RATIONAL EXPONENTS

Laws of Exponents

- Multiplication Properties
  - Product of powers
  - Power to a power
  - Power of a product
- Zero and negative exponents
- Division properties of exponents
  - Quotient of powers
  - Power of a quotient

Basic Terminology

Exponent – In Exponential notation, the number of times the base is used as a factor.

Base – In Exponential notation, the number or Variable that undergoes repeated MULTIPLICATION.

EXPONENT

$$2^4$$

means

$$2\times 2\times 2\times 2=16$$

BASE

Important Examples

IMPORTANT EXAMPLES

$$-3^4 \ \textit{means} \ -(3 \times 3 \times 3 \times 3) = -81$$

$$(-3)^4 \ \textit{means} \ (-3) \times (-3) \times (-3) \times (-3) = 81$$

$$-3^3 \ \textit{means} \ -(3 \times 3 \times 3) = -27$$

$$(-3)^3 \ \textit{means} \ (-3) \times (-3) \times (-3) = -27$$

Variable Examples

Variable Expressions

x^5 *means* (*use parentheses for multiplication*) $(x)(x)(x)(x)(x)$

y^3 *means* $(y)(y)(y)$

Substitution and Evaluating

STEPS

1. Write out the original problem.
2. Show the substitution with parentheses.
3. Work out the problem.

$$Example: Solve\ if\ x = 4; \quad x^3$$

$$Solve\ if\ x = 4; \quad x^3$$

$$(4)^3 = 64$$

More Examples

Evaluate the variable expression when x = 1, y = 2, and w = -3

$$(x)^2 + (y)^2$$

↓ Step 1

$$(x)^2 + (y)^2$$

↓ Step 2

$$(1)^2 + (2)^2$$

↓ Step 3

$$1 + 4 = 5$$

$$(x + y)^2$$

↓ Step 1

$$(x + y)^2$$

↓ Step 2

$$\left((1) + (2)\right)^2$$

↓ Step 3

$$(3)^2 = 9$$

$$wx^y$$

↓ Step 1

$$wx^y$$

↓ Step 2

$$(-3)(1)^2$$

↓ Step 3

$$(-3)(1) = -3$$

MULTIPLICATION PROPERTIES

PRODUCT OF POWERS

This property is used to combine 2 or more exponential expressions with the SAME base.

$$2^3 \times 2^5 \rightarrow (2 \times 2 \times 2)(2 \times 2 \times 2 \times 2 \times 2) \rightarrow 2^{3+5} \rightarrow 2^8 \rightarrow 256$$

$$(x^3)(x^4) \rightarrow \big((x)(x)(x)\big)\big((x)(x)(x)(x)\big) \rightarrow x^{3+4} \rightarrow x^7$$

MULTIPLICATION PROPERTIES

POWER TO A POWER

This property is used to write and exponential expression as a single power of the base.

$$(5^2)^3 \longrightarrow (5^2)(5^2)(5^2) \longrightarrow 5^6$$

$$(x^2)^4 \longrightarrow (x^2)(x^2)(x^2)(x^2) \longrightarrow x^8$$

MULTIPLICATION PROPERTIES

POWER OF PRODUCT

This property combines the first 2 multiplication properties to simplify exponential expressions.

$$(-6 \times 5)^2 \longrightarrow (-6)^2 \times (5^2) \longrightarrow 36 \times 25 = 900$$

$$(5xy)^3 \longrightarrow (5^3)(x^3)(y^3) \longrightarrow 125x^3y^3$$

$$(4x^2)^3 \bullet x^5 \longrightarrow (4^3)(x^2)^3 \bullet x^5 \longrightarrow (64)\big((x^2)(x^2)(x^2)\big) \bullet x^5$$

$$(64)(x^6) \bullet x^5 \longrightarrow 64x^{11}$$

MULTIPLICATION PROPERTIES

SUMMARY

PRODUCT OF POWERS

ADD THE EXPONENTS

$$x^a \bullet x^b = x^{a+b}$$

POWER TO A POWER

MULTIPLY THE EXPONENTS

$$\left(x^a\right)^b = x^{a \bullet b}$$

POWER OF PRODUCT

DISTRIBUTE THE EXPONENT

$$(xy)^a = x^a y^a$$

ZERO AND NEGATIVE EXPONENTS

ANYTHING TO THE ZERO POWER IS 1.

$$3^3 = 27$$

$$3^2 = 9$$

$$3^1 = 3$$

$$3^0 = 1$$

$$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$2x^{-2} = 2\left(\frac{1}{x^2}\right) = \frac{2}{x^2}$$

$$(2x)^{-2} = \frac{1}{(2x)^2} = \frac{1}{2^2x^2} = \frac{1}{4x^2}$$

$$\frac{1}{3^{-4}} = 1 \times \frac{3^4}{1} = 3^4 = 81$$

DIVISION PROPERTIES

QUOTIENT OF POWERS

This property is used when dividing two or more exponential expressions with the same base.

$$\frac{x^5}{x^3} = \frac{(\cancel{x})(\cancel{x})(\cancel{x})(x)(x)}{(\cancel{x})(\cancel{x})(\cancel{x})} = \frac{(x)(x)}{1} = x^2$$

$$\frac{X^{-3}}{X^4} = \frac{1}{X^3} \div X^4 = \frac{1}{X^3} \times \frac{1}{X^4} = \frac{1}{X^7}$$

DIVISION PROPERTIES

POWER OF A QUOTIENT

$$\left(\frac{x^2}{y^3}\right)^4 = \frac{(x^2)^4}{(y^3)^4} = \frac{x^8}{y^{12}}$$

Hard Example

$$\left(\frac{2xy^{-2}}{3x^3y^{-4}}\right)^3 = \frac{(2xy^{-2})^3}{(3x^3y^{-4})^3} = \frac{2^3x^3y^{-6}}{3^3x^9y^{-12}} = \frac{8x^3y^{12}}{27x^9y^6} =$$

$$\frac{8x^3y^{12}}{27x^9y^6} = \frac{8y^6}{27x^6}$$

Summary of Zero, Negative, and Division Properties

ZERO, NEGATIVE, AND DIVISION PROPERTIES

Zero power $(x)^0 = 1$

Negative Exponents

$$x^{-a} = \frac{1}{x^a}$$

$$and$$

$$\frac{1}{x^{-a}} = x^a$$

Quotient of powers

$$\frac{x^a}{x^b} = x^{a-b}$$

Power of a quotient

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$