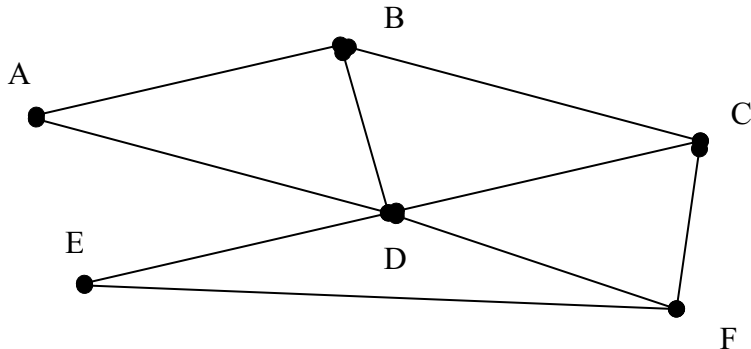


Short Answer – You must answer all questions.

The degree of a vertex is the number of lines that meet the vertex.

1) Determine the degree of each vertex in the graph below.

- A. 2
- B. 3
- C. 3
- D. 5
- E. 2
- F. 3



An Euler Path is a path that goes through each edge of a graph only once.

An Euler Circuit is a circuit that passes through each edge of a graph exactly one time.

2) How can you quickly determine whether a graph has an Euler circuit? (Hint: Euler's first theorem)

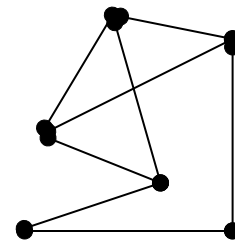
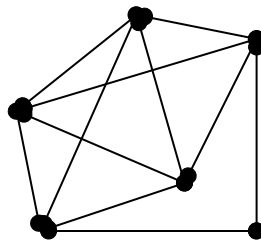
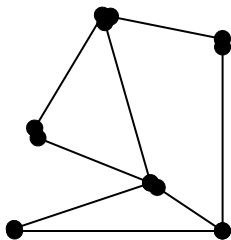
all degrees must be even

an Euler Path?

must be 2 or fewer odd degrees

3) Determine whether each graph below has an Euler Circuit, and Euler Path, or Neither.

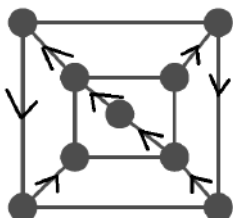
- A) Euler Path B) Euler Circuit C) Neither



4) A **Hamilton Path** is a path that contains each vertex of a graph exactly one time. A **Hamilton Circuit** passes through all vertices of the graph exactly once and ends where it started. Draw a:

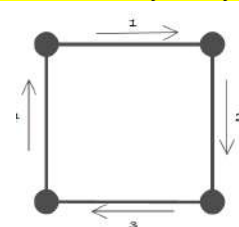
a) Hamilton Path that is not a Hamilton Circuit.

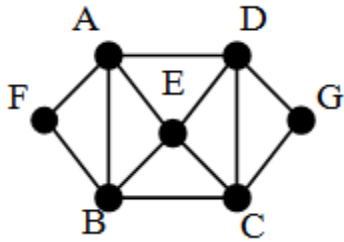
Answer May Vary:



b) Hamilton Circuit

Answer May Vary:





5) Find

A. a Hamilton path that begins at A and ends at E.

AFBCGDE

B. a Hamilton circuit that starts at A and ends with the pair of vertices E, A.

AFBCGDEA

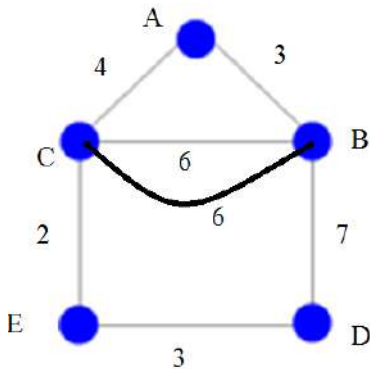
C. Find a Hamilton path that begins at F and ends at G.

Answers may vary. FABEDCG or FABECDG

6) What is the difference between an Euler Circuit and a Hamiltonian Circuit?

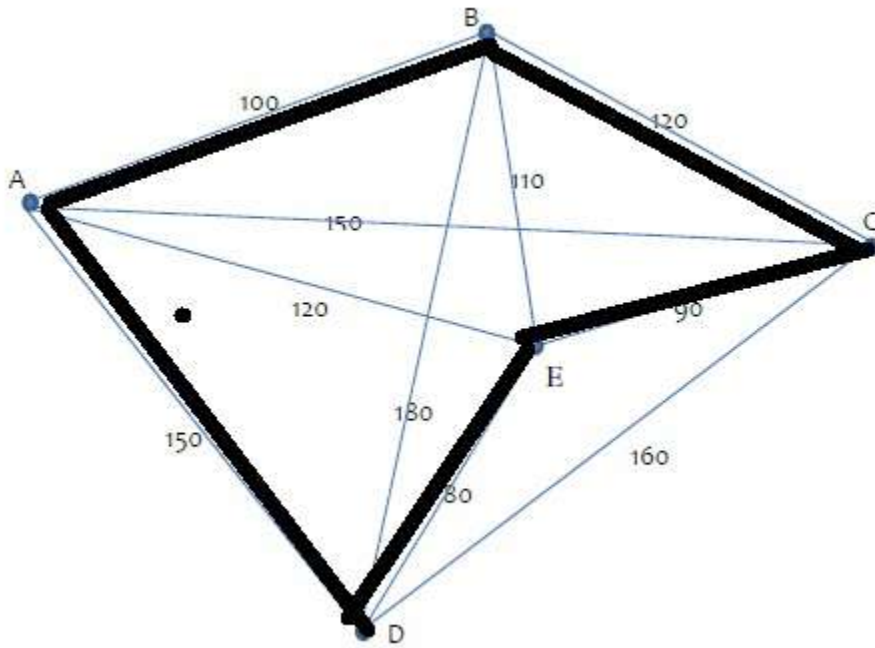
Euler Circuits use every edge and Hamiltonian Circuits use every vertex

7) What is the minimum number of hours it would take for a snowplow driver to plow all the roads of this town if they want to return to their starting place at B? Show the path that you took to complete your task. Explain how you arrived at your final answer.



Since B and C have odd vertices there is no Euler Circuit. Create an extra edge on shortest path. Now an Euler Circuit exists with many paths. Many Paths exist!! A Sample Path

- 8) You need to find the Hamiltonian Circuit with the **minimum time**, starting and ending at vertex B, for the graph below. Show all work (including the two circuits) and be specific in your answer. Include the path of your chosen circuit, listing vertices to name it.



Minimum Time = $100 + 150 + 80 + 90 + 120 = 540$

Sample Routes : BCEDAB or BADECB

- 9) Make a graph of the map below and color it using the colored graph process. Do NOT consider the ocean a separate region. Identify the minimum number of colors needed and WRITE THE FIRST INITIAL OF COLOR ON THE COUNTRY.

Color Choices:

R: Red

B: Blue

G: Green

Y: Yellow

P: Purple

O: Orange

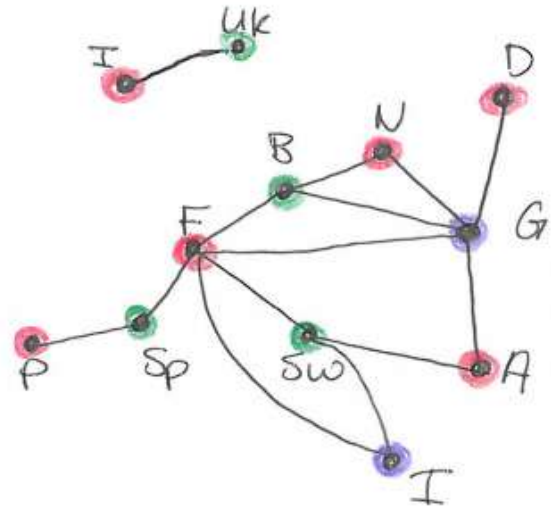
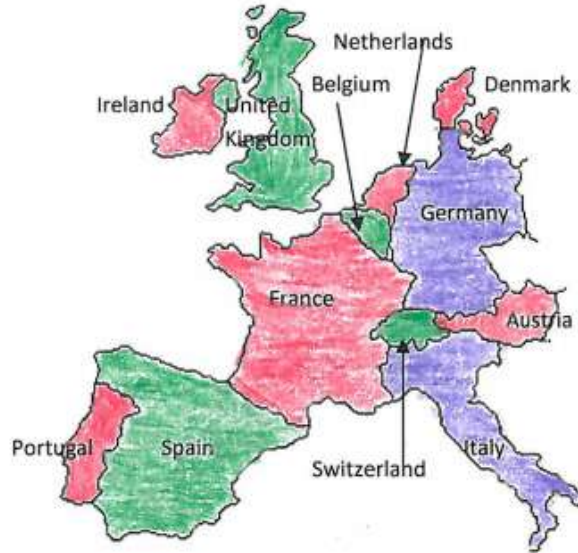
I: Indigo

J: Jade

M: Magenta

S: Silver

Should only need 3 colors but no more than 4!!!. Answers may vary.

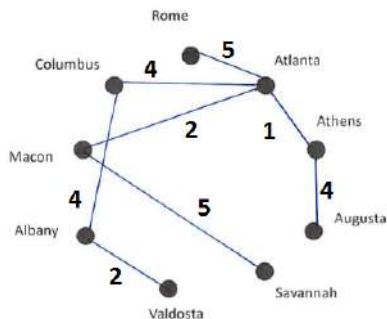


- 10) Georgia is planning to build a high-speed commuter rail system to connect major cities in the state. The table below gives the cost (in millions of dollars) to connect each city.

	Albany	Athens	Atlanta	Augusta	Columbus	Macon	Rome	Savannah	Valdosta
Albany		12	9	14	4	6	22	12	2
Athens	12		1	4	7	6	12	11	16
Atlanta	9	1		5	4	2	5	18	10
Augusta	14	4	5		8	6	20	5	10
Columbus	4	7	4	8		6	7	19	13
Macon	6	6	2	6	6		15	5	6
Rome	22	12	5	20	7	15		32	28
Savannah	12	11	18	5	19	5	32		9
Valdosta	2	16	10	10	13	6	28	9	

Use Kruskal's algorithm to create a minimal spanning tree for the state showing which connections should be made to minimize the cost of the project yet still connect every city.

Show all of your work. You do not need to make a graph of the entire set of connections, only a graph of your minimal spanning tree.



What is the length of your minimal spanning tree? 27