Inscribed angle and intercepted arc

Objectives

Find the measure of an inscribed angle.

Use inscribed angles and their properties to solve problems.

Vocabulary

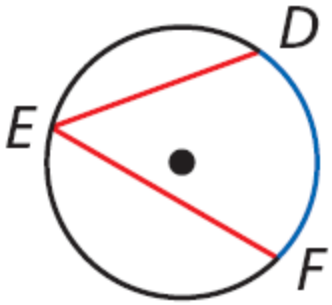
inscribed angle

intercepted arc

subtend

String art often begins with pins or nails that are placed around the circumference of a circle. A long piece of string is then wound from one nail to another. The resulting pattern may include hundreds of *inscribed angles*.

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle. An **intercepted arc** consists of endpoints that lie on the sides of an inscribed angle and all the points of the circle between them. A chord or arc **subtends** an angle if its endpoints lie on the sides of the angle.



$\angle DEF$ is an **inscribed angle**.

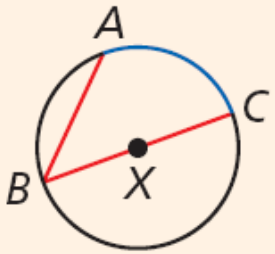
$\overset{\frown}{DF}$ is the **intercepted arc**.

$\overset{\frown}{DF}$ subtends $\angle DEF$.

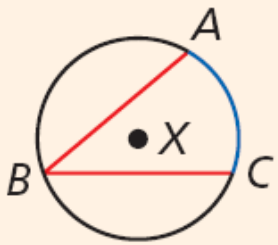
Theorem 11-4-1 Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

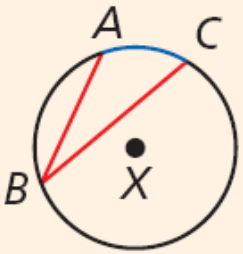
$$m\angle ABC = \frac{1}{2}m\overset{\frown}{AC}$$



Case 1



Case 2

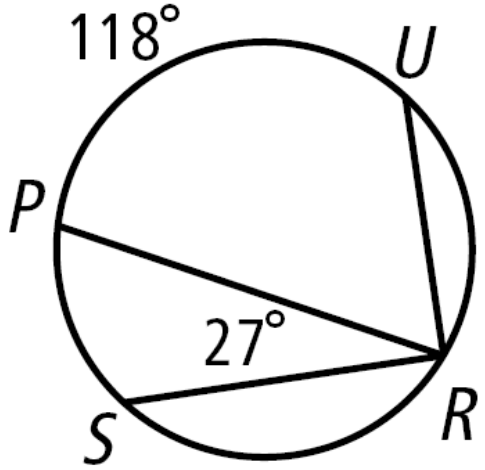


Case 3

Example 1A: Finding Measures of Arcs and Inscribed Angles

Find each measure.

m∠*PRU*

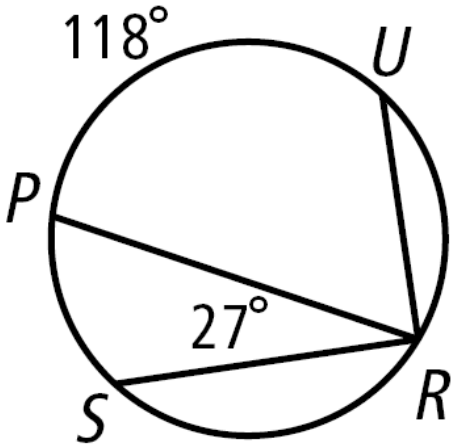


$$m\angle PRU = \frac{1}{2}m\overset{\frown}{PU}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(118°) = 59°$$ *Substitute 118 for $m\overset{\frown}{PU}$.*

Example 1B: Finding Measures of Arcs and Inscribed Angles

Find each measure.



$m\overset{\frown}{SP}$

$m\angle SRP = \frac{1}{2} m\overset{\frown}{SP}$ *Inscribed ∠ Thm.*

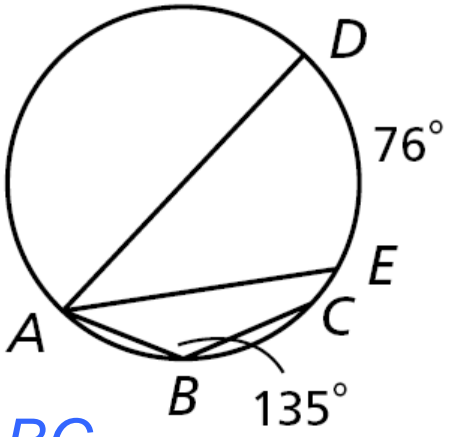
$27° = \frac{1}{2} m\overset{\frown}{SP}$ *Substitute 27 for m∠ SRP.*

$m\overset{\frown}{SP} = 54°$ *Multiply both sides by 2.*

Check It Out! Example 1a

Find each measure.

m$\overset{\frown}{ADC}$



$m\angle ABC = \frac{1}{2} m\overset{\frown}{ADC}$ *Inscribed ∠ Thm.*

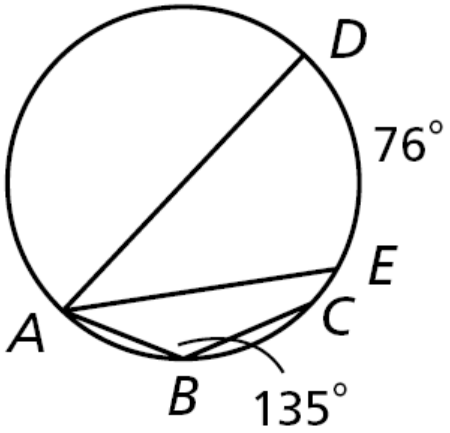
$135° = \frac{1}{2} m\overset{\frown}{ADC}$ *Substitute 135 for m∠ ABC.*

$270° = m\overset{\frown}{ADC}$ *Multiply both sides by 2.*

Check It Out! Example 1b

Find each measure.

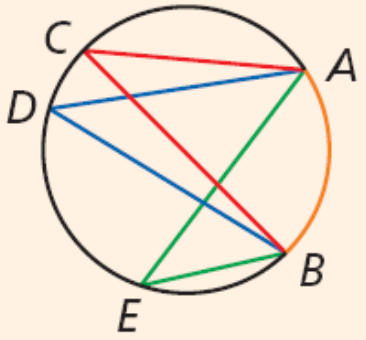
m∠*DAE*



$$m\angle DAE = \frac{1}{2}m\overset{\frown}{DE}$$ *Inscribed ∠ Thm.*

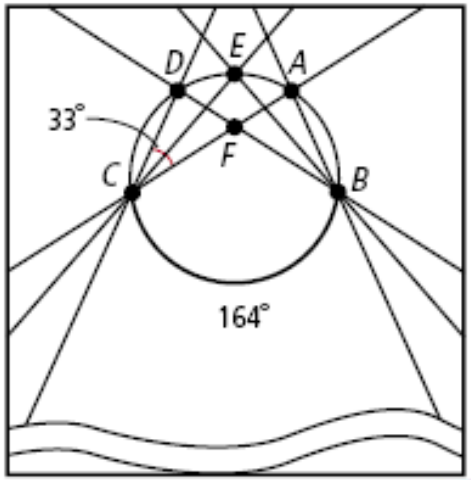
$$= \frac{1}{2}(76°) = 38°$$ *Substitute 76 for $m\overset{\frown}{DE}$.*

Corollary 11-4-2

COROLLARY	HYPOTHESIS	CONCLUSION
If inscribed angles of a circle intercept the same arc or are subtended by the same chord or arc, then the angles are congruent.	 $\angle ACB$, $\angle ADB$, and $\angle AEB$ intercept $\overset{\frown}{AB}$.	$\angle ACB \cong \angle ADB \cong \angle AEB$ (and $\angle CAE \cong \angle CBE$)

Example 2: Hobby Application

An art student turns in an abstract design for his art project.



Find m∠*DFA*.

$m\angle DFA = m\angle DCF + m\angle CDF$ *Ext ∠ Thm.*

$= m\angle DCF + \frac{1}{2}m\overset{\frown}{BC}$ *Inscribed ∠ Thm.*

$= 33° + \frac{1}{2}(164°)$ *Substitute.*

$= 115°$ *Simplify.*

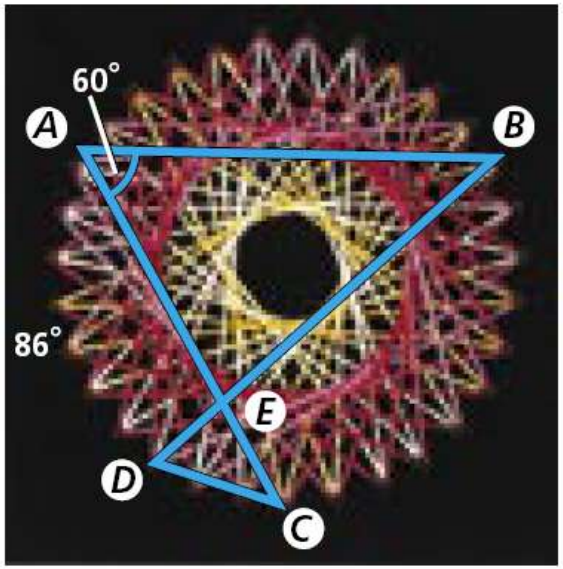
Check It Out! Example 2

Find $m\angle ABD$ and $m\overset{\frown}{BC}$ in the string art.

$$m\angle ABD = \frac{1}{2}m\overset{\frown}{DA}$$ *Inscribed ∠ Thm.*

$$= \frac{1}{2}(86°)$$ *Substitute.*

$$= 43°$$



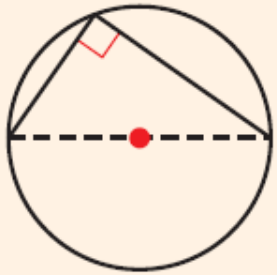
$$m\angle CAB = \frac{1}{2}m\overset{\frown}{BC}$$ *Inscribed ∠ Thm.*

$$60° = \frac{1}{2}m\overset{\frown}{BC}$$ *Substitute.*

$$m\overset{\frown}{BC} = 120°$$

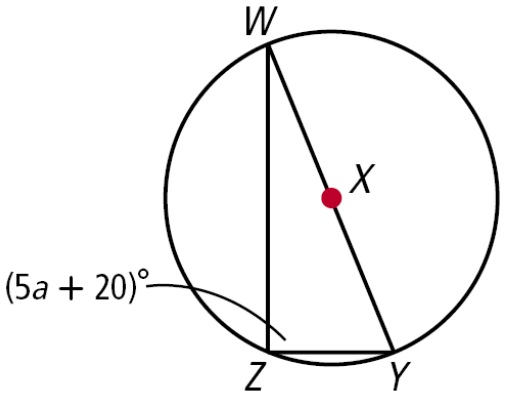
Theorem 11-4-3

An inscribed angle subtends a semicircle if and only if the angle is a right angle.



Example 3A: Finding Angle Measures in Inscribed Triangles

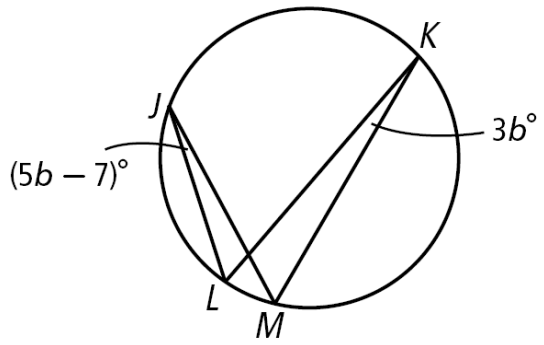
Find *a*.



$\angle WZY$ is a right angle	*$\angle WZY$ is inscribed in a semicircle.*
$m\angle WZY = 90°$	*Def of rt. $\angle$*
$5a + 20 = 90$	*Substitute $5a + 20$ for $m\angle WZY$.*
$5a = 70$	*Subtract 20 from both sides.*
$a = 14$	*Divide both sides by 5.*

Example 3B: Finding Angle Measures in Inscribed Triangles

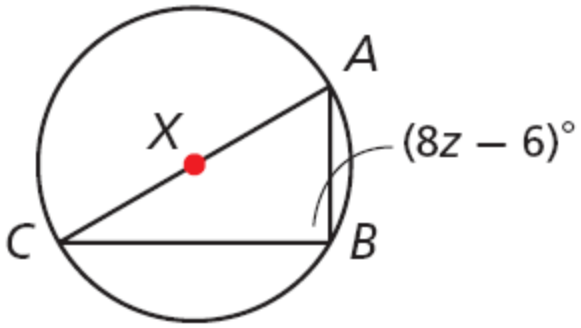
Find m∠*LJM*.



$m\angle LJM = m\angle LKM$	*m∠LJM and m∠LKM both intercept $\overset{\frown}{LM}$.*
$5b - 7 = 3b$	*Substitute the given values.*
$2b - 7 = 0$	*Subtract 3b from both sides.*
$2b = 7$	*Add 7 to both sides.*
$b = 3.5$	*Divide both sides by 2.*
$m\angle LJM = 5(3.5) - 7 = 10.5°$	*Substitute 3.5 for b.*

Check It Out! Example 3a

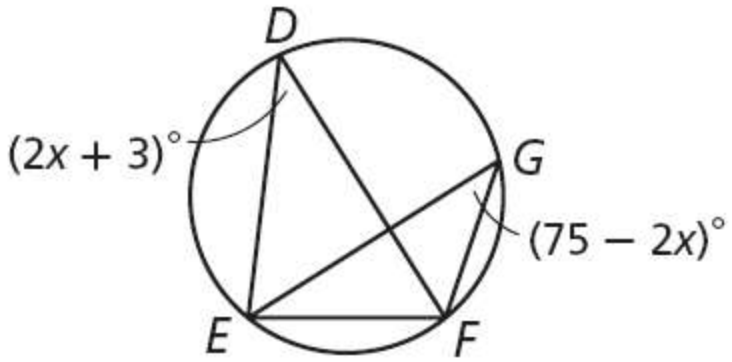
Find *z*.



$\angle ABC$ is a right angle	*$\angle ABC$ is inscribed in a semicircle.*
$m\angle ABC = 90°$	*Def of rt. $\angle$*
$8z - 6 = 90$	*Substitute.*
$8z = 96$	*Add 6 to both sides.*
$z = 12$	*Divide both sides by 8.*

Check It Out! Example 3b

Find m∠*EDF*.



$m\angle EDF = m\angle EGF$ — *m∠EGF and m∠EDF both intercept $\overset{\frown}{EF}$.*

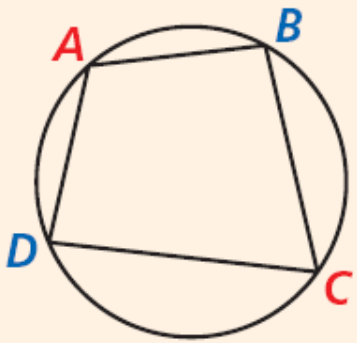
$2x + 3 = 75 - 2x$ — *Substitute the given values.*

$4x = 72$ — *Add 2x and subtract 3 from both sides.*

$x = 18$ — *Divide both sides by 4.*

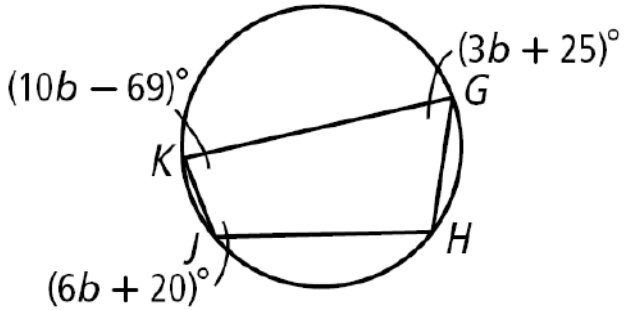
$m\angle EDF = 2(18) + 3 = 39°$

Theorem 11-4-4

THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.	 *ABCD* is inscribed in ⊙*E*.	∠*A* and ∠*C* are supplementary. ∠*B* and ∠*D* are supplementary.

Example 4: Finding Angle Measures in Inscribed Quadrilaterals

Find the angle measures of *GHJK*.

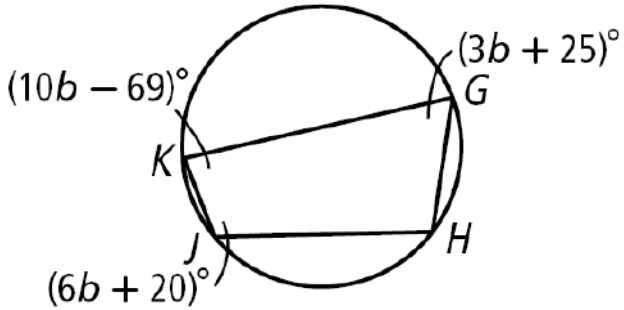


Step 1 Find the value of *b*.

$m\angle G + m\angle J = 180°$	*GHJK is inscribed in a ⊙.*
$3b + 25 + 6b + 20 = 180$	*Substitute the given values.*
$9b + 45 = 180$	*Simplify.*
$9b = 135$	*Subtract 45 from both sides.*
$b = 15$	*Divide both sides by 9.*

Example 4 Continued

Step 2 Find the measure of each angle.



$m\angle G = 3(15) + 25 = 70°$ *Substitute 15 for b*

$m\angle J = 6(15) + 20 = 110°$ *in each expression.*

$m\angle K = 10(15) - 69 = 81°$

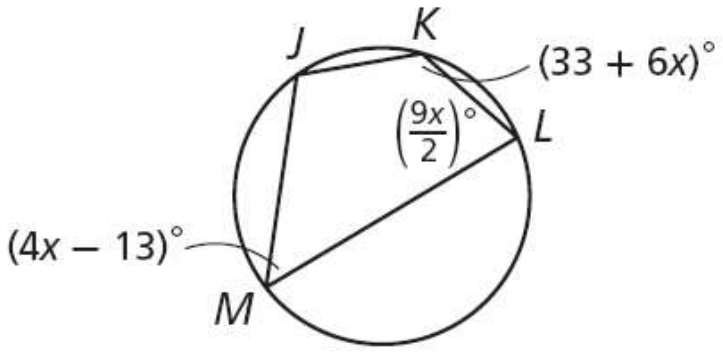
$m\angle H + m\angle K = 180°$ *∠H and ∠K are supp.*

$m\angle H + 81° = 180°$ *Substitute 81 for m∠K.*

$m\angle H = 99°$ *Subtract 81 from both sides*

Check It Out! Example 4

Find the angle measures of *JKLM*.



Step 1 Find the value of *b*.

$m\angle M + m\angle K = 180°$ *JKLM is inscribed in a ⊙.*

$4x - 13 + 33 + 6x = 180$ *Substitute the given values.*

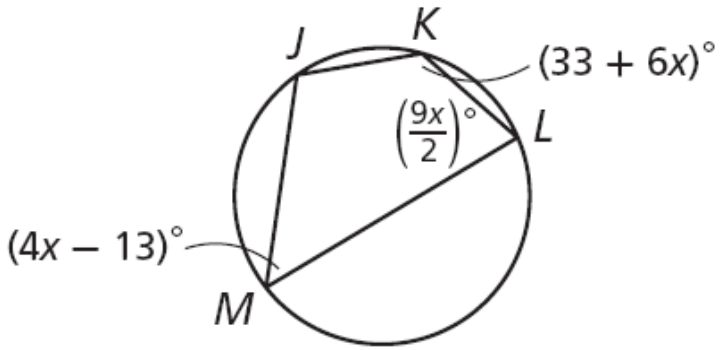
$10x + 20 = 180$ *Simplify.*

$10x = 160$ *Subtract 20 from both sides.*

$x = 16$ *Divide both sides by 10.*

Check It Out! Example 4 Continued

Find the angle measures of *JKLM*.



Step 2 Find the measure of each angle.

$m\angle M = 4(16) - 13 = 51°$

$m\angle K = 33 + 6(16) = 129°$

$$m\angle L = \frac{9(16)}{2} = 72°$$

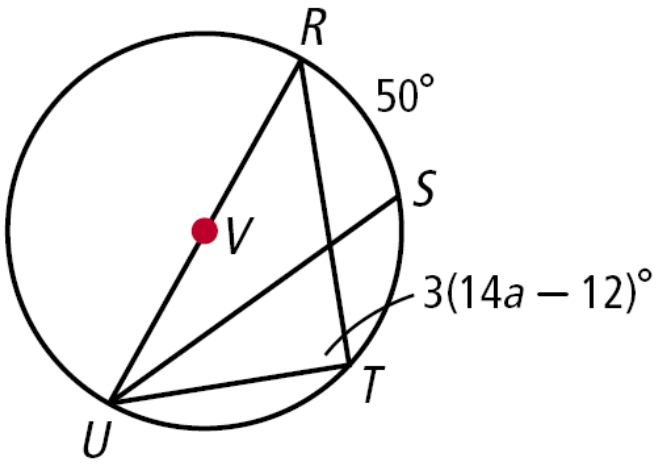
$m\angle J = 360° - 252° = 108°$

Lesson Quiz: Part I

Find each measure.

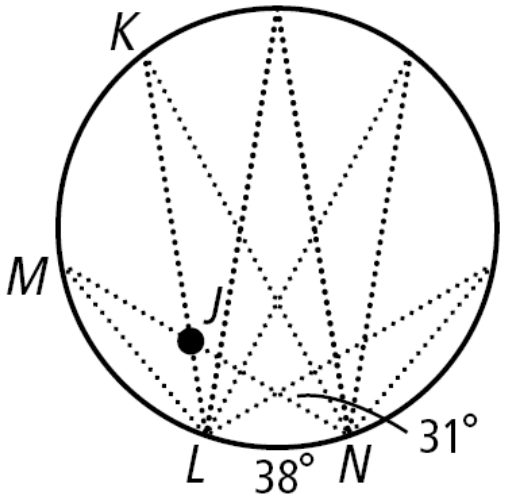
1. $\angle RUS$ 25°

2. a 3



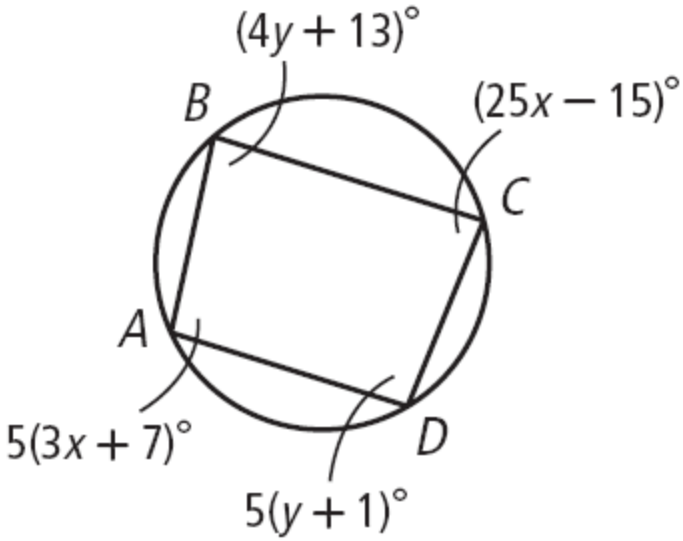
Lesson Quiz: Part II

3. A manufacturer designs a circular ornament with lines of glitter as shown. Find m∠*KJN.*



130°

4. Find the angle measures of *ABCD*.



m∠ *A* = 95°
m∠ *B* = 85°
m∠ *C* = 85°
m∠ *D* = 95°