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Linear Equations Part 1

7 Steps for solving Linear Equations

1. If there are fractions multiply by the LCD.
2. If there are brackets use the distributive law to eliminate them.
3. Simplify both sides if possible.
4. Choose which side the variable terms (letters) are going on and which side the constant terms (numbers) are going on.
5. By adding or subtracting (Inverse operations) "move" all the variable terms to the chosen side.
6. By adding or subtracting (Inverse operations) "move" all the constant terms to the other side.
7. By dividing (Inverse operation) by the co-efficient of the variable (number in front of the letter) get it by itself.

This means that the equation is now solved.

*These are my seven steps for solving equations. They will work for every example except 2 specific cases which I will cover at the end of Part 2 of Linear Equations.

In my answers I label the steps that I'm using. This is not necessary. It is just there to help with the explanation.

Lesson 1: Co-efficients only

YouTube Video: https://youtu.be/9-aPfxnAXHQ

Exercise 1

Solve the following equations using only **Step 7**:

e.g. A
$$5x = 15$$
$$5x^{\div 5} = 15^{\div 5}$$
$$x = 3$$

e.g. B
$$27 = 3y$$
$$27^{\div 3} = 3y^{\div 3}$$
$$9 = y$$

e.g. C
$$3x = -36$$
$$3x^{\div 3} = -36^{\div 3}$$
$$x = -12$$

e.g. D
$$18 = -6y$$
$$18^{\div(-6)} = -6y^{\div(-6)}$$
$$-3 = y$$

1. $4z = 16$
2. $48 = 6y$
3. $3a = -24$
4. $25 = -5b$
5. $-2c = -30$
6. $3 = \frac{1}{2}d$
7. $1{,}5e = 4{,}5$
8. $-4 = \frac{2}{3}f$

Lesson 2: Constants on both sides with variables only on one side.

YouTube Video: https://youtu.be/MzjuUiQ9pBE

Exercise 2

Solve the following equations using only **Step 6 and 7**:

e.g. A
$x + 15 = 5$
$x + 15^{-15} = 5^{-15}$ {Step 6}
$x = -10$ {Step 7 not necessary}

e.g. B
$1 = y - 3$
$1^{+3} = y - 3^{+3}$ {Step 6}
$4 = y$ {Step 7 not necessary}

e.g. C
$3x + 4 = 10$
$3x + 4^{-4} = 10^{-4}$ {Step 6}
$3x = 6$ *
$3x^{\div 3} = 6^{\div 3}$ {Step 7}
$x = 2$

e.g. D
$3 = 5 - 2y$
$3^{-5} = 5^{-5} - 2y$ {Step 6}
$-2 = -2y$ *
$-2^{\div(-2)} = -2y^{\div(-2)}$ {Step 7}
$1 = y$

1	$z - 3 = 13$	2	$0 = y + 1$
3	$2g + 2 = -4$	4	$12 = -5h - 3$
5	$-2j + 1 = 1$	6	$0{,}2 = 0{,}5k - 0{,}4$
7	$5m - 2 = \frac{1}{2}$	8	$-1{,}5 = \frac{1}{3}n + 2{,}5$

Lesson 3: Variables on both sides and constants on one side.

YouTube Video: https://youtu.be/ssMYQnRQVHY

Exercise 3

Solve the following equations using only **Step 5 and 7**:

e.g. A
$2x + 15 = 3x$
$2x^{-2x} + 15 = 3x^{-2x}$ {Step 6}
$15 = x$ {Step 7 not necessary}

e.g. B
$2y = y - 3$
$2y^{-y} = y^{-y} - 3$ {Step 6}
$y = -3$ {Step 7 not necessary}

e.g. C
$3x + 4 = 5x$
$3x^{-3x} + 4 = 5x^{-3x}$ {Step 6}
$4 = 2x$
$4^{\div 2} = 2x^{\div 2}$ {Step 7}
$2 = x$

e.g. D
$2y = 10 - 3y$
$2y^{+3y} = 10 - 3y^{+3y}$ {Step 6}
$5y = 10$
$5y^{\div 5} = 10^{\div 2}$ {Step 7}
$1 = y$

1	$3z - 3 = 4z$	2	$-2y = -3y + 2$
3	$2p + 12 = 5p$	4	$q = 3 - 2q$
5	$-2r + 15 = r$	6	$3s = 10s - 14$
7	$5t - 9 = \frac{1}{2}t$	8	$6u = u + 2\frac{1}{2}$

Lesson 4: Variables and constants on both sides.

YouTube Video: https://youtu.be/NrJiA32Kw-g

Exercise 4

Solve the following equations using **Steps 4 to 7**:

e.g. A

C **V** {Step 4}

$3x + 4 = 5x - 2$

$3x^{-3x} + 4 = 5x^{-3x} - 2$ {Step 5}

$4 = 2x - 2$

$4^{+2} = 2x - 2^{+2}$ {Step 6}

$6 = 2x$

$6^{\div 2} = 2x^{\div 2}$ {Step 7}

$3 = x$

$3x$ and $5x$ are my variables with $3 < 5$ so my variables will be on the Right Hand Side and constants on the Left Hand Side.

e.g. B

V **C** {Step 4}

$x - 7 = -x + 3$

$x^{+x} - 7 = -x^{+x} + 3$ {Step 5}

$2x - 7 = 3$

$2x - 7^{+7} = 3^{+7}$ {Step 6}

$2x = 10$

$2x^{\div 2} = 10^{\div 2}$ {Step 7}

$x = 5$

x and $-x$ are my variables with $1 < -1$ so my variables will be on the Right Hand Side and constants on the Left Hand Side.

1	$3a - 3 = 4a + 4$	2	$5 - 2b = -1 - 3b$
3	$2c + 2 = 6c + 14$	4	$d - 5 = 4d + 1$
5	$-2e + 7 = e + 1$	6	$3f - 2 = 14 - f$
7	$2g - 9 = 5g + 3$	8	$6h - 5 = 2h + 1$

Lesson 5: Checking your answers without redoing the equation.

YouTube Video: https://youtu.be/xnfc_E7zUuU

Exercise 5

Check to see if the given answer to the equation is correct without solving the equation:

e.g. A $2x + 15 = 3x$
$15 = x$

LHS	RHS
$2x + 15$	$3x$
$= 2(15) + 15$	$= 3(15)$
$= 45$	$= 45$

$\text{LHS} = \text{RHS}$
$\therefore x = 15$ is a solution

e.g. B $2y = y - 3$
$y = 3$

LHS	RHS
$2y$	$y - 3$
$= 2(3)$	$= (3) - 3$
$= 6$	$= 0$

$\text{LHS} \neq \text{RHS}$
$\therefore y = 3$ is not a solution

e.g. C $3x + 4 = 5x - 6$
$x = 4$

LHS	RHS
$3x + 4$	$5x - 6$
$= 3(4) + 4$	$= 5(4) - 6$
$= 16$	$= 14$

$\text{LHS} \neq \text{RHS}$
$\therefore x = 4$ is not a solution

e.g. D $2y + 5 = 10 - 3y$
$y = -1$

LHS	RHS
$2y + 5$	$10 - 3y$
$= 2(-1) + 5$	$= 10 - 3(-1)$
$= 3$	$= 13$

$\text{LHS} \neq \text{RHS}$
$\therefore y = -1$ is not a solution

1 $5x = 10$
$x = 2$

2 $x - 4 = 2x$
$x = 0$

3 $-3 = 4a + 4$
$a = 1$

4 $5 - 2b = -1 - 3b$
$b = -5$

5 $2c + 2 = 6c + 14$
$c = 1$

6 $d - 5 = 4d + 1$
$d = -2$

7 $-2e + 7 = -1$
$e = -1$

8 $3f - 2 = 14 - f$
$f = 4$

Answers

Exercise 1

1
$$4z = 16$$
$$4z^{\div 4} = 16^{\div 4}$$
$$z = 4$$

2
$$48 = 6y$$
$$48^{\div 6} = 6y^{\div 6}$$
$$8 = y$$

3
$$3a = -24$$
$$3a^{\div 3} = -24^{\div 3}$$
$$a = -8$$

4
$$25 = -5b$$
$$25^{\div(-5)} = -5b^{\div(-5)}$$
$$-5 = b$$

5
$$-2c = -30$$
$$-2c^{\div(-2)} = -30^{\div(-2)}$$
$$c = 15$$

6
$$3 = \frac{1}{2}d$$
$$3^{\div\frac{1}{2}} = \frac{1}{2}d^{\div\frac{1}{2}}$$
$$6 = d$$

7
$$1{,}5e = 4{,}5$$
$$1{,}5e^{\div 1{,}5} = 4{,}5^{\div 1{,}5}$$
$$e = 3$$

8
$$-4 = \frac{2}{3}f$$
$$-4^{\div\frac{2}{3}} = \frac{2}{3}f^{\div\frac{2}{3}}$$
$$-6 = f$$

Exercise 2

1
$$z - 3 = 13$$
$$z - 3^{+3} = 13^{+3} \quad \{\text{Step 6}\}$$
$$z = 16 \quad \{\text{Step 7 not necessary}\}$$

2
$$0 = y + 1$$
$$0^{-1} = y + 1^{-1} \quad \{\text{Step 6}\}$$
$$-1 = y \quad \{\text{Step 7 not necessary}\}$$

3
$$2g + 2 = -4$$
$$2g + 2^{-2} = -4^{-2} \quad \{\text{Step 6}\}$$
$$2g = -6$$
$$2g^{\div 2} = -6^{\div 2} \quad \{\text{Step 7}\}$$
$$g = -3$$

4
$$12 = -5h - 3$$
$$12^{+3} = -5h - 3^{+3} \quad \{\text{Step 6}\}$$
$$15 = -5h$$
$$15^{\div(-5)} = -5h^{\div(-5)} \quad \{\text{Step 7}\}$$
$$-3 = h$$

5
$$-2j + 1 = 1$$
$$-2j + 1^{-1} = 1^{-1} \quad \{\text{Step 6}\}$$
$$-2j = 0$$
$$-2j^{\div(-2)} = 0^{\div(-2)} \quad \{\text{Step 7}\}$$
$$j = 0$$

6
$$0{,}2 = 0{,}5k - 0{,}4$$
$$0{,}2^{+0{,}4} = 0{,}5k - 0{,}4^{+0{,}4} \quad \{\text{Step 6}\}$$
$$0{,}6 = 0{,}5k$$
$$0{,}6^{\div(0{,}5)} = 0{,}5k^{\div(0{,}5)} \quad \{\text{Step 7}\}$$
$$1{,}2 = k$$

7
$$5m - 2 = \frac{1}{2}$$
$$5m - 2^{+2} = \frac{1}{2}^{+2} \quad \{\text{Step 6}\}$$
$$5m = 2\frac{1}{2}$$
$$5m^{\div 5} = 2\frac{1}{2}^{\div 5} \quad \{\text{Step 7}\}$$
$$m = \frac{1}{2}$$

8
$$-1{,}5 = \frac{1}{3}n + 2{,}5$$
$$-1{,}5^{-2{,}5} = \frac{1}{3}n + 2{,}5^{-2{,}5} \quad \{\text{Step 6}\}$$
$$-4 = \frac{1}{3}n$$
$$-4^{\div(\frac{1}{3})} = \frac{1}{3}n^{\div(\frac{1}{3})} \quad \{\text{Step 7}\}$$
$$-12 = n$$

Exercise 3

1 $3z - 3 = 4z$

$3z^{-3z} - 3 = 4z^{-3z}$ {Step 5}

$-3 = z$ {Step 7 not necessary}

2 $-2y = -3y + 2$

$-2y^{+3y} = -3y^{+3y} + 2$ {Step 5}

$y = 2$ {Step 7 not necessary}

3 $2p + 12 = 5p$

$2p^{-2p} + 12 = 5p^{-2p}$ {Step 5}

$12 = 3p$

$12^{\div 3} = 3p^{\div 3}$ {Step 7}

$4 = p$

4 $q = 3 - 2q$

$q^{+2q} = 3 - 2q^{+2q}$ {Step 5}

$3q = 3$

$3q^{\div 3} = 3^{\div 3}$ {Step 7}

$q = 1$

5 $-2r + 15 = r$

$-2r^{+2r} + 15 = r^{+2r}$ {Step 5}

$15 = 3r$

$15^{\div 3} = 3r^{\div 3}$ {Step 7}

$5 = r$

6 $3s = 10s - 14$

$3s^{-10s} = 10s^{-10s} - 14$ {Step 5}

$-7s = -14$

$-7s^{\div(-7)} = -14^{\div(-7)}$ {Step 7}

$s = 2$

7 $5t - 9 = \frac{1}{2}t$

$5t^{-5t} - 9 = \frac{1}{2}t^{-5t}$ {Step 5}

$-9 = -4\frac{1}{2}t$

$-9^{\div(-4\frac{1}{2})} = -4\frac{1}{2}t^{\div(-4\frac{1}{2})}$ {Step 7}

$2 = t$

8 $6u = u + 2\frac{1}{2}$

$6u^{-u} = u^{-u} + 2\frac{1}{2}$ {Step 5}

$5u = 2\frac{1}{2}$

$5u^{\div 5} = 2\frac{1}{2}^{\div 5}$ {Step 7}

$u = \frac{1}{2}$

Exercise 4

1 **C** **V** {Step 4}

$3a - 3 = 4a + 4$

$3a^{-3a} - 3 = 4a^{-3a} + 4$ {Step 5}

$-3 = a + 4$

$-3^{-4} = a + 4^{-4}$ {Step 6}

$-7 = a$ {Step 7 not necessary}

2 **V** **C** {Step 4}

$5 - 2b = -1 - 3b$

$5 - 2b^{+3b} = -1 - 3b^{+3b}$ {Step 5}

$5 + b = -1$

$5^{-5} + b = -1^{-5}$ {Step 6}

$b = -6$ {Step 7 not necessary}

3 **C** **V** {Step 4}

$2c + 2 = 6c + 14$

$2c^{-2c} + 2 = 6c^{-2c} + 14$ {Step 5}

$2 = 4c + 14$

$2^{-14} = 4c + 14^{-14}$ {Step 6}

$-12 = 4c$

$-12^{\div 4} = 4c^{\div 4}$ {Step 7}

$-3 = c$

4 **C** **V** {Step 4}

$d - 5 = 4d + 1$

$d^{-d} - 5 = 4d^{-d} + 1$ {Step 5}

$-5 = 3d + 1$

$-5^{-1} = 3d + 1^{-1}$ {Step 6}

$-6 = 3d$

$-6^{\div 3} = 3d^{\div 3}$ {Step 7}

$-2 = d$

5 **C** **V** {Step 4}

$-2e + 7 = e + 1$

$-2e^{+2e} + 7 = e^{+2e} + 1$ {Step 5}

$7 = 3e + 1$

$7^{-1} = 3e + 1^{-1}$ {Step 6}

$6 = 3e$

$6^{\div 2} = 3e^{\div 3}$ {Step 7}

$2 = e$

6 **V** **C** {Step 4}

$3f - 2 = 14 - f$

$3f^{+f} - 2 = 14 - f^{+f}$ {Step 5}

$4f - 2 = 14$

$4f - 2^{+2} = 14^{+2}$ {Step 6}

$4f = 16$

$4f^{\div 4} = 16^{\div 4}$ {Step 7}

$f = 4$

7 **C** **V** {Step 4}

$2g - 9 = 5g + 3$

$2g^{-2g} - 9 = 5g^{-2g} + 3$ {Step 5}

$-9 = 3g + 3$

$-9^{-3} = 3g + 3^{-3}$ {Step 6}

$-12 = 3g$

$-12^{\div 3} = 3g^{\div 3}$ {Step 7}

$-4 = g$

8 **V** **C** {Step 4}

$6h - 5 = 2h + 1$

$6h^{-2h} - 5 = 2h^{-2h} + 1$ {Step 5}

$4h - 5 = 1$

$4h - 5^{+5} = 1^{+5}$ {Step 6}

$4h = 6$

$4h^{\div 4} = 6^{\div 4}$ {Step 7}

$h = 1\frac{1}{2}$

Exercise 5

1 $5x = 10$

$x = 2$

LHS	RHS
$5x$	10
$= 5(2)$	
$= 10$	

LHS = RHS

$\therefore x = 2$ is a solution

2 $x - 4 = 2x$

$x = 0$

LHS	RHS
$x - 4$	$2x$
$= (0) - 4$	$= 2(0)$
$= -4$	$= 0$

LHS ≠ RHS

$\therefore x = 0$ is not a solution

3 $-3 = 4a + 4$

$a = 1$

LHS	RHS
-3	$4a + 4$
	$= 4(1) + 4$
	$= 8$

LHS ≠ RHS

$\therefore a = 1$ is not a solution

4 $5 - 2b = -1 - 3b$

$b = -5$

LHS	RHS
$5 - 2b$	$-1 - 3b$
$= 5 - 2(-5)$	$= -1 - 3(-5)$
$= 15$	$= 14$

LHS ≠ RHS

$\therefore b = -5$ is not a solution

5 $2c + 2 = 6c + 14$

$c = 1$

LHS	RHS
$2c + 2$	$6c + 14$
$= 2(1) + 2$	$= 6(1) + 14$
$= 4$	$= 20$

LHS ≠ RHS

$\therefore c = 1$ is not a solution

6 $d - 5 = 4d + 1$

$d = -2$

LHS	RHS
$d - 5$	$4d + 1$
$= (-2) - 5$	$= 4(-2) + 1$
$= -7$	$= -7$

LHS = RHS

$\therefore d = -2$ is a solution

7 $-2e + 7 = -1$

$e = -1$

LHS	RHS
$-2e + 7$	-1
$= -2(-1) + 7$	
$= 10$	

LHS ≠ RHS

$\therefore e = -1$ is not a solution

8 $3f - 2 = 14 - f$

$f = 4$

LHS	RHS
$3f - 2$	$14 - f$
$= 3(4) - 2$	$= 14 - (4)$
$= 10$	$= 10$

LHS = RHS

$\therefore f = 4$ is a solution