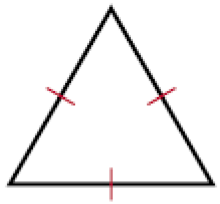
Isosceles and equilateral triangles

Warm Up

1. Find each angle measure.



60°; 60°; 60°

True or False. If false explain.

2. Every equilateral triangle is isosceles.

True

3. Every isosceles triangle is equilateral.

False; an isosceles triangle can have only two congruent sides.

Objectives

Prove theorems about isosceles and equilateral triangles.

Apply properties of isosceles and equilateral triangles.

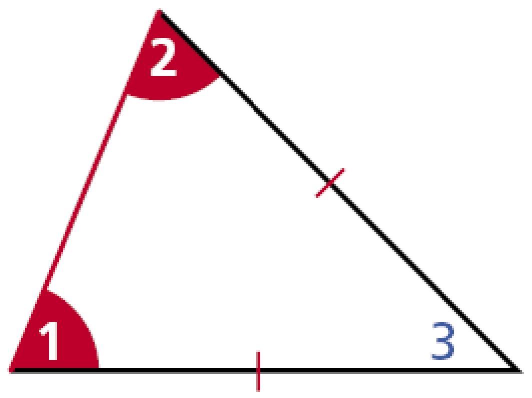
Vocabulary

legs of an isosceles triangle
vertex angle
base
base angles

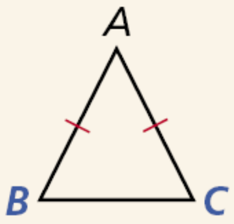
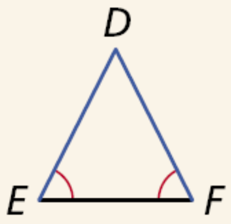
Recall that an isosceles triangle has at least two congruent sides. The congruent sides are called the **legs**. The **vertex angle** is the angle formed by the legs. The side opposite the vertex angle is called the **base**, and the **base angles** are the two angles that have the base as a side.

$\angle 3$ is the vertex angle.

$\angle 1$ and $\angle 2$ are the base angles.



Theorems — Isosceles Triangle

	THEOREM	HYPOTHESIS	CONCLUSION
4-8-1	**Isosceles Triangle Theorem** If two sides of a triangle are congruent, then the angles opposite the sides are congruent.		$\angle B \cong \angle C$
4-8-2	**Converse of Isosceles Triangle Theorem** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.		$\overline{DE} \cong \overline{DF}$

Reading Math

The Isosceles Triangle Theorem is sometimes stated as “Base angles of an isosceles triangle are congruent.”

Example 1: Astronomy Application

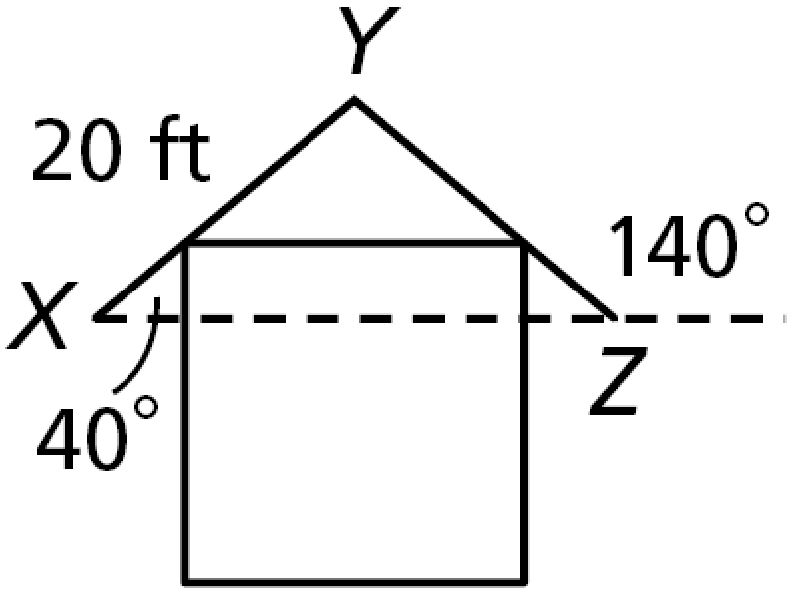
The length of $\overline{YX}$ is 20 feet.

Explain why the length of $\overline{YZ}$ is the same.

The $m\angle YZX = 180 - 140$, so $m\angle YZX = 40°$.

Since $\angle YZX \cong \angle X$, ΔXYZ is isosceles by the Converse of the Isosceles Triangle Theorem.

Thus $YZ = YX = 20$ ft.



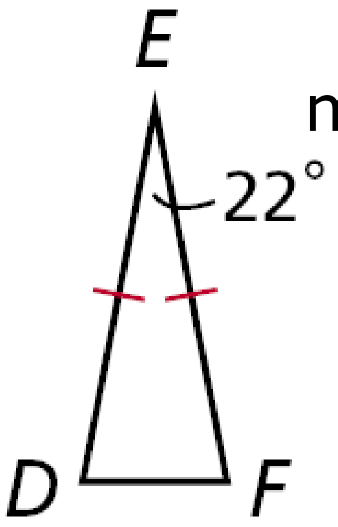
Check It Out! Example 1

If the distance from Earth to a star in September is 4.2×10^{13} km, what is the distance from Earth to the star in March? Explain.

4.2×10^{13}; since there are 6 months between September and March, the angle measures will be approximately the same between Earth and the star. By the Converse of the Isosceles Triangle Theorem, the triangles created are isosceles, and the distance is the same.

Example 2A: Finding the Measure of an Angle

Find m∠*F*.



m∠*F* = m∠*D* = *x*° *Isosc. Δ Thm.*

m∠*F* + m∠*D* + m∠*A* = 180 *Δ Sum Thm.*

x + *x* + 22 = 180 *Substitute the given values.*

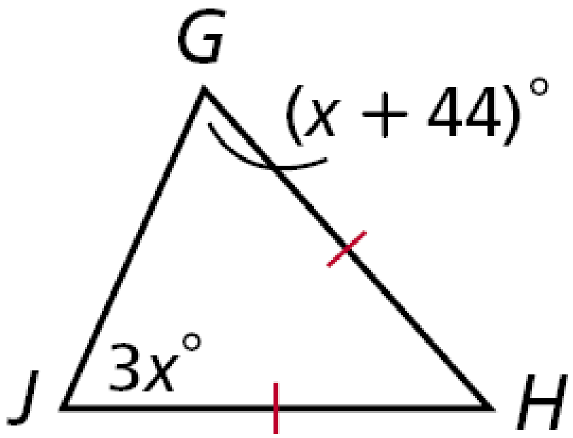
2*x* = 158 *Simplify and subtract 22 from both sides.*

x = 79° *Divide both sides by 2.*

Thus m∠*F* = 79°

Example 2B: Finding the Measure of an Angle

Find m∠*G*.



$m\angle J = m\angle G$	*Isosc. Δ Thm.*
$(x + 44)^\circ = 3x^\circ$	*Substitute the given values.*
$44 = 2x$	*Simplify x from both sides.*
$x = 22^\circ$	*Divide both sides by 2.*

Thus $m\angle G = 22^\circ + 44^\circ = 66^\circ$.

Check It Out! Example 2A

Find m∠H.

$m\angle H = m\angle G = x°$ *Isosc. Δ Thm.*

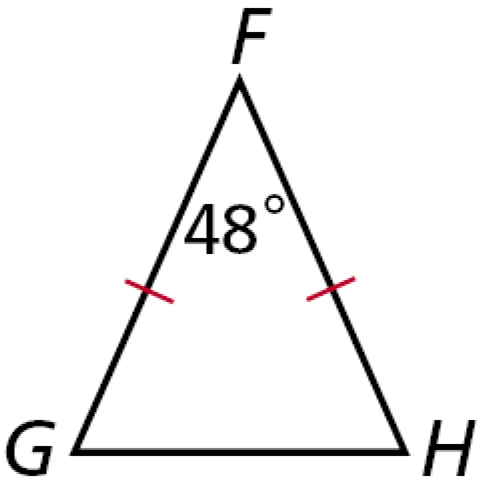
$m\angle H + m\angle G + m\angle F = 180$ *Δ Sum Thm.*

$x + x + 48 = 180$ *Substitute the given values.*

$2x = 132$ *Simplify and subtract 48 from both sides.*

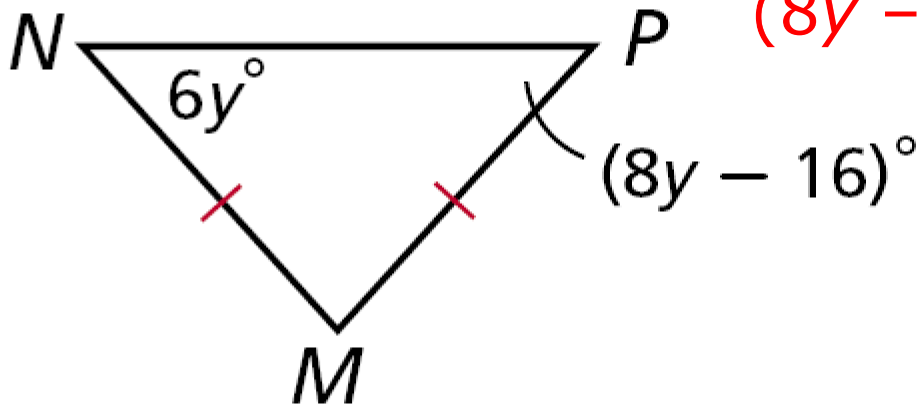
$x = 66°$ *Divide both sides by 2.*

Thus $m\angle H = 66°$



Check It Out! Example 2B

Find m∠*N*.



$m\angle P = m\angle N$ *Isosc. Δ Thm.*

$(8y - 16)° = 6y°$ *Substitute the given values.*

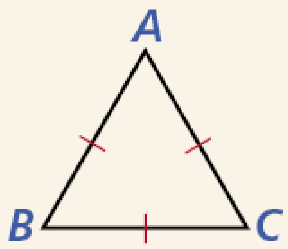
$2y = 16$ *Subtract 6y and add 16 to both sides.*

$y = 8°$ *Divide both sides by 2.*

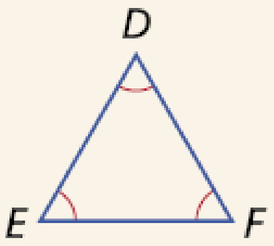
Thus $m\angle N = 6(8) = 48°$.

The following corollary and its converse show the connection between equilateral triangles and equiangular triangles.

Corollary 4-8-3 **Equilateral Triangle**

COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equilateral, then it is equiangular. (equilateral $\triangle \rightarrow$ equiangular $\triangle$)		$\angle A \cong \angle B \cong \angle C$

Corollary 4-8-4 Equiangular Triangle

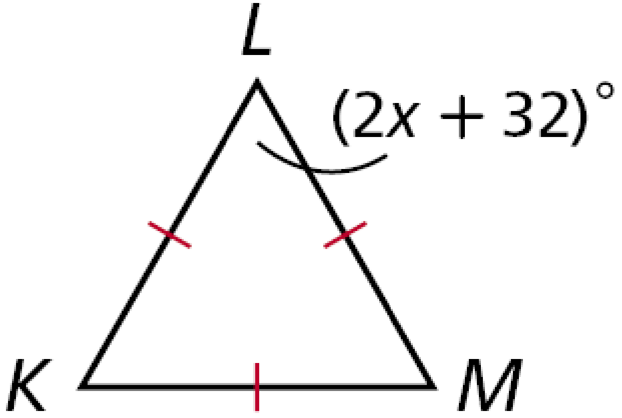
COROLLARY	HYPOTHESIS	CONCLUSION
If a triangle is equiangular, then it is equilateral. (equiangular $\triangle \rightarrow$ equilateral $\triangle$)		$\overline{DE} \cong \overline{DF} \cong \overline{EF}$

Example 3A: Using Properties of Equilateral Triangles

Find the value of *x*.

ΔLKM is equilateral.

Equilateral Δ → equiangular Δ



$(2x + 32)° = 60°$ *The measure of each ∠ of an equiangular Δ is 60°.*

$2x = 28$ *Subtract 32 both sides.*

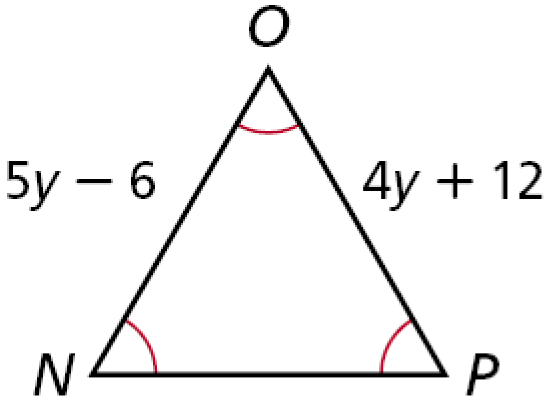
$x = 14$ *Divide both sides by 2.*

Example 3B: Using Properties of Equilateral Triangles

Find the value of *y*.

Δ*NPO* is equiangular.

Equiangular Δ → equilateral Δ

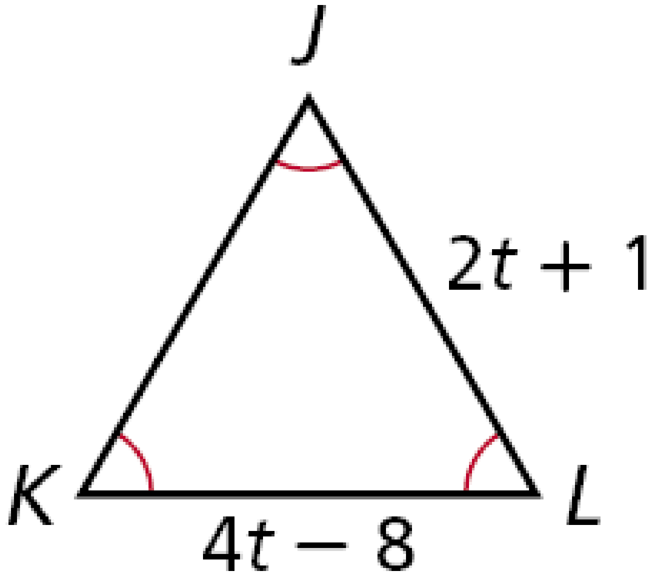


$5y - 6 = 4y + 12$ *Definition of equilateral Δ.*

$y = 18$ *Subtract 4y and add 6 to both sides.*

Check It Out! Example 3

Find the value of JL.



Δ*JKL* is equiangular.

Equiangular Δ → equilateral Δ

$4t - 8 = 2t + 1$ *Definition of equilateral Δ.*

$2t = 9$ *Subtract 4y and add 6 to both sides.*

$t = 4.5$ *Divide both sides by 2.*

Thus $JL = 2(4.5) + 1 = 10$.

Remember!

A coordinate proof may be easier if you place one side of the triangle along the *x*-axis and locate a vertex at the origin or on the *y*-axis.

Example 4: Using Coordinate Proof

Prove that the segment joining the midpoints of two sides of an isosceles triangle is half the base.

Given: In isosceles ΔABC, X is the mdpt. of $\overline{AB}$, and Y is the mdpt. of $\overline{AC}$.

Prove: $XY = \frac{1}{2}AC$.

Example 4 Continued

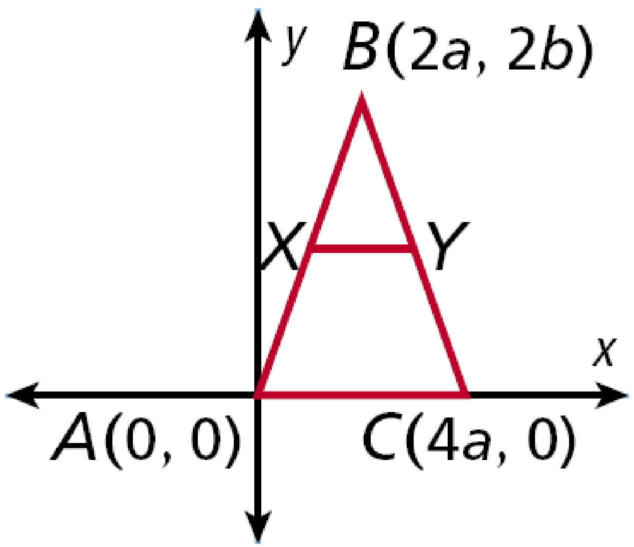
Proof:

Draw a diagram and place the coordinates as shown.

By the Midpoint Formula, the coordinates of *X* are (a, b), and *Y* are $(3a, b)$.

By the Distance Formula, $XY = \sqrt{4a^2} = 2a$, and $AC = 4a$.

Therefore $XY = \frac{1}{2}AC$.

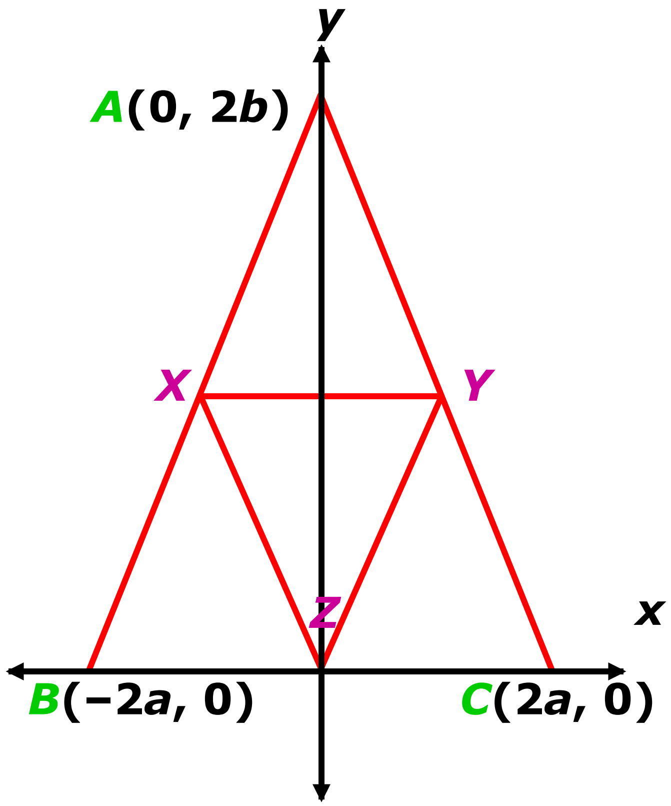


Check It Out! Example 4

What if…? The coordinates of isosceles Δ*ABC* are *A*(0, 2*b*), *B*(-2*a*, 0), and *C*(2*a*, 0). *X* is the midpoint of *AB*, and *Y* is the midpoint of *AC*. Prove Δ*XYZ* is isosceles.

Proof:

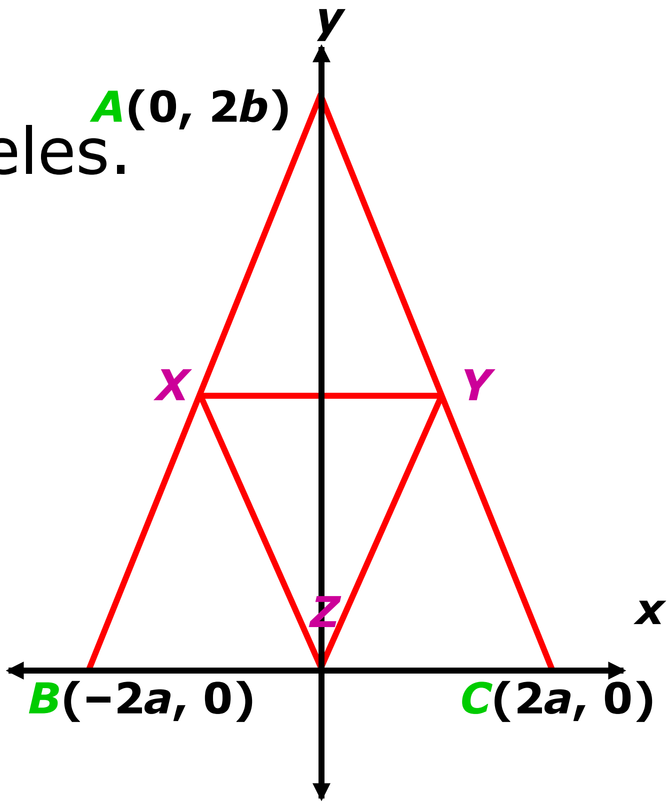
Draw a diagram and place the coordinates as shown.



Check It Out! Example 4 Continued

By the Midpoint Formula, the coordinates. of X are $(-a, b)$, the coordinates. of Y are (a, b), and the coordinates of Z are $(0, 0)$. By the Distance Formula, $XZ = YZ = \sqrt{a^2+b^2}$.

So $\overline{XZ} \cong \overline{YZ}$ and ΔXYZ is isosceles.

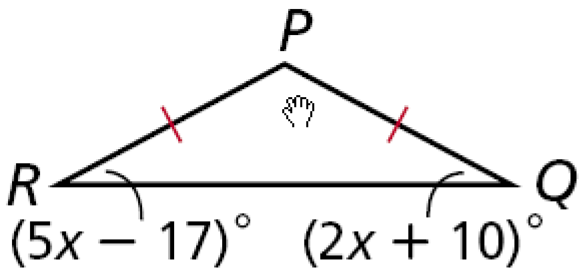


Lesson Quiz: Part I

Find each angle measure.

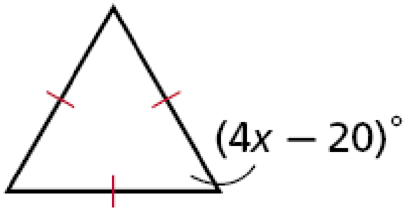
1. m∠*R** 28°

2. m∠*P* 124°



Find each value.

3. *x*



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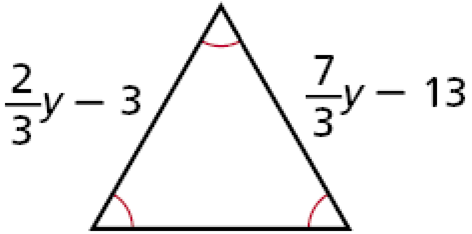
4. *y*



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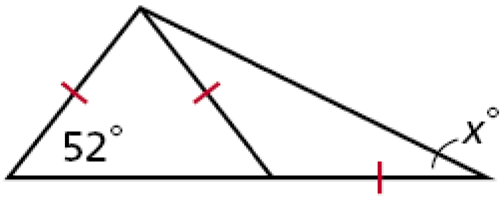
5. *x*



26°

Lesson Quiz: Part II

6. The vertex angle of an isosceles triangle measures (a + 15)°, and one of the base angles measures 7a°. Find a and each angle measure.

a = 11; 26°; 77°; 77°