Lesson 5:

Sequences are Functions

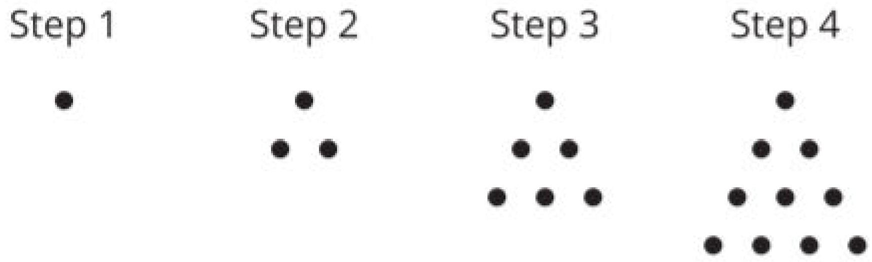
Learning Targets:

- I can define arithmetic and geometric sequences recursively using function notation.

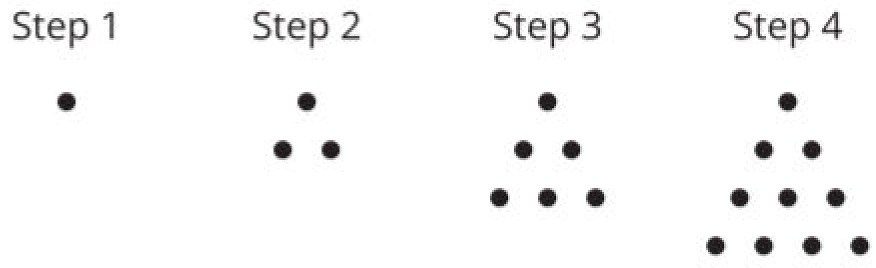
Let's learn how to define a sequence recursively.

5.1 Bowling for Triangles (Part 1)

Describe how to produce one step of the pattern from the previous step.



Activity Synthesis



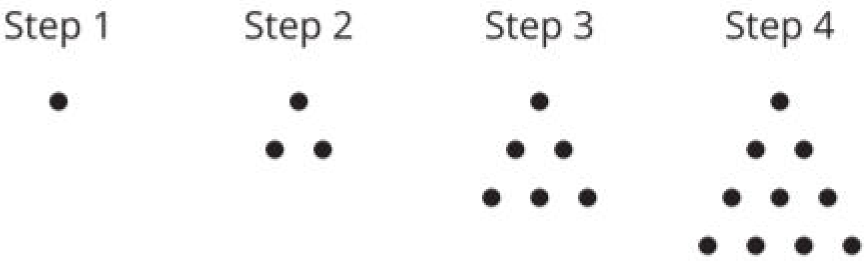
How can you produce one step of the pattern from the previous step?

- The number of dots in a step is a function of the step number.
  - Let's call this function *D*.

- What is the value of *D(4)* and *D(5)* ?

5.2 Bowling for Triangles (Part 2)

Here is a visual pattern of dots. The number of dots $D(n)$ is a function of the step number n.



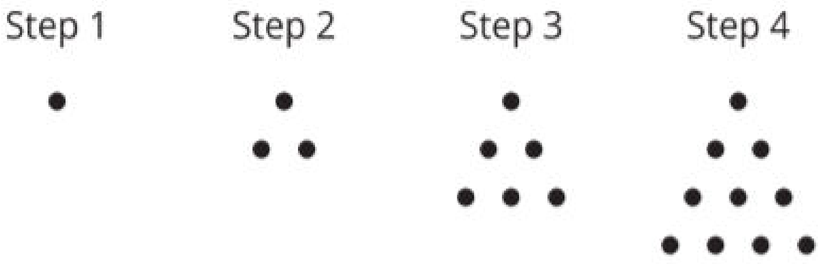
1. What values make sense for n in this situation? What values don't make sense for ?

2. Complete the table for Steps 1 to 5.

n	$D(n)$
1	1
2	$D(1) + 2 = 3$
3	$D(2) + 3 = 6$
4	
5	

Activity Synthesis

Here is a visual pattern of dots. The number of dots $D(n)$ is a function of the step number n.



- What values make sense for n?
- What values do not make sense for n?
- What is your equation for $D(n)$?

Recursive definition: it describes a repeated, or recurring, process for getting the values of D.

**The two other pieces needed for this type of definition are what value to start at and what values can n be.*

5.3 Let's Define Some Sequences

Use the first 5 terms of each sequence to state if the sequence is arithmetic, geometric, or neither. Next, define the sequence recursively using function notation

Quiet Work Time

1. A: 30, 40, 50, 60, 70, . . .

2. B: 80, 40, 20, 10, 5, 2.5, . . .

3. C: 1, 2, 4, 8, 16, 32, . . .

4. D: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

5. E: 20, 13, 6, -1, -8, . . .

6. F: 1, 3, 7, 15, 31, . . .

Activity Synthesis

1. A: 30, 40, 50, 60, 70, . . .

2. B: 80, 40, 20, 10, 5, 2.5, . . .

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4. D: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

5. E: 20, 13, 6, -1, -8, . . .

6. F: 1, 3, 7, 15, 31, . . .

Lesson Synthesis:

Geometric (exponential function)

Multiply each term by the growth factor.

Example sequence: 2, 6, 18, 54, . . .

Growth factor: 3

Recursive: $f(1) = 2, f(n) = 3 \cdot f(n-1)$, $n \geq 2$

n^{th} term: $f(n) = 2 \cdot 3^{n-1}, n \geq 1$

Example sequence: 160, 40, 10, 2.5, . . .

Growth factor: $\frac{1}{4}$

Recursive: $h(1) = 160, h(n) = \frac{1}{4} \cdot h(n-1)$, $n \geq 2$

n^{th} term: $h(n) = 160 \cdot \frac{1}{4}^{n-1}, n \geq 1$

Arithmetic (linear function)

Add to each term the rate of change.

Example sequence: 2, 7, 12, 17, . . .

Rate of change: 5

Recursive: $g(1) = 2, g(n) = 5 + g(n-1)$, $n \geq 2$

n^{th} term: $g(n) = 2 + 5(n-1), n \geq 1$

Example sequence: 9, 5, 1, -3, . . .

Rate of change: -4

Recursive: $k(1) = 9, k(n) = -4 + k(n-1)$, $n \geq 2$

n^{th} term: $k(n) = 9 - 4(n-1), n \geq 1$