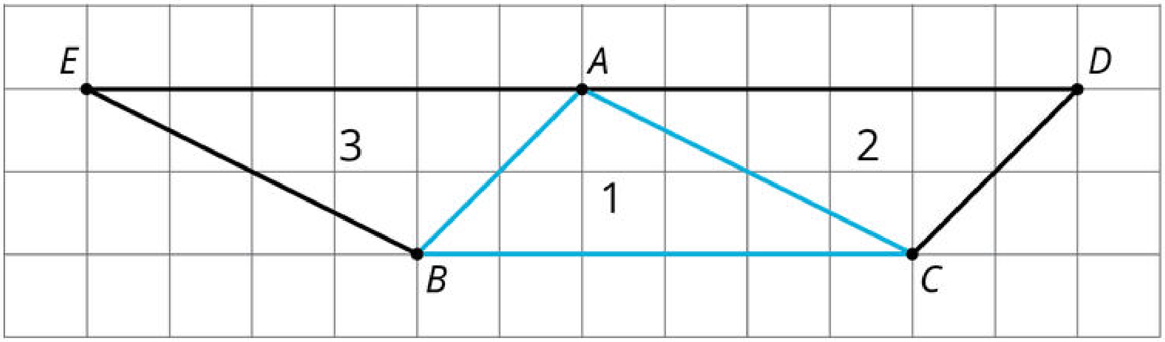
Here is triangle ABC.



1. Draw midpoint M of side AC.
2. Rotate triangle ABC 180 degrees using center M to form triangle CDA.
3. What kind of quadrilateral is ABCD? Explain how you know.

Triangle Plus Two

Using the applet, (click here) to help you answer the questions..



1. Describe a rigid transformation that takes Triangle 1 to Triangle 2. What points in Triangle 2 correspond to points A, B, and C in the original triangle?

1. Describe a rigid transformation that takes Triangle 1 to Triangle 3. What points in Triangle 3 correspond to points A, B, and C in the original triangle?

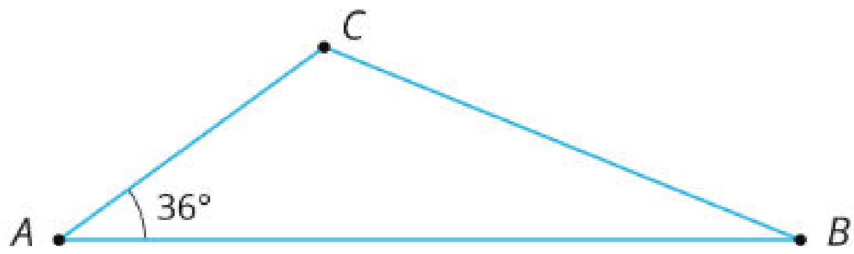
1. Find two pairs of line segments in the diagram that are the same length, and explain how you know they are the same length.

1. Find two pairs of angles in the diagram that have the same measure, and explain how you know they have the same measure.

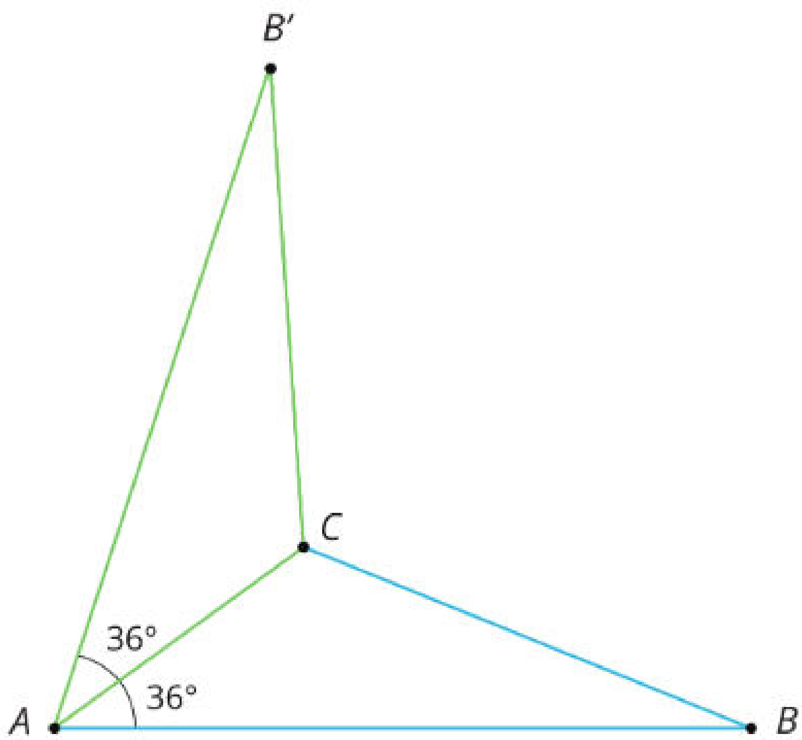
Lesson 10 Summary

Earlier, we learned that if we apply a sequence of rigid transformations to a figure, then corresponding sides have equal length and corresponding angles have equal measure. These facts let us figure out things without having to measure them!

For example, here is triangle ABC.



We can reflect triangle ABC across side AC to form a new triangle:



Because points A and C are on the line of reflection, they do not move. So the image of triangle ABC is $AB'C$. We also know that:

- Angle $B'AC$ measures 36° because it is the image of angle BAC.
- Segment AB' has the same length as segment AB.

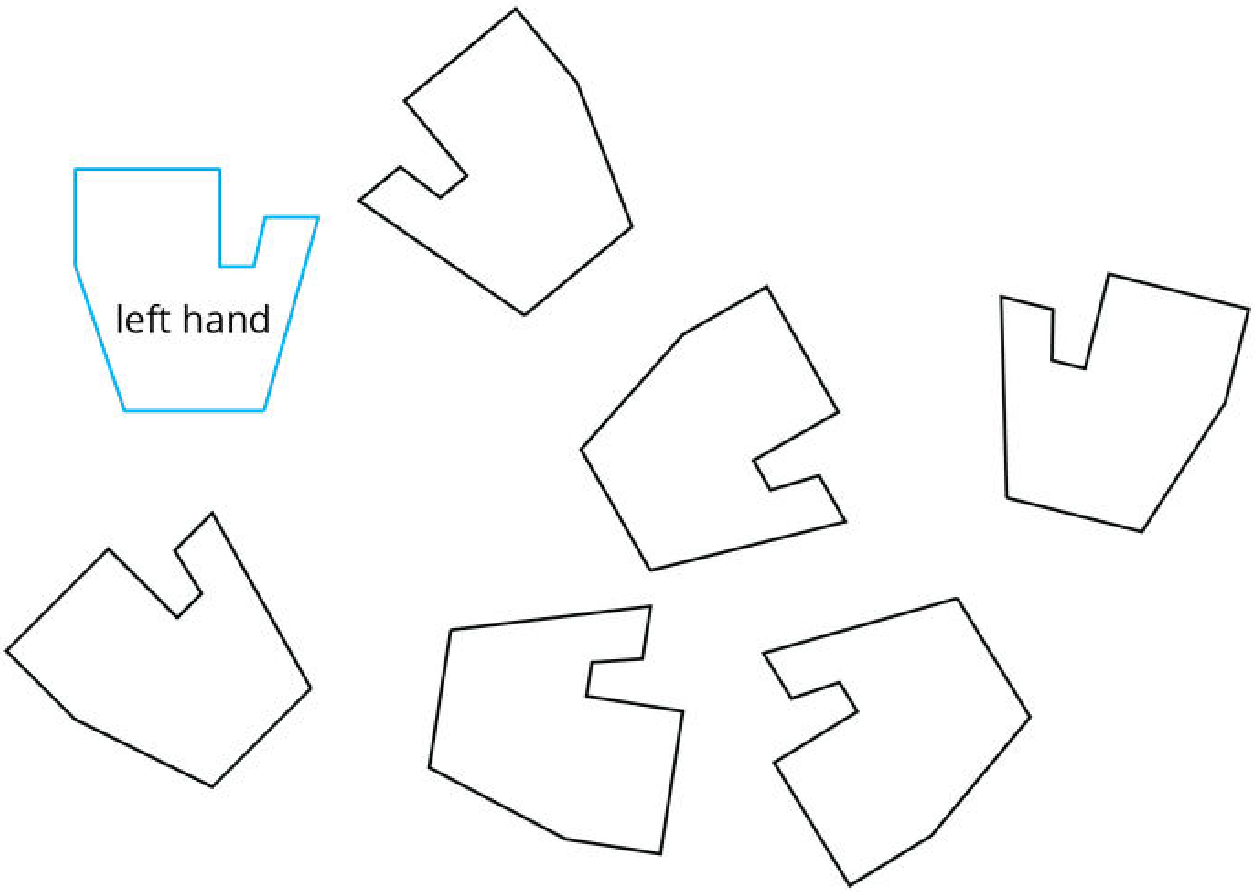
When we construct figures using copies of a figure made with rigid transformations, we know that the measures of the images of segments and angles will be equal to the measures of the original segments and angles.

Lesson 11

Lesson 11: What is the Same?

Find the Right Hands

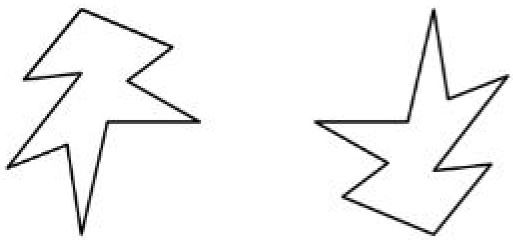
A person's hands are mirror images of each other. In the diagram, a left hand is labeled. Places Xs on all of the right hands.



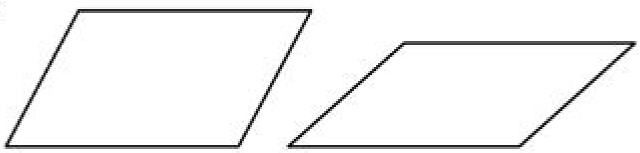
Are They the Same?

For each pair of shapes, decide whether or not they are the same.

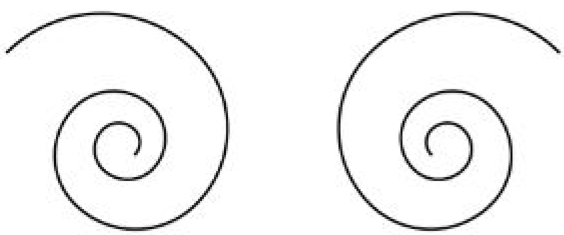
A



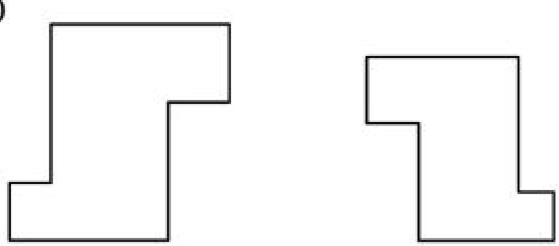
B



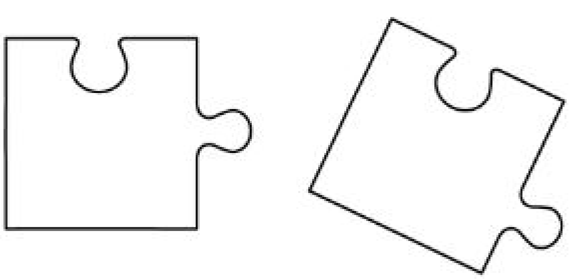
C



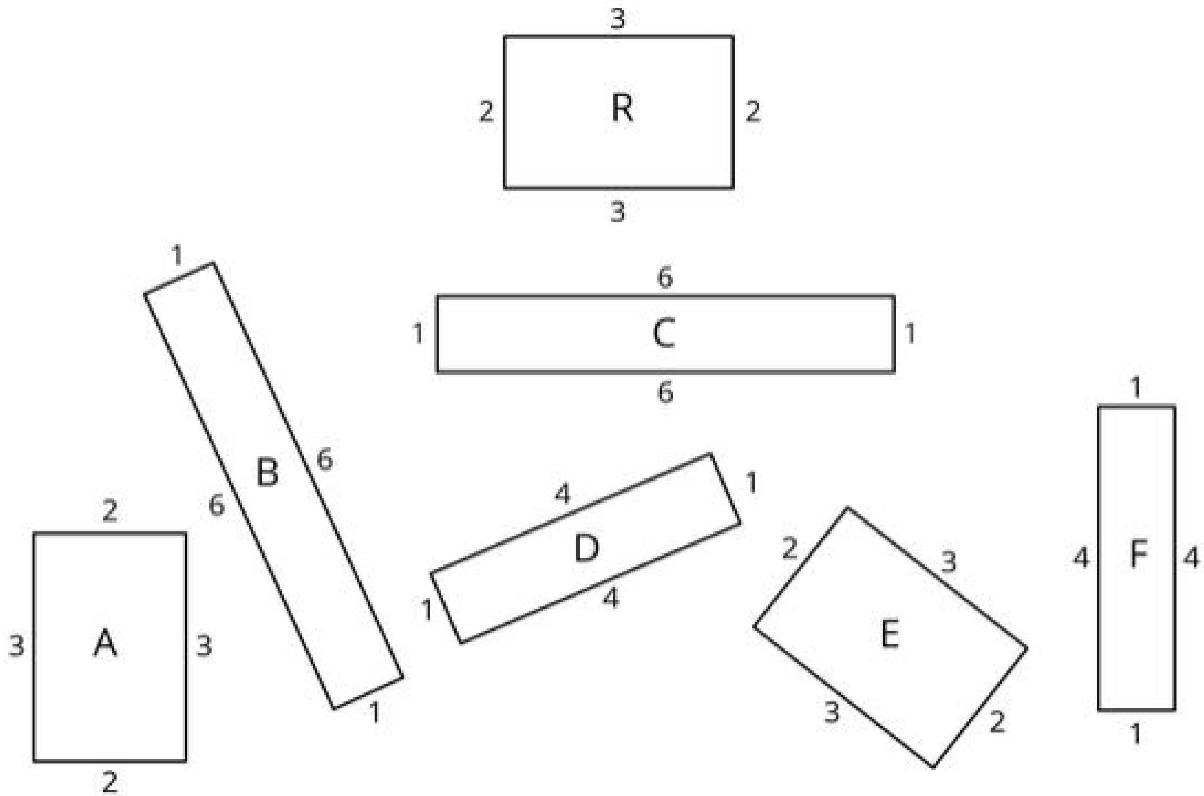
D



E



Area, Perimeter, and Congruence



1. Which of these rectangles have the same area as Rectangle R but different perimeter?

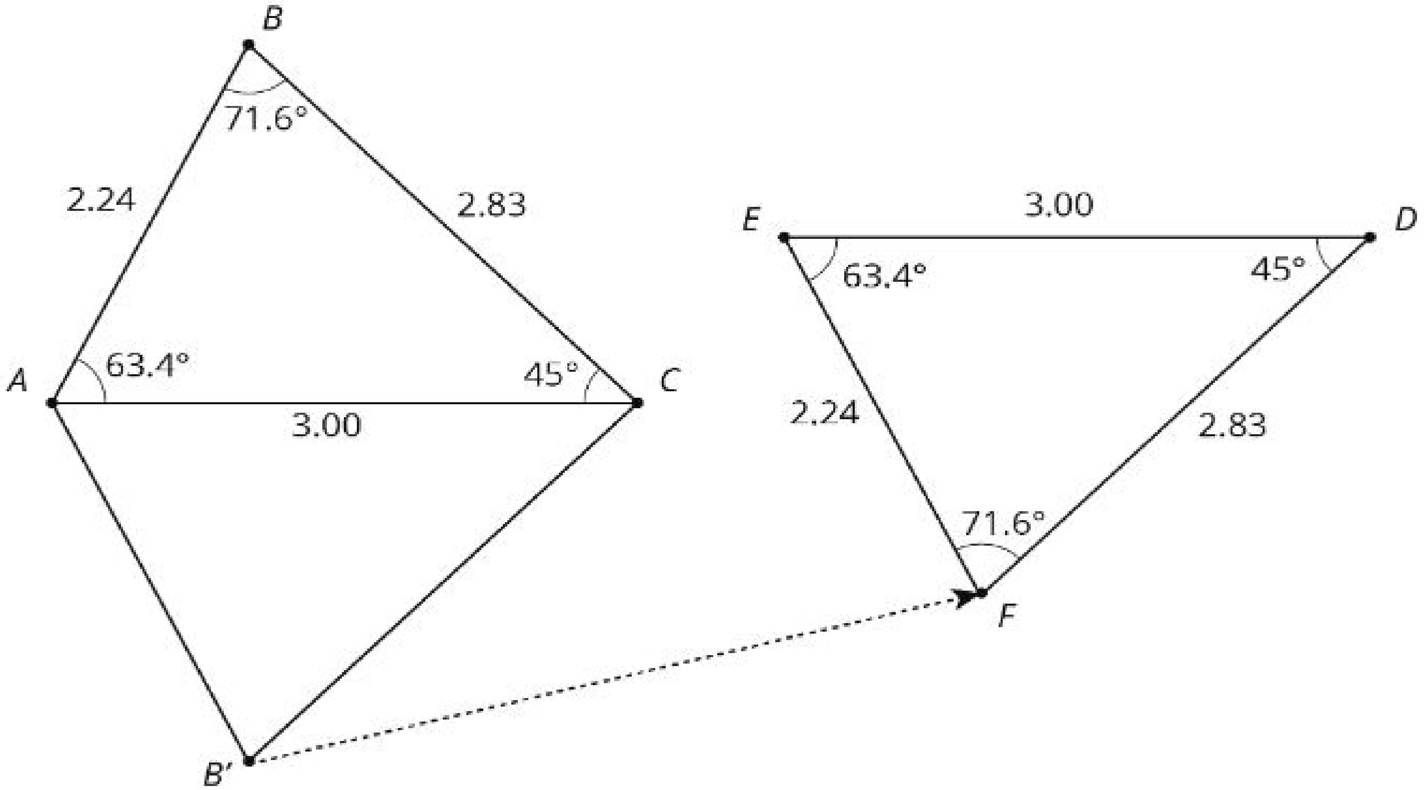
1. Which rectangles have the same perimeter but different area?

1. Which have the same area and the same perimeter?

1. Decide which rectangles are congruent and highlight them with the same color.

Lesson 11 Summary

Congruent is a new term for an idea we have already been using. We say that two figures are congruent if one can be lined up exactly with the other by a sequence of rigid transformations. For example, triangle EFD is congruent to triangle ABC because they can be matched up by reflecting triangle ABC across AC followed by the translation shown by the arrow. Notice that all corresponding angles and side lengths are equal.



Here are some other facts about congruent figures:

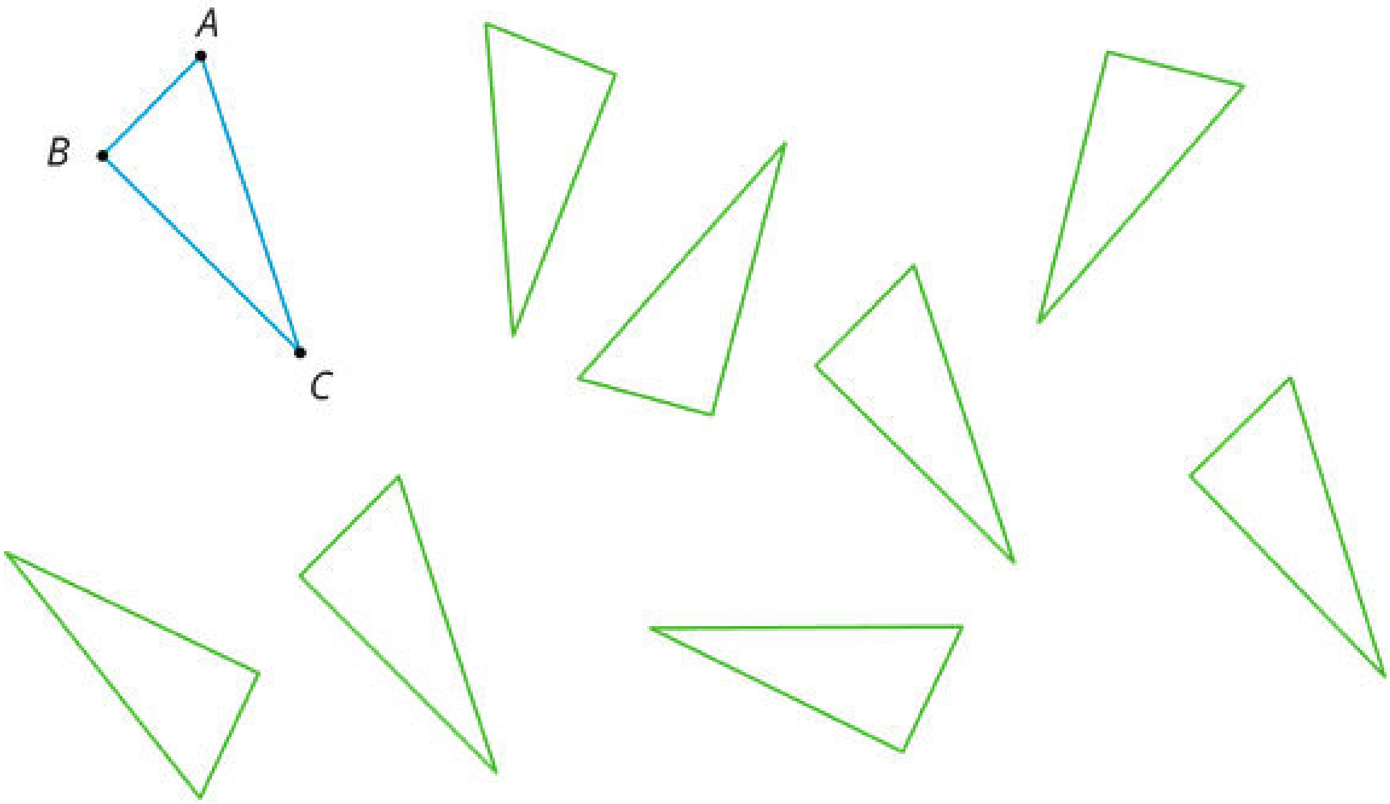
- We don't need to check all the measurements to prove two figures are congruent; we just have to find a sequence of rigid transformations that match up the figures.
- A figure that looks like a mirror image of another figure can be congruent to it. This means there must be a reflection in the sequence of transformations that matches up the figures.
- Since two congruent polygons have the same area and the same perimeter, one way to show that two polygons are *not* congruent is to show that they have a different perimeter or area.

Lesson 12

Lesson 12: Congruent Polygons

Translated Images

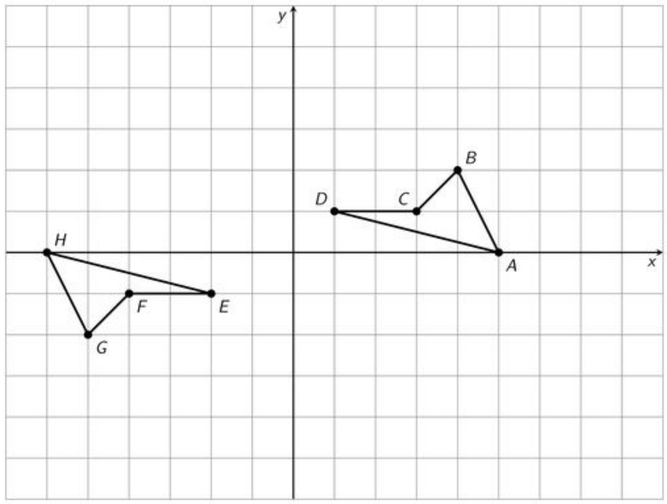
All of these triangles are congruent. Sometimes we can take on figure to another with a translation. Highlight the triangles that are images of triangle *ABC* under a translation.



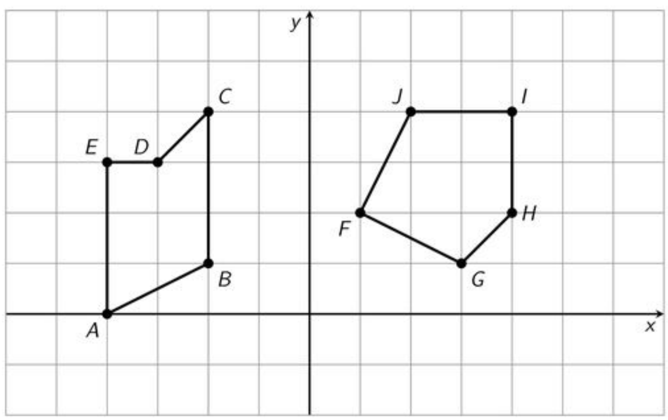
Congruent Pairs (Part 1)

For each of the following pairs of shapes, decide whether or not they are congruent. Explain your reasoning.

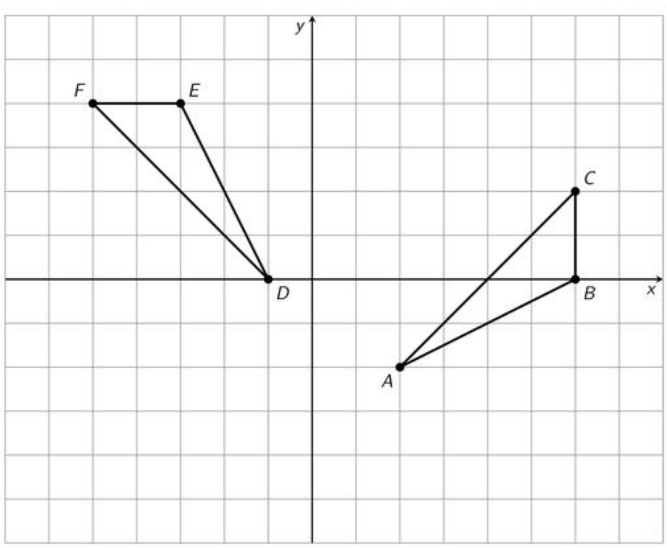
1.



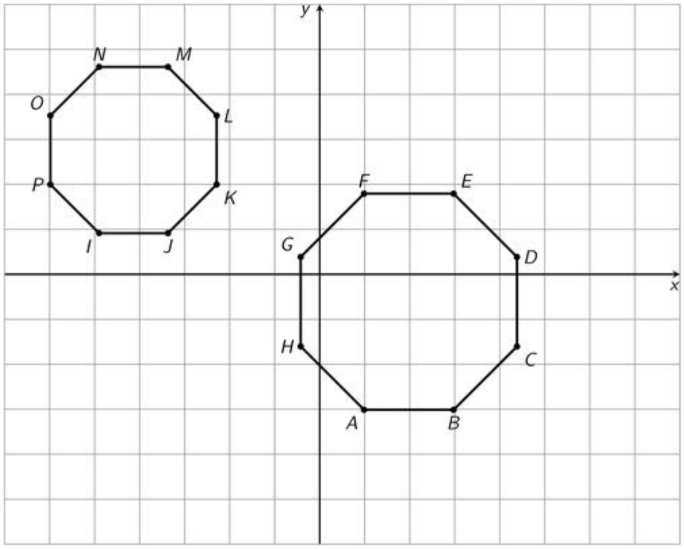
2.



3.



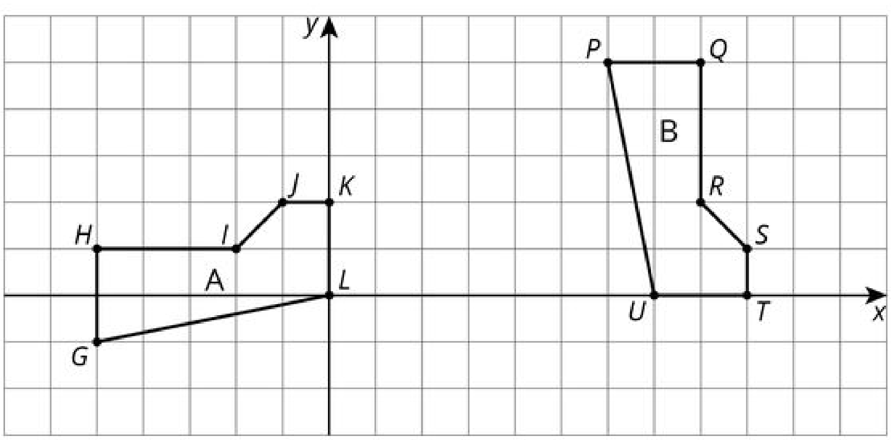
4.



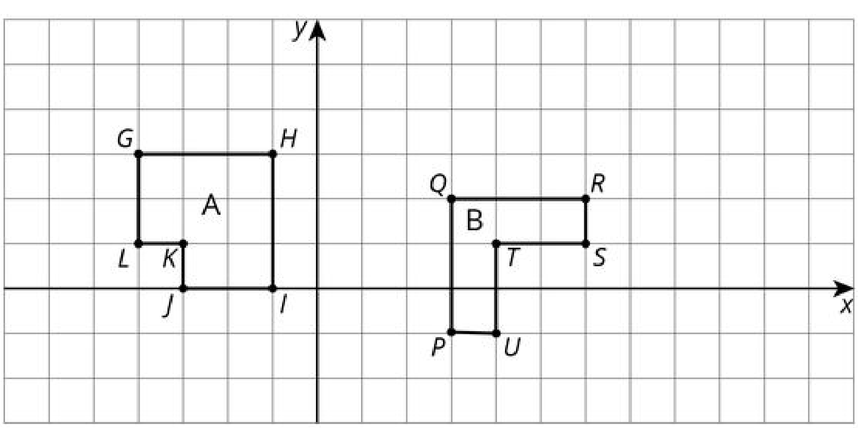
Congruent Pairs (Part 2)

For each pair of shapes, decide whether or not Shape A is congruent to Shape B. Explain how you know.

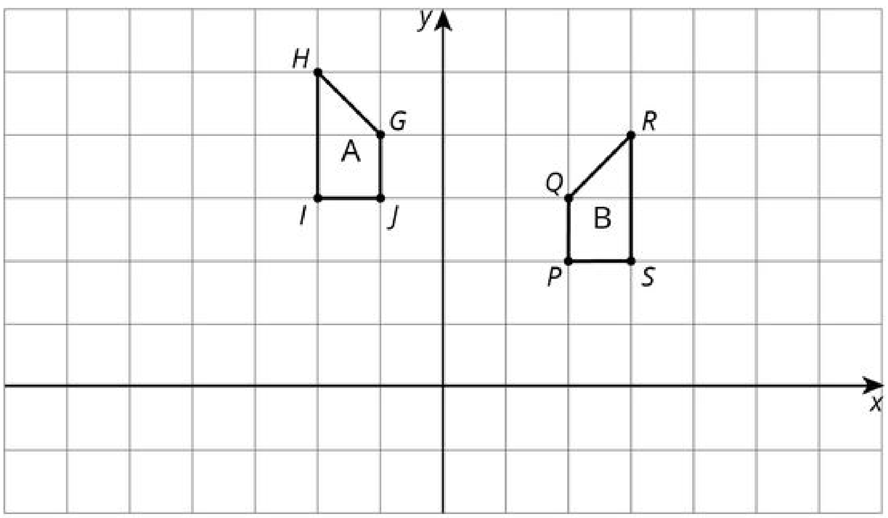
1.



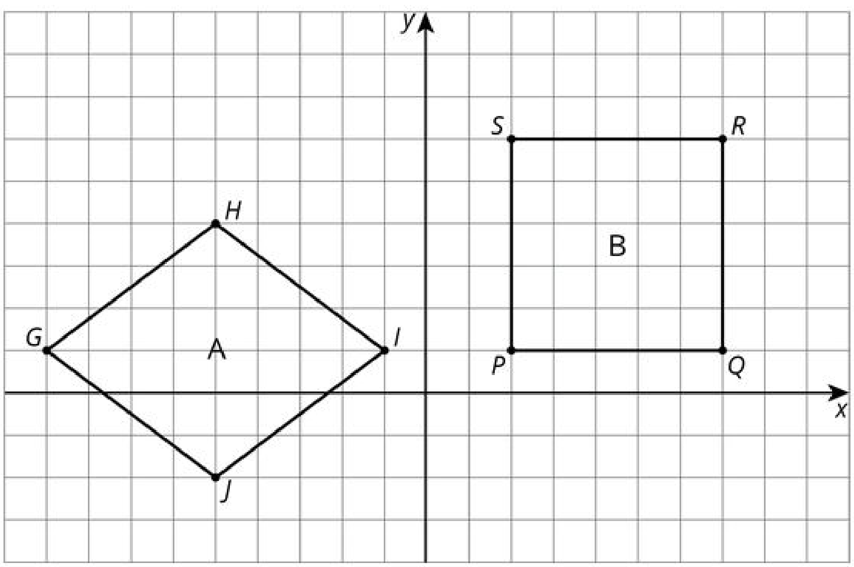
2.



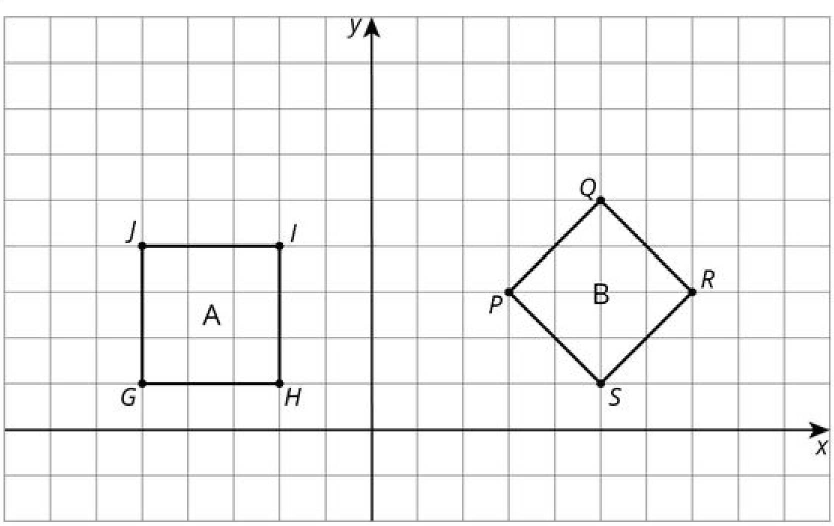
3.



4.



5.



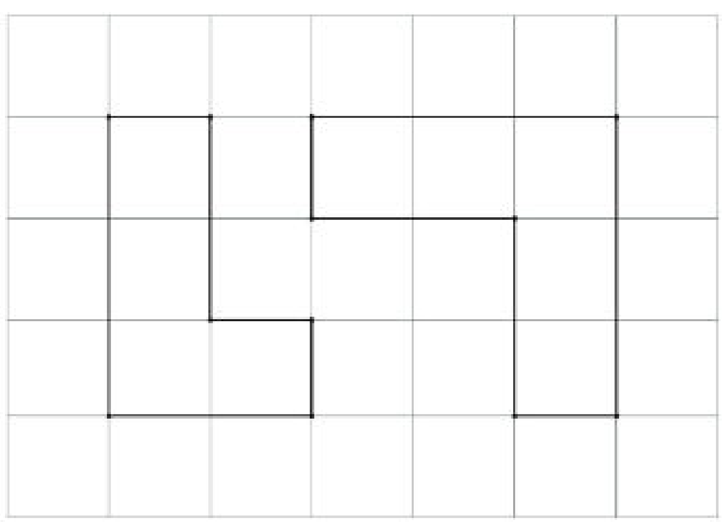
Lesson 12 Summary

How do we know if two figures are congruent?

- If we copy one figure on tracing paper and move the paper so the copy covers the other figure exactly, then that suggests they are congruent.
- We can prove that two figures are congruent by describing a sequence of translations, rotations, and reflections that move one figure onto the other so they match up exactly.

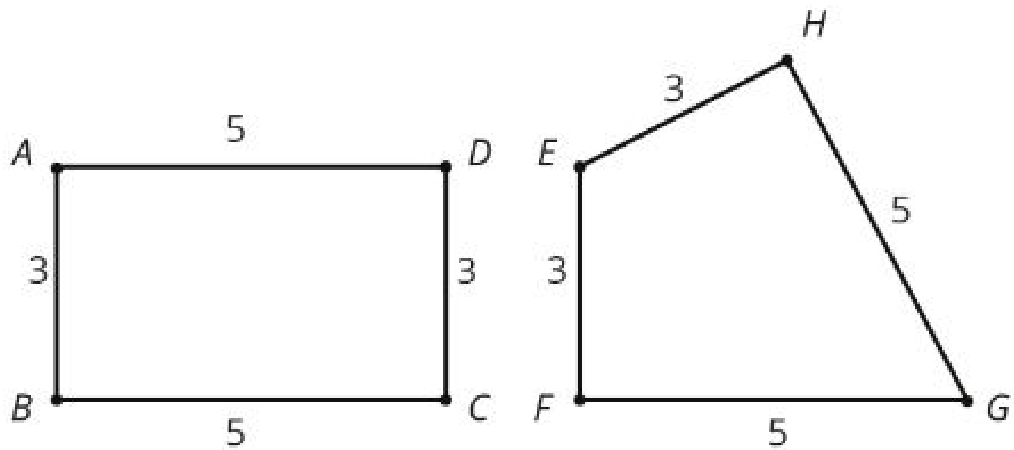
How do we know that two figures are *not* congruent?

- If there is no correspondence between the figures where the parts have equal measure, that proves that the two figures are *not* congruent. In particular,
    - If two polygons have different sets of side lengths, they can't be congruent. For example, the figure on the left has side lengths 3, 2, 1, 1, 2, 1. The figure on the right has side lengths 3, 3, 1, 2, 2, 1. There is no way to make a correspondence between them where all corresponding sides have the same length.

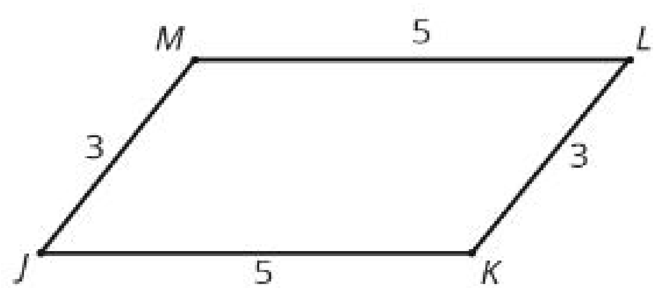


Lesson 12 Summary

- If two polygons have the same side lengths, but their orders can't be matched as you go around each polygon, the polygons can't be congruent. For example, rectangle $ABCD$ can't be congruent to quadrilateral $EFGH$. Even though they both have two sides of length 3 and two sides of length 5, they don't correspond in the same order. In $ABCD$, the order is 3, 5, 3, 5 or 5, 3, 5, 3; in $EFGH$, the order is 3, 3, 5, 5 or 3, 5, 5, 3 or 5, 5, 3, 3.



- If two polygons have the same side lengths, in the same order, but different corresponding angles, the polygons can't be congruent. For example, parallelogram $JKLM$ can't be congruent to rectangle $ABCD$. Even though they have the same side lengths in the same order, the angles are different. All angles in $ABCD$ are right angles. In $JKLM$, angles J and L are less than 90 degrees and angles K and M are more than 90 degrees.



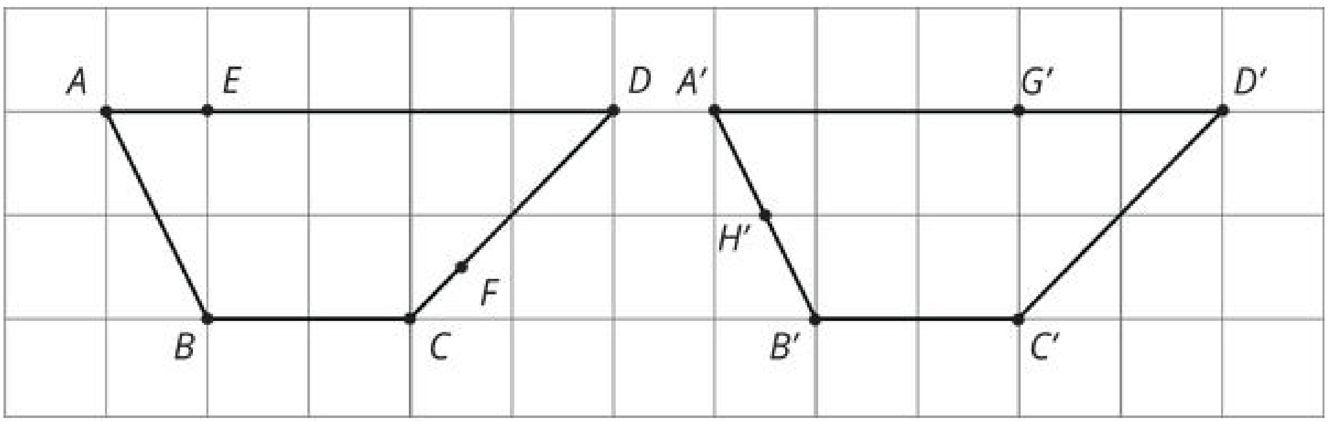
Lesson 13

Lesson 13: Congruence

Not Just the Vertices

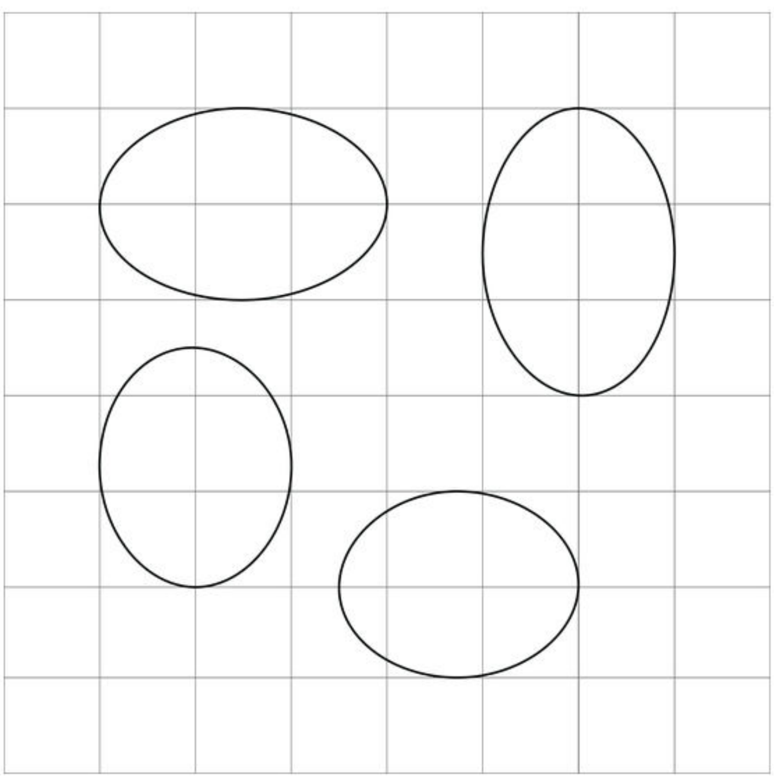
Trapezoids *ABCD* and *A'B'C'D'* are congruent.

- Place and label points on A'B'C'D' that correspond to E and F.
- Place and label points on ABCD that correspond to G' and H'.
- Place and label at least three more pairs of corresponding points.



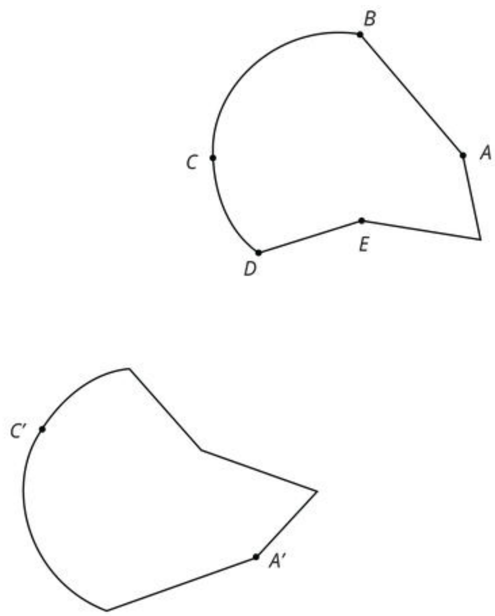
Congruent Ovals

Are any of the ovals congruent to one another? Explain how you know.



Corresponding Points in Congruent Figures

Here are two congruent shapes with some corresponding points labeled.



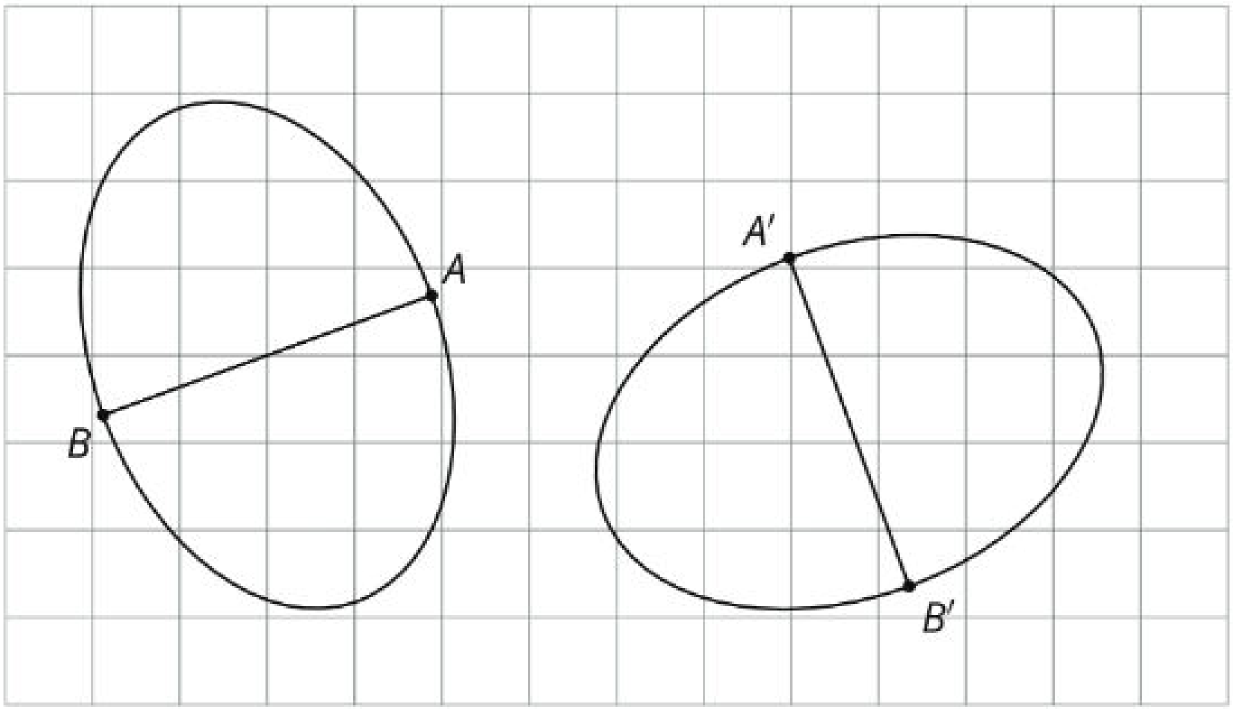
Click on the image to go to the activity and complete the following:

1. Draw the points corresponding to B, D, and E, and label them B', D', and E'.
2. Draw line segments AD and A'D' and measure them. Do the same for segments BC and B'C' and for segments AE and A'E'. What do you notice?

1. Do you think there could be a pair of corresponding segments with different lengths? Explain.

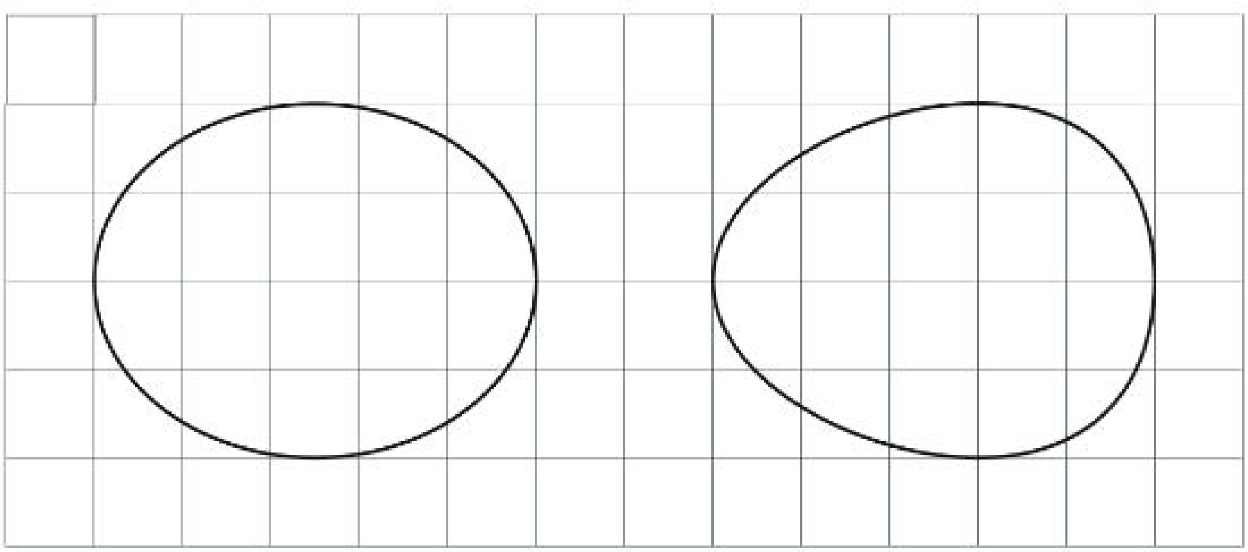
Lesson 13 Summary

To show two figures are congruent, you align one with the other by a sequence of rigid transformations. This is true even for figures with curved sides. Distances between corresponding points on congruent figures are always equal, even for curved shapes. For example, corresponding segments AB and $A'B'$ on these congruent ovals have the same length:



To show two figures are not congruent, you can find parts of the figures that should correspond but that have different measurements.

For example, these two ovals don't look congruent.



Lesson 13 Summary

On both, the longest distance is 5 units across, and the longest distance from top to bottom is 4 units. The line segment from the highest to lowest point is in the middle of the left oval, but in the right oval, it's 2 units from the right end and 3 units from the left end. This proves they are not congruent.