Lesson Plan: 8.NS.A.1: Are You Rational or Irrational?

(This lesson should be adapted, including instructional time, to meet the needs of your students.)

Background Information	
Content/Grade Level	Number Systems/Grade 8
Unit/Cluster	Know that there are numbers that are not rational, and approximate them by rational numbers.
Essential Questions/Enduring Understandings Addressed in the Lesson	**Essential Questions:** How can you determine if a number is rational or irrational? What are the characteristics of a rational number? What are the characteristics of an irrational number? **Enduring Understandings**: Rational numbers include integers, fractions, terminating and repeating decimals. Rational numbers can be represented in multiple ways. Any rational number can be expressed as a fraction in an infinite number of ways. All real numbers, which include rational and irrational numbers, can be plotted on a number line. An irrational number cannot be represented as a simple fraction. Irrational numbers are those real numbers that cannot be represented as terminating or repeating decimals.
Standards Addressed in This Lesson	**8.NS.A.1:** Understand informally that every number has a decimal expansion; the rational numbers are those with decimal expansions that terminate in 0s or eventually repeat. Know that other numbers are called irrational. It is critical that the Standards for Mathematical Practices are incorporated in ALL lesson activities throughout the unit as appropriate. It is not the expectation that all eight Mathematical Practices will be evident in every lesson. The Standards for Mathematical Practices make an excellent framework on which to plan your instruction. Look for the infusion of the Mathematical Practices throughout this unit.
Lesson Topic	Identifying rational and irrational numbers.

Relevance/Connections	**8.NS.A.2:** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). **8.EE.A.2**: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{}$ is irrational. **8.G.B.7:** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.G.B.8:** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Student Outcomes	Students will be able to identify and describe rational and irrational numbers.
Prior Knowledge Needed to Support This Learning	**7.NS.A.2:** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **2b.** Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. **2d.** Convert a rational number to a decimal using long division; and know that the decimal form of a rational number terminates in 0s or eventually repeats.
Method for determining student readiness for the lesson	Use the warm-up as a starting point to determine students' abilities to identify characteristics of numbers.

Learning Experience		
Component	*Details*	*Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?*
Warm Up	**Teacher Notes:** **Definition of Natural Numbers:** Natural numbers are 1, 2, 3, 4, 5, 6… **Definition of Whole Numbers:** Whole numbers are 0, 1, 2, 3, 4, 5, 6… **Definition of Integers:** A positive or negative number that can be written without fractions or decimal components. …-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, …	

Learning Experience		
Component	***Details***	***Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?***
	Definition of Rational Numbers: A rational number is any number that can be expressed as the quotient or fraction of $\frac{p}{q}$ of two integers, with the denominator q not equal to zero. **Definition of Irrational Numbers:** An irrational number is any real number that cannot be expressed as a ratio $\frac{a}{b}$, where a and b are integers and b is non-zero. Irrational numbers include π, e, nonterminating and nonrepeating decimals. Warm-Up: • Make a set of number cards for each group of students. (Attachment #4) • Put students into groups of three or four. Give each group a set of warm-up number cards. • Have the students work together to sort the numbers using any attributes they choose. (for example, positive, negative, fractions, decimals, whole numbers, integers) • Lead a discussion which will allow the students to share and discuss their sorting methods. Questions to Summarize: • What are whole numbers? • What are integers? • What is the difference between whole numbers and integers? • Why do we have different ways to represent numbers? • Resort the fractions into more than one group. Explain. • Resort the decimals into more than one group. Explain.	

	Learning Experience	
Component	*Details*	*Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?*
	• Explain why fractions and decimals can be in the same group. • Are whole numbers integers? Explain. • Are decimals or fractions part of the integer group? Why do you think that?	
Motivation	We want the students to be able to identify and describe rational and irrational numbers. Use the comic strip to capture their interest. Tell them that by the end of class they will understand the meaning behind the comic. (Attachment #5)	
Activity 1 UDL Components • Multiple Means of Representation • Multiple Means for Action and Expression • Multiple Means for Engagement Key Questions Formative Assessment Summary	UDL Components: • Principle I: Representation is present in the activity as students write their conclusions on chart paper for the gallery walk. • Principle II: Expression is present in the activity as the students use the Think, Pair, Share activity to express in their own words what they perceive to be the ways to group numbers. • Principle III: Engagement is present in the activity as they explain to others what they are thinking about the activity. Use the warm-up activity to summarize the different ways to organize the numbers using Think-Pair-Share. (Attachment # 1)	Students compare rational and irrational numbers using the Think, Pair, Share activity and persevere until the whole group agrees. (SMP#1) Students need to communicate clearly on the posters so that everyone understands each others' thinking. (SMP#6)

Learning Experience		
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	After Think-Pair-Share use a Gallery Walk to summarize them for the class. (Note to teacher: See directions below.) **Gallery Walk/Run Instructions:** ● **Write Topics-**-Before class time, write the Gallery Walk titles-Decimals, Integers, Fractions, and Miscellaneous on large sheets of self adhering chart, post-it paper, self supporting flip charts, whiteboards (34" x 24"), or simply write on pieces of normal loose leaf paper. ● **Post Sheets** -- Post the sheets on the wall around the class, giving sufficient separation space between sheets. ● **Group Students and Assign Roles** -- Arrange students into teams of three to five. Provide each group with a different colored marker, pen, or crayon. Ask that each group member introduce themselves. If cooperative learning techniques will be used, assign roles like leader, reporter, monitor, and recorder. ● **Begin Gallery Walk** -- Direct teams to different charts or "stations." Upon arriving at the station, each team writes comments for the question posed at the station. ● **Rotate to New Station and Add Content** -- After a short period of time, say three to five minutes but the exact time will depend upon the nature of the question, say "rotate." The group then rotates, clockwise, to the next station. At the new station the group adds new comments and responds to	

Learning Experience		
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	comments left by the previous group. To involve all group members, switch recorders at each station. ● **Instructor Monitors Progress** -- As groups rotate, the instructor nurtures student discussion and involves all group members. Be ready to rephrase questions or to provide hints if students either don't understand or misinterpret questions; be ready to provide instructions for those that still don't understand how to conduct a Gallery Walk. ● **Return to Starting Point** -- Teams continue to review the answers already contributed by previous groups, adding their own comments. ● **Report** -- In the "Report" stage, the group synthesizes what has been written about their original discussion question. Allow about ten minutes for the group to synthesize comments. The "reporter" chosen earlier, summarizes the group's comments with the help of other group members and makes an oral presentation to the class using the blackboard or on an overhead projector. ● **Gauge for Student Understanding** -- During "Report" stage, the instructor reinforces correctly expressed concepts and corrects for misconceptions and errors. What, for example, did students seem to readily understand? What did they find difficult and how can I adjust my teaching to accommodate students? The teacher will summarize the lists and post them across the front of the classroom to be used for the next activity.	

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Activity 2
 	
UDL Components
• Multiple Means of Representation
• Multiple Means for Action and Expression
• Multiple Means for Engagement
Key Questions
Formative Assessment
Summary
	UDL Components:
	• Principle I: Representation is present in the activity as students use their conclusions from the gallery walk to put information into categories.
	
	• Principle II: Expression is present in the activity as the students discuss the new categories and the placement of the cards.
	
	• Principle III: Engagement is present in the activity as they explain to others what they are thinking about the activity of sorting into categories.
	
	Exploration Activity:
	• Copy the numbers cards for activity 2 and cut out enough so that each group has their own set. (Attachment #6)
	• Using the information that we have summarized in the chart paper the students will work together again to sort the numbers into the following categories:
	   o positive
	   o negative
	   o fractions
	   o decimals
	   o whole numbers
	   o integers
	   o square roots
	   o non-repeating and non-terminating decimals
	• Ask the students to share how they categorized their numbers.
		Students make sense of problems and persevere in solving them as they see various representations in both decimals and fractions and notice that they can both be grouped in the same category.
		(SMP#1)
		
		Students need to communicate clearly on the posters so that everyone understands each others' thinking.
		(SMP#6)

	Learning Experience	
Component	**Details**	**Which Standards for Mathematical Practice(s) does this address? How is the Practice used to help students develop proficiency?**
	• Using a graphic organizer, the teacher and the students will summarize the properties Rational Numbers, Integers, Whole Numbers, Natural Numbers and Irrational Numbers. (Attachment #2) (Answer Key: Attachment #3) • Now that the graphic organizer has been created, student groups will go back and re-sort the number cards into the appropriate place. • Have students complete the Rational and Irrational Independent Practice Worksheet. (Attachment # 7) (Answer key Attachment #8) • **Note to Teacher**: During middle school, fractions are augmented by negative fractions to form the rational numbers. In grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the Real Numbers. Have a short discussion with the students about Real Numbers.	
Closure	• The teacher should summarize the graphic organizer again with the student. • Show the Comic Strip Again (attachment #5). Have one of the students explain the meaning behind the comic. • Exit Ticket (attachment #9) Note there are three exit tickets on a page. Answer to Exit Ticket is in Attachment #10.	

Supporting Information	
Interventions/Enrichments	Students with Disabilities/Struggling Learners

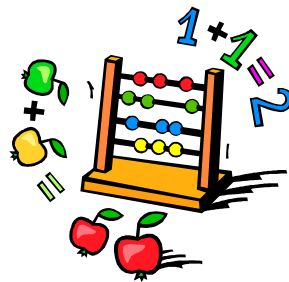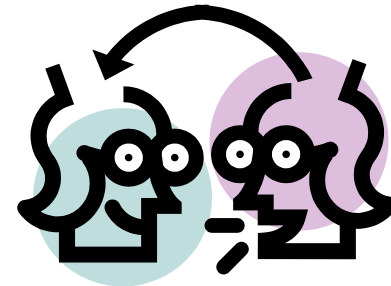|	
• Students with Disabilities/Struggling Learners • ELL • Gifted and Talented	• Teacher should create groups before class and not use random grouping in these activities. Peer tutoring should evolve. • Create a vocabulary wall/list and discuss meanings of words. ELL • Create a vocabulary wall/list and discuss meanings of words. Gifted and Talented • Students should create their own comic strip describing the difference between rational and irrational numbers.
Materials	• Comic Strip • Warm up Number Cards • Think Pair Share Summary Sheet • The Real Number System: Rational and Irrational Graphic Organizer • The Real Number System: Rational and Irrational Graphic Organizer Answer Key • Activity #2 Number Cards • Rational and Irrational Independent Practice Worksheet • Answer Key to Rational and Irrational Independent Practice Worksheet • Exit Ticket • Answer to Exit Ticket • Post It Chart Paper, flip chart or white boards • Markers
Technology	Document camera
Resources	Van de Walle, John, *Teaching Student-Centered Mathematics*, Grades 5-8.

Think-Pair-Share

Summary Sheet

Think	Pair	Share

Graphic Organizer

Real Numbers

Rational Numbers

Integers

Whole Numbers

Natural Numbers

Irrational Numbers

Attachment #3

Graphic Organizer
Answer Key

Real Numbers

Rational Numbers

Fractions, Terminating and Repeating Decimals: .5, .33333…, 2.6, 2/3, 7/2

Integers

Whole Numbers and their opposites: …-3, -2, -1, 0, 1,2,3 … (no fractions or decimals)

Whole Numbers

0, 1,2,3,4,5,6…

Natural Numbers

1,2,3,4,5,6…

Irrational Numbers

Nonterminating and nonrepeating decimals: $\pi, \sqrt{3}, \sqrt{2}$, 3.121221222….

Warm Up Number Cards

$3\frac{2}{3}$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{21}{9}$
0.24	-17.8	$\sqrt{2}$	15.123
$0.\overline{2}$	π	$0.\overline{27}$	$4.15\overline{7}$
0	-2	15	$\frac{12}{12}$

$\sqrt{16}$	$\dfrac{3}{8}$	0.123456324...	$\sqrt{\dfrac{9}{25}}$

Attachment #5

Activity 2 Number Cards – There are two pages of cards

$4\frac{2}{3}$	$\frac{1}{4}$	$-\frac{7}{8}$	$\frac{17}{3}$
$\frac{20}{5}$	-16.3	$1.21221222...$	23.64
$0.\overline{5}$	π	$0.\overline{83}$	$9.87\overline{5}$
0	$\frac{-2}{3}$	$\frac{6}{-1}$	$\frac{-1}{-1}$
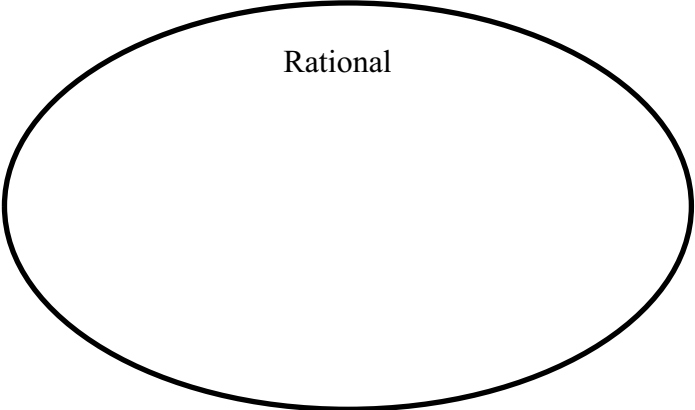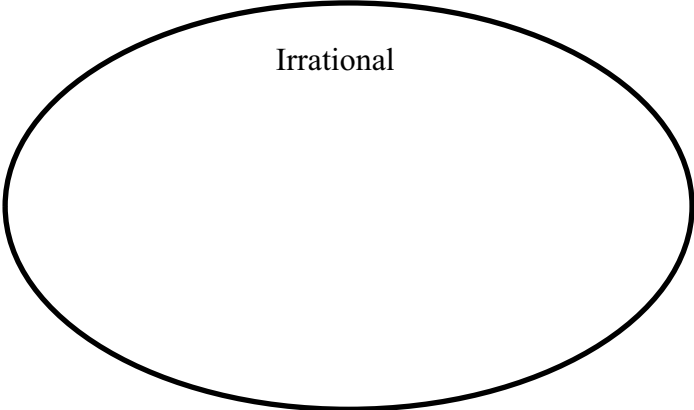
$\sqrt{25}$	$\sqrt{144}$	$\sqrt{26}$	$-\sqrt{81}$
$\sqrt{100}$	$\sqrt{50}$	$\sqrt{17}$	$-\sqrt{56}$
$\sqrt{1}$	$\sqrt{2}$	$\sqrt{\dfrac{4}{9}}$	$\sqrt{25}$

Attachment #7

Rational and Irrational Numbers   Name _____

Independent Practice

1. Sort the numbers into 2 groups, rational or irrational. Write the numbers in the appropriate bubble.

0.8 $\sqrt{64}$ 0 $\sqrt{32}$ -19 $-\sqrt{100}$ 2.343443444…

   $\dfrac{3}{7}$ $\sqrt{75}$ $6\dfrac{2}{7}$ $12.6\overline{7}$ $\sqrt{121}$ $\dfrac{12}{5}$ π

Rational

Irrational

2. Graph and label each number on the number line below. You may label the number with the letter.

A   0.75
B $\sqrt{3}$
C $\sqrt{9}$
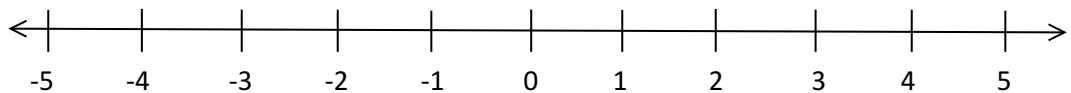D $-2\dfrac{1}{2}$
E $-\dfrac{15}{10}$
F $2.\overline{6}$
G $-\sqrt{2}$
H π

-5   -4   -3   -2   -1   0   1   2   3   4   5

Attachment #8

Rational and Irrational Numbers   Name _____

Independent Practice   ANSWER KEY

1. Sort the numbers into 2 groups, rational or irrational. Write the numbers in the appropriate bubble.

0.8 $\qquad$ $\sqrt{64}$ $\qquad$ 0 $\qquad$ $\sqrt{32}$ $\qquad$ -19 $\qquad$ $-\sqrt{100}$ $\qquad$ $2.343443444\ldots$

$\dfrac{3}{7}$ $\qquad$ $\sqrt{75}$ $\qquad$ $6\dfrac{2}{7}$ $\qquad$ $12.6\overline{7}$ $\qquad$ $\sqrt{121}$ $\qquad$ $\dfrac{12}{5}$ $\qquad$ π

Rational

0.8 $\quad$ $\sqrt{64}$ $\quad$ 0 $\quad$ -19 $\quad$ $12.6\overline{7}$

$-\sqrt{100}$ $\quad$ $\dfrac{3}{7}$ $\quad$ $6\dfrac{2}{7}$ $\quad$ $\sqrt{121}$ $\quad$ $\dfrac{12}{5}$

Irrational

$2.343443444\ldots$ $\quad$ $\sqrt{32}$ $\quad$ $\sqrt{75}$

π

π

2. Graph and label each number on the number line below. You may label the number with the letter.

A 0.75
B $\sqrt{3}$
C $\sqrt{9}$
D $-2\dfrac{1}{2}$
E $-\dfrac{15}{10}$
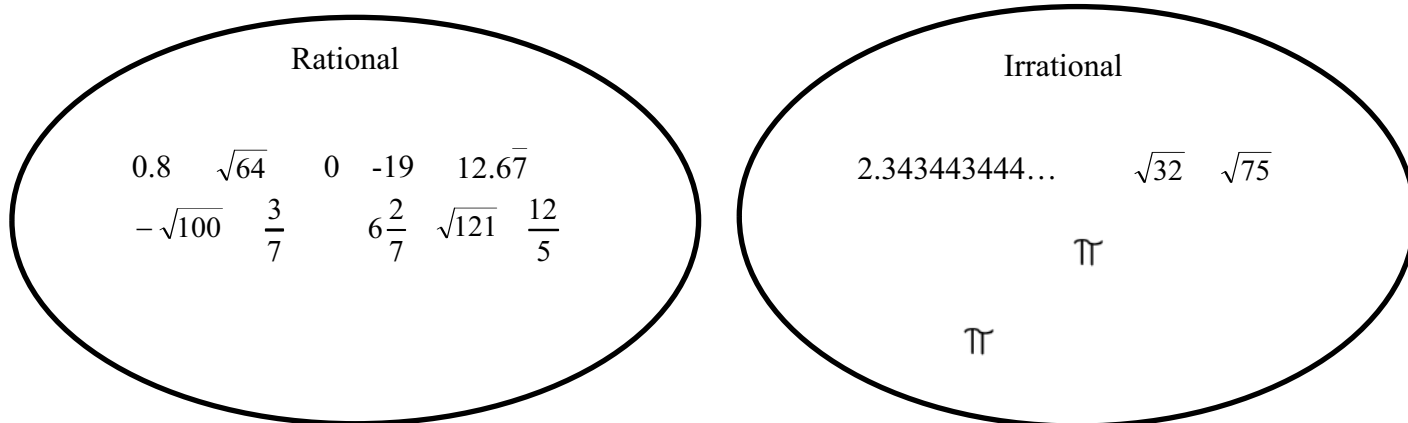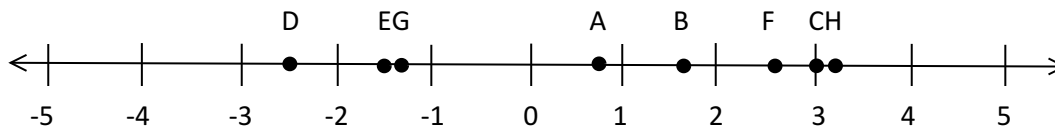F $2.\overline{6}$
G $-\sqrt{2}$
H π

Number line from -5 to 5 with points labeled: D (near -3.5), EG (near -1.5), A (near 0.75), B (near 1.7), F (near 2.6), CH (near 3)

Exit Ticket – Rational and Irrational Numbers   Name _____

Describe the difference between rational and irrational numbers. You can use examples to help support your answer.

Identify each number as rational or irrational.

3.676776777….   12.6$\overline{7}$ $\sqrt{32}$ $\sqrt{49}$ $\dfrac{7}{-5}$

Exit Ticket – Rational and Irrational Numbers   Name _____

Describe the difference between rational and irrational numbers. You can use examples to help support your answer.

Identify each number as rational or irrational.

3.676776777….   12.6$\overline{7}$ $\sqrt{32}$ $\sqrt{49}$ $\dfrac{7}{-5}$

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3.676776777….   12.6$\overline{7}$ $\sqrt{32}$ $\sqrt{49}$ $\dfrac{7}{-5}$

Attachment #10
Exit Ticket – Rational and Irrational Numbers   Name _____

Describe the difference between rational and irrational numbers. You can use examples to help support your answer.

Sample answer: *Rational numbers are numbers that can be written as ratios. This includes fractions, terminating decimals, repeating decimals, and integers. Irrational numbers can't be written as ratios. Irrational numbers include square roots that don't work out to be ratios (no perfect answers) and decimals that don't repeat but that never end.*
a

Identify each number as rational or irrational.

3.676776777….	$12.6\overline{7}$	$\sqrt{32}$	$\sqrt{49}$	$\dfrac{7}{-5}$
Irrational	Rational	Irrational	Rational	Rational