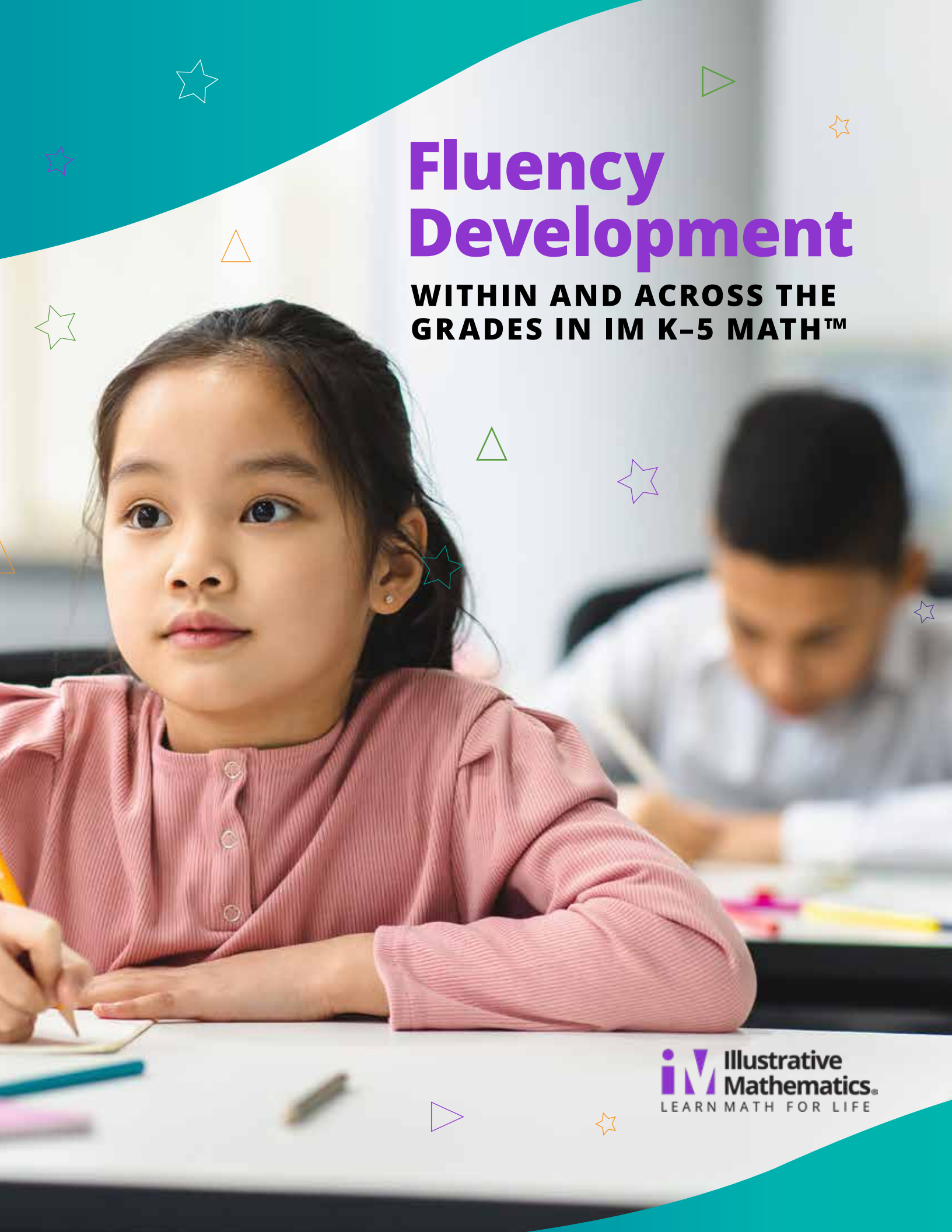
Fluency Development

WITHIN AND ACROSS THE GRADES IN IM K–5 MATH™



Illustrative Mathematics®
LEARN MATH FOR LIFE

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PART 1

FLUENCY DEVELOPMENT WITHIN AND ACROSS THE GRADES IN IM K–5 MATH™

Addition and Subtraction

By Sarah Caban, Grade 5 Lead Writer and Dionne Aminata, Grade 2–5 Lead Writer

"When do students practice their math facts? How is math fluency assessed?"

Current and prospective IM K-5 Math™ teachers everywhere

We appreciate these questions for a couple of reasons: 1) We took great care in thinking through fluency development in the design of the curriculum, and we are eager to share how we did it, and 2) We realize that the way we have incorporated fluency development is different from most curricula, and it may be difficult to recognize.

To outline how the IM K–5 Math™ curriculum focuses on fluency, we have a four-part series of blog posts. This first post is a deep dive into our Grade 1 course to show how students develop addition and subtraction fluency

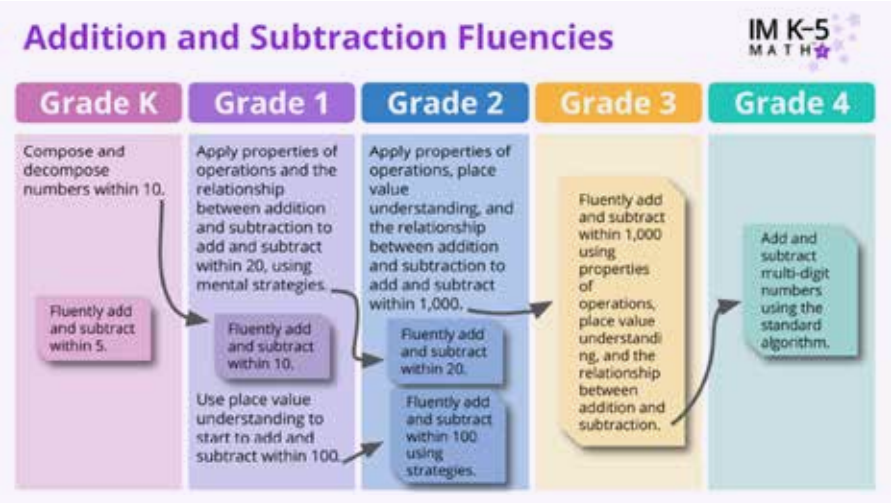
within 10. The next post will highlight the development of procedural fluency with addition and subtraction algorithms. The final post will show the progression of fluency development in multiplication and division.

We hope that this series of blog posts serves to bring to light the intentional design that supports students' progress toward fluency over time, both within and across the grades. Also included are ways for teachers track and assess students' math fluency development over the course of the school year.

We define procedural fluency as "using procedures flexibly, accurately, efficiently, and appropriately."

We define procedural fluency as **"using procedures flexibly, accurately, efficiently, and appropriately."** (**Adding It Up: Helping Children Learn Mathematics** from the National Research Council, 2001) In the IM K–5 Math™ curriculum, the representations, strategies, and algorithms that are used are purposefully designed to build a coherent progression where conceptual understanding and procedural fluency develop in parallel. This progression develops within and across grade levels.

The progression of procedural fluency with addition and subtraction of whole numbers spans Kindergarten through fourth grade. Each of these grade levels has specific expectations for procedural fluency that are interrelated.



The coherence of the curriculum is designed for students of any given grade to work towards accurately and efficiently operating with one set of numbers in the context of building conceptual understanding within a larger set of numbers. This coherence is evident in the way IM K–5 Math intends for students to come to know their facts at any given grade level.

1. Students learn the meaning of the operations and the relationships between them.
2. Students start to learn their facts.
3. Students relate more complicated facts to simpler ones.
4. Students know their facts.

FLUENCY WITH ADDITION AND SUBTRACTION WITHIN 10

To better understand the intentional design of the curriculum, we will share how we thought about fluency development in grade 1. Let's look at how students come to know their facts within 10.

1. Students learn the meaning of the operations and the relationships between them.

The foundation for fluently adding and subtracting within 10 is built in kindergarten, when students first learn about the meaning of addition and subtraction through story problems. The fluency focus of kindergarten is to learn to accurately and efficiently add and subtract within 5, which supports the grade 1 work of composing and decomposing numbers within 10.

The foundation for fluently adding and subtracting within 10 is built in kindergarten, when students first learn about the meaning of addition and subtraction through story problems.

In Unit 5 of our Kindergarten course, students compose numbers to 10 in ways that make sense to them. For example, in **Lesson 13, Activity 2,** students are given a number less than 10, and work with a partner to determine the part needed to make 10. They then write an equation for each 10 they make using the frame 10 = ___ + ___. Students are encouraged to use their fingers, counters, or 10-frames to help them determine the parts needed to make 10.

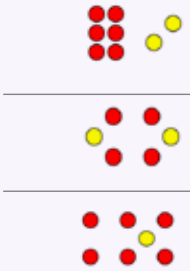
During the synthesis of this activity, it is suggested that teachers highlight the different strategies used based on the given number. This important step supports students' development of flexibility. As they hear other students sharing their strategies, they may learn a strategy that is more efficient, and add that strategy to their repertoire.

2. Students start to learn their facts.

With a foundational understanding of addition and subtraction, students begin to know a few facts. Unit 1 of our Grade 1 course begins with activities and centers focused on addition and subtraction within 10. The work in this unit allows teachers to assess students' understanding of addition and subtraction, as well as their fluency with facts within 5, a kindergarten goal. To develop fluency within 10, students start with

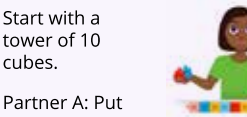
activities where they add or subtract 1 or 2, which not only encourages students to use strategies such as counting on or counting back, but also helps them relate addition and subtraction to counting.

Here are some examples of how students continue to practice sums within 10 during warm-up routines throughout Units 1 and 2. As displayed below, by Unit 2, students are noticing and using relationships between facts to mentally find differences.

Warm-Ups		
How Many Do You See? *Unit 1 Lesson 9*	*Number Talk* *Unit 1 Lesson 14*	*Number Talk* *Unit 2 Lesson 16*
	Find the value of each expression mentally. • 4 + 2 • 7 – 2 • 9 – 2 • 8 + 2	Find the value of each expression mentally. • 9 – 7 • 9 – 2 • 8 – 7 • 8 – 2

The centers within our materials play an important role in developing fluency. Centers afford students opportunities to practice by recalling facts, computing mentally, and writing true equations. In the examples that follow, students work with either cubes or number cards to make 10 with one part given. As students share their thinking and write the equations, they begin to learn new facts.

Centers afford students opportunities to practice by recalling facts, computing mentally, and writing true equations.

Centers	
What's Behind My Back? (Stage 2)	*Find the Pair (Stage 2)*
Directions: • Start with a tower of 10 cubes. • Partner A: Put the tower behind your back, and break off some cubes. Show your partner the rest of the tower. • Partner B: Record an addition equation with a blank to represent the missing cubes.  • Partner A: Ask "How many are behind my back? How do you know?" • Switch roles and repeat.	Directions: • Take 5 cards and put the rest in a pile face down. • Partner A: ○ Ask your partner for a number that can be added to one of your cards to make 10. ○ If they have the card, put the pair of cards down and fill in the equation. ____ + ____ = 10 ○ If they don't have that card, pick a card from a pile. • Take turns asking for cards. The partner with the most pairs wins the game.

In IM K–5 Math™, the overall design builds conceptual understanding while simultaneously developing students' procedural fluency.

3. Students relate more complicated facts to simpler ones.

According to NCTM, "effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems." (**Principles to Actions** NCTM, 2014) It's no secret that IM aims to help students develop conceptual understanding in mathematics. In IM K–5 Math™, the overall design builds conceptual understanding while simultaneously developing students' procedural fluency. Students practice their facts in ways that allow them to show or explain their thinking. They analyze and learn to use representations, strategies, and algorithms in a purposeful and coherent way.

An example in the curriculum of the parallel development of procedural fluency and conceptual understanding is the way students transition from knowing a few facts to relating the facts they know to more complex facts. In Unit 3 of our Grade 1 course, students begin by naming the sums within 10 that they know. In Lesson 1, Activity 2, students practice their facts using cards with different addition expressions, such as 2 + 5 and 4 + 4. They sort them into the ones they can do right away and the ones they are still working on.

got it	not yet

During the activity synthesis of this lesson, teachers name the year-long fluency expectations for addition within 10, and support students in reflecting on the facts they know and the facts they still need to practice. This process supports students in building positive math identities and encourages them to connect new learning to what they already know.

Activity Synthesis
• "What patterns do you see in the sums you know? Why?" (I know all the +1 sums. I know all the sums of 10.) • "Which sums are more difficult to find?" (The sums that have two bigger numbers, like 4 + 5, are harder.) • "By the end of the year, you are going to be able to quickly tell the sum for all the addition expressions within 10."

As the unit continues, students deepen their understanding of the commutative property, equivalence, and the relationship between addition and subtraction, and apply this understanding to use simpler facts to find more complex ones.

4. Students know their facts.

Over time, after a variety of experiences where they have shared their developing thinking, learned new facts, and practiced relating more complex facts to simpler ones, students begin to know more facts.

Throughout Units 4–8 in our Grade 1 course, students continue to practice their addition and subtraction facts during centers and warm-up activities. They also have multiple opportunities to reflect on which facts they know and which facts they are still learning.

The example below shows this practice continuing through the final unit of the course. In **Lessons 1** and **2** of Unit 8, students make note of the sums and differences they know from memory, and create cards to practice the facts they don't know yet.

Grade 1, Unit 8, Lesson 2, Activity 3

Color the differences you know the value of from memory.

10-0	10-1	10-2	10-3	10-4	10-5	10-6	10-7	10-8	10-9	10-10
9-0	9-1	9-2	9-3	9-4	9-5	9-6	9-7	9-8	9-9	
8-0	8-1	8-2	8-3	8-4	8-5	8-6	8-7	8-8		
7-0	7-1	7-2	7-3	7-4	7-5	7-6	7-7			
6-0	6-1	6-2	6-3	6-4	6-5	6-6				
5-0	5-1	5-2	5-3	5-4	5-5					
4-0	4-1	4-2	4-3	4-4						
3-0	3-1	3-2	3-3							
2-0	2-1	2-2								
1-0	1-1									

- Write each difference that is not colored, on an index card.
- Use the cards to practice these differences.
- Write an addition equation on the card that can help you find the value of the difference.

ASSESSING FLUENCY

Because fluency is developed over time and practice takes place in many different ways in our curriculum, student progress with fact fluency may seem difficult to measure. To address this challenge, IM K–5 Math™ includes monitoring sheets for each grade level that align with the section goals of each unit. The example below shows the progression of grade 1 monitoring sheets over the course of the year that support teachers with assessing students' progress toward fluency within 10.

UNIT 1 Students start to learn their facts	*Build toward fluency by adding and subtracting within 10, in a way that makes sense to them.* **Possible Addition Methods:** • Recognize the number of dots without counting. • Count all to find the sum. • Count on to find the sum • Use their knowledge of the count sequence to know certain sums. • Know certain sums. **Possible Subtraction Methods:** • Represent all, then cross off or remove to find the difference. • Count back to find the difference. • Use their knowledge of the count sequence to know certain differences. • Know certain differences.
UNIT 3 Students relate more complicated facts to simpler ones.	*Build toward fluency with adding and subtracting within 10.* • Count on to find the sum • Know certain sums. • Take away to find the difference. • Count up to find the difference. • Know certain differences. • Use the relationship between addition and subtraction to find the difference. • Use known sums to adjust expressions and find the sum on difference.
UNIT 8 Students know their facts.	*Fluently add and subtract within 10.* • Add fluently within 10. • Subtract fluently within 10.

A THOUGHTFUL APPROACH TO DEVELOPING STUDENTS' MATH FLUENCY

To our current and prospective IM K–5 Math™ teachers, we hope we were able to display the care that went into designing a curriculum that intentionally distributes opportunities for students to practice their math facts in activities, centers, practice problems, and warm-ups. (Read more about **distributed practice**.) Furthermore, with monitoring sheets that align with the fluency requirements for each grade level, teachers can track and assess student progress.

Our approach to math fluency is aligned with our vision "to create a world where all learners know, use, and enjoy mathematics."

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NEXT STEPS

How do students develop fluency with addition and subtraction within 5? Within 20? We encourage you to look at the curriculum for similar progressions in **Kindergarten** and **Grade 2**.

Watch **this video** where lead writer Dionne Aminata shares the intentional design of the curriculum and students show their accuracy, efficiency, and flexibility with math facts. You can also learn more about **fluency in IM K-5 Math™** on the IM website.

PART 2

FLUENCY DEVELOPMENT WITHIN AND ACROSS THE GRADES IN IM K–5 MATH™

Addition and Subtraction

By Sarah Caban, Grade 5 Lead Writer and Dionne Aminata, Grade 2–5 Lead Writer

"We have to break from the notion that learning mathematics must be a linear and procedural endeavor mastered through rote practice and memorization. Instead, we must recognize and emphasize that interconnected concepts lead to stronger foundations in mathematics and stronger personal and mathematical identities."

TODOS: Mathematics for ALL (2020)

Because of the problem-based structure of our curriculum, students' ideas play an integral role in the progression of learning in IM K–5 Math™. Starting with the belief that all students are capable learners with prior skills and funds of knowledge, we designed our curriculum to give teachers opportunities to gauge what students already know by allowing them to play with mathematical ideas before formalizing concepts. Students learn math by doing math, even to develop procedural fluency with algorithms.

During the IM K-5 Math Beta Pilot, **Kaylee Rivera-Ramirez interviewed her mom** as a part of an assignment in Meghan Codere's 3rd grade class. Students were invited to ask a family member how they learned to subtract.

Starting the addition and subtraction unit in this way celebrates the knowledge of students' families, promotes positive math identities, and provides opportunities for teachers to build cultural knowledge.

In our **last post** we shared how the curriculum design supports the development of fact fluency. Focusing on the fluency requirement in grade 1—fluently add and subtract within 10—we highlighted places in our Grade 1 course where students learn and practice their facts, building from the facts they know and their conceptual understanding of numbers and operations. In this post we shed light on how the curriculum supports students to develop procedural fluency with addition and subtraction algorithms across grades 2–5.

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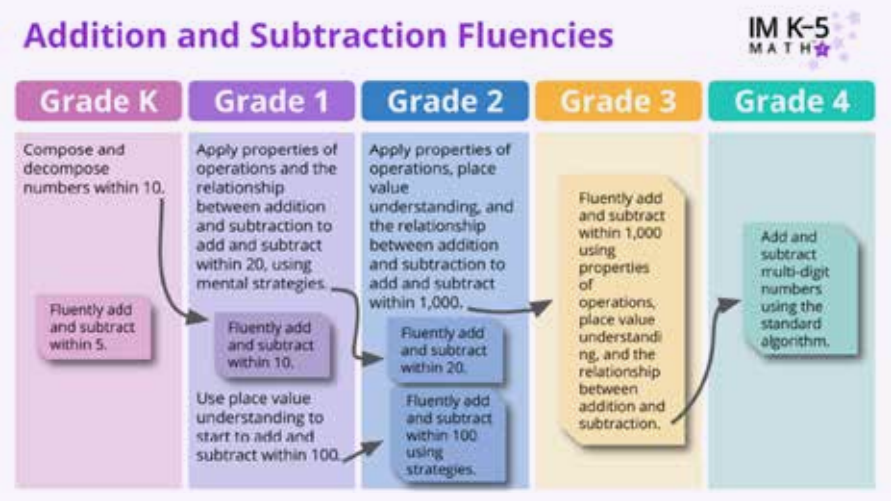
FLUENCY WITH ADDITION AND SUBTRACTION ALGORITHMS IN GRADES 2–5

Beginning in grade 2, as students develop fluency with facts within 20, there is a coinciding progression towards developing fluency with the standard algorithm for addition and subtraction of multi-digit whole numbers by the end of grade 4.



In IM K–5 Math™, the process for developing fluency with an algorithm is as follows:

1. Students operate in ways that make sense to them.
2. Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.
3. Students know and use the standard algorithm.

With this process, students begin with what they know and build from there. They make sense of new strategies, representations, and algorithms before being asked to use them. Below we highlight the progression towards procedural fluency with the addition and subtraction algorithms in the curriculum across grades 2–5.

1. Students operate in ways that make sense to them.

When students enter grade 2, they have a developing understanding of place value, the properties of operations, and the relationship between addition and subtraction. They apply this knowledge to accurately and efficiently add and subtract within 100. As they work towards this goal, they also learn to use strategies based on place value, the properties of operations, and the relationship between addition and subtraction to add and subtract within 1,000.

This progression of fluency development in our Grade 2 course can be most clearly seen by looking at the warm-up activities throughout the year. Students are encouraged to think flexibly and use strategies that make sense to them. Teachers can use these instructional routines to understand student thinking and assess math fluency.

Grade 2 Progression of Warm-up Activities

Unit 2 ***Lesson 4***	**Unit 5** ***Lesson 3***	**Unit 5** ***Lesson 5***	**Unit 7** ***Lesson 1***	**Unit 8** ***Lesson 9***
Find the value of each expression mentally. 6 – 3 66 – 3 66 – 30 66 – 33	Find the value of each expression mentally. 42 + 42 21 + 63 50 + 34 48 + 36	Decide if each statement is true or false. Be prepared to explain your reasoning. 800 + 90 + 7 = 897 156 = 50 + 100 + 6 407 = 70 + 400 632 = 22 + 10 + 600	Find the value of each expression mentally. 586 – 6 586 – 8 434 – 5 352 – 4	Find the value of each expression mentally. 9 + 5 20 + 30 29 + 35 229 + 435

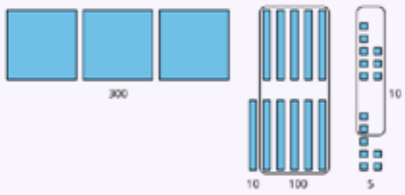
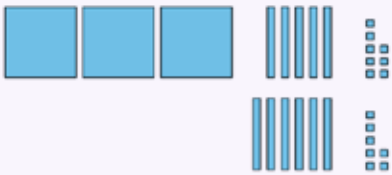
2. Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.

After several units of deep study, building numeracy and flexibility with strategies for operating with large numbers, students begin to analyze and try written methods. In Unit 7 of Grade 2, students use base-ten blocks and diagrams to connect what they know about place value, the properties of operations, and the relationship between addition and subtraction to equations. They use these tools to represent adding hundreds to hundreds, tens to tens, and ones to ones, and learn that when they add or subtract, they may need to compose or decompose a ten, a hundred, or both.

In the example below, students analyze two different strategies to find 358 + 67. They make connections between composing a ten and a hundred using a base-ten diagram, and using a written method.

Grade 2, Unit 7, Lesson 8

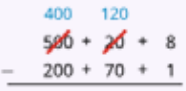
Priya and Lin were asked to find the value of 358 + 67.
What do you notice about their work?

Priya's Work	Lin's Work
	

IM–5 Math™ is structured so conceptual understanding can be developed alongside, or even after, students have learned how to execute procedures.

These concepts continue to be developed in grade 3, when students practice using algorithms that are based on place value and support the progression towards adding and subtracting larger numbers and decimals. In our Grade 3 course, students start with algorithms that show expanded form, and then move toward algorithms that are more efficient. Students may come to know and use the standard algorithms for addition and subtraction at different times. IM–5 Math™ is structured so conceptual understanding can be developed alongside, or even after, students have learned how to execute procedures.

The examples below are from our Grade 2 and Grade 3 courses. They show how students examine algorithms by making connections to the strategies they know. Students use their understanding of place value and properties of operations to make sense of the algorithms and then compute.

Grade 2, Unit 7, Lesson 15	Grade 3, Unit 3, Lesson 10
Elena's thinking:  7 hundreds 2 tens 6 ones – 5 hundreds 5 tens 8 ones 1. Use base-ten blocks or a base-ten diagram to show Elena's steps. 2. Finish Elena's work to find the value of 726 – 558. 3. What is another way you could use numbers or equations to show subtracting by place to find the value of 726 – 558?	Andre and Clare found the value of 528 – 271. How they started their work is shown. Andre's algorithm  Clare's algorithm  1. Complete both algorithms to find the difference. 2. Andre and Clare started their subtraction in different ways. How did their way of starting affect the steps needed to find the difference?

3. Students know and use the standard algorithm.

In grade 4, students build from the written methods learned in grades 2 and 3 to understand and use the standard algorithm to add and subtract multi-digit numbers. In Unit 4 of our Grade 4 course, students note the similarities and differences between the algorithm in expanded form and the standard algorithm. They make sense of the standard algorithm by analyzing it before being asked to try it for themselves.

Grade 4, Unit 4, Lesson 18

Analyze the strategies. Discuss with your partner:

- What is happening in each strategy?
- How are they alike? How are they different?

Strategy A:

$$\begin{array}{rrrrrrrrrr} & 10{,}000 & + & 7{,}000 & + & 300 & + & 70 & + & 5 \\ + & 10{,}000 & + & 4{,}000 & + & 0 & + & 20 & + & 4 \\ \hline & 20{,}000 & + & 11{,}000 & + & 300 & + & 90 & + & 9 \end{array} = 31{,}399$$

Strategy B:

$$\begin{array}{r} {\scriptstyle 1} \\ 17{,}375 \\ +\;14{,}024 \\ \hline 31{,}399 \end{array}$$

Grade 4, Unit 4, Lesson 20

Use the standard algorithm to find the value of the difference.

$$\begin{array}{r} 173{,}225 \\ -\;114{,}329 \\ \hline \end{array}$$

Students consider different strategies for approaching multi-digit subtraction, including leveraging the relationship between addition and subtraction.

ASSESSING FLUENCY

Beyond reviewing student computations, teachers can assess students' procedural fluency with an algorithm in a number of ways. Students can be tasked with analyzing the (correct or incorrect) work of a fictional student, or students can be asked to compare and contrast two different computational methods.

In the Grade 4 example below, students attend to potential errors in using the algorithm, particularly when it is necessary to decompose or compose a base-ten unit multiple times, as in the case when subtracting from a number with zeros. Students consider different strategies for approaching multi-digit subtraction, including leveraging the relationship between addition and subtraction.

Grade 4, Unit 9, Lesson 4

To find the value of 20,000 – 472, Priya and Han set up their calculations differently.

Priya's work

$$\begin{array}{r} 20,000 \\ -\quad 472 \\ \hline \end{array}$$

Han's work

$$\begin{array}{r} 472 \\ +\quad\quad\quad \\ \hline 20,000 \end{array}$$

1. Use both methods to find the difference of 20,000 and 472.
2. Which method do you prefer? Or do you prefer another way? Explain your reasoning.
3. Find the value of 50,400 – 1,389. Show your reasoning.

Across the grades you can see how students progress from using strategies that make sense to them to using strategies and written methods based on generalizable concepts. The process for developing procedural fluency with algorithms is evident in the look-fors on the monitoring sheets that teachers can use to track student progress.

GRADE 2, UNIT 7 Students operate in ways that make sense to them.	• Add and subtract numbers within 1.000 without composition or decomposition and use strategies based on the relationship between addition and subtraction and the properties of operations. • Add numbers within 1,000 using strategies based on place value understanding, including composing a ten or hundred. • Subtract numbers within 1,000 using strategies based on place value understanding, including composing a ten or hundred.
GRADE 3, UNIT 3 Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.	• Fluently add within 1,000 using algorithms based on place value and properties of operations. • Use place value understanding to compose and decompose numbers. • Fluently subtract within 1,000 using algorithms based on place value, properties of operations, and the relationship between addition and subtraction.
GRADE 4, UNIT 4 Students know and use the standard algorithm.	• Add and subtract multi-digit whole numbers using the standard algorithm.

BUILDING FLEXIBILITY WITH PROCEDURES

The development of fluency with addition and subtraction in kindergarten through grade 4 is critical for operations with decimals in grade 5. While developing an understanding of place value to the right of the decimal, students learn that they can apply what they know about properties of operations, place value understanding, and the relationship between addition and subtraction with whole numbers to flexibly add and subtract decimals to the hundredths place. They use place value understanding to decide whether sums and differences are reasonable and to ensure that the digits in the numbers are aligned correctly when using the standard algorithm.

as students learn to add and subtract multi-digit numbers using the standard algorithm with accuracy and efficiency, they continue to practice and gain flexibility with mental math during warm-up routines.

Throughout this progression, as students learn to add and subtract multi-digit numbers using the standard algorithm with accuracy and efficiency, they continue to practice and gain flexibility with mental math during warm-up routines. This practice not only helps students build and maintain numeracy, but it also reminds students that the standard algorithm may not always be the most appropriate or efficient strategy.

Number Talks with Multi-digit Numbers		
Grade 3, Unit 6, Lesson 14	*Grade 4, Unit 4, Lesson 10*	*Grade 5, Unit 5, Lesson 12*
Find the value of each expression mentally. • 306 + 199 • 318 + 297 • 275 + 325 • 275 + 329	Find the value of each expression mentally. • 650 + 75 • 5,650 + 75 • 50,650 + 75 • 500,650 + 75	Find the value of each expression mentally. • 1.00 + 0.99 + 0.02 • 1.99 + 0.02 • 1.99 + 0.03 • 1.99 + 0.13

A COHERENT PATHWAY TOWARDS ACCURACY, EFFICIENCY, AND FLEXIBILITY

The progression towards fluency with addition and subtraction in IM K–5 Math™ is carefully developed to empower students to connect what they know to a coherent story about addition and subtraction concepts and procedures.

Along the way, we support the development of positive mathematical identities by reminding students of the knowledge they bring to each lesson as they engage in warm-up routines. These routines allow them to flex their mental math muscles and continue to use strategies that make sense to them. This consistent practice, in tandem with a coherent progression of learning, provides students with the tools they need to compute fluently. Not only do they become equipped to use procedures accurately, efficiently, and flexibly, but they also gain the ability to determine, from the variety of strategies at their disposal, which ones are most appropriate for a given problem.

NEXT STEPS

Watch **this video** where lead writer Dionne Aminata shares the intentional design of the curriculum and students show their accuracy, efficiency, and flexibility with math facts.

We invite you to take a look at **the curriculum** to see how it supports fluency development. In our next blog post we will discuss the progression towards fluency with multiplication and division procedures and algorithms.

You can also learn more about **fluency in IM K-5 Math™** on the IM website. ★

PART 3

FLUENCY DEVELOPMENT WITHIN AND ACROSS THE GRADES IN IM K–5 MATH™

Multiplication and Division

By Sarah Caban, Grade 5 Lead Writer and Dionne Aminata, Grade 2–5 Lead Writer

"The idea of being a 'math person' is based on two pervasive world views; first, the so-called 'fixed' mindset, that what one knows about math cannot change meaningfully over time through study and practice. Second, it is based on a narrow understanding of what it means to know and do math; 'being a math person' means memorizing formulas, computing quickly in your head, and being fluent with algebraic procedures only."

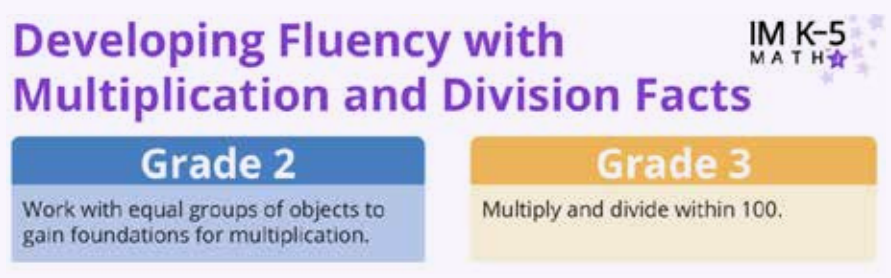
Rolanda Baldwin, "What's Your Math Identity?" (2020)

When designing IM K–5 Math™, we thought deeply about the potential impact of our curriculum on students' mathematical identities. We believe all students have valuable experiences and ideas, and the capacity to learn grade-level math. Throughout each course, as students develop their understanding of what it means to know and do math, they have opportunities to compute in ways that make sense to them, build their knowledge of concepts through analysis and collaboration, and apply their knowledge to compute with accuracy, flexibility, and efficiency.

In this post and the one that follows, we intend to share how students' mathematical identities play a role as they develop multiplication and division fluency within and across grades 2–5. In our last two posts—**part 1** and **part 2** on fluency development in addition and subtraction—we focused on addition and subtraction fluency development across grades K–5. We highlighted the processes for developing fluency with math facts and algorithms. Here we will show how these processes are evident in the design of the curriculum for developing fact fluency with multiplication and division.

FLUENCY WITH MULTIPLICATION AND DIVISION FACTS

By the end of grade 3, students develop fluency with multiplication and division facts within 100. As shown in the table, the process of developing this fluency begins in grade 2 as students work with equal groups and arrays.



Much like in the story of addition and subtraction, fluency with multiplication and division facts is developed through an intentionally coherent process in IM K–5 Math™.

1. Students learn the meaning of the operations and the relationships between them.
2. Students start to learn their facts.
3. Students relate more complicated facts to simpler ones.
4. Students know their facts.

During this process, students have opportunities to learn new concepts by building from what they already know, which is the nature of our problem-based curriculum. Our intention is that through these experiences, they begin to see themselves as valued contributors to the progression of learning in their classrooms and develop positive mathematical identities.

Below, we analyze student work from beta pilot classrooms at each stage of the process. By doing so, we can get a sense of how fact fluency and mathematical identities can be developed with IM K–5 Math™.

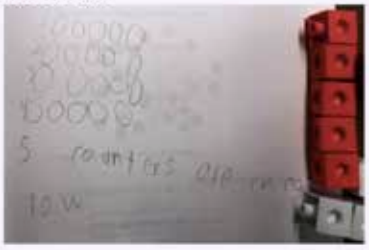
1. Students learn the meaning of the operations and the relationships between them.

Beginning in grade 2, students work with equal groups and sums of equal addends to build a foundation for understanding multiplication and division in grade 3. In the IM K–5 Math™ Grade 2 course, this work starts in Unit 8. Students first make sense of equal groups through a study of odd and even. They then use their developing understanding of equal groups to learn about arrays.The problem below is from Lesson 7, the first lesson on arrays. To offer students space to make sense of new ideas, they may represent their thinking in different ways. IM Certified Facilitator Maureen O'Connell taught this lesson to a group of second graders in Ipswich, MA. The table displays the work of four of her students

Grade 2, Unit 8, Lesson 7

Use 20 counters to make 4 rows with the same number in each row. How many counters are in each row?

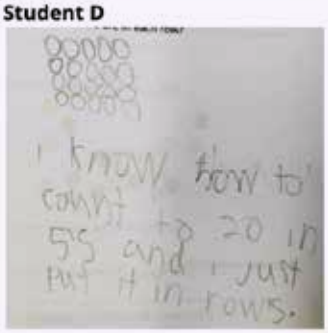
Student A



Student B

Student C

Student D



In the example above, each student created an array with 4 rows and a total of 20 counters. Because they could use what they already knew to respond in ways that made sense to them, students represented their arrays by drawing counters or using physical objects. Student A used notation that showed how they counted the number of rows, while Student B shared that their prior knowledge of skip-counting helped them draw the array with 5 in each row. During the activity synthesis, students are invited to share their developing thinking about the structure of arrays, and begin to recognize that 20 can be made with a sum of equal addends, 5 + 5 + 5 + 5.

Looking back at the prompt, only one student, Student A, has a written response to the question, "How many counters are in each row?" The activity synthesis also suggests that teachers ask this question to uncover student thinking. While the focus of this question is to help students understand the structure of arrays as having the same amount in each row, students begin to develop ideas about equal groups, which will support understanding of multiplication in grade 3. This question also serves as a precursor to understanding the relationship between multiplication and division.

Students start to learn their facts as they develop an understanding of the meaning of multiplication.

2. Students start to learn their facts.

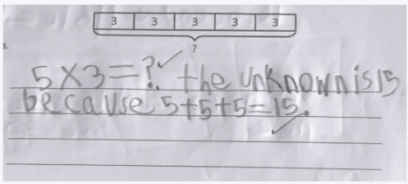
Students start to learn their facts as they develop an understanding of the meaning of multiplication. In Unit 1 of Grade 3, students learn that multiplication can mean finding the total number of objects in a groups of b objects each, and can be represented by a x b. To build from what students know about skip-counting from earlier grades, the curriculum starts grade 3 students with learning twos, fives, and tens.

In the example below from the beta pilot, students from Andrea Welch's class in Ipswich, MA make sense of a diagram showing 5 groups of 3. While both students write an equation with a symbol for the unknown and find the correct solution of 15, their reasoning is very different. Student A relates 5 x 3 to the sum 5 + 5 + 5 = 15. Student B decomposes 3 into 2 + 1, first counting each box by twos, then by one to get 10 and 5 more.

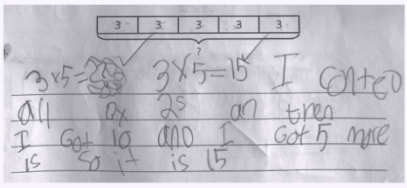
Grade 3, Unit 1, Beta Pilot

Write an equation with a symbol for the unknown to represent the diagram. Find the number that makes the equation true. Show your reasoning.

Student A



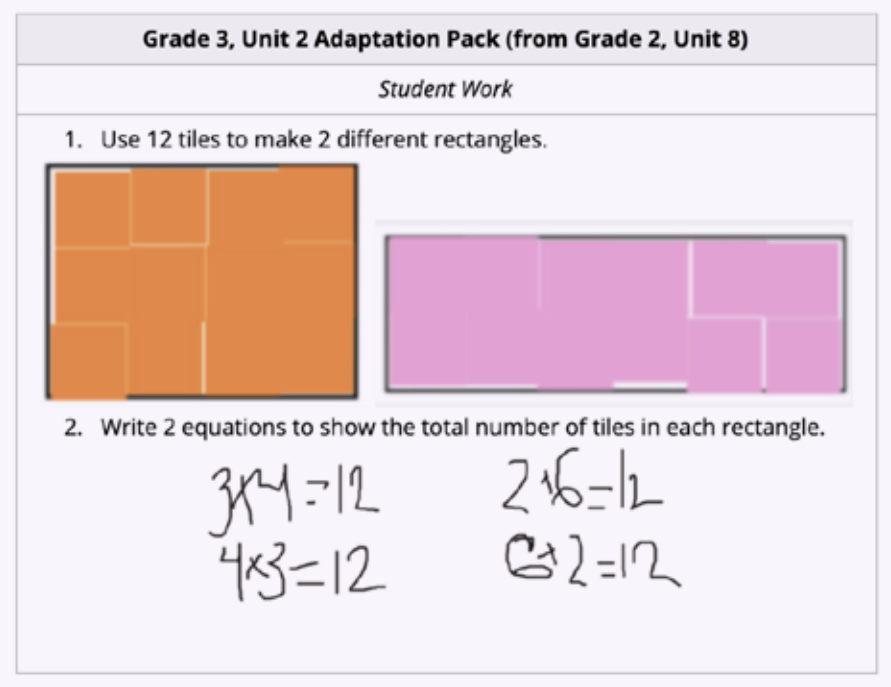
Student B



Without further investigation, one may conclude that both students are still developing an understanding that a x b represents a groups of b objects.

Student A reasons about 5 x 3 using 5 + 5 + 5 instead of 3 + 3 + 3 + 3 + 3, and Student B interprets the diagram with 5 groups of 3 as 3 x 5 = 15 instead of 5 x 3 = 15. Students are not expected to adhere to the convention of writing a groups of b objects as a x b rather than b x a, but if they describe groups, they are expected to do so accurately. As shown in Student A's work, knowledge of the commutative property starts to develop in Unit 1 of grade 3, and students are encouraged to use that reasoning to find products.

This next example is from the Grade 3, Unit 2 Adaptation Pack, which teachers used during the beta pilot to support unfinished instruction due to the pandemic. The lesson activity, which can be found in Grade 2, Unit 8, sets the stage for area understanding and a deeper understanding of multiplication. In working with arrays, this student from Amy Gookin's class in Portland, ME begins to notice the commutative property of multiplication.



While grade 2 students would likely write equations involving sums of equal addends, grade 3 students can write multiplication equations when this lesson is positioned after an introduction to the multiplication symbol in Grade 3, Unit 1.

In Unit 2, students relate the area of rectangles to multiplication. They see that rectangles can be tiled with squares in equal-size rows (or columns), so if the rectangle is 3 units by 4 units, there are 3 groups of 4 tiles or 4 groups of 3 tiles. The number of square units is 12, which can be represented by the equations 3 x 4 = 12 or 4 x 3 = 12. In later lessons, students come to understand that multiplying the side lengths of a rectangle gives the same number of squares as counting them.

3. Students relate more complicated facts to simpler ones.

As students come to know some facts, they can use their conceptual understanding to learn new facts. The progression of Number Talks below highlights how students practice this skill in grade 3 to develop fact fluency.

Multiplication Number Talks Progression in Grade 3, Units 1–3

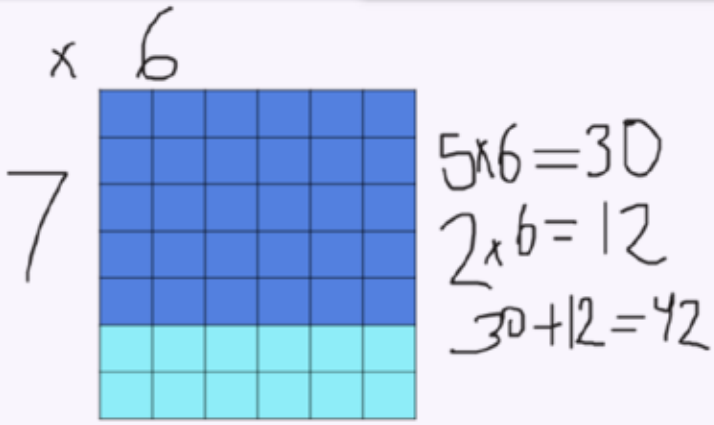
Unit 1, Lesson 14	*Unit 1, Lesson 15*	*Unit 1, Lesson 19*	*Unit 2, Lesson 10*	*Unit 3, Lesson 12*	*Unit 3, Lesson 16*	*Unit 3, Lesson 20*
1 x 5	1 x 10	10 x 2	5 x 2	2 x 6	5 x 7	20 + (2 x 3)
2 x 5	2 x 10	9 x 2	6 x 2	3 x 6	4 x 7	30 + (4 x 3)
3 x 5	3 x 10	8 x 2	5 x 6	2 x 7	6 x 7	50 + (8 x 3)
4 x 5	4 x 10	7 x 2	6 x 6	3 x 7	4 x 8	99 + (8 x 3)

After practicing their twos, fives, and tens, students conceptualize one more group, one less group, doubling, and halving to learn and practice their threes, fours, sixes, and then the rest of their facts.

In this example from Unit 4 Lesson 10, a grade 3 student from Amy Gookin's class in Portland, ME finds 6 x 7 by decomposing (or breaking apart) the 7 into 5 and 2, and then adding the two products.

Grade 3, Unit 4, Lesson 10 Cool Down

1. *Mark or shade the rectangle in a way that would help you find its area.*
2. *Write one or more expressions that can represent your work on the diagram and show how you find the area.*



The curriculum allows students to analyze and use area diagrams to develop understanding.

The curriculum allows students to analyze and use area diagrams to develop understanding.

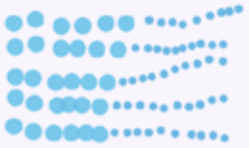
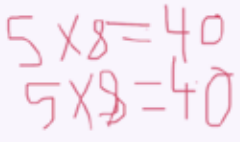
The curriculum allows students to analyze and use area diagrams to develop understanding. This strategy of relating more complex facts to simpler ones becomes helpful as students learn to multiply larger numbers. A student from Tabitha Eutsler's class in Springfield, MO, shared just that in an added reflection on this Lesson 13 cool-down.

Students begin to develop positive mathematical identities when we allow them to make sense of mathematics through problem solving, and give them space to use strategies that build from what they already know. The student above showed appreciation for a strategy that helped with efficiency. As students learn the importance of fact fluency, they develop a sense that certain strategies are more efficient than others. They become more confident when they can approach new problems and find solutions even when they don't know all of their facts from memory.

Throughout Unit 4, students use the relationship between multiplication and division to develop fluency with multiplication and division facts within 100. By Lesson 20, students are using the strategies they know to divide larger numbers. Below, students from Maureen O'Connell's class in Ipswich, MA show very different ways for finding the value of 80 ÷ 5.

Grade 3, Unit 4, Lesson 20

Find the value of 80 ÷ 5. Explain or show your reasoning. Organize it so it can be followed by others.

Student A	Student B	Student C
80 ÷ 5 = 16 	80 ÷ 5 = 16  1 dot = 2 I counted them by 2s	80 ÷ 5 = 16 

The activity synthesis for the activity above asks teachers to consider polling the class on the strategy they used. This teacher move offers students a moment to deepen conceptual understanding of the relationship between multiplication and division, and the properties of operations. Connecting their strategies to those of their peers also gives students room to celebrate their thinking and see themselves as doers of mathematics. As students share and discuss strategies, they begin to make decisions about the

strategies they would like to try, as some strategies may be more appropriate or more efficient than what they had previously used.

Deepening conceptual understanding in this way encourages flexible thinking. Students are eventually able to use these strategies mentally during instructional routines and centers. The Number Talk routines in the table below are strategically placed throughout the rest of the Grade 3 course to allow students to continue to practice these strategies and develop fact fluency.

Deepening conceptual understanding in this way encourages flexible thinking.

Number Talks Progression Relating Multiplication and Division in Grade 3, Units 4–8						
Unit 4, Lesson 8	*Unit 4, Lesson 20*	*Unit 5, Lesson 8*	*Unit 6, Lesson 8*	*Unit 6, Lesson 15*	*Unit 7, Lesson 11*	*Unit 8, Lesson 11*
4 x 10 40 ÷ 4 40 ÷ 10 60 ÷ 6	3 x 5 6 x 5 10 x 5 65 ÷ 5	12 ÷ 4 24 ÷ 4 60 ÷ 4 72 ÷ 4	30 ÷ 3 60 ÷ 3 63 ÷ 3 54 ÷ 3	80 ÷ 8 72 ÷ 8 96 ÷ 8 96 ÷ 4	5 x 5 10 x 5 2 x 5 85 ÷ 5	48 ÷ 2 48 ÷ 3 48 ÷ 4 48 ÷ 6

4. Students know their facts.

Throughout grade 3, students may be in different places along the continuum of fluency. Some students may be more efficient with certain facts. As we mentioned in our earlier post about developing fluency with addition and subtraction facts, conceptual understanding can be developed as a foundation for fact fluency or even after fluency has been achieved.

During Unit 4, students sort facts cards and reflect on the facts they know right away and the facts they are still working on. They continue to use these cards throughout the year to intentionally practice their facts and monitor their own progress. (Read more about how students come to know their multiplication facts in **this post**.)

Grade 3, Unit 4, Lesson 8 Card Sort		
1. Sort your products into one of these columns. a. know it right away b. can find it quickly c. don't know it yet		
2. Review your strategies and practice the products that are in the "don't know it yet" column.		
know it right away	can find it quickly	don't know it yet
Multiplication expressions I'm going to practice: 1. 2. 3. 4. 5.		

As they practice their facts and develop fluency during lessons, centers, and warm-ups, they take ownership of their learning and strengthen their mathematical identities. In **the video clip**, third-grader Viviana Ramirez from Katy, TX uses facts she knows from memory to accurately, flexibly, and efficiently divide within 100.

When Viviana pauses, she is taking time to think flexibly and use mental strategies. These pauses signal perseverance, and should not be characterized as having a lack of division fluency. Her perseverance is evidence of a positive mathematical identity, as she believes that she is capable of arriving at a correct response on her own, given time to think it through. The joyful smile on her face is also evidence.

DESIGNED TO DEVELOP POSITIVE MATHEMATICAL IDENTITIES AND FLUENCY

Flexible thinking is important for the development of positive mathematical identities. Opening up activities and discussions to include different ways of thinking gives students freedom to explore their ideas. Students build confidence as they practice using strategies based on their prior and developing knowledge of numbers and operations.

While it remains to be seen if each student who goes through our courses will eventually identify as a "math person", we are confident that our intentional design will help students realize that they already have important mathematical ideas that they can use to solve problems and learn new mathematical concepts.

NEXT STEPS

For more on multiplication fluency, read **this post** by Grade 3 Lead Author, Zack Hill.

How do students develop fact fluency with addition and subtraction? This post is part 3 of a four-part series on developing fluency with IM K–5 Math™.

In case you missed it, read **part 1** and **part 2** on fluency development in addition and subtraction.You can also learn more about fluency in IM K–5 Math™ on the IM website. ★

PART 4

FLUENCY DEVELOPMENT WITHIN AND ACROSS THE GRADES IN IM K–5 MATH™

Multiplication and Division

By Sarah Caban, Grade 5 Lead Writer and Dionne Aminata, Grade 2–5 Lead Writer

"Some students are never given the opportunity to engage with mathematics in rich and meaningful ways that emphasize critical thinking and problem solving. Moreover, educators often use mathematics assessments to make definitive judgments about students' competencies and abilities."

The Impact of Identity in K-8 Mathematics Learning and Teaching, Aguirre, et al., 2013

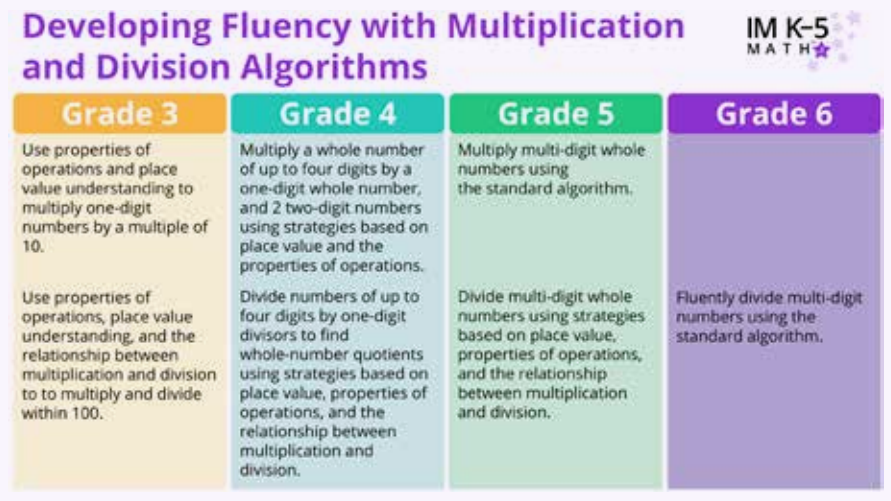
As a problem-based curriculum, IM K–5 Math™ aims to offer all students opportunities to problem-solve and think critically about new learning. By design, students are expected to approach each lesson using the mathematical knowledge and cultural experiences they already have to solve problems. The guidance given in the teacher-facing materials support teachers with using this student knowledge as a launching pad for new learning. This basic design holds true for each mathematical concept in the curriculum, even in the development of procedural skills and fluency.

Often labeled asset-based or student-centered, this problem-based approach can have a lasting and positive impact on students' mathematical identities. In this post we will highlight how procedural fluency with multiplication and division algorithms are developed within and across grades 3–5 in IM K-5 Math™. Along the way we will describe ways in which the lessons intend to position students as doers of mathematics and support the development of positive mathematical identities.

FLUENCY WITH MULTIPLICATION AND DIVISION ALGORITHMS

As students shift from multiplying and dividing single-digit to multi-digit numbers, they begin to see the need for increased accuracy, flexibility, and efficiency with operations. Unfortunately, students who demonstrate limited knowledge of basic facts are often prevented from engaging in more complex tasks. The practice of using mathematical gatekeepers such as basic facts assessments to limit student access to grade-level content disproportionately affects Black and Brown students and can adversely affect students' mathematical identities. (NCSM, TODOS 2016)

The IM K–5 Math™ curriculum intends to mitigate this practice through a coherent progression of units and lessons within and across the grades that allows students to develop procedural skills and conceptual understanding simultaneously. The Common Core Standards offer students two years of deep study to fluently use the standard algorithms for multiplication and division. The table below highlights the progression of learning from grade 3 to grade 6. Students start using the properties of operations and place value understanding to multiply in grade 3, which supports students to develop fluency with the standard algorithm for multiplication by the end of grade 5.



Fluency with the division standard algorithm is not expected until the end of grade 6, but students begin to use place value representations, the properties of operations, and their understanding of the relationship between multiplication and division to divide a multi-digit whole number by a single-digit number in grade 4.

In IM K–5 Math™, the process for developing fluency with an algorithm is the same for the four operations. In part 2 of this blog post series, we outlined the following steps:

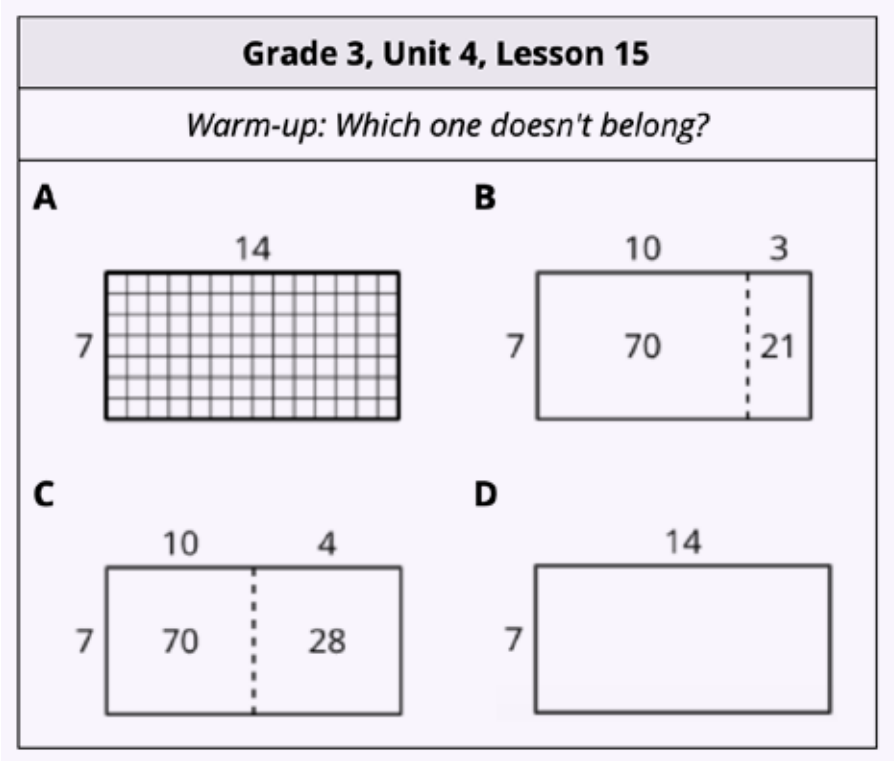
1. Students operate in ways that make sense to them.
2. Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.
3. Students know and use the standard algorithm.

These steps help to foster a positive mathematical identity because we start with what students know. We then center their ideas as they analyze new strategies, and give them space to make sense of algorithms before expecting fluency.

1. Students operate in ways that make sense to them.

Students begin to multiply multi-digit numbers in grade 3. In Unit 4 of our Grade 3 course, students see that rectangles can help them reason about and use strategies based on place value understanding and the distributive property to multiply. In an earlier unit they learned that the side lengths of a rectangle can represent the two factors and its area can represent the product.

This warm-up in Lesson 15 helps students recall these concepts and begin to articulate how place value is used to decompose factors when multiplying larger numbers.

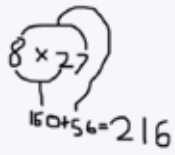


As students come to know these concepts, they gain flexibility in using them to approach more complex problems. In the example below, grade 4 students in a class in Portland, ME use place value reasoning and the properties of operations to find the value of 27 x 8. They use the strategy that makes the most sense to them.

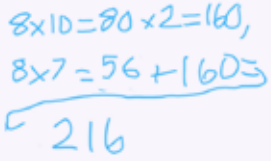
Grade 4, Unit 6, Lesson 5

A theater has 8 rows of seats and 27 seats in each row. How many seats are in the theater? Show your reasoning.

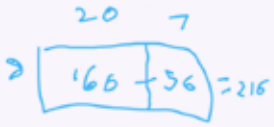
Student A



Student B



Student C



Each student arrived at the answer accurately and efficiently, using strategies that decompose 27 into 20 and 7 to find the product. As students share their strategies and reasoning, they can make connections between strategies, build flexibility, and solidify their conceptual understanding.

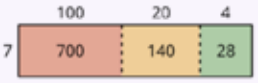
2. Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.

Beginning in Grade 4 of our curriculum, students learn an algorithm that records partial products vertically. As they analyze this new written method, they connect it to how they recorded partial products in rectangular diagrams. In Unit 6, Lesson 9 students try this new method only after deep analysis and discussion.

Grade 4, Unit 6, Lesson 9

Noah drew a diagram and wrote expressions to show his thinking as he multiplied two numbers. Later, Noah learned another way to record the multiplication.

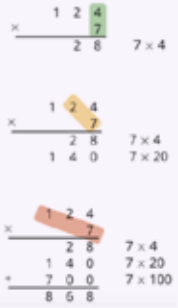
1. How does each expression represent Noah's diagram? Be prepared to share your thinking with a partner.



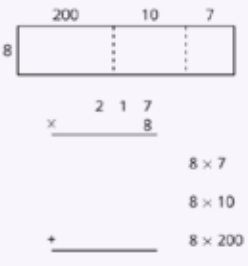
700 + 140 + 28 = 868

7×124
$7 \times (100 + 20 + 4)$
$(7 \times 100) + (7 \times 20) + (7 \times 4)$
$700 + 140 + 28$

2. Make sense of each step of the calculations and record your thoughts. Be prepared to explain Noah's steps to a partner.



3. Complete the diagram to find the value of 217 x 8. Use Noah's recording method to check your work.



In Unit 4 of our Grade 5 course, students engage in a similar process to make sense of the standard algorithm for multiplication. They recognize that they can use place value understanding to record partial products in a more condensed way.

In Lesson 6 students are given time to analyze, make connections, and build conceptual understanding of the standard algorithm before being asked to use it to find the value of the product.

Grade 5, Unit 4, Lesson 6

1. Find the value of 241 x 23.
2. Lin used the standard algorithm to find the value of 241 x 23. Here is her work:

$$\begin{array}{r} {\scriptstyle 1} \\ 241 \\ \times\ \ 23 \\ \hline {\scriptstyle 1} \\ 723 \\ +\ 4,820 \\ \hline 5,543 \end{array}$$

   a. Where do you see 241 x 3 in Lin's work?

   b. Where do you see 241 x 20 in Lin's work?

   c. What does the 1 above 241 represent in Lin's calculation?

3. Use the standard algorithm to find the value of 182 x 41.

As stated earlier, offering students space to use what they already know to learn new concepts is key to developing positive mathematical identities, and is a foundational part of the design of our problem-based curriculum. We see this play out in our approach to developing students' procedural fluency with division algorithms.

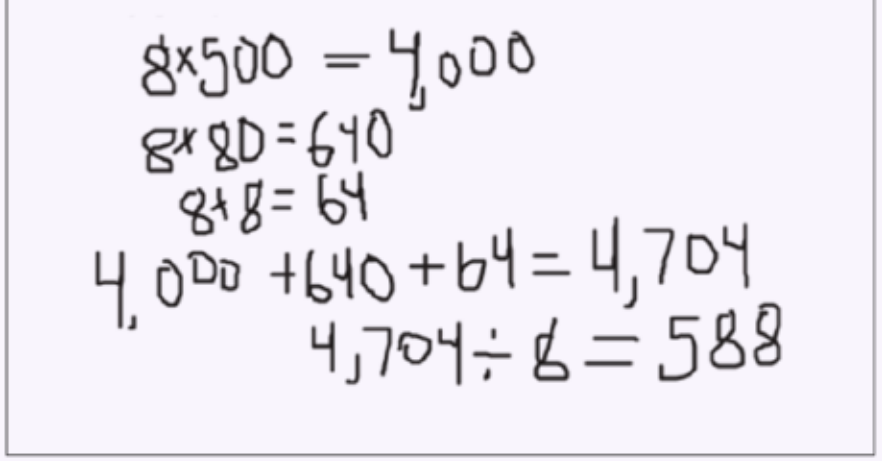
Offering students space to use what they already know to learn new concepts is key to developing positive mathematical identities, and is a foundational part of the design of our problem-based curriculum.

Beginning in Grade 4, students solve division problems in various situations, including those about equal-size groups, factors and multiples, and area of rectangles. These experiences reinforce students' understanding of the relationship between multiplication and division.

The example below opens the section on division in our Grade 5 course. Here, a student in a class in Portland, ME uses what they know about partial products to solve a division problem.

Grade 5, Unit 4, Lesson 10

There were 4,704 people at the record-breaking folk dance. How many groups of 8 dancers are there?



To begin the work towards procedural fluency with the standard algorithm for division by the end of grade 6, our Grade 4 and 5 courses allow students to continue using what they know about place value, the properties of operations, and the relationships between operations to divide multi-digit numbers. They learn to use a vertical recording method to organize partial quotients by first analyzing and discussing the new method, then trying it for themselves.

Grade 4, Unit 6, Lesson 18

Tyler uses a different method to find the value of 465 ÷ 5. Let's compare Priya's and Tyler's work.

Priya's method

$400 \div 5 = 80$
$60 \div 5 = 12$
$5 \div 5 = 1$
$465 \div 5 = 93$

Tyler's method

```
  93
   1
  12
  80
5)465
- 400   5 × 80
  65
- 60    5 × 12
   5
-  5    5 × 1
   0
```

1. How are Priya and Tyler's methods alike? How are they different? List as many similarities and differences as you can find.
2. Why do you think Tyler uses subtraction in his method?
3. Show how Tyler might record the process of finding the value of 428 ÷ 4.

Grade 5, Unit 4, Lesson 12

1. Find the value of 448 ÷ 16. Show your thinking. Organize it so it can be followed by others.
2. This is Elena's work. Describe the steps Elena took to find the value of 448 ÷ 16.

```
  28
   3
   5
  20
16)448
-320   (20 × 16)
 128
- 80   (5 × 16)
  48
- 48   (3 × 16)
   0
```

Activity Synthesis:
MLR1 Stronger and Clearer Each Time
"Share your description of how Elena found the value of 448 ÷ 16 with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."

In the Grade 5 example above, students have the opportunity to engage in the Mathematical Language Routine Stronger and Clearer Each Time, in which they get immediate feedback from their peers on their developing ideas. This routine is not only a powerful way for students to develop their academic language, but to also collaboratively solidify conceptual understanding of complex topics, both of which help foster positive mathematical identities. Mathematically, students come to see that some decompositions may be more helpful than others for finding whole-number quotients. They can use this insight to make sense of algorithms and use partial quotients that are more complex.

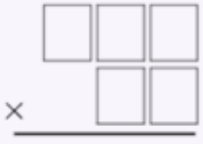
Mathematically, students come to see that some decompositions may be more helpful than others for finding whole-number quotients. They can use this insight to make sense of algorithms and use partial quotients that are more complex.

3. Students know and use the standard algorithm.

The algorithms using partial quotients in our Grade 4 and 5 courses follow a coherent progression that allows students to make sense of the standard algorithm for division that they will learn and come to know in grade 6. To meet the grade 5 requirement of fluency with the standard algorithm for multiplication, students have opportunities to practice the algorithm in Units 6 and 8 of our Grade 5 course.

Grade 5, Unit 6, Lesson 8

Using the digits 3, 5, 6, 8, and 9 make a product whose value is close to 50,000.



Grade 5, Unit 8, Lesson 2

Find the value of each product.

$$\begin{array}{r} 260 \\ \times\ 35 \\ \hline \end{array}$$

$$\begin{array}{r} 2,600 \\ \times\ 35 \\ \hline \end{array}$$

ASSESSING FLUENCY DEVELOPMENT

Teachers can use the look-fors on our section-level monitoring sheets to track student progress with fluency development. Looking across the grades, this tool shows how students transition from operating in ways that make sense to them to using and knowing the standard algorithm.

Monitoring Student Progress Across the Grades		
Grade, Unit	**Multiplication**	**Division**
Grade 3, Unit 4 *Students operate in ways that make sense to them.*	Use properties of operations and place value understanding to develop strategies to multiply within 100 and to multiply one-digit numbers by a multiple of 10.	Use properties of operations, place value understanding, and the relationship between multiplication and division to divide within 100.
Grade 4, Unit 6 *Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.*	Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.	Divide numbers of up to four digits by one-digit divisors to find whole-number quotients and remainders, using strategies based on place value, properties of operations, and the relationship between multiplication and division.
Grade 5, Unit 4 *Students know and use the standard algorithm.* *Students analyze and try strategies and algorithms based on place value understanding, the properties of operations, and the relationships between operations.*	Multiply multi-digit whole numbers using the standard algorithm.	Divide multi-digit whole numbers using strategies based on place value, properties of operations, and the relationship between multiplication and division.

MAINTAINING FLEXIBILITY WITH MENTAL MATH STRATEGIES

Even with a deep focus on the progression towards fluency with the standard algorithms, IM K–5 Math continues to offer students opportunities to practice their facts and demonstrate flexibility with multiplication and division through instructional routines.

Number Talk *Grade 4, Unit 6*
5 x 40 8 x 40 10 x 40 15 x 40

In this **video clip**, Savannah Sanders, a fourth grader in Saugus, CA explains how she efficiently reasoned through 15 x 40, the last expression in the Number Talk shown.

As a fourth grader, Savannah can use diagrams and written methods to solve problems like 15 x 40, but these Number Talk routines help students to think flexibly about the facts they know to compute mentally using their conceptual understanding of place value, the properties of operations, and the relationships between operations.

As students become comfortable with using mental strategies and the standard algorithms, they are able to determine whether it is appropriate to use the standard algorithm to find the value of a given product or quotient, or if a different strategy is more efficient.

A GATEWAY FOR FLUENCY, CONCEPTUAL UNDERSTANDING AND POSITIVE MATHEMATICAL IDENTITIES

In this four-part blog post series we have shown our curriculum to be a coherent progression of concepts within and across grades, leading to fluency in the four operations with facts and algorithms. We have also shown how the curriculum's problem-based structure creates a gateway for students to engage in mathematics in rich and meaningful ways. Our intentional design supports students to connect the ideas they already have to new learning, engage as doers of mathematics, and develop positive mathematical identities.

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NEXT STEPS

Take a look at the story of decimals in the curriculum in grades 4–6 to see if you notice a similar progression towards fluency with the standard algorithms for decimal operations.

ICYMI for further reading, check out the earlier posts our four-part series on Fluency with parts **1**, **2** and **3**.

You can also learn more about fluency in **IM K–5 Math on the IM website**.

MEET THE AUTHORS

Dionne Aminata

IM LEAD CURRICULUM WRITER, GRADES 2–5 AND SENIOR DIRECTOR, STRATEGIC INITIATIVES

Dionne Aminata has served over 20 years in education as a teacher, instructional coach, math content specialist, curriculum writer, and curriculum implementation specialist, and has taught at the elementary, middle school, undergraduate, and graduate levels.

At Illustrative Mathematics (IM), Dionne is one of the Lead Authors of the IM K–5 Math curriculum, playing a key role in embedding curricular supports for culturally responsive teachers. She successfully supported LAUSD's beta pilot of IM K–5 Math in 2020–21, and now serves as the Senior Director of Strategic Initiatives at IM in the Marketing department. In her current position she uses her marketing and curriculum implementation expertise to oversee research and outreach efforts to IM's community of users. As a part of this work, she supports math leaders nation-wide to successfully implement IM K–12 Math through the use of impact measurement tools, cycles of observation and feedback, and protocols to collaborate with peers to discuss problems of practice and share best practices.

In 2020, she launched MathTrust, a consulting firm dedicated to supporting educators to seek the brilliance of Black students and trust all students to do the math. In 2022, she joined the Council of Chief State School Officers (CCSSO) as a Math Advisor for the Math Collaborative, a network of state math leaders dedicated to affecting change in systems and policies in order to promote mathematics as a subject for all students.

Dionne earned her undergraduate degree in Sociology and Business at UCLA. She has an MBA In Entrepreneurship and Global Marketing from Babson College, and a Masters in Math Education from CUNY Brooklyn College.

MEET THE AUTHORS

Sarah Caban

IM LEAD CURRICULUM WRITER, GRADE 5 AND SPECIALIST, K–5 PROFESSIONAL LEARNING

Sarah started her teaching career in a K-8 one room schoolhouse on an island off the coast of Maine. She then spent 10 years as a teacher, district math coach and coordinator.

At Illustrative Mathematics, she was one of the lead authors of the grade 5 curriculum for IM K-5 and she is currently a senior specialist of professional learning. In her roles at IM, Sarah collaborates within and across teams to develop curriculum and professional learning experiences that position teachers and students as author's of important mathematical ideas.

Sarah earned her undergraduate degree in English from Hofstra University and her Masters in Education for Antioch New England Graduate School. She also has an Advanced Certificate in Math Leadership from University of Maine Farmington.

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