Georgia
Standards of Excellence
Grade Level
Curriculum Overview

Mathematics

GSE Kindergarten

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Grade Level Overview

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These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts and standards addressed in earlier units. All units include the Mathematical Practices and indicate skills to maintain.						

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades K-2 Key: CC = Counting and Cardinality, G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, OA = Operations and Algebraic Thinking.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.
The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Students are expected to:
1. Make sense of problems and persevere in solving them.
Mathematically proficient students in Kindergarten begin to develop effective dispositions toward problem solving. In rich settings in which informal and formal possibilities for solving problems are numerous, young children develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). Using both verbal and nonverbal means, kindergarten students begin to explain to themselves and others the meaning of a problem, look for ways to solve it, and determine if their thinking makes sense or if another strategy is needed. As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, kindergarten students begin to reason as they become more conscious of what they know and how they solve problems.

2. Reason abstractly and quantitatively.
Mathematically proficient students in Kindergarten begin to use numerals to represent a specific amount (quantity). For example, a student may write the numeral "11" to represent an amount of objects counted, select the correct number card "17" to follow "16" on the calendar, or build a pile of counters depending on the number drawn. In addition, kindergarten students begin to draw pictures, manipulate objects, use diagrams or charts, etc. to express quantitative ideas such as a joining situation (Mary has 3 bears. Juanita gave her 1 more bear. How many bears does Mary have altogether?), or a separating situation (Mary had 5 bears. She gave some to Juanita. Now she has 3 bears. How many bears did Mary give Juanita?). Using the language developed through numerous joining and separating scenarios, kindergarten students begin to understand how symbols (+, -, =) are used to represent quantitative ideas in a written format.

3. Construct viable arguments and critique the reasoning of others.
In Kindergarten, mathematically proficient students begin to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Through opportunities that encourage exploration, discovery, and discussion, kindergarten students begin to learn how to express opinions, become skillful at listening to others, describe their reasoning and respond to others' thinking and reasoning. They begin to develop the ability to reason and

analyze situations as they consider questions such as, "Are you sure…?", "Do you think that would happen all the time…?", and "I wonder why…?"

4. Model with mathematics.
Mathematically proficient students in Kindergarten begin to experiment with representing real-life problem situations in multiple ways such as with numbers, words (mathematical language), drawings, objects, acting out, charts, lists, and number sentences. For example, when making toothpick designs to represent the various combinations of the number "5", the student writes the numerals for the various parts (such as "4" and "1") or selects a number sentence that represents that particular situation (such as 5 = 4 + 1)*.

Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required. However, please note that it is not until First Grade when "Understand the meaning of the equal sign" is an expectation (1.OA.7).

5. Use appropriate tools strategically.
In Kindergarten, mathematically proficient students begin to explore various tools and use them to investigate mathematical concepts. Through multiple opportunities to examine materials, they experiment and use both concrete materials (e.g. 3- dimensional solids, connecting cubes, ten frames, number balances) and technological materials (e.g., virtual manipulatives, calculators, interactive websites) to explore mathematical concepts. Based on these experiences, they become able to decide which tools may be helpful to use depending on the problem or task. For example, when solving the problem, "There are 4 dogs in the park. 3 more dogs show up in the park. How many dogs are in the park?", students may decide to act it out using counters and a story mat; draw a picture; or use a handful of cubes.

6. Attend to precision.
Mathematically proficient students in Kindergarten begin to express their ideas and reasoning using words. As their mathematical vocabulary increases due to exposure, modeling, and practice, kindergarteners become more precise in their communication, calculations, and measurements. In all types of mathematical tasks, students begin to describe their actions and strategies more clearly, understand and use grade-level appropriate vocabulary accurately, and begin to give precise explanations and reasoning regarding their process of finding solutions. For example, a student may use color words (such as blue, green, light blue) and descriptive words (such as small, big, rough, smooth) to accurately describe how a collection of buttons is sorted.

7. Look for and make use of structure.
Mathematically proficient students in Kindergarten begin to look for patterns and structures in the number system and other areas of mathematics. For example, when searching for triangles around the room, kindergarteners begin to notice that some triangles are larger than others or come in different colors- yet they are all triangles. While exploring the part-whole relationships of a number using a number balance, students begin to realize that 5 can be broken down into sub-parts, such as 4 and 1 or 3 and 2, and still remain a total of 5.

8. Look for and express regularity in repeated reasoning.
In Kindergarten, mathematically proficient students begin to notice repetitive actions in geometry, counting, comparing, etc. For example, a kindergartener may notice that as the number of sides increase on a shape, a new shape is created (triangle has 3 sides, a rectangle has 4 sides, a pentagon has 5 sides, a hexagon has 6 sides). When counting out loud to 100, kindergartners may recognize the pattern 1-9 being repeated for each decade (e.g., Seventy-ONE, Seventy-TWO, Seventy- THREE… Eighty-ONE, Eighty-TWO, Eighty-THREE…). When joining one more cube to a pile, the child may realize that the new amount is the next number in the count sequence.

Mathematical Practices 1 and 6 should be evident in EVERY lesson

CONTENT STANDARDS

COUNTING AND CARDINALITY (CC)

CLUSTER #1: KNOW NUMBER NAMES AND THE COUNT SEQUENCE.

MGSEK.CC.1 Count to 100 by ones and by tens.

This standard calls for students to rote count by starting at one and count to 100. When students count by tens they are only expected to master counting on the decade (0, 10, 20, 30, 40 …). This objective does not require recognition of numerals. It is focused on the rote number sequence.

MGSEK.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

This standard includes numbers 0 to 100. This asks for students to begin a rote forward counting sequence from a number other than 1. Thus, given the number 4, the student would count, "4, 5, 6 …" This objective does not require recognition of numerals. It is focused on the rote number sequence.

MGSEK.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

This standard addresses the writing of numbers and using the written numerals (0-20) to describe the amount of a set of objects. Due to varied development of fine motor and visual development, a reversal of numerals is anticipated for a majority of the students. While reversals should be pointed out to students, the emphasis is on the use of numerals to represent quantities rather than the correct handwriting formation of the actual numeral itself.

In addition, the standard asks for students to represent a set of objects with a written numeral. The number of objects being recorded should not be greater than 20. Students can record the quantity of a set by selecting a number card or tile (numeral recognition) or writing the numeral. Students can also create a set of objects based on the numeral presented.

CLUSTER #2: COUNT TO TELL THE NUMBER OF OBJECTS.

Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects and comparing sets or numerals.

MGSEK.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

This standard asks students to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated numbers or sets. These connections are higher-level skills that require students to analyze, to reason about, and to explain relationships between numbers and sets of objects. This standard should first be addressed using numbers 1-5 with teachers building to the numbers 1-10 later in the year.

The expectation is that students are comfortable with these skills with the numbers 1-10 by the end of Kindergarten.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. (one-to-one correspondence)
This standard reflects the ideas that students implement correct counting procedures by pointing to one object at a time (one-to-one correspondence) using one counting word for every object (one-to-one tagging/synchrony), while keeping track of objects that have and have not been counted. This is the foundation of counting.

b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. (cardinality)
This standard calls for students to answer the question "How many are there?" by counting objects in a set and understanding that the last number stated when counting a set (…8, 9, 10) represents the total amount of objects: "There are 10 bears in this pile" (cardinality). It also requires students to understand that the same set counted three different times will end up being the same amount each time. Thus, a purpose of keeping track of objects is developed. Therefore, a student who moves each object as it is counted recognizes that there is a need to keep track in order to figure out the amount of objects present. While it appears that this standard calls for students to have conservation of number, (regardless of the arrangement of objects, the quantity remains the same), conservation of number is a developmental milestone of which some Kindergarten children will not have mastered. The goal of this objective is for students to be able to count a set of objects; regardless of the formation those objects are placed.

c. Understand that each successive number name refers to a quantity that is one larger.
This standard represents the concept of "one more" while counting a set of objects. Students are to make the connection that if a set of objects was increased by one more object then the number name for that set is to be increased by one as well. Students are asked to understand this concept with and without objects. For example, after counting a set of 8 objects, students should be able to answer the question, "How many would there be if we added one more object?"; and answer a similar question when not using objects, by asking hypothetically, "What if we have 5 cubes and added one more. How many cubes would there be then?" This concept should be first taught with numbers 1-5 before building to numbers 1-10. Students are expected to be comfortable with this skill with numbers to 10 by the end of Kindergarten.

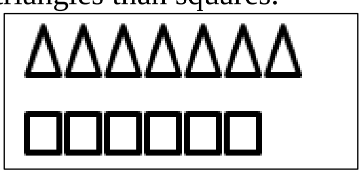
MGSEK.CC.5 Count to answer 'how many?" questions.

a. Count to answer "how many?" questions about as many as 20 things arranged in a variety of ways (a line, a rectangular array, or a circle), or as many as 10 things in a scattered configuration.

b. Given a number from 1-20, count out that many objects.

c. Identify and be able to count pennies within 20. (Use pennies as manipulatives in multiple mathematical contexts.)
This standard addresses various counting strategies. Based on early childhood mathematics experts, such as Kathy Richardson, students go through a progression of four general ways to count. These counting strategies progress from least difficult to most difficult. First, students move objects and count them as they move them. The second strategy is that students line up the objects and count them. Third, students have a scattered arrangement and they touch each object as they count. Lastly, students have a scattered arrangement and count them by visually scanning without touching them. Since the scattered arrangements are the most challenging for students, MGSEK.CC.5 calls for students to only count 10 objects in a scattered arrangement, and count up to 20 objects in a line, rectangular array, or circle. Out of these 3 representations, a line is the easiest type of arrangement to count.

MGSEK.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
This standard expects mastery of up to ten objects. Students can use matching strategies (Student 1), counting strategies or equal shares (Student 3) to determine whether one group is greater than, less than, or equal to the number of objects in another group (Student 2).

Student 1	Student 2	Student 3
I lined up one square and one triangle. Since there is one extra triangle, there are more triangles than squares. 	I counted the squares and I got 8. Then I counted the triangles and got 9. Since 9 is bigger than 8, there are more triangles than squares.	I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means there are more triangles than squares.

MGSEK.CC.7 Compare two numbers between 1 and 10 presented as written numerals.
This standard calls for students to apply their understanding of numerals 1-10 to compare one from another. Thus, looking at the numerals 8 and 10, a student must be able to recognize that the numeral 10 represents a larger amount than the numeral 8. Students should begin this standard by having ample experiences with sets of objects (MGSEK.K.CC.3 and MGSEK.K.CC.6) before completing this standard with just numerals. Based on early childhood research, students should not be expected to be comfortable with this skill until the end of kindergarten.

OPERATIONS AND ALGEBRAIC THINKING (OA)

CLUSTER #1: UNDERSTAND ADDITION AS PUTTING TOGETHER AND ADDING TO, AND UNDERSTAND SUBTRACTION AS TAKING APART AND TAKING FROM.

All standards in this cluster should only include numbers through 10. Students will model simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

MGSEK.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings , sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

This standard asks students to demonstrate the understanding of how objects can be joined (addition) and separated (subtraction) by representing addition and subtraction situations in various ways. This objective is primarily focused on understanding the concept of addition and subtraction, rather than merely reading and solving addition and subtraction number sentences (equations).

MGSEK.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

This standard asks students to solve problems presented in a story format (context) with a specific emphasis on using objects or drawings to determine the solution. This objective builds upon their understanding of addition and subtraction from K.OA.1, to solve problems. Once again, numbers should not exceed 10.

Teachers should be cognizant of the three types of problems. There are three types of addition and subtraction problems: Result Unknown, Change Unknown, and Start Unknown. These types of problems become increasingly difficult for students. Research has found that Result Unknown problems are easier than Change and Start Unknown problems. Kindergarten students should have experiences with all three types of problems. The level of difficulty can be decreased by using smaller numbers (up to 5) or increased by using larger numbers (up to 10). Please see Appendix, Table 1 for additional examples.

MGSEK.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation. (drawings need not include an equation).

This standard asks students to understand that a set of (5) objects can be broken into two sets (3 and 2) and still be the same total amount (5). In addition, this objective asks students to realize that a set of objects (5) can be broken in multiple ways (3 and 2; 4 and 1). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set (inclusion). This should be developed in context before moving into how to represent decomposition with symbols (+, -, =).

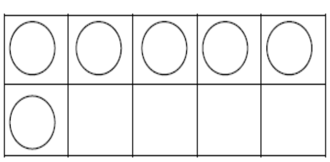
Example:
"Bobby Bear is missing 5 buttons on his jacket. How many ways can you use blue and red buttons to finish his jacket? Draw a picture of all your ideas. Students could draw pictures of:

•4 blue and 1 red button	•3 blue and 2 red buttons
•2 blue and 3 red buttons	•1 blue and 4 red buttons

After the students have had numerous experiences with decomposing sets of objects and recording with pictures and numbers, the teacher eventually makes connections between the drawings and symbols: 5=4+1, 5=3+2, 5=2+3, and 5=1+4.
The number sentence only comes after pictures or work with manipulatives, and students should never give the number sentence without a mathematical representation.

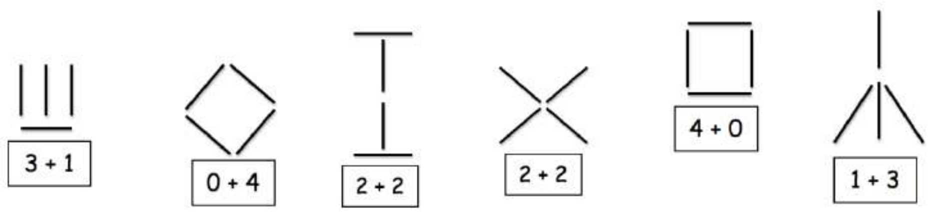
<u>MGSEK.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.</u>

This standard builds upon the understanding that a number can be decomposed into parts (K.OA.3). Once students have had experiences breaking apart ten into various combinations, this asks students to find a missing part of 10.
Example:
"A full case of juice boxes has 10 boxes. There are only 6 boxes in this case. How many juice boxes are missing?

Student 1	**Student 2**	**Student 3**
Using a Ten-Frame	*Think Addition*	*Basic Fact*
I used 6 counters for the 6 boxes of juice still in the case. There are 4 blank spaces, so 4 boxes have been removed. This makes sense since 6 and 4 more equals 10. 	I counted out 10 cubes because I knew there needed to be ten. I pushed these 6 over here because there were in the container. These are left over. So there's 4 missing.	I know that it's 4 because 6 and 4 is the same amount as 10.

<u>MGSEK.OA.5 Fluently add and subtract within 5.</u>

Students are fluent when they display accuracy (correct answer), efficiency (a reasonable amount of steps in about 3 seconds without resorting to counting), and flexibility (using strategies such as the distributive property). Students develop fluency by understanding and internalizing the relationships that exist between and among numbers. Oftentimes, when children think of each "fact" as an individual item that does not relate to any other "fact", they are attempting to memorize separate bits of information that can be easily forgotten. Instead, in order to fluently add and subtract, children must first be able to see sub-parts within a number (inclusion, K.CC.4.c). Once they have reached this milestone, children need repeated experiences with many different types of concrete materials (such as cubes, chips, and buttons) over an extended amount of time in order to recognize that there are only particular sub-parts for each number. Therefore, children will realize that if 3 and 2 is a combination of 5, then 3 and 2 cannot be a combination of 6. For example, after making various arrangements with toothpicks, students learn that only a certain number of sub-parts exist within the number 4:



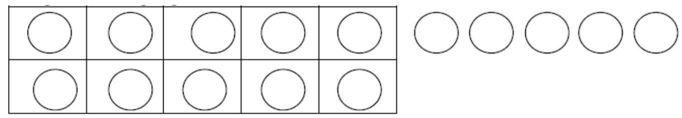
Then, after numerous opportunities to explore, represent and discuss "4", a student becomes able to fluently answer problems such as, "One bird was on the tree. Three more birds came. How many are on the tree now?" and "There was one bird on the tree. Some more came. There are now 4 birds on the tree. How many birds came?" Traditional flash cards or timed tests have not been proven as effective instructional strategies for developing fluency. Rather, numerous experiences with breaking apart actual sets of objects help children internalize parts of number

NUMBERS AND OPERATIONS IN BASE TEN (NBT)

CLUSTER #1: WORK WITH NUMBERS 11–19 TO GAIN FOUNDATIONS FOR PLACE VALUE.

<u>MGSEK.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones to understand that these numbers are composed of ten ones and one, two, three, four, five, six , seven, eight, or nine ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8)</u>

This standard is the first time that students move beyond the number 10 with representations, such as objects (manipulatives) or drawings. The spirit of this standard is that students separate out a set of 11-19 objects into a group of ten objects with leftovers. This ability is a pre-cursor to later grades when they need to understand the complex concept that a group of 10 objects is also one ten (unitizing). Ample experiences with ten frames will help solidify this concept. Research states that students are not ready to unitize until the end of first grade. Therefore, this work in Kindergarten lays the foundation of composing tens and recognizing leftovers.



MEASUREMENT AND DATA (MD)

CLUSTER #1: DESCRIBE AND COMPARE MEASURABLE ATTRIBUTES.

MGSEK.MD.1 Describe several measurable attributes of an object, such as length or weight. For example, a student may describe a shoe as, "This shoe is heavy! It is also really long!"

This standard calls for students to describe measurable attributes of objects, such as length, weight, and size. For example, a student may describe a shoe as "This shoe is heavy! It's also really long." This standard focuses on using descriptive words and does not mean that students should sort objects based on attributes. Sorting appears later in the Kindergarten standards.

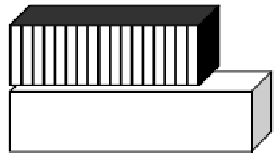
MGSEK.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.

This standard asks for direct comparisons of objects. Direct comparisons are made when objects are put next to each other, such as two children, two books, two pencils. For example, a student may line up two blocks and say, "This block is a lot longer than this one." Students are not comparing objects that cannot be moved and lined up next to each other.

Through ample experiences with comparing different objects, children should recognize that objects should be matched up at the end of objects to get accurate measurements. Since this understanding requires conservation of length, a developmental milestone for young children, children need multiple experiences to move beyond the idea that …. "Sometimes this block is longer than this one and sometimes it's shorter (depending on how I lay them side by side) and that's okay." "This block is always longer than this block (with each end lined up appropriately)."

Before conservation of length: The striped block is longer than the plain block when they are lined up like this. But when I move the blocks around, sometimes the plain block is longer than the striped block.

After conservation of length: I have to line up the blocks to measure them. The plain block is always longer than the striped block.



CLUSTER #2: CLASSIFY OBJECTS AND COUNT THE NUMBER OF OBJECTS IN EACH CATEGORY.

MGSEK.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)

This standard asks students to identify similarities and differences between objects (e.g., size, color, shape) and use the identified attributes to sort a collection of objects. Once the objects are sorted, the student counts the amount in each set. Once each set is counted, then the student is asked to sort (or group) each of the sets by the amount in each set.

For example, when given a collection of buttons, the student separates the buttons into different piles based on color (all the blue buttons are in one pile, all the orange buttons are in a different pile, etc.). Then the student counts the number of buttons in each pile: blue (5), green (4), orange (3), purple (4). Finally, the student organizes the groups by the quantity in each group (Orange buttons (3), Green buttons next (4), Purple buttons with the green buttons because purple also had (4), Blue buttons last (5).

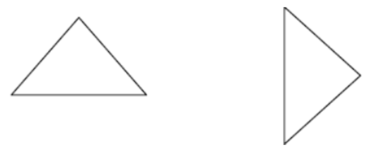
This objective helps to build a foundation for data collection in future grades. In later grades, students will transfer these skills to creating and analyzing various graphical representations.

GEOMETRY (G)

CLUSTER #1: IDENTIFY AND DESCRIBE SHAPES (SQUARES, CIRCLES, TRIANGLES, RECTANGLES, HEXAGONS, CUBES, CONES, CYLINDERS, AND SPHERES).

This entire cluster asks students to understand that certain attributes define what a shape is called (number of sides, number of angles, etc.) and other attributes do not (color, size, orientation). Then, using geometric attributes, the student identifies and describes particular shapes listed above. Throughout the year, Kindergarten students move from informal language to describe what shapes look like (e.g., "That looks like an ice cream cone!") to more formal mathematical language (e.g., "That is a triangle. It has three sides").

In Kindergarten, students need ample experiences exploring various forms of the shapes (e.g., size: big and small; types: triangles, equilateral, isosceles, scalene; orientation: rotated slightly to the left, "upside down") using geometric vocabulary to describe the different shapes. In addition, students need numerous experiences comparing one shape to another, rather than focusing on one shape at a time. This type of experience solidifies the understanding of the various attributes and how those attributes are different- or similar- from one shape to another. Students in Kindergarten typically recognize figures by appearance alone, often by comparing *them to a known example of a shape, such as the triangle on the left. For example, students in Kindergarten typically recognize that the figure on the left is a triangle, but claim that the figure on the right is not a triangle, since it does not have a flat bottom. The properties of a figure are not recognized or known. Students make decisions on identifying and describing shapes based on perception, not reasoning.*



MGSEK.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind, and next to*.

This standard expects students to use positional words (such as those italicized above) to describe objects in the environment. Kindergarten students need to focus first on location and position of two-and-three-dimensional objects in their classroom prior to describing location and position of two-and-three-dimensional representations on paper.

MGSEK.G.2 Correctly name shapes regardless of their orientations or overall size.

This standard addresses students‟ identification of shapes based on known examples. Students at this level do not yet recognize triangles that are turned upside down as triangles, since they don't "look like" triangles. Students need ample experiences looking at and manipulating shapes with various typical and atypical orientations. Through these experiences, students will begin to move beyond what a shape "looks like" to identifying particular geometric attributes that define a shape.

MGSEK.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid").

This standard asks students to identify flat objects (2 dimensional) and solid objects (3 dimensional). This standard can be taught by having students sort flat and solid objects, or by having students describe the appearance or thickness of shapes.

CLUSTER #2: ANALYZE, COMPARE, CREATE, AND COMPOSE SHAPES.

MGSEK.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

This standard asks students to note similarities and differences between and among 2-D and 3-D shapes using informal language. These experiences help young students begin to understand how 3-dimensional shapes are composed of 2-dimensional shapes (e.g.., the base and the top of a cylinder is a circle; a circle is formed when tracing a sphere).

MGSEK.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

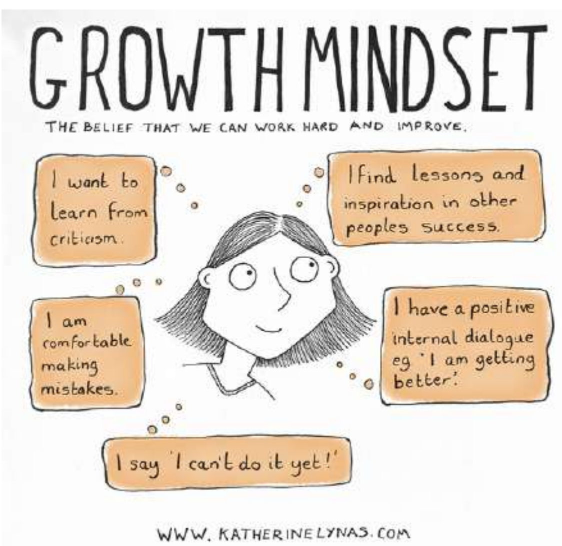
This standard asks students to apply their understanding of geometric attributes of shapes in order to create given shapes. For example, a student may roll a clump of play-dough into a sphere or use their finger to draw a triangle in the sand table, recalling various attributes in order to create that particular shape.

MGSEK.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

This standard moves beyond identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. This concept begins to develop as students first move, rotate, flip, and arrange puzzle pieces. Next, students use their experiences with puzzles to move given shapes to make a design (e.g., "Use the 7 tangram pieces to make a fox."). Finally, using these previous foundational experiences, students manipulate simple shapes to make a new shape.

MINDSET AND MATHEMATICS

Growth mindset was pioneered by Carol Dweck, Lewis and Virginia Eaton Professor of Psychology at Stanford University. She and her colleagues were the first to identify a link between growth mindset and achievement. They found that students who believed that their ability and intelligence could grow and change, otherwise known as growth mindset, outperformed those who thought that their ability and intelligence were fixed. Additionally, students who were taught that they could grow their intelligence actually did better over time. Dweck's research showed that an increased focus on the process of learning, rather than the outcome, helped increase a student's growth mindset and ability.
(from WITH+MATH=I CAN)



Jo Boaler, Professor of Mathematics Education at the Stanford Graduate School of Education and author of *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*, was one of the first to apply growth mindset to math achievement.
You can learn how to use the power of growth mindset for yourself and your students here:

https://www.amazon.com/gp/withmathican

https://www.mindsetkit.org/topics/about-growth-mindset

https://www.youcubed.org/

Growth and Fixed Mindset images courtesy of Katherine Lynas (katherinelynas.com). Thank you, Katherine!

VERTICAL UNDERSTANDING OF THE MATHEMATICS LEARNING TRAJECTORY

Why does it matter if you know what happens in mathematics in the grades before and after the one you teach? Isn't it enough just to know and understand the expectations for your grade?

There are many reasons to devote a bit of your time to the progression of standards.

You'll:

- Deepen your understanding of how development of algebraic thinking has proven to be a critical element of student mathematics success as they transition from elementary to middle school. Elementary and middle school teachers must understand how algebraic thinking develops prior to their grade, in their grade, and beyond their grade in order to support student algebraic thinking
- Know what to expect when students show up in your grade because you know what they should understand from the years before
- Understand how conceptual understanding develops, making it easier to help students who have missing bits and pieces
- Be able to help students to see the connections between ideas in mathematics in your grade and beyond, helping them to connect to what they already know and what is to come
- Assess understanding more completely, and develop better assessments
- Know what the teachers in the grades to come expect your students to know and understand
- Plan more effectively with same-grade and other-grade colleagues
- Deepen your understanding of the mathematics of your grade

We aren't asking you to take a month off to study up, just asking that you reference the following resources when you want to deepen your understanding of where students are in their mathematics learning, understand why they are learning what they are learning in your grade, and understand the mathematical ideas and connections within your grade and beyond.

Resources:

The Coherence Map:

http://achievethecore.org/page/1118/coherence-map This resource diagrams the connections between standards, provides explanations of standards, provides example tasks for many standards, and links to the progressions document when further detail is required.

A visual learning trajectory of:

Addition and Subtraction - http://gfletchy.com/2016/03/04/the-progression-of-addition-and-subtraction/
Multiplication - http://gfletchy.com/2015/12/18/the-progression-of-multiplication/
Division - http://gfletchy.com/2016/01/31/the-progression-of-division/
(Many thanks to Graham Fletcher, the genius behind these videos)

The Mathematics Progression Documents:

http://math.arizona.edu/~ime/progressions/

Learning Trajectories in Mathematics-

http://www.cpre.org/images/stories/cpre_pdfs/learning%20trajectories%20in%20math_ccii%20report.pdf

RESEARCH OF INTEREST TO MATHEMATICS TEACHERS:

Social Emotional Learning and Math-
http://curry.virginia.edu/uploads/resourceLibrary/Teachers_support_for_SEL_contributes_to_improved_math_teaching_and_learning,_et_al.pdf

Why how you teach math is important- https://www.youcubed.org/

GLOSS AND IKAN

GloSS and IKAN information can be found here: http://ccgpsmathematics6-8.wikispaces.com/GLoSS+%26+IKAN While this page 'lives' on the 6-8 math wiki, the information provided is useful to teachers of grades K-8.

The GloSS and IKAN professional learning video found here: https://www.georgiastandards.org/Georgia-Standards/Pages/FOA/Foundations-of-Algebra-Day-1.aspx provides an in-depth look at the GloSS and IKAN. While it was created for teachers of Foundations of Algebra, the information is important for teachers of grades K- 12.

The GloSS and IKAN prezi found on georgiastandards.org, here:
https://www.georgiastandards.org/Georgia-Standards/Pages/Global-Strategy-Stage-GloSS-and-Individual-Knowledge-Assessment-of-Number-IKAN.aspx

FLUENCY

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.

Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.

Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see:

http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and:

https://bhi61nm2cr3mkdgk1dtaov18-wpengine.netdna-ssl.com/wp-content/uploads/nctm-timed-tests.pdf

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing- there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task
Clarify expectations for products or behavior
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task
- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Give 'em a chance
Students- Grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals.
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe, assess, and repeat.

Closing: Best Learning Happens Here
Students- Share answers, justify thinking, clarify understanding, explain thinking, question each other, identify patterns and make generalizations, make additional connections to real world
Teacher- Listen attentively to all ideas, asks for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Take away from lesson- anchor charts and journal reflections using number, pictures, and words
Read Van de Walle K-3, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units and tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

1. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

2. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks, centers, or math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html (this is a for-profit site with several free resources)
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

3. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, *that does not mean that these are the only standards addressed in the task*. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the specified materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school, most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practice (SMP) should be at the forefront of every mathematics lesson and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMP is to construct lessons built on context or through story problems. Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and to develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around a lot of problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult however, "kindergarten students should be expected to solve word problems" (Van de Walle, K-3).

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 11).

Use of Manipulatives

"It would be difficult for you to have become a teacher and not at least heard that the use of manipulatives, or a "hands-on approach," is the recommended way to teach mathematics. There is no doubt that these materials can and should play a significant role in your classroom. Used correctly they can be a positive factor in children's learning. But they are not a cure-all that some educators seem to believe them to be. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas. We can't just give students a ten-frame or bars of Unifix cubes and expect them to develop the mathematical ideas that these manipulatives can potentially represent. When a new model or new use of a familiar model is

introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use. ”
(Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 6).

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty.” (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 8-9)
- Modeling also includes the use of mathematical symbols to represent/model the concrete mathematical idea/thought process/situation. This is a very important, yet often neglected step along the way. Modeling can be concrete, representational, and abstract. Each type of model is important to student understanding. Modeling also means to “mathematize” a situation or problem, to take a situation which might at first glance not seem mathematical, and view it through the lens of mathematics. For example, students notice that the cafeteria is always out of their favorite flavor of ice cream on ice cream days. They decide to survey their schoolmates to determine which flavors are most popular, and share their data with the cafeteria manager so that ice cream orders reflect their findings. The problem: Running out of ice cream flavors. The solution: Use math to change the flavor amounts ordered.

Use of Strategies and Effective Questioning

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking ‘Guess what is in my head’ questions;

- focusing on just a small number of learners;
- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A 'no hands' approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient 'wait time' between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

0-99 Chart or 1-100 Chart

(Adapted information from About Teaching Mathematics A K–8 RESOURCE MARILYN BURNS 3rd edition and Van de Walle)

Both the 0-99 Chart and the 1-100 Chart are valuable tools in the understanding of mathematics. Most often these charts are used to reinforce counting skills. Counting involves two separate skills: (1) ability to produce the standard list of counting words (i.e. one, two, three) and (2) the ability to connect the number sequence in a one-to-one manner with objects (Van de Walle, 2007). The counting sequence is a rote procedure. The ability to attach meaning to counting is "the key conceptual idea on which all other number concepts are developed" (Van de Walle, p. 122). Children have greater difficulty attaching meaning to counting than rote memorization of the number sequence. Although both charts can be useful, the focus of the 0-99 chart should be at the forefront of number sense development in early elementary.

A 0-99 Chart should be used in place of a 1-100 Chart when possible in early elementary mathematics for many reasons, but the overarching argument for the 0-99 is that it helps to develop a deeper understanding of place value. Listed below are some of the benefits of using the 0-99 Chart in your classroom:

- A 0-99 Chart begins with zero where as a hundred's chart begins with 1. It is important to include zero because it is a digit and just as important as 1-9.
- A 1-100 chart puts the decade numerals (10, 20, 30, etc.) on rows without the remaining members of the same decade. For instance, on a hundred's chart 20 appears at the end of the teens' row. This causes a separation between the number 20 and the numbers 21-29. The number 20 is the beginning of the 20's family; therefore it should be in the beginning of the 20's row like in a 99's chart to encourage students to associate the quantities together.
- A 0-99 chart ends with the last two digit number, 99, this allows the students to concentrate their understanding using numbers only within the ones' and tens' place values. A hundred's chart ends in 100, introducing a new place value which may change the focus of the places.

- The understanding that 9 units fit in each place value position is crucial to the development of good number sense. It is also very important that students recognize that zero is a number, not merely a placeholder. This concept is poorly modeled by a typical 1-100 chart, base ten manipulatives, and even finger counting. We have no "zero" finger, "zero" block, or "zero" space on typical 1-100 number charts. Whereas having a zero on the chart helps to give it status and reinforces that zero holds a quantity, a quantity of none. Zero is the answer to a question such as, “How many elephants are in the room?”.
- Including zero presents the opportunity to establish zero correctly as an even number, when discussing even and odd. Children see that it fits the same pattern as all of the other even numbers on the chart.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

While there are differences between the 0-99 Chart and the 1-100 Chart, both number charts are valuable resources for your students and should be readily available in several places around the classroom. Both charts can be used to recognize number patterns, such as the increase or decrease by multiples of ten. Provide students the opportunity to explore the charts and communicate the patterns they discover.

The number charts should be placed in locations that are easily accessible to students and promote conversation. Having one back at your math calendar or bulletin board area provides you the opportunity to use the chart to engage students in the following kinds of discussions. Ask students to find the numeral that represents:

- the day of the month
- the month of the year
- the number of students in the class
- the number of students absent or any other amount relevant to the moment.

Using the number is 21, give directions and ask questions similar to those below.

- Name a number greater than 21.
- Name a number less than 21.
- What number is 3 more than or less than 21?
- What number is 5 more than or less than 21?
- What number is 10 more than or less than 21?
- Is 21 even or odd?
- What numbers live right next door to 21?

Ask students to pick an even number and explain how they know the number is even. Ask students to pick an odd number and explain how they know it is odd. Ask students to count by 2's, 5's or 10's. Tell them to describe any patterns that they see. *(Accept any patterns that students are able to justify. There are many right answers!)*

Number Lines

The use of number lines in elementary mathematics is crucial in students' development of number and mathematical proficiency. While the MGSEK explicitly state use number lines in grades 2-5, number lines should be used in all grade levels and in multiple settings.

According to John Van de Walle,

> A number line is also a worthwhile model, but can initially present conceptual difficulties for children below second grade and students with disabilities (National Research Council Committee, 2009) This is partially due to their difficulty in seeing the unit, which is a challenge when it appears in a continuous line. A number line is also a shift from counting a number of individual objects in a collection to continuous length units. There are, however, ways to introduce and model number lines that support young learners as they learn this representation. Familiarity with a number line is essential because third grade students will use number lines to locate fractions and add and subtract time intervals, fourth graders will locate decimals and use them for measurement, and fifth graders will use perpendicular number lines in coordinate grids (CCSSO,2010).
>
> A number line measures distance from zero the same way a ruler does. If you don't actually teach the use of the number line through emphasis on the unit (length), students may focus on the hash marks or numerals instead of the spaces (a misunderstanding that becomes apparent when their answers are consistently off by one). At first students can build a number path by using a given length, such as a set of Cuisenaire rods of the same color to make a straight line of multiple single units (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 106-107)

Open number lines are particularly useful for building students' number sense. They can also form the basis for discussions that require the precise use of vocabulary and quantities, and are therefore a good way to engage students in the Standards for Mathematical Practice.

While the possibilities for integrating number lines into the mathematics classroom are endless, the following are some suggestions/ideas:

- On a bulletin board, attach a string which will function as an open number line. Each morning (or dedicated time for math routines) put a new number on each student's desk. Using the some type of adhesive (thumb tack, tape, etc.), students will place the number in the appropriate location on the string. In the beginning of the year, provide students with numbers that are more familiar to them. As the year progresses, move through more complex problems such as skip counting, fractions, decimals or other appropriate grade

level problems. Through daily integration, the number line becomes part of the routine. Following the number placement, have a brief discussion/debriefing of the reasoning used by students to place the numbers.
- In the 3-Act tasks placed throughout the units, students will be provided opportunities to use an open number line to place estimates that are too low, too high and just right as related to the posed problem. Similar opportunities can also be used as part of a daily routine.

Math Maintenance Activities

In addition to instruction centered on the current unit of study, the math instructional block should include time devoted to reviewing mathematics that have already been taught, previewing upcoming mathematics, and developing mental math and estimation skills. There is a saying that if you don't use it, you'll lose it. If students don't have opportunities to continuously apply and refine the math skills they've learned previously, then they may forget how to apply what they've learned. Unlike vocabulary words for literacy, math vocabulary words are not used much outside math class, so it becomes more important to use those words in discussions regularly. Math maintenance activities incorporate review and preview of math concepts and vocabulary and help students make connections across domains. It's recommended that 15 to 30 minutes of the math instructional block be used for these math maintenance activities each day. It's not necessary nor is it recommended that teachers do every activity every day. Teachers should strive for a balance of math maintenance activities so that over the course of a week, students are exposed to a variety of these activities. Math maintenance time may occur before or after instruction related to the current math unit, or it can occur at a different time during the day.

The goals of this maintenance time should include:
- Deepening number sense, including subitizing, flexible grouping of quantities, counting forward and backward using whole numbers, fractions, decimals and skip counting starting at random numbers or fractional amounts
- Developing mental math skills by practicing flexible and efficient numerical thinking through the use of operations and the properties of operations
- Practicing estimation skills with quantities and measurements such as length, mass, and liquid volume, depending on grade level
- Practicing previously-taught skills so that students deepen and refine their understanding
- Reviewing previously-taught concepts that students struggled with as indicated on their assessments, including gaps in math concepts taught in previous grade levels
- Using a variety of math vocabulary terms, especially those that are used infrequently
- Practicing basic facts using strategies learned in previous grade levels or in previous units to develop or maintain fluency
- Previewing prerequisite skills for upcoming math units of study

- Participating in mathematical discussions with others that require students to construct viable arguments and critique the reasoning of others

To accomplish these goals, math maintenance activities can take many different forms. Some activities include:
- Number Corner or Calendar Time
- Number Talks
- Estimation Activities/Estimation 180
- Problem of the Day or Spiraled Review Problems

In addition, math discussions, math journals and math games are appropriate not only for the current unit of study, but also for maintaining math skills that were previously taught.

Although there are commercially-available materials to use for math maintenance activities, there are also many excellent websites and internet resources that are free for classroom use. Here is a partial list of some recommended resources. A more detailed explanation of some of these components follows below.

Math Maintenance Activity	**Possible Resources**
Number Corner or Calendar Time	• http://teachelemmath.weebly.com/calendar.html • Every Day Counts Calendar Math from Houghton Mifflin Harcourt • Number Corner from The Math Learning Center
Number Talks	• *Number Talks* by Sherry Parrish • *http://kentuckymathematics.org/number_talk_resources.php*
Estimation Activities/Estimation 180	• http://www.estimation180.com/
Problem of the Day/Spiraled Review Problems	• www.insidemathematics.org • http://nzmaths.co.nz/teaching-material • http://www.k-5mathteachingresources.com/ • http://mathlearnnc.sharpschool.com/cms/One.aspx?portalId=4507283&pageId=5856325 • *Children's Mathematics: Cognitively Guided Instruction* by Thomas Carpentar, et. Al. • *Extending Children's Mathematics: Fractions and Decimals* by Epson and Levi

Number Corner/Calendar Time

Number Corner is a time set aside to go over mathematics skills (Standards for calendar can be found in Social Studies) during the primary classroom day. This should be an interesting and motivating time for students. A calendar board or corner can be set up and there should be several elements that are put in place. The following elements should be set in place for students to succeed during Number Corner:

1. a safe environment
2. concrete models or math tools
3. opportunities to think first and then discuss
4. student interaction

Number Corner should relate several mathematics concepts/skills to real life experiences. This time can be as simple as reviewing the months, days of the week, temperature outside, and the schedule for the day, but some teachers choose to add other components that integrate more standards. Number Corner should be used as a time to engage students in a discussion about events which can be mathematized, or as a time to engage in Number Talks.

- Find the number ___ .
- If I have a nickel and a dime, how much money do I have? (any money combination)
- What is ___ more than ___?
- What is ___ less than ___?
- Mystery number: Give clues and they have to guess what number you have.
- This number has ___tens and ___ ones. What number am I?
- What is the difference between ___ and ____?
- What number comes after ___? before ___?
- Tell me everything you know about the number ____. (Anchor Chart)

Number Corner is also a chance to familiarize your students with Data Analysis. This creates an open conversation to compare quantities, which is a vital process that must be explored before students are introduced to addition and subtraction.

- At first, choose questions that have only two mutually exclusive answers, such as yes or no (e.g., Are you a girl or a boy?), rather than questions that can be answered yes, no, or maybe (or sometimes). This sets up the part-whole relationship between the number of responses in each category and the total number of students present and it provides the easiest comparison situation (between two numbers; e.g., Which is more? How much more is it?). Keep in mind that the concept of less than (or fewer) is more difficult than the concept of greater than (or more). Be sure to frequently include the concept of less in your questions and discussions about comparisons.
- Later, you can expand the questions so they have more than two responses. Expected responses may include maybe, I'm not sure, I don't know or a short, predictable list of categorical responses (e.g., In which season were you born?).
- Once the question is determined, decide how to collect and represent the data. Use a variety of approaches, including asking students to add their response to a list of names or tally marks, using Unifix cubes of two colors to accumulate response sticks, or posting 3 x 5 cards on the board in columns to form a bar chart.
- The question should be posted for students to answer. For example, "Do you have an older sister?" Ask students to contribute their responses in a way that creates a simple visual representation of the data, such as a physical model, table of responses, bar graph, etc.

- Each day, ask students to describe, compare, and interpret the data by asking questions such as these: "What do you notice about the data? Which group has the most? Which group has the least? How many more answered [this] compared to [that]? Why do you suppose more answered [this]?" Sometimes ask data gathering questions: "Do you think we would get similar data on a different day? Would we get similar data if we asked the same question in another class? Do you think these answers are typical for first graders? Why or why not?"
- Ask students to share their thinking strategies that justify their answers to the questions. Encourage and reward attention to specific details. Focus on relational thinking and problem solving strategies for making comparisons. Also pay attention to identifying part-whole relationships; and reasoning that leads to interpretations.
- Ask students questions about the ideas communicated by the representation used. What does this graph represent? How does this representation communicate this information clearly? Would a different representation communicate this idea better?
- The representation, analysis, and discussion of the data are the most important parts of the routine (as opposed to the data gathering process or the particular question being asked). These mathematical processes are supported by the computational aspects of using operations on the category totals to solve part-whole or "compare" problems.

Number Talks

In order to be mathematically proficient, students must be able to compute accurately, efficiently, and flexibly. Daily classroom number talks provide a powerful avenue for developing "efficient, flexible, and accurate computation strategies that build upon the key foundational ideas of mathematics." (Parrish, 2010) Number talks involve classroom conversations and discussions centered upon purposefully planned computation problems.

In Sherry Parrish's book, Number Talks: Helping Children Build Mental Math and Computation Strategies, teachers will find a wealth of information about Number Talks, including:
- Key components of Number Talks
- Establishing procedures
- Setting expectations
- Designing purposeful Number Talks
- Developing specific strategies through Number Talks

There are four overarching goals upon which K-2 teachers should focus during Number Talks. These goals are:
1. Developing number sense
2. Developing fluency with small numbers
3. Subitizing
4. Making Tens

Number talks are a great way for students to use mental math to solve and explain a variety of math problems. A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. A great

place to introduce a Number Talk is during Number Corner/Calendar Time. It is important to keep Number Talks short, as they **are not intended to replace current curriculum or take up the majority of the time spent on mathematics**. In fact, teachers only need to spend 5 to 15 minutes on Number Talks. Number Talks are most effective when done every day. The primary goal of Number Talks is computational fluency. Children develop computational fluency while thinking and reasoning like mathematicians. When they share their strategies with others, they learn to clarify and express their thinking, thereby developing mathematical language. This in turn serves them well when they are asked to express their mathematical processes in writing. In order for children to become computationally fluent, they need to know particular mathematical concepts that go beyond what is required to memorize basic facts or procedures.
Students will begin to understand major characteristics of numbers, such as:

- Numbers are composed of smaller numbers.
- Numbers can be taken apart and combined with other numbers to make new numbers.
- What we know about one number can help us figure out other numbers.
- What we know about parts of smaller numbers can help us with parts of larger numbers.
- Numbers are organized into groups of tens and ones (and hundreds, tens and ones and so forth).
- What we know about numbers to 10 helps us with numbers to 100 and beyond.

All Number Talks follow a basic six-step format. The format is always the same, but the problems and models used will differ for each number talk.

1. **Teacher presents the problem.** Problems are presented in many different ways: as dot cards, ten frames, sticks of cubes, models shown on the overhead, a word problem or a written algorithm. Strategies are *not explicitly taught* to students, instead the problems presented lead to various strategies.
2. **Students figure out the answer.** Students are given time to figure out the answer. To make sure students have the time they need, the teacher asks them to give a “thumbs-up” when they have determined their answer. The thumbs up signal is unobtrusive- a message to the teacher, not the other students.
3. **Students share their answers**. Four or five students volunteer to share their answers and the teacher records them on the board.
4. **Students share their thinking.** Three or four students volunteer to share how they got their answers. (Occasionally, students are asked to share with the person(s) sitting next to them.) The teacher records the student's thinking.
5. **The class agrees on the "real" answer for the problem.** The answer that together the class determines is the right answer is presented as one would the results of an experiment. The answer a student comes up with initially is considered a conjecture. Models and/or the logic of the explanation may help a student see where their thinking went wrong, may help them identify a step they left out, or clarify a point of confusion. There should be a sense of confirmation or clarity rather than a feeling that each problem is a test to see who is right and who is wrong. A student who is still unconvinced of an answer should be encouraged to keep thinking and to keep trying to understand. For some students, it may take one more experience for them to understand what is happening with the numbers and for others it may be out of reach for some time. The mantra should be, "If you are not sure or it doesn't make sense yet, keep thinking."
6. **The steps are repeated for additional problems**.

Similar to other procedures in your classroom, there are several elements that must be in place to ensure students get the most from their Number Talk experiences. These elements are:

1. A safe environment
2. Problems of various levels of difficulty that can be solved in a variety of ways
3. Concrete models
4. Opportunities to think first and then check
5. Interaction
6. Self-correction

Estimation 180

Estimation is a skill that has many applications, such as checking computation answers quickly. Engaging in regular estimation activities will develop students' reasoning skills, number sense, and increase their repertoire of flexible and efficient strategies. As students gain more experiences with estimation, their accuracy will improve.

According to John Van de Walle, there are three types of estimation that students should practice:

- Measurement estimation – determining an approximate measurement, such as weight, length, or capacity
- Quantity estimation – approximating the number of items in a collection
- Computational estimation – determining a number that is an approximation of a computation

One resource which provides contexts for all three types of estimation is Andrew Stadel's website, http://www.estimation180.com/. In his website, Mr. Stadel has posted daily estimation contexts. Here are his directions for using his website:

1. Click on a picture.
2. Read the question.
3. Look for context clues.
4. Make an estimate.
5. Tell us how confident you are.
6. Share your reasoning (what context clues did you use?).
7. See the answer.
8. See the estimates of others.

The most important part is step #6. After you make an estimate, feel free to give a brief description. It's so valuable to a classroom when students share their logic or use of context clues when formulating an estimate.

Andrew Stadel has collaborated with Michael Fenton to create a recording sheet for students to use with the estimation contexts on the website. The recording sheet can also be found at http://www.estimation180.com/. Here are his directions for the recording sheet:

Day #	Description	⬇ Too Low	Too High ⬆	My Estimate	My Reasoning	Answer	Error	|Error| as %
Ex. A	Tyler's age (months)	24	36	30	He looks a little older than my cousin (who is 2)	26	+ – 4	4/26 ≈ 15%
Ex. B	*Bohemian Rhapsody*	4:00	5:00	4:30	10% of song = 30 sec 300 sec total = 5 min	5:56	+ – 86	86/356 ≈ 24%
							+ –	

Column use descriptions from Andrew Stadel:

Day #
In Estimation 180's first year, I was just trying to keep up with creating these estimation challenges in time for use in my own classroom. There really wasn't a scope and sequence involved. That said, now that there are over 160 estimation challenges available, teachers and students can use them at any time throughout the school year and without completing them in sequential order. Therefore, use the *Day #* column simply to number your daily challenges according to the site. Tell your students or write it up on the board that you're doing the challenge from Day 135 even though you might be on the fifth day of school.

Description
In my opinion, this column is more important than the *Day #* column. Don't go crazy here. Keep it short and sweet, but as specific as possible. For example, there's a lot of scattered height estimates on the site. Don't write down "How tall?" for Day 110. Instead write "Bus height" because when you get to Day 111, I'd write in "Parking structure height". I believe the teacher has the ultimate say here, but it can be fun to poll your students for a short description in which you all can agree. Give students some ownership, right? If unit measurement is involved, try and sneak it in here. Take Day 125 for instance. I'd suggest entering "Net Wt. (oz.) of lg Hershey's bar." Keep in mind that Day 126 asks the same question, but I'd suggest you encourage your class to use pounds if they don't think of it.
*By the way, sometimes unit measurement(s) are already included in the question. Use discretion.

Too Low
Think of an estimate that is too low.
Don't accept one (1), that's just rubbish, unless one (1) is actually applicable to the context of the challenge. Stretch your students. Think of it more as an answer that's too low, but reasonably close. After all, this is a site of estimation challenges, not gimmes.

Too High
Refer to my notes in *Too Low*. Just don't accept 1 billion unless it's actually applicable. Discuss with students the importance of the *Too Low* and *Too High* sections: we are trying to eliminate wrong answers while creating a range of possible answers.

My Estimate
This is the place for students to fill in their answer. If the answer requires a unit of measurement, we better see one. Not every estimation challenge is "How many..." marshmallows? or Christmas lights? or cheese balls?
Even if a unit of measurement has already been established (see the *Description* notes), I'd still encourage your students to accompany their numerical estimate with a unit of measurement.

For example, on Day 41, "What's the height of the Giant [Ferris] Wheel?" use what makes sense to you, your students and your country's customary unit of measurement. Discuss the importance of unit measurements with students. Don't accept 108. What does that 108 represent? Pancakes? Oil spills? Bird droppings? NO! It represents 108 feet.

My Reasoning
The *My Reasoning* section is the most recent addition to the handout and I'm extremely thrilled about it. This is a student's chance to shine! Encourage their reasoning to be short and sweet. When a student writes something down, they'll be more inclined to share it or remember it. Accept bullet points or phrases due to the limited space. We don't need students to write paragraphs. However, we are looking for students to identify any context clues they used, personal experiences, and/or prior knowledge. Hold students accountable for their reasoning behind the estimate of the day.

Don't let student reasoning go untapped! If you're doing a sequence of themed estimation challenges, don't accept, "I just guessed" after the first day in the sequence. For example, if you're doing the flight distance themed estimate challenges starting on Day 136, you will establish the distance across the USA on the first day. Sure, go ahead and guess on Day 136, but make sure you hold students accountable for their reasoning every day thereafter.
Have students share their reasoning before and after revealing the answer. Utilize Think-Pair-Share. This will help create some fun conversations *before* revealing the answer. After revealing the answer, get those who were extremely close (or correct) to share their reasoning. I bet you'll have some great mathematical discussions. I'm also curious to hear from those that are way off and how their reasoning could possibly be improved.

I'd say the *My Reasoning* section was born for Mathematical Practice 3: *Construct viable arguments and critique the reasoning of others.* Keep some of these thoughts in mind regarding Mathematical Practice 3:

- Explain and defend your estimate.
- Construct a detailed explanation referencing context clues, prior knowledge, or previous experiences.
- Invest some confidence in it.
- Try to initiate a playful and respectful argument in class.
- Ask "Was anyone convinced by this explanation? Why? Why not?" or"Are you guys going to let [student name] off the hook with that explanation?"

There's reasoning behind every estimate (not guess).

- Find out what that reasoning is!
- DON'T let student reasoning go untapped!

Answer
Jot down the revealed answer. I'd also encourage students to write down the unit of measurement used in the answer. The answer might use a different unit of measurement than what you and your class agreed upon. Take the necessary time to discuss the most relative unit of measurement. I might be subjectively wrong on some of the answers posted. As for more thoughts on unit of measurement, refer to the *My Estimate* notes above. Continue having mathematical discussion after revealing the answer. Refer to my notes regarding the use of Mathematical Practice 3 in the *My Reasoning* section.

Error
Find the difference between *My Estimate* and *Answer*. Have students circle either the "+" or the "-" if they didn't get it exactly correct.
+ Your estimate was **greater** than (above) the actual answer.
- Your estimate was **less** than (below) the actual answer.

Mathematize the World through Daily Routines
The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), daily questions, 99 chart questions, and calendar activities. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, how to open and close a door, etc. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of the routines are important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

Workstations and Learning Centers
It is recommended that workstations be implemented to create a safe and supportive environment for problem solving in a standards based classroom. These workstations typically occur during

the “exploring” part of the lesson, which follows the mini-lesson. Your role is to introduce the concept and allow students to identify the problem. Once students understand what to do and you see that groups are working towards a solution, offer assistance to the next group.

Groups should consist of 2-5 students and each student should have the opportunity to work with all of their classmates throughout the year. Avoid grouping students by ability. Students in the lower group will not experience the thinking and language of the top group, and top students will not hear the often unconventional but interesting approaches to tasks in the lower group (28, Van de Walle and Lovin 2006).

In order for students to work efficiently and to maximize participation, several guidelines must be in place (Burns 2007):
1. You are responsible for your own work and behavior.
2. You must be willing to help any group member who asks.
3. You may ask the teacher for help only when everyone in your group has the same question.

These rules should be explained and discussed with the class so that each student is aware of the expectations you have for them as a group member. Once these guidelines are established, you should be able to successfully lead small groups, which will allow you the opportunity to engage with students on a more personal level while providing students the chance to gain confidence as they share their ideas with others.

The types of activities students engage in within the small groups will not always be the same. Facilitate a variety of tasks that will lead students to develop proficiency with numerous concepts and skills. Possible activities include: math games, related previous Framework tasks, problems, and computer-based activities. With all tasks, regardless if they are problems, games, etc. include a recording sheet for accountability. This recording sheet will serve as a means of providing you information of how a child arrived at a solution or the level at which they can explain their thinking (Van de Walle 2006).

Games

“A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience. However, the few examples just mentioned and many others do have children thinking through ideas that are not easily developed in one or two lessons. In this sense, they fit the definition of a problem-based task.

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned. Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used.” (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 26)

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student’s thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? How did they assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Can you think of a different way to do the calculation that may be more efficient?
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

Representing

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.

- How could you represent this idea algebraically? Graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” ”estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret” could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK cont'd…

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, “We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.”

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from “doing it myself!”

DIFFERENTIATION

Information and instructional strategies for gifted students, English Language Learners and students with disabilities is available at http://education.ohio.gov/Topics/Academic-Content-Standards/New-Learning-Standards.

Problem Solving Rubric (K-2)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using above-grade-level appropriate vocabulary in their process of finding solutions.
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

Many thanks to Richmond County Schools for sharing this rubric!

SUGGESTED LITERATURE

- *The Greedy Triangle* by Marilyn Burns
- *When a Line Begins…A Shape Begins* by Rhonda Gowler Greene
- *Round Trip* by Ann Jonas
- *Eight Hands Round* by Ann Whitford Paul
- *Pattern* by Henry Arthur Pluckrose
- *Shape* by Henry Arthur Pluckrose
- *The Shape of Things* by Dayle Ann Dobbs
- *Grandfather Tang's Story* by Ann Tompert and Robert Andrew Parker
- *Three Pigs, One Wolf and Seven Magic Shapes* by Maccarone and Neuhaus and
- *The Tangram Magician* by Lisa Campbell Ernst.
- *Measuring Penny* by Loreen Leedy or
- *Me and the Measure of Things* by Joan Sweeney
- *The Best Bug Parade* by Stuart J. Murphy
- *Chrysanthemum* by Kevin Henkes
- *Mighty Maddie* by Stuart Murphy
- *If You Give a Mouse a Cookie* by Laura Numeroff
- *Ten Black Dots* by Donald Crews
- *Anno's Counting House* by Mitsumasa Anno
- *The Penny Pot* by Barbara Derubertis
- *Ten Flashing Fireflies* by Philemon Sturges
- *12 Ways to Get to 11* by Eve Merriam
- *One Is a Snail, Ten Is a Crab* by April Pulley Sayre and Jeff Sayre
- *Two of Everything* by Lily Toy Hong

Additional Resources:

http://kentuckymathematics.org/pimser_printables.php

TECHNOLOGY LINKS

http://catalog.mathlearningcenter.org/apps
http://nzmaths.co.nz/digital-learning-objects
https://www.georgiastandards.org/resources/Pages/Tools/LearningVillage.aspx
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

RESOURCES CONSULTED

Content:

Mathematics Progressions Documents
http://ime.math.arizona.edu/progressions/
Illustrative Mathematics
https://www.illustrativemathematics.org/
Ohio DOE
http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704
Arizona DOE
http://www.azed.gov/standards-practices/mathematics-standards/
NZmaths
http://nzmaths.co.nz/

Teacher/Student Sense-making:

http://www.youtube.com/user/mitcccnyorg?feature=watch
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx or
http://secc.sedl.org/common_core_videos/

Journaling:

http://www.kindergartenkindergarten.com/2010/07/setting-up-math-problemsolving-notebooks.html

Community of Learners:

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