COMPOSITION BOOK

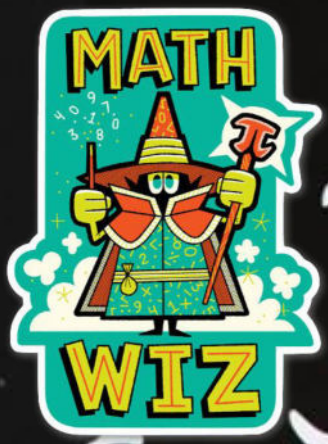
Ms. Forbes'

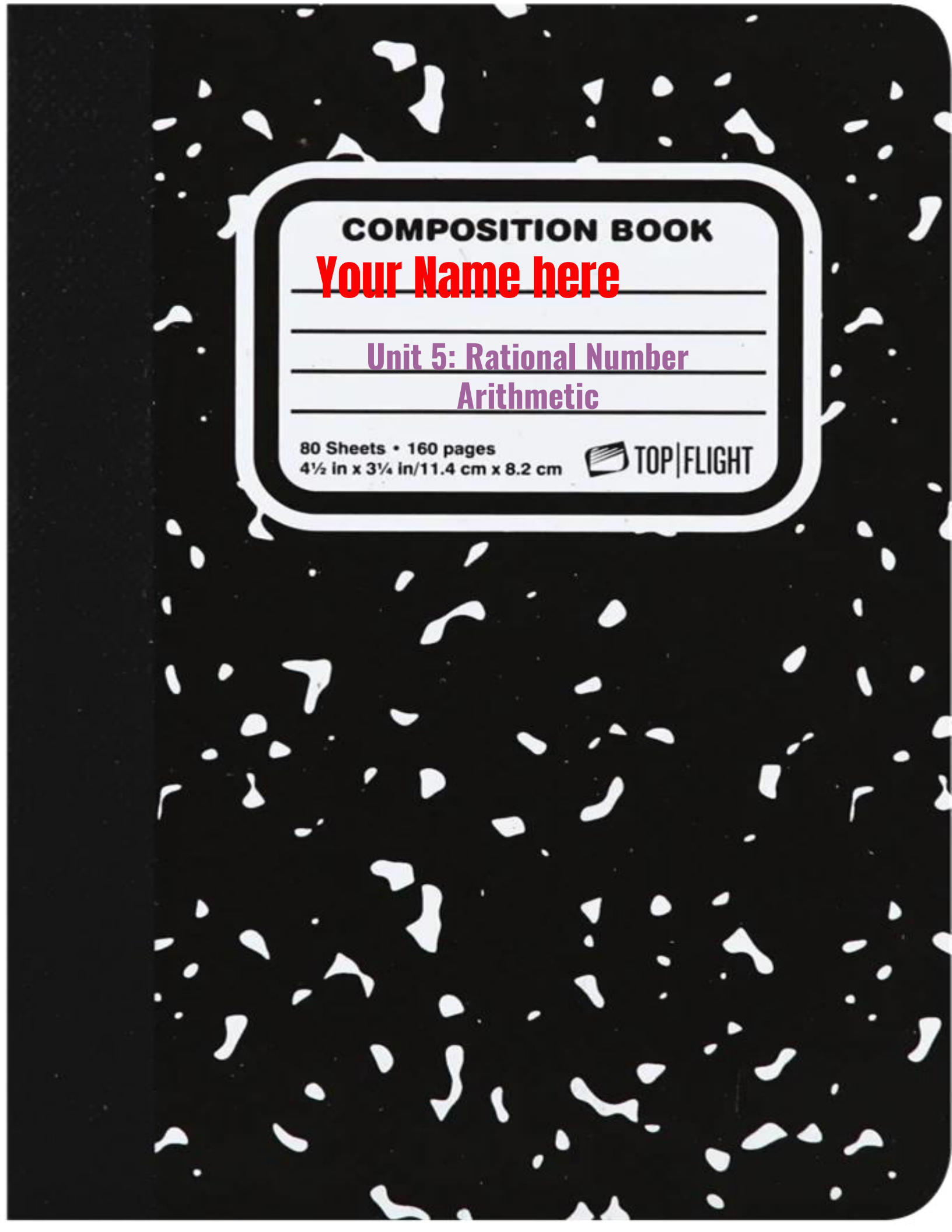
Math 7 Journal

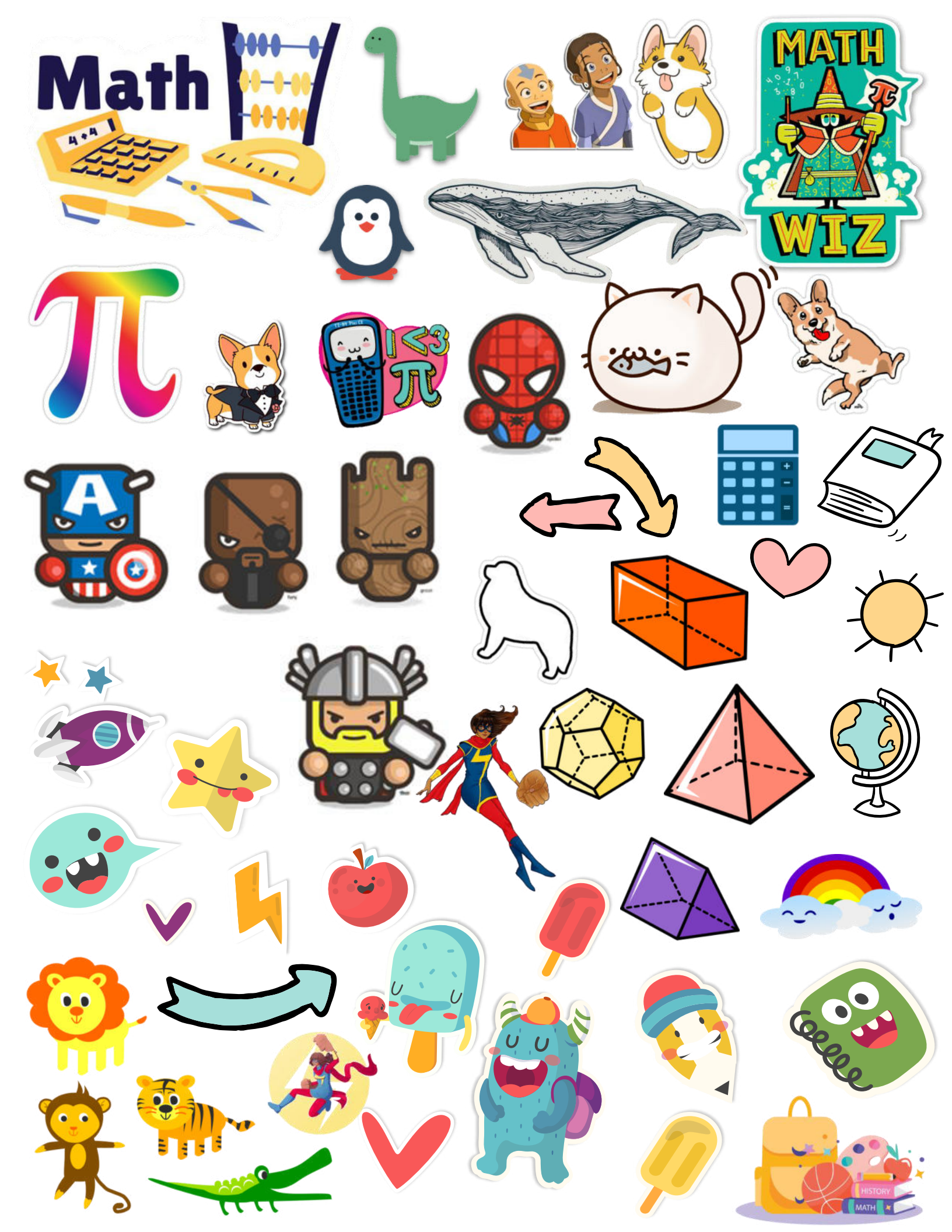
Unit 5: Rational Number Arithmetic

80 Sheets • 160 pages
4½ in x 3¼ in/11.4 cm x 8.2 cm

TOP|FLIGHT




COMPOSITION BOOK
Your Name here
Unit 5: Rational Number Arithmetic
80 Sheets • 160 pages
4½ in x 3¼ in/11.4 cm x 8.2 cm
TOP|FLIGHT


Math
MATH
WIZ
I<3
HISTORY
MATH



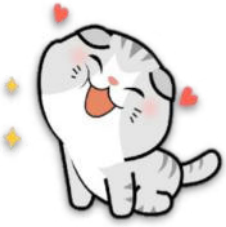




OK!










STRANGER
THINGS





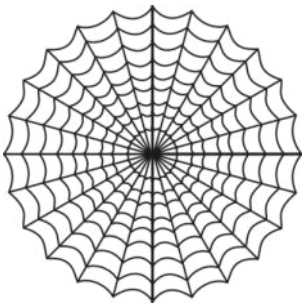
NOPE

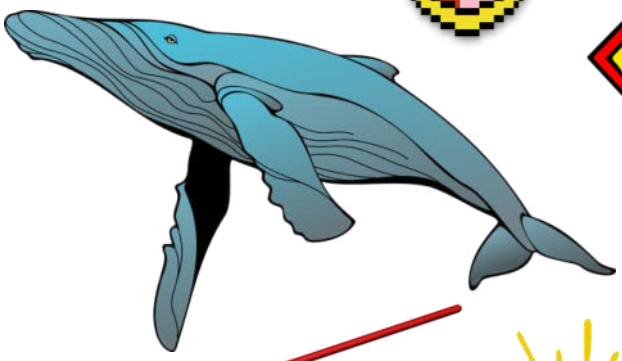
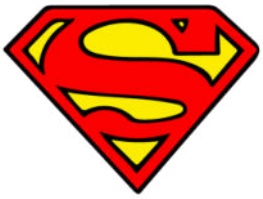


COMPOSITION




LOL!


STAR
WARS



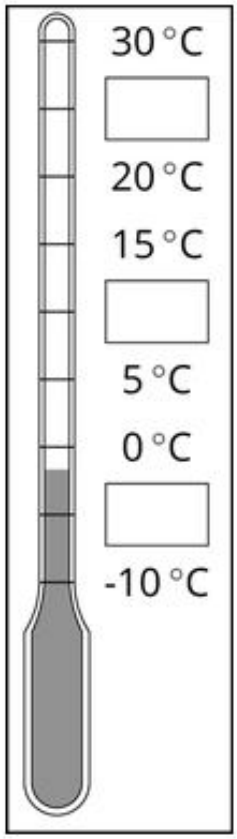
Table of Contents

Lesson 1

Lesson 1: Interpreting Negative Numbers

Using the Thermometer

Here is a weather thermometer. Three of the numbers have been left off.

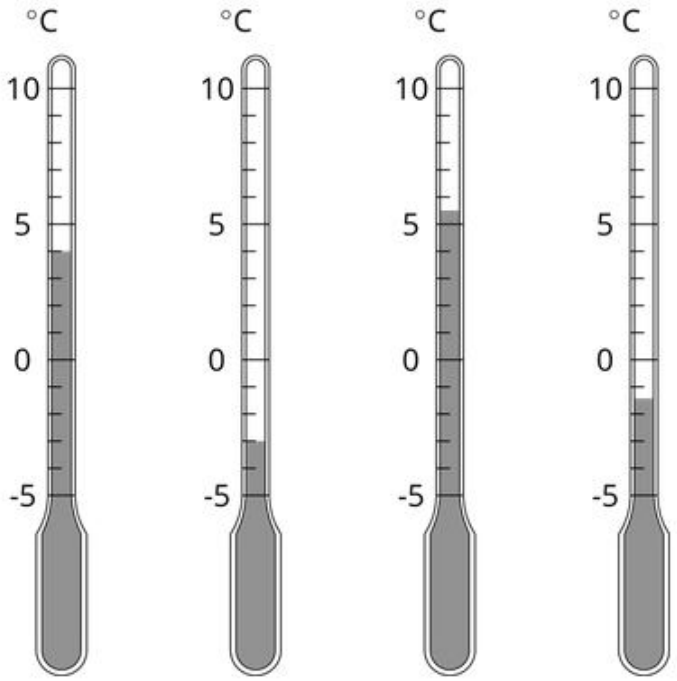


1. What numbers go in the boxes?

.

1. What temperature does the thermometer show?

Fractions of a Degree



1. What temperature is shown on each thermometer?

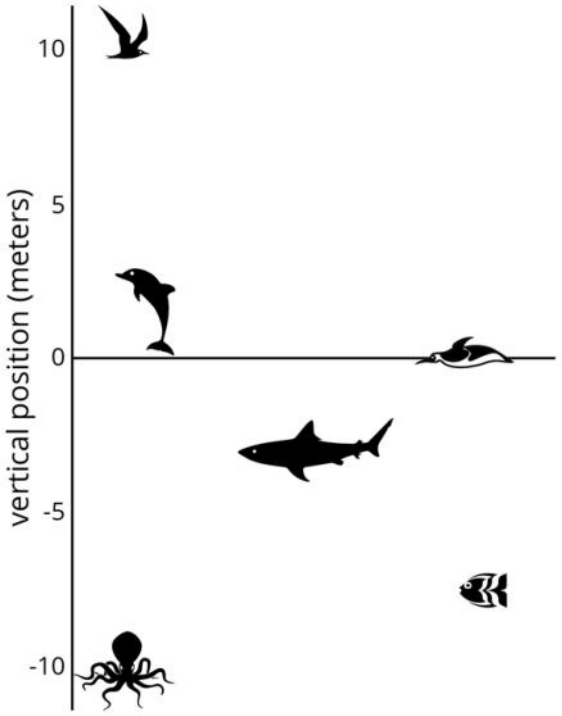
1. Which thermometer shows the highest temperature?

1. Which thermometer shows the lowest temperature?

1. Suppose the temperature outside is -4°C. Is that colder or warmer than the coldest temperature shown? How do you know?

Seagulls Soar, Sharks Swim

Here is a picture of some sea animals. The number line on the left shows the vertical position of each animal above or below sea level, in meters.



1. How far above or below sea level is each animal? Measure to their eye level.

2. A mobula ray is 3 meters above the surface of the ocean. How does its distance from the surface of the ocean compare to the vertical distance from the eyes of:

   The jumping dolphin?
   The flying seagull?
   The octopus?

3. An albatross is 5 meters above the surface of the ocean. How does its distance from the surface compare to the vertical distance from the eyes of:

   The jumping dolphin?
   The flying seagull?
   The octopus?

4. A clownfish is 2 meters below the surface of the ocean. How does its distance from the surface compare to the vertical distance from the eyes of:

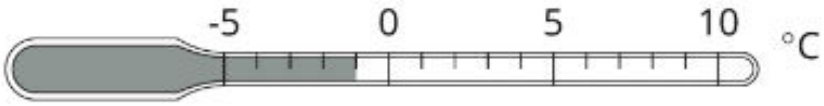
   The jumping dolphin?
   The flying seagull?
   The octopus?

5. The vertical distance of a new dolphin from the dolphin in the picture is 3 meters. What is its distance from the surface of the ocean?

Lesson 1 Summary

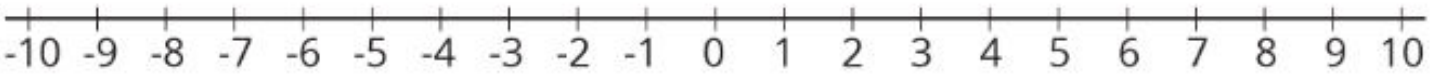
We can use positive and negative numbers to represent temperature and elevation.

When numbers represent temperatures, positive numbers indicate temperatures that are warmer than zero and negative numbers indicate temperatures that are colder than zero. This thermometer shows a temperature of -1 degree Celsius, which we write $-1^{\circ}\mathrm{C}$.



When numbers represent elevations, positive numbers indicate positions above sea level and negative numbers indicate positions below sea level.

We can see the order of signed numbers on a number line.



A number is always less than numbers to its right. So $-7 < -3$.

We use absolute value to describe how far a number is from 0. The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites*. They are on opposite sides of 0 on the number line, but the same distance from 0.

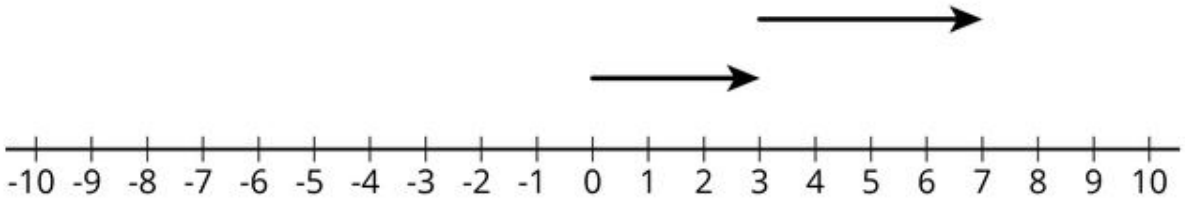
Lesson 2

Lesson 2: Changing Temperatures

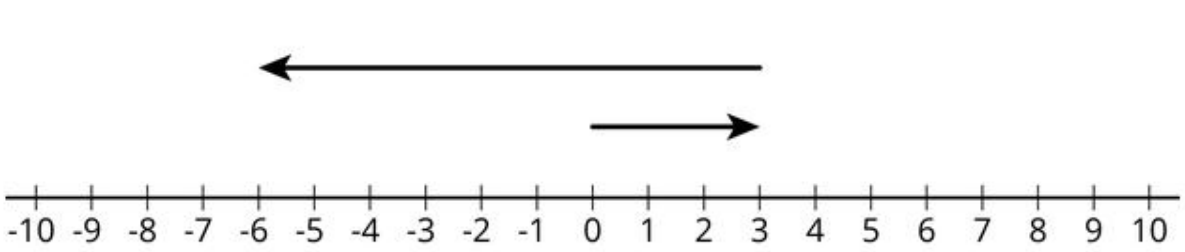
Which One Doesn't Belong: Arrows

Which pair of arrows doesn't belong?

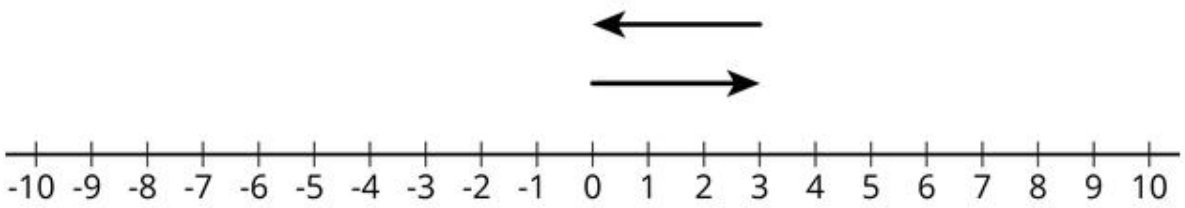
1.



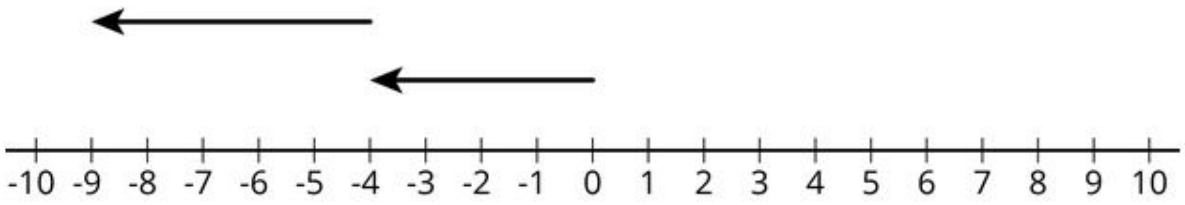
2.



3.



4.

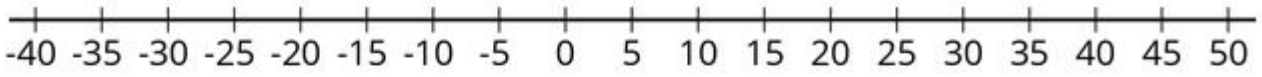


Warmer and Colder

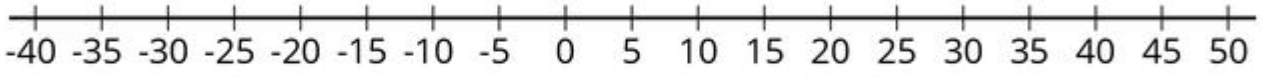
Complete the table and draw a number line diagram for each situation.

	start (°C)	change (°C)	final (°C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder		
c	+40	30 degrees colder		
d	+40	40 degrees colder		
e	+40	50 degrees colder		

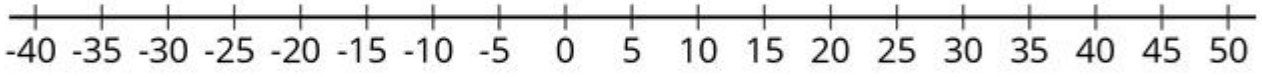
a.



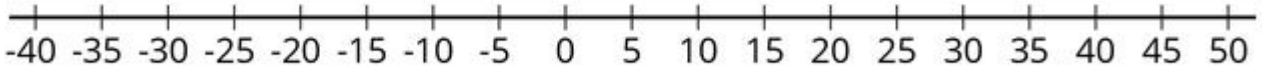
b.



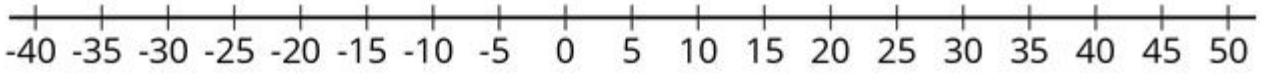
c.



d.



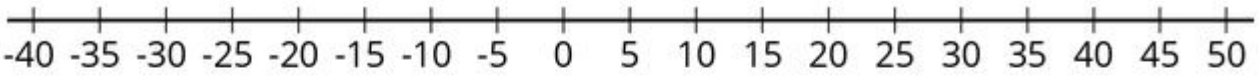
e.



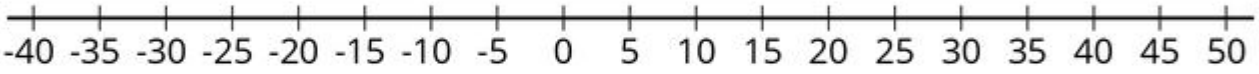
Complete the table and draw a number line diagram for each situation.

	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer		
b	-20	35 degrees warmer		
c	-20	15 degrees warmer		
d	-20	15 degrees colder		

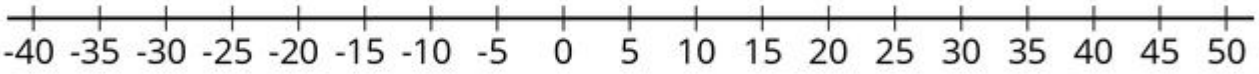
a.



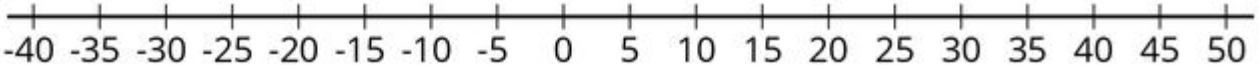
b.



c.



d.



Winter Temperatures

One winter day, the temperature in Houston is 8° Celsius. Find the temperatures in these other cities. Explain or show your reasoning.

1. In Orlando, it is 10° warmer than it is in Houston.

1. In Salt Lake City, it is 8° colder than it is in Houston.

1. In Minneapolis, it is 20° colder than it is in Houston.

1. In Fairbanks, it is 10° colder than it is in Minneapolis.

1. Write an addition equation that represents the relationship between the temperature in Houston and the temperature in Fairbanks.

Lesson 2 Summary

If it is 42° outside and the temperature increases by 7°, then we can add the initial temperature and the change in temperature to find the final temperature.

$$42 + 7 = 49$$

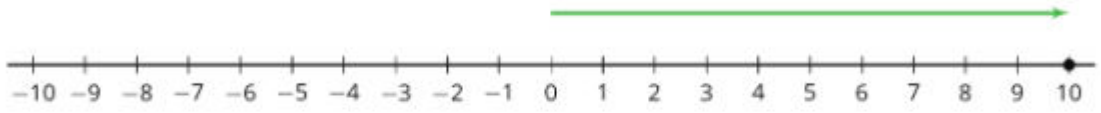
If the temperature decreases by 7°, we can either subtract $42 - 7$ to find the final temperature, or we can think of the change as -7°. Again, we can add to find the final temperature.

$$42 + (-7) = 35$$

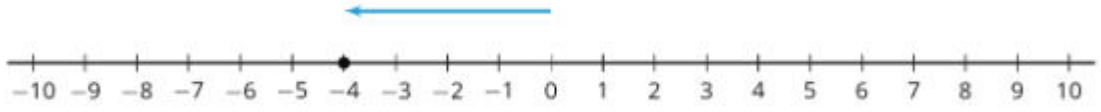
In general, we can represent a change in temperature with a positive number if it increases and a negative number if it decreases. Then we can find the final temperature by adding the initial temperature and the change. If it is 3° and the temperature decreases by 7°, then we can add to find the final temperature.

$$3 + (-7) = -4$$

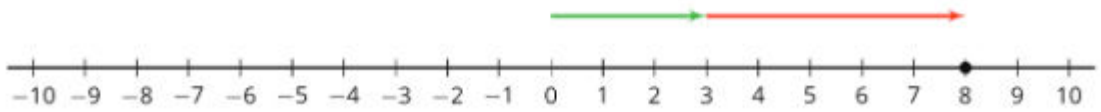
We can represent signed numbers with arrows on a number line. We can represent positive numbers with arrows that start at 0 and points to the right. For example, this arrow represents +10 because it is 10 units long and it points to the right.



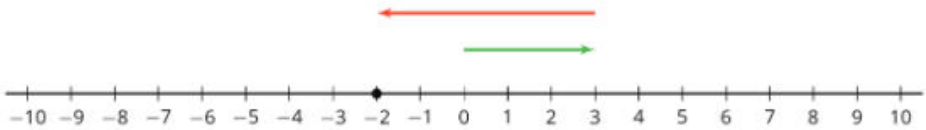
We can represent negative numbers with arrows that start at 0 and point to the left. For example, this arrow represents -4 because it is 4 units long and it points to the left.



To represent addition, we put the arrows "tip to tail." So this diagram represents $3 + 5$:



And this represents $3 + (-5)$:



Lesson 3

Lesson 3: Changing Elevation

That's the Opposite

. Draw arrows on a number line to represents these situations:

a. The temperature was -5 degrees. Then the temperature rose 5 degrees.

b. A climber was 30 feet above sea level. Then she descended 30 feet.

. What's the opposite?

a. Running 150 feet east.

b. Jumping down 10 steps.

c. Pouring 8 gallons into a fish tank.

Cliffs and Caves

A mountaineer is climbing on a cliff. She is 400 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.

a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	
B	+400	150 down	
C	+400	400 down	
D	+400		+50

b. Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.

A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.

a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	
B	-200	100 up	
C	-200	200 up	
D	-200	250 up	
E	-200		-500

b. Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

c. What does the expression $-75 + 100$ tell us about the spelunker? What does the value of the expression tell us?

Adding Rational Numbers

Find the sums.

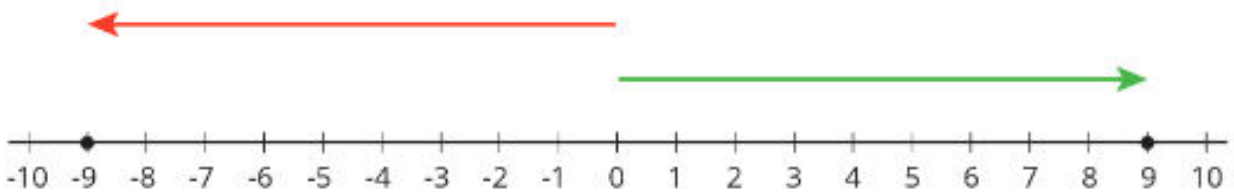
1. $-35 + (30 + 5)$

2. $-0.15 + (-0.85) + 12.5$

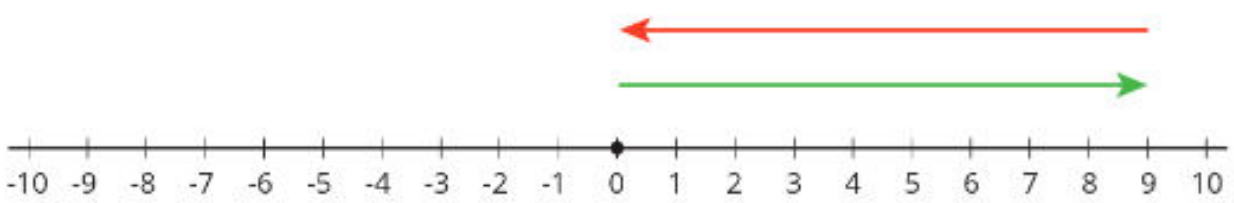
3. $\frac{1}{2} + (-\frac{3}{4})$

Lesson 3 Summary

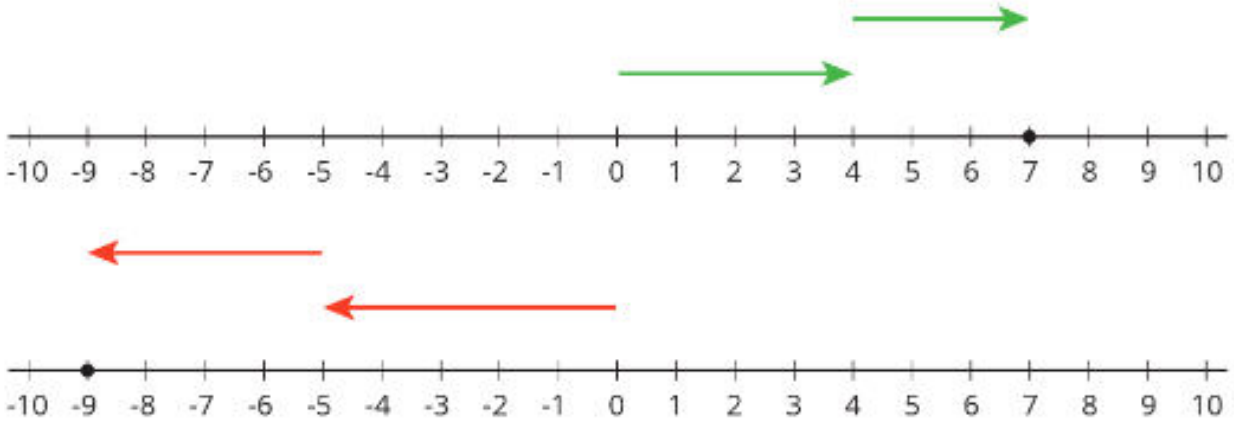
The opposite of a number is the same distance from 0 but on the other side of 0.



The opposite of -9 is 9. When we add opposites, we always get 0. This diagram shows that $9 + -9 = 0$.

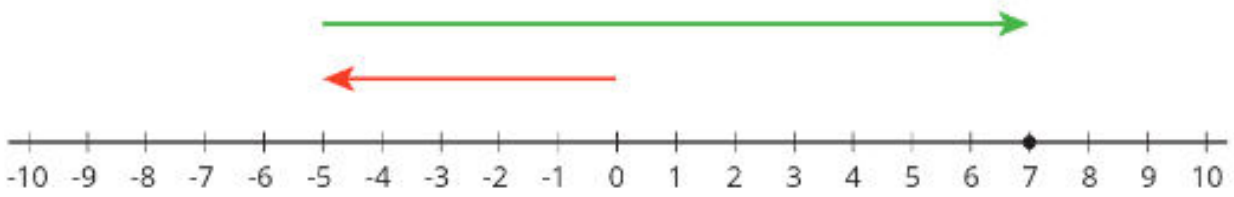


When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign.



To find the sum, we add the magnitudes and give it the correct sign. For example, $(-5) + (-4) = -(5 + 4)$.

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and give it the sign of the number with the larger magnitude. For example, $(-5) + 12 = +(12 - 5)$.



Lesson 4

Lesson 4: Money and Debts

Concert Tickets

Priya wants to buy three tickets for a concert. She has earned $135 and each ticket costs $50. She borrows the rest of the money she needs from a bank and buys the tickets.

1. How can you represent the amount of money that Priya has after buying the tickets?

2. How much more money will Priya need to earn to pay back the money she borrowed from the bank?

3. How much money will she have after she pays back the money she borrowed from the bank?

Cafeteria Food Debt

At the beginning of the month Kiran had $24 in his school cafeteria account. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then, represent each transaction on a number line. What is the unknown quantity in each case?

1. In the first week he spent $16 on lunches. How much was in his account then?

2. Then he deposited some more money and his account balance was $28. How much did he deposit?

3. Then he spent $34 on lunches the next week. How much was in his account then?

4. Then he deposited enough money to pay off his debt to the cafeteria. How much did he deposit?

5. Explain why it makes sense to use a negative number to represent Kiran's account balance when he owes money.

Bank Statement

Here is a bank statement.

Responsible Bank
210 2nd Street
Anytown, MH 06930

Checking Account Statement
Page: 1 of 1

Andre Person
1729 Euclid Ave
Anytown, MH 06930

Statement Period	Account No.
2017-10-01 to 2017-11-01	1120635978

Date	Description	Withdrawals	Deposits	Balance
2017-10-03	Previous Balance			39.87
2017-10-05	Check Number 256	28.50		11.37
2017-10-06	ATM Deposit - Cash		45.00	56.37
2017-10-10	Wire Transfer	37.91		18.46
2017-10-17	Point of Sale - Grocery Store	16.43		2.03
2017-10-25	Funds Transfer from Savings		50.00	52.03
2017-10-28	Check Number 257	42.00		10.03
2017-10-29	Online Payment - Phone Services	72.50		-62.47

1. If we put withdrawals and deposits in the same column, how can they be represented?

2. Andre withdraws $40 to buy a music player. What is his new balance?

3. If Andre deposits $100 in this account, will he still be in debt? How do you know?

Lesson 4 Summary

Banks use positive numbers to represent money that gets put into an account and negative numbers to represent money that gets taken out of an account. When you put money into an account, it is called a **deposit**. When you take money out of an account, it is called a **withdrawal**.

People also use negative numbers to represent debt. If you take out more money from your account than you put in, then you owe the bank money, and your account balance will be a negative number to represent that debt. For example, if you have $200 in your bank account, and then you write a check for $300, you will owe the bank $100 and your account balance will be -$100.

starting balance	deposits and withdrawals	new balance
0	50	$0 + 50$
50	150	$50 + 150$
200	-300	$200 + (-300)$
-100		

In general, you can find a new account balance by adding the value of the deposit or withdrawal to it. You can also tell quickly how much money is needed to repay a debt using the fact that to get to zero from a negative value you need to add its opposite.

Lesson 5

Lesson 5: Representing Subtraction

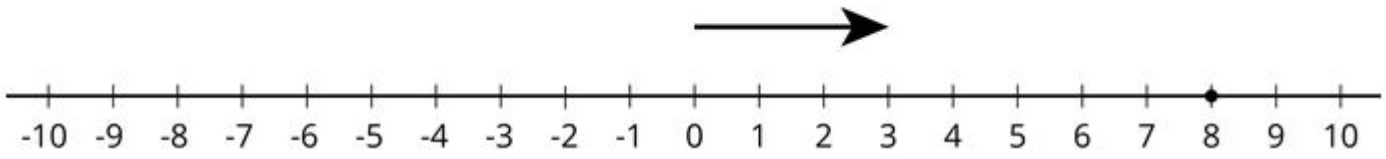
Equivalent Equations

For the equations in the second and third columns, write two more equations using the same numbers that express the same relationship in a different way. If you get stuck, consider looking at the examples in the first column.

$2 + 3 = 5$	$9 + (-1) = 8$	$-11 + x = 7$
$3 + 2 = 5$		
$5 - 3 = 2$		
$5 - 2 = 3$		

Subtraction with Number Lines

Here is an unfinished number line diagram that represents a sum of 8.

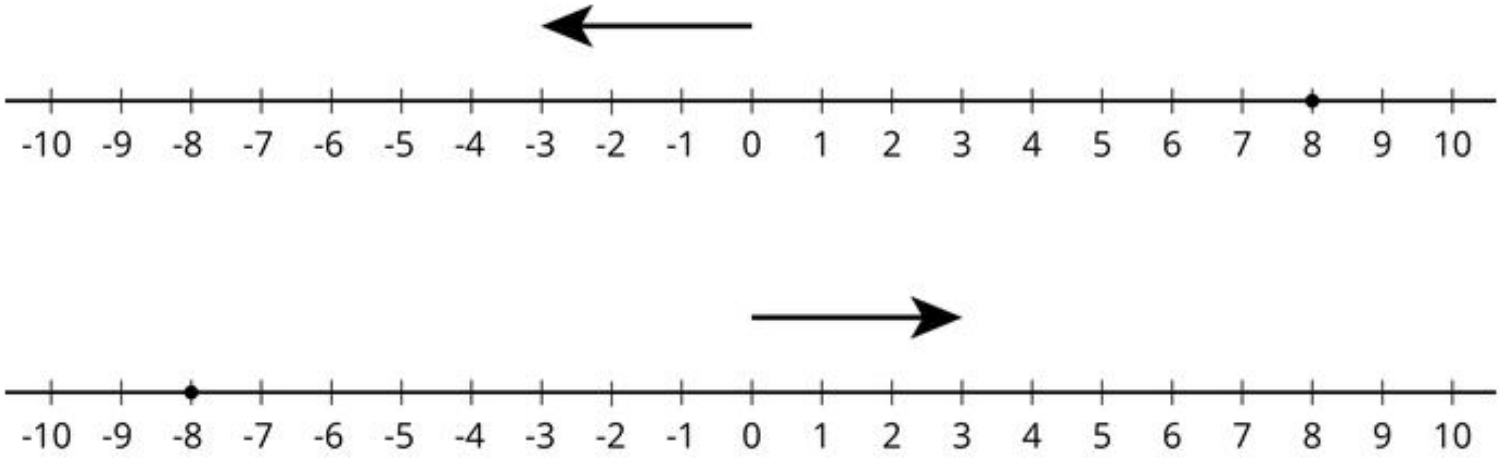


a. How long should the other arrow be?

b. For an equation that goes with this diagram, Mai writes $3 + ? = 8$. Tyler writes $8 - 3 = ?$. Do you agree with either of them?

c. What is the unknown number? How do you know?

Here are two more unfinished diagrams that represent sums.



For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

d. What number would complete each equation? Be prepared to explain your reasoning.

Draw a number line diagram for $(-8) - (-3) = ?$ What is the unknown number? How do you know?

We Can Add Instead

Match each diagram to one of these expressions:

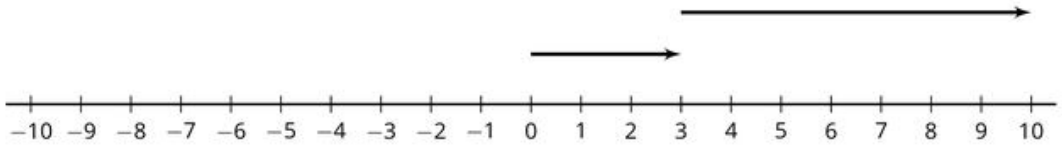
$3 + 7$

$3 - 7$

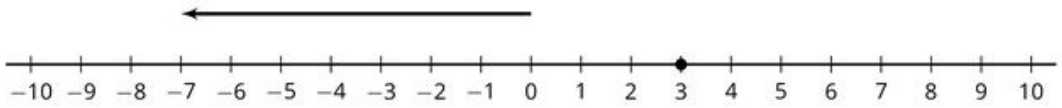
$3 + (-7)$

$3 - (-7)$

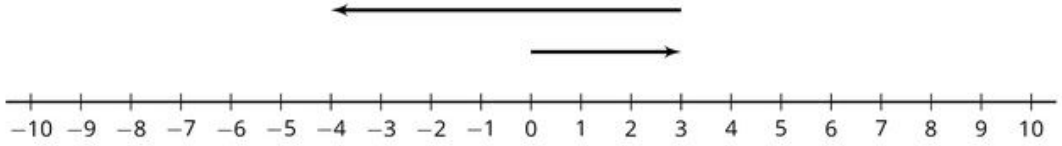
A.



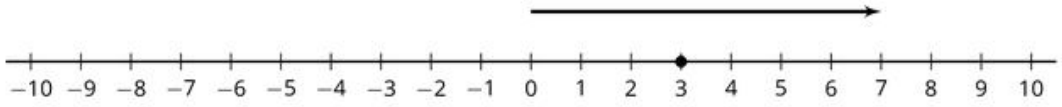
B.



C.



D.



Which expressions in the first question have the same value? What do you notice?

Complete each of these tables. What do you notice?

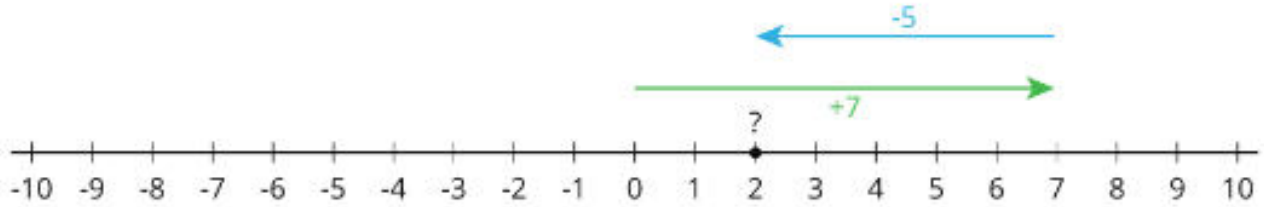
expression	value
$8 + (-8)$	
$8 - 8$	
$8 + (-5)$	
$8 - 5$	
$8 + (-12)$	
$8 - 12$	

expression	value
$-5 + 5$	
$-5 - (-5)$	
$-5 + 9$	
$-5 - (-9)$	
$-5 + 2$	
$-5 - (-2)$	

Lesson 5 Summary

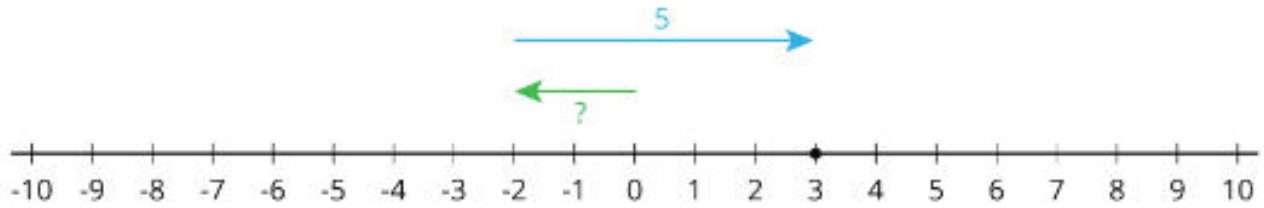
The equation $7 - 5 = ?$ is equivalent to $? + 5 = 7$. The diagram illustrates the second equation.

Notice that the value of $7 + (-5)$ is 2.



We can solve the equation $? + 5 = 7$ by adding -5 to both sides. This shows that $7 - 5 = 7 + (-5)$

Likewise, $3 - 5 = ?$ is equivalent to $? + 5 = 3$.



Notice that the value of $3 + (-5)$ is -2.

We can solve the equation $? + 5 = 3$ by adding -5 to both sides. This shows that $3 - 5 = 3 + (-5)$

In general:

$$a - b = a + (-b)$$

If $a - b = x$, then $x + b = a$. We can add $-b$ to both sides of this second equation to get that $x = a + (-b)$

Lesson 6

Lesson 6: Subtracting Rational Numbers

Expressions with Altitude

A mountaineer is changing elevations. Write an expression that represents the difference between the final elevation and beginning elevation. Then write the value of the change. The first one is done for you.

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50		
+400	-120		
-200	+610		
-200	-50		
-200	-500		
-200	0		

Does the Order Matter?

Find the value of each subtraction expression.

A	B
$3 - 2$	$2 - 3$
$5 - (-9)$	$(-9) - 5$
$(-11) - 2$	$2 - (-11)$
$(-6) - (-3)$	$(-3) - (-6)$
$(-1.2) - (-3.6)$	$(-3.6) - (-1.2)$
$(-2\frac{1}{2}) - (-3\frac{1}{2})$	$(-3\frac{1}{2}) - (-2\frac{1}{2})$

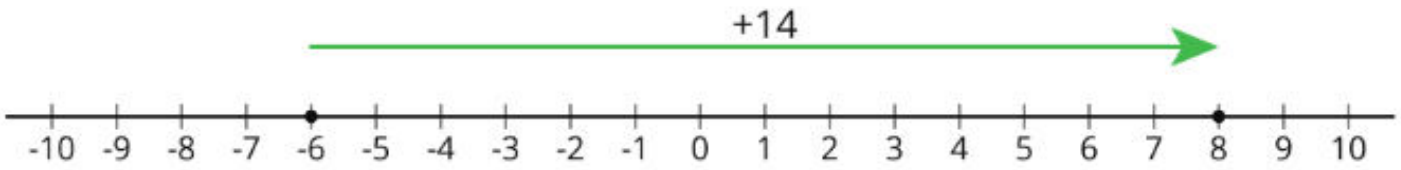
What do you notice about the expressions in Column A compared to Column B?

What do you notice about their values?

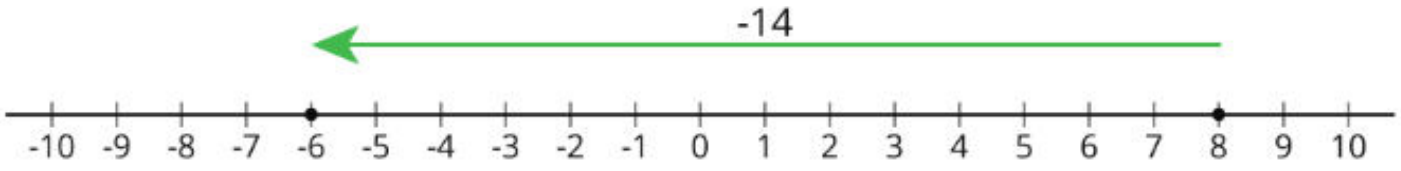
Lesson 6 Summary

When we talk about the difference of two numbers, we mean, "subtract them." Usually, we subtract them in the order they are named. For example, the difference of +8 and -6 is $8 - (-6)$.

The difference of two numbers tells you how far apart they are on the number line. 8 and -6 are 14 units apart, because $8 - (-6) = 14$:



Notice that if you subtract them in the opposite order, you get the opposite number:



$(-6) - 8 = -14$

In general, the distance between two numbers a and b on the number line is $|a - b|$. Note that the *distance* between two numbers is always positive, no matter the order. But the *difference* can be positive or negative, depending on the order.

Lesson 7

Lesson 7: Adding and Subtracting to Solve Problems

Phone Inventory

A store tracks the number of cell phones it has in stock and how many phones it sells. The table shows the inventory for one phone model at the beginning of each day last week. The inventory changes when they sell phones or get shipments of phones into the store.

	inventory	change
Monday	18	-2
Tuesday	16	-5
Wednesday	11	-7
Thursday	4	-6
Friday	-2	20

1. What do you think it means when the change is positive? Negative?

2. What do you think it means when the inventory is positive? Negative?

3. Based on the information in the table, what do you think the inventory will be at on Saturday morning? Explain your reasoning.

4. What is the difference between the greatest inventory and the least inventory?

Solar Power

Han's family got a solar panel. Each month they get a credit to their account for the electricity that is generated by the solar panel. The credit they receive varies based on how sunny it is.

In January they used $83.56 worth of electricity and generated $6.75 worth of electricity. Here is their electricity bill from January.

Current charges: $83.56

Solar Credit: -$6.75

Amount due: $74.81

In July they were traveling away from home and only used $19.24 worth of electricity. Their solar panel generated $22.75 worth of electricity. What was their amount due in July?

The table shows the value of the electricity they used and the value of the electricity they generated each week for a month. What amount is due for this month?

	used ($)	generated ($)
week 1	13.45	-6.33
week 2	21.78	-8.94
week 3	18.12	-7.70
week 4	24.05	-5.36

What is the difference between the value of the electricity generated in week 1 and week 2? Between week 2 and week 3?

Lesson 7 Summary

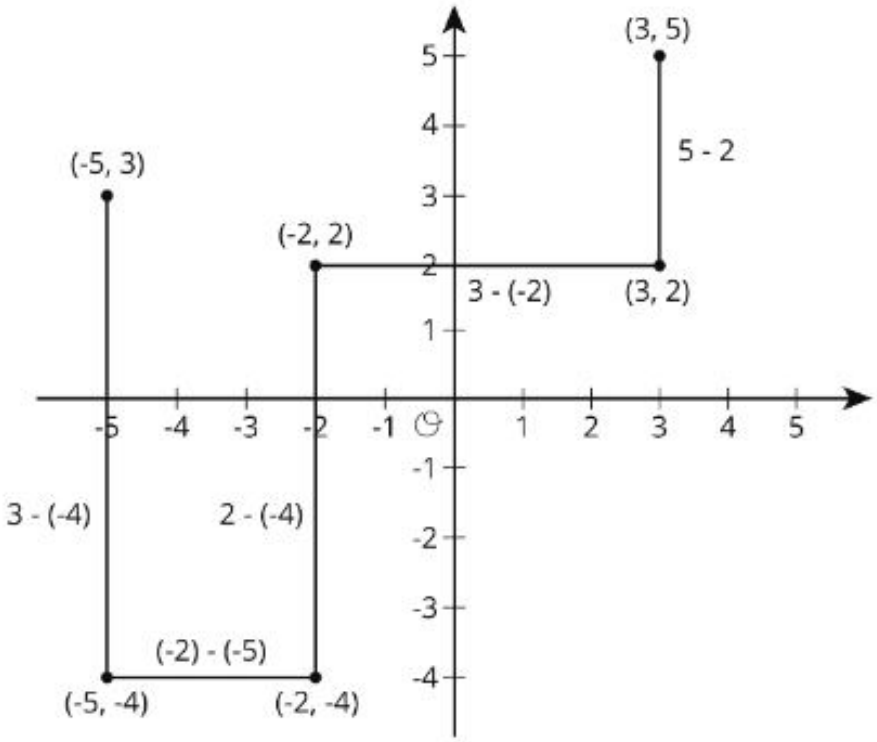
Sometimes we use positive and negative numbers to represent quantities in context. Here are some contexts we have studied that can be represented with positive and negative numbers:

- temperature
- elevation
- inventory
- an account balance
- electricity flowing in and flowing out

In these situations, using positive and negative numbers, and operations on positive and negative numbers, helps us understand and analyze them. To solve problems in these situations, we just have to understand what it means when the quantity is positive, when it is negative, and what it means to add and subtract them.

When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.

When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their y-coordinates.



Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

Lesson 8

Lesson 8: Position, Speed, and Direction

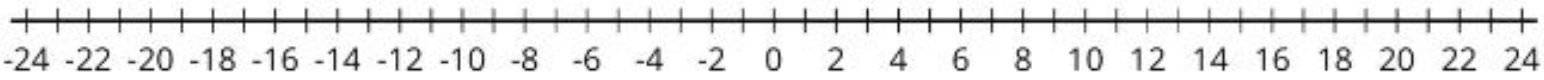
Distance, Rate, Time

1. An airplane moves at a constant speed of 120 miles per hour for 3 hours. How far does it go?

1. A train moves at constant speed and travels 6 miles in 4 minutes. What is its speed in miles per minute?

1. A car moves at a constant speed of 50 miles per hour. How long does it take the car to go 200 miles?

Going Left, Going Right



After each move, record your location in the table. Then write an expression to represent the ending position that uses the starting position, the speed, and the time. The first row is done for you.

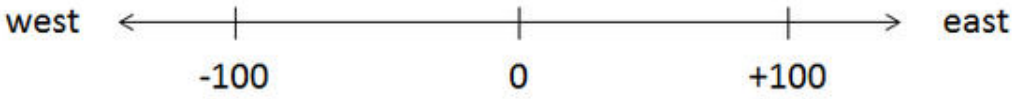
starting position	direction	speed (units per second)	time (seconds)	ending position (units)	expression
0	right	5	3	+15	$0+5\cdot 3$
0	left	4	6		
0	right	2	8		
0	right	6	2		
0	left	1.1	5		

How can you see the *direction* of movement in the expression?

Using a starting position p, a speed s, and a time t, write two expressions for an ending position. One expression should show the result of moving right, and one expression should show the result of moving left.

Velocity

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.



Vehicles that are traveling towards the east have a positive velocity, and vehicles that are traveling towards the west have a negative velocity.

Complete the table with the position of each vehicle if the vehicle is traveling at a constant speed for the indicated time period. Then write an equation.

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30		
+32	+40		
-35	+20		
+28	0		

If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply two positive numbers, is the result positive or negative?

If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply a positive and a negative number, is the result positive or negative?

Lesson 8 Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point, and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.

When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the right of zero. In other words, $5 \cdot 10 = 50$.

If you start at zero and move -5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the *left* of zero. In other words,

$$-5 \cdot 10 = -50$$

In general, a negative number times a positive number is a negative number.

Lesson 9

Lesson 9: Multiplying Rational Numbers

Before and After

Where was the girl

1. 5 seconds *after* this picture was taken? Mark her approximate location on the picture.

2. 5 seconds *before* this picture was taken? Mark her approximate location on the picture.

Backwards in Time

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.

Here are some positions and times for one car:

position (feet)	-180	-120	-60	0	60	120
time (seconds)	-3	-2	-1	0	1	2

a. In what direction is this car traveling?

b. What is its velocity?

2. a. What does it mean when the time is zero?

b. What could it mean to have a negative time?

Here are the positions and times for a different car whose velocity is -50 feet per second:

position (feet)				0	-50	-100
time (seconds)	-3	-2	-1	0	1	2

a. Complete the table with the rest of the positions.

b. In what direction is this car traveling? Explain how you know.

4. Complete the table for several different cars passing the camera.

	velocity (meters per second)	time after passing camera (seconds)	ending position (meters)	equation
car C	+25	+10	+250	$25 \cdot 10 = 250$
car D	-20	+30		
car E	+32	-40		
car F	-35	-20		
car G	-15	-8		

5. a. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

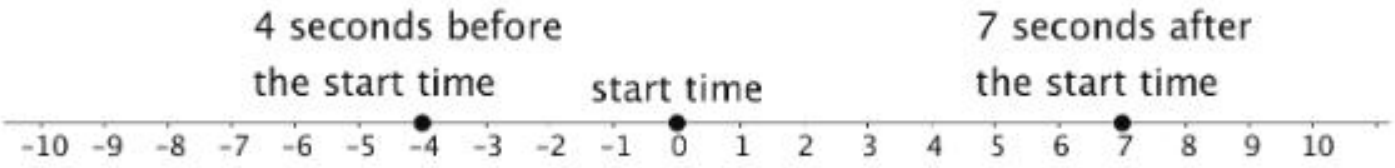
b. If we multiply a negative number and a positive number, is the result positive or negative?

6. a. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

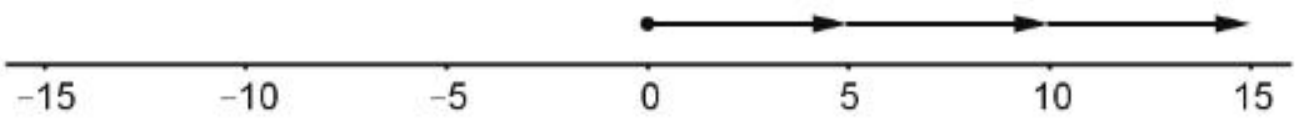
b. If we multiply two negative numbers, is the result positive or negative?

Lesson 9 Summary

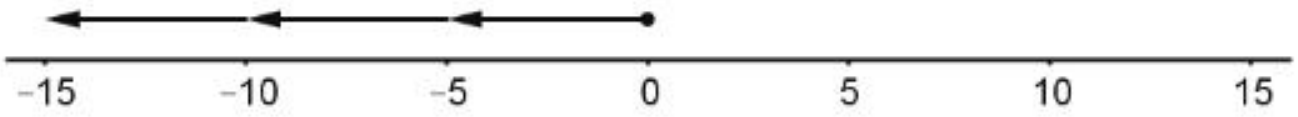
We can use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.



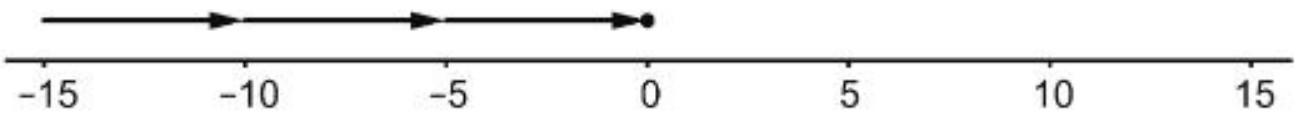
If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position. A positive times a positive is positive.



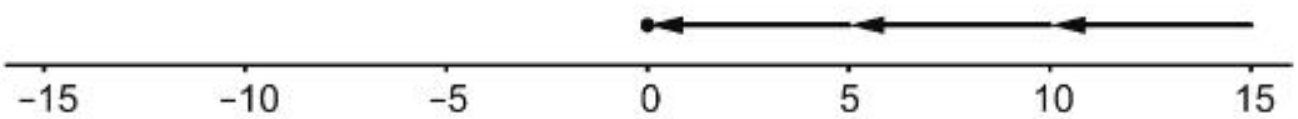
If a car is at position 0 and is moving in a negative direction, then for times after that (positive times), it will have a negative position. A negative times a positive is negative.



If a car is at position 0 and is moving in a positive direction, then for times *before* that (negative times), it must have had a negative position. A positive times a negative is negative.



If a car is at position 0 and is moving in a negative direction, then for times *before* that (negative times), it must have had a positive position. A negative times a negative is positive.



Lesson 9 Summary

Here is another way of seeing this:
We can think of $3 \cdot 5$ as $5 + 5 + 5$, which has a value of 15.

We can think of $3 \cdot (-5)$ as $(-5) + (-5) + (-5)$, which has a value of -15.

We can multiply positive numbers in any order:

$$3 \cdot 5 = 5 \cdot 3$$

If we can multiply signed numbers in any order, then $-5 \cdot 3 = -15$.

We can find $-5 \cdot (3 + (-3))$ two ways:

- $-5 \cdot 0 = 0$
- $-5 \cdot 3 + (-5) \cdot (-3)$ (this is the distributive property)

That means that

$$-5 \cdot 3 + (-5) \cdot (-3) = 0$$

Which is the same as

$$-15 + (-5) \cdot (-3) = 0$$

So

$$-5 \cdot (-3) = 15$$

There was nothing special about these particular numbers. This always works!

- A positive times a positive is always positive
- A negative times a positive or a positive times a negative is always negative
- A negative times a negative is always positive

Lesson 11

Lesson 11: Dividing Rational Numbers

Tell Me Your Sign

Consider the equation: $-27x = -35$

Without computing:

1. Is the solution to this equation positive or negative?

2. Are either of these two numbers solutions to the equation?

$\frac{35}{27}$ $-\frac{35}{27}$

Multiplication and Division

1. Find the missing values in the equations

a. $-3 \cdot 4 = ?$

b. $-3 \cdot ? = 12$

c. $3 \cdot ? = 12$

d. $? \cdot -4 = 12$

e. $? \cdot 4 = -12$

2. Rewrite the unknown factor problems as division problems.

3. Complete the sentences. Be prepared to explain your reasoning.

a. The sign of a positive number divided by a positive number is always:

b. The sign of a positive number divided by a negative number is always:

c. The sign of a negative number divided by a positive number is always:

d. The sign of a negative number divided by a negative number is always:

4. Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.
    - Han's velocity is 4 feet per second.
    - Clare's velocity is -5 feet per second.

a. Where is each person 10 seconds before they meet up?

b. When is each person at the position -10 feet from the meeting place?

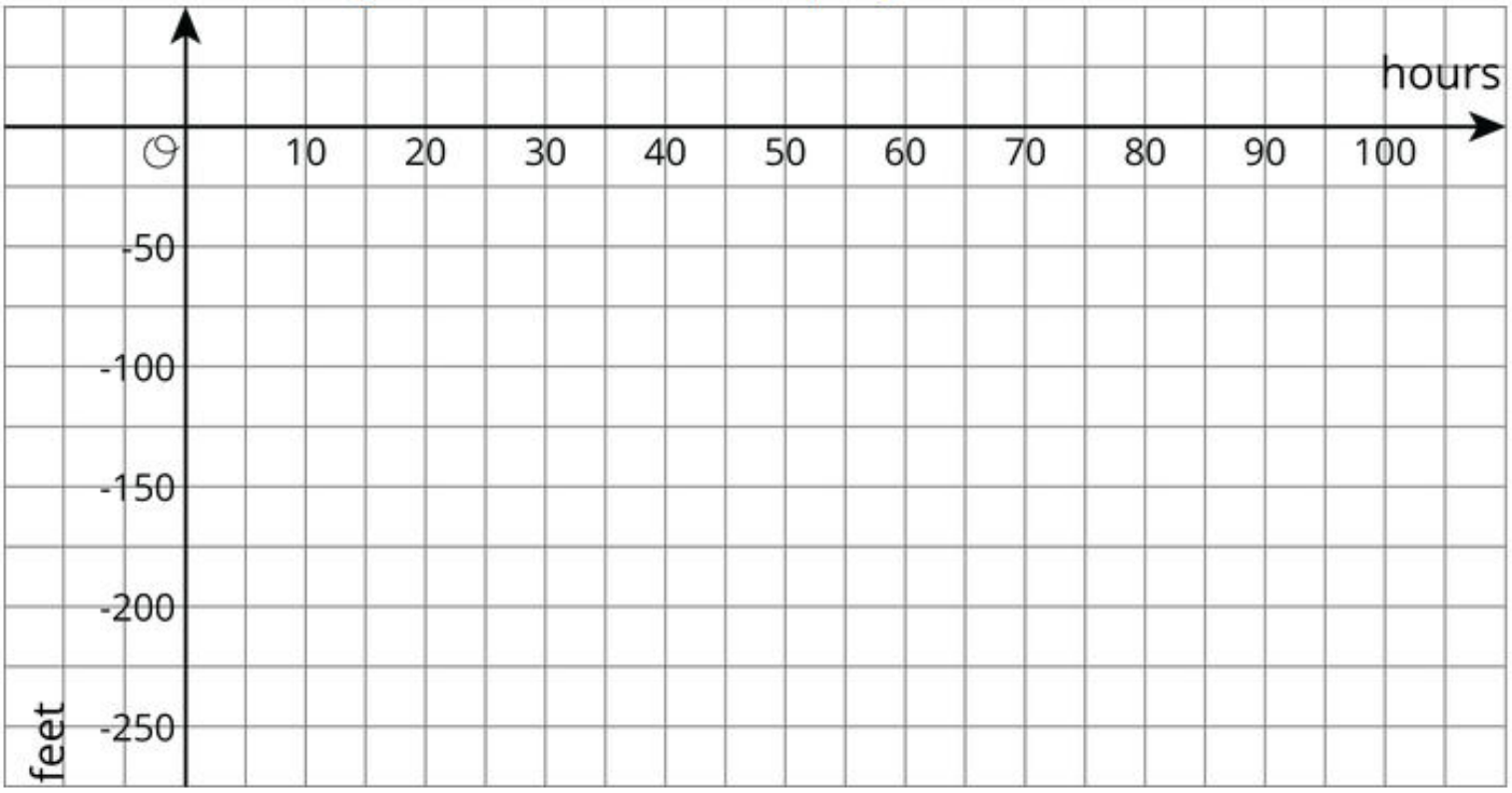
Drilling Down

A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.

1. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

2. If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

3. Use the coordinate grid to show the drill's progress.



4. At this rate, how many hours will it take until the drill reaches -250 feet?

Lesson 11 Summary

Any division problem is actually a multiplication problem:

- $6 \div 2 = 3$ because $2 \cdot 3 = 6$
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$

Because we know how to multiply signed numbers, that means we know how to divide them.

- The sign of a positive number divided by a negative number is always negative.
- The sign of a negative number divided by a positive number is always negative.
- The sign of a negative number divided by a negative number is always positive.

Lesson 12

Lesson 12: Negative Rates

Grapes per Minute

1. If you eat 5 grapes per minute for 8 minutes, how many grapes will you eat?

1. If you hear 9 new songs per day for 3 days, how many new songs will you hear?

1. If you run 15 lapes per practice, how many practices will it take you to run 30 laps?

Water Level in the Aquarium

1. A large aquarium should contain 10,000 liters of water when it is filled correctly. It will overflow if it gets up to 12,000 liters. The fish will get sick if it gets down to 4,000 liters. The aquarium has an automatic system to help keep the correct water level. If the water level is too low, a faucet fills it. If the water level is too high, a drain opens.

   One day, the system stops working correctly. The faucet starts to fill the aquarium at a rate of 30 liters per minute, and the drain opens at the same time, draining the water at a rate of 20 liters per minute.

   a. Is the water level rising or falling? How do you know?

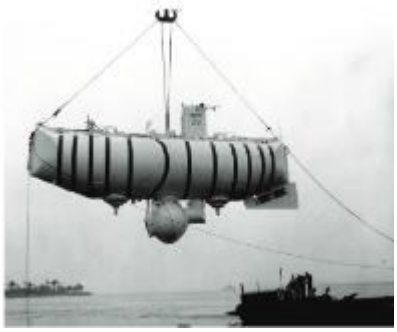
   b. How long will it take until the tank starts overflowing or the fish get sick?

2. A different aquarium should contain 15,000 liters of water when filled correctly. It will overflow if it gets to 17,600 liters.

   One day there is an accident and the tank cracks in 4 places. Water flows out of each crack at a rate of $\frac{1}{2}$ liter per hour. An emergency pump can re-fill the tank at a rate of 2 liters per minute. How many minutes must the pump run to replace the water lost each hour?

Up and Down with the Piccards

Challenger Deep is the deepest known point in the ocean, at 35,814 feet below sea level. In 1960, Jacques Piccard and Don Walsh rode down in the Trieste and became the first people to visit the Challenger Deep.



If sea level is represented by 0 feet, explain how you can represent the depth of a submarine descending from sea level to the bottom of Challenger Deep.

Trieste's descent was a change in depth of -3 feet per second. We can use the relationship $y = -3x$ to model this, where y is the depth (in feet) and x is the time (in seconds). Using this model, how much time would the Trieste take to reach the bottom?

It took the Trieste 3 hours to ascend back to sea level. This can be modeled by a different relationship $y = kx$. What is the value of k in this situation?

2. The design of the Trieste was based on the design of a hot air balloon built by Auguste Piccard, Jacques's father. In 1932, Auguste rode in his hot-air balloon up to a record-breaking height.

a. Auguste's ascent took 7 hours and went up 51,683 feet. Write a relationship $y = kx$ to represent his ascent from his starting location.

b. Auguste's descent took 3 hours and went down 52,940 feet. Write another relationship to represent his descent.

c. Did Auguste Piccard end up at a greater or lesser altitude than his starting point? How much higher or lower?

Lesson 12 Summary

We saw earlier that we can represent speed with direction using signed numbers. Speed with direction is called *velocity*. Positive velocities always represent movement in the opposite direction from negative velocities.

We can do this with vertical movement: moving up can be represented with positive numbers, and moving down with negative numbers. The magnitude tells you how fast, and the sign tells you which direction. (We could actually do it the other way around if we wanted to, but usually we make up positive and down negative.)

Lesson 13

Lesson 13: Expressions with Rational Numbers

Card Sort: The Same but Different

You will need to group the set of cards into pairs of expressions that have the same value.

$1-2$

$1+2$

$-10+7$

$-10-(-7)$

$1 \cdot 4$

$-10+(-7)$

$1+(-2)$

$1-(-2)$

$-15 \div (-6)$

$-1 \cdot 4$

$-10-7$

$8 \cdot (-\frac{1}{4})$

$8 \div 4$

$15 \div (-6)$

$1 \div \frac{1}{4}$

$8 \div (-4)$

$15 \cdot \frac{1}{6}$

$1 \div (-\frac{1}{4})$

$8 \cdot \frac{1}{4}$

$-15 \cdot \frac{1}{6}$

Near and Far From Zero

a	b	$-a$	$-4b$	$-a+b$	$a \div -b$	a^2	b^3
$-\frac{1}{2}$	6						
$\frac{1}{2}$	-6						
-6	$-\frac{1}{2}$						

1. For each set of values for a and b, evaluate the given expressions and record your answers in the table.

2. When $a = -\frac{1}{2}$ and $b = 6$, which expression:

has the largest value?
has the smallest value?
is the closest to zero?

3. When $a = \frac{1}{2}$ and $b = -6$, which expression:

has the largest value?
has the smallest value?
is the closest to zero?

4. When $a = -6$ and $b = -\frac{1}{2}$, which expression:

has the largest value?
has the smallest value?
is the closest to zero?

Lesson 13 Summary

We can represent sums, differences, products, and quotients of rational numbers, and combinations of these, with numerical and algebraic expressions.

Sums:	Differences:	Products:	Quotients:
$\frac{1}{2} + (-9)$	$\frac{1}{2} - (-9)$	$(\frac{1}{2})(-9)$	$(\frac{1}{2}) \div (-9)$
$-8.5 + x$	$-8.5 - x$	$-8.5x$	$\frac{-8.5}{x}$

We can write the product of two numbers in different ways.

- By putting a little dot between the factors, like this: $-8.5 \cdot x$.
- By putting the factors next to each other without any symbol between them at all, like this: $-8.5x$.

We can write the quotient of two numbers in different ways as well.

- By writing the division symbol between the numbers, like this: $-8.5 \div x$.
- By writing a fraction bar between the numbers like this: $\frac{-8.5}{x}$.

When we have an algebraic expression like $\frac{-8.5}{x}$ and are given a value for the variable, we can find the value of the expression. For example, if x is 2, then the value of the expression is -4.25, because $-8.5 \div 2 = -4.25$.

Lesson 14

Lesson 14: Solving Problems with Rational Numbers

Which One Doesn't Belong: Equations

Which equation doesn't belong?

$\frac{1}{2}x = -50$

$x + 90 = -100$

$-60t = 30$

$-0.01 = -0.001x$

Draining and Filling a Tank

A tank of water is being drained. Due to a problem, the sensor does not start working until some time into the draining process. The sensor starts its recording at time zero when there are 770 liters in the tank.

1. Given that the drain empties the tank at a constant rate of 14 liters per minute, complete the table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
0	0	$770 + (0)(-14)$	770
1	-14	$770 + (1)(-14)$	756
5	-70		
10			

2. Later, someone wants to use the data to find out how long the tank had been draining before the sensor started. Complete this table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
1	-14	$770 + (1)(-14)$	756
0	0	$770 + (0)(-14)$	770
-1	14	$770 + (-1)(-14)$	784
-2	28		
-3			
-4			
-5			

3. If the sensor started working 15 minutes into the tank draining, how much was in the tank to begin with?

Lesson 14 Summary

We can apply the rules for arithmetic with rational numbers to solve problems

In general:

$$a - b = a + (-b)$$

If $a - b = x$, then $x + b = a$. We can add $-b$ to both sides of this second equation to get that $x = a + (-b)$

Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

And when multiplying or dividing:

- The sign of a positive number multiplied or divided by a negative number is always negative.
- The sign of a negative number multiplied or divided by a positive number is always negative.
- The sign of a negative number multiplied or divided by a negative number is always positive.

Lesson 15

Lesson 15: Solving Equations with Rational Numbers

Match Solutions

Match each equation to its solution. Be prepared to explain your reasoning.

A. $\frac{1}{2}x = -5$

B. $-2x = -9$

C. $-\frac{1}{2}x = \frac{1}{4}$

D. $-2x = 7$

E. $x + (-2) = -6.5$

F. $-2 + x = \frac{1}{2}$

1. $x = -4.5$

2. $x = -\frac{1}{2}$

3. $x = -10$

4. $x = 4.5$

5. $x = 2\frac{1}{2}$

6. $x = -3.5$

Trips to the Mountains

The Hiking Club is on a trip to hike up a mountain.

1. The members increased their elevation 290 feet during their hike this morning. Now they are at an elevation of 450 feet.

   a. Explain how to find their elevation before the hike.

   b. Han says the equation $e + 290 = 450$ describes the situation. What does the variable e represent?

   c. Han says that he can rewrite his equation as $e = 450 + (-290)$ to solve for e. Compare Han's strategy to your strategy for finding the beginning elevation.

2. The temperature fell 4 degrees in the last hour. Now it is 21 degrees. Write and solve an equation to find the temperature it was 1 hour ago.

3. There are 3 times as many students participating in the hiking trip this year than last year. There are 42 students on the trip this year.

a. Explain how to find the number of students that came on the hiking trip last year.

b. Mai says the equation $3s = 42$ describes the situation. What does the variable s represent?

c. Mai says that she can rewrite her equation as $s = \frac{1}{3} \cdot 42$ to solve for s. Compare Mai's strategy to your strategy for finding the number of students on last year's trip.

4. The cost of the hiking trip this year is $\frac{2}{3}$ of the cost of last year's trip. This year's trip cost $32. Write and solve an equation to find the cost of last year's trip.

Card Sort: Matching Inverses

You will be given a set of cards with numbers on them.

1. Match numbers with their additive inverses.
2. Next, match numbers with their multiplicative inverses.
3. What do you notice about the numbers and their inverses?

Card Sort: Matching Inverses

$\frac{5}{10}$

Card Sort: Matching Inverses

$\frac{2}{1}$

Card Sort: Matching Inverses

4

Card Sort: Matching Inverses

0.2

Card Sort: Matching Inverses

0.25

Card Sort: Matching Inverses

10

Card Sort: Matching Inverses

$-\frac{10}{1}$

Card Sort: Matching Inverses

$-\frac{4}{100}$

Card Sort: Matching Inverses

0.1

Card Sort: Matching Inverses

-2

Card Sort: Matching Inverses

-0.5

Card Sort: Matching Inverses

-25

Card Sort: Matching Inverses

$-\frac{1}{4}$

Card Sort: Matching Inverses

-4

Card Sort: Matching Inverses

$\frac{10}{2}$

Card Sort: Matching Inverses

3

Card Sort: Matching Inverses

$\frac{1}{3}$

Card Sort: Matching Inverses

$-\frac{1}{10}$

Card Sort: Matching Inverses

$\frac{4}{100}$

Card Sort: Matching Inverses

25

Card Sort: Matching Inverses

-0.2

Card Sort: Matching Inverses

-3

Card Sort: Matching Inverses

$-\frac{1}{3}$

Card Sort: Matching Inverses

-5

Lesson 15 Summary

To solve the equation $x + 8 = -5$, we can add the opposite of 8, or -8, to each side:

$$\begin{aligned} x + 8 &= -5 \\ (x + 8) + -8 &= (-5) + -8 \\ x &= -13 \end{aligned}$$

Because adding the opposite of a number is the same as subtracting that number, we can also think of it as subtracting 8 from each side.

We can use the same approach for this equation:

$$\begin{aligned} -12 &= t + -\frac{2}{9} \\ (-12) + \frac{2}{9} &= \left(t + -\frac{2}{9}\right) + \frac{2}{9} \\ -11\frac{7}{9} &= t \end{aligned}$$

To solve the equation $8x = -5$, we can multiply each side by the reciprocal of 8, or $\frac{1}{8}$:

$$\begin{aligned} 8x &= -5 \\ \frac{1}{8}(8x) &= \frac{1}{8}(-5) \\ x &= -\frac{5}{8} \end{aligned}$$

Because multiplying by the reciprocal of a number is the same as dividing by that number, we can also think of it as dividing by 8. We can use the same approach for this equation:

$$\begin{aligned} -12 &= -\frac{2}{9}t \\ -\frac{9}{2}(-12) &= -\frac{9}{2}\left(-\frac{2}{9}t\right) \\ 54 &= t \end{aligned}$$

Lesson 16

Lesson 16: Representing Contexts with Equations

Warmer or Colder than Before?

For each situation,

- Find *two* equations that could represent the situation from the bank of equations. (Some equations will not be used.)
- Explain what the variable v represents in the situation.
- Determine the value of the variable that makes the equation true, and explain your reasoning.

Bank of equations:

$-3v = 9$	$v = -16 + 6$	$v = \frac{1}{3} \cdot (-6)$	$v + 12 = 4$
$-4 \cdot 3 = v$	$v = 4 + (-12)$	$v = -16 - (6)$	$v = 9 + 3$
$-4 \cdot -3 = v$	$-3v = -6$	$-6 + v = -16$	$-4 = \frac{1}{3}v$
$v = -\frac{1}{3} \cdot 9$	$v = -\frac{1}{3} \cdot (-6)$	$v = 4 + 12$	$4 = 3v$

1. Between 6 a.m. and noon, the temperature rose 12 degrees Fahrenheit to 4 degrees Fahrenheit.

2. At midnight the temperature was -6 degrees. By 4 a.m. the temperature had fallen to -16 degrees.

3. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is -6 degrees at a certain time.

4. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is 9 degrees at a certain time.

5. The temperature at 9 p.m. is one third the temperature at midnight.

Equations Tell a Story

Your teacher will assign your group *one* of these situations. Create a visual display about your situation that includes:

- An equation that represents your situation
- What your variable and each term in the equation represent
- How the operations in the equation represent the relationships in the story
- How you use inverses to solve for the unknown quantity
- The solution to your equation

1. As a $7\frac{1}{4}$ inch candle burns down, its height decreases $\frac{3}{4}$ inch each hour. How many hours does it take for the candle to burn completely?

2. On Monday $\frac{1}{9}$ of the enrolled students in a school were absent. There were 4,512 students present. How many students are enrolled at the school?

3. A hiker begins at sea level and descends 25 feet every minute. How long will it take to get to an elevation of -750 feet?

4. Jada practices the violin for the same amount of time every day. On Tuesday she practices for 35 minutes. How much does Jada practice in a week?

5. The temperature has been dropping $2\frac{1}{2}$ degrees every hour and the current temperature is $-15°\mathrm{F}$. How many hours ago was the temperature $0°\mathrm{F}$?

6. The population of a school increased by 12%, and now the population is 476. What was the population before the increase?

7. During a 5% off sale, Diego pays $74.10 for a new hockey stick. What was the original price?

8. A store buys sweaters for $8 and sells them for $26. How many sweaters does the store need to sell to make a profit of $990?

Lesson 16 Summary

We can use variables and equations involving signed numbers to represent a story or answer questions about a situation.

For example, if the temperature is $-3^\circ C$ and then falls to $-17^\circ C$, we can let x represent the temperature change and write the equation:

$$-3 + x = -17$$

We can solve the equation by adding 3 to each side. Since $-17 + 3 = -14$, the change is $-14^\circ C$.

Here is another example: if a starfish is descending by $\frac{3}{2}$ feet every hour then we can solve

$$-\frac{3}{2}h = -6$$

to find out how many hours h it takes the starfish to go down 6 feet.

We can solve this equation by multiplying each side by $-\frac{2}{3}$. Since $-\frac{2}{3} \cdot -6 = 4$, we know it will take the starfish 4 hours to descend 6 feet.