Illustrative Mathematics

About These Materials

Each course contains nine units. Each of the first eight are anchored by a few big ideas in grade-level mathematics. Units contain between 11 and 23 lesson plans. Each unit has a diagnostic assessment for the beginning of the unit (Check Your Readiness) and an end-of-unit assessment. Longer units also have a mid-unit assessment. The last unit in each course is structured differently, and contains optional lessons that help students apply and tie together big ideas from the year.

The time estimates in these materials refer to instructional time. Each lesson plan is designed to fit within a class period that is at least 45 minutes long. Some lessons contain optional activities that provide additional scaffolding or practice for teachers to use at their discretion.

What is a "Problem-Based" Curriculum?

What students should know and be able to do

Our ultimate purpose is to impact student learning and achievement. First, we define the attitudes and beliefs about mathematics and mathematics learning we want to cultivate in students, and what mathematics students should know and be able to do.

Attitudes and Beliefs We Want to Cultivate

Many people think that mathematical knowledge and skills exclusively belong to “math people.” Yet research shows that students who believe that hard work is more important than innate talent learn more mathematics.[1] We want students to believe anyone can do mathematics and that persevering at mathematics will result in understanding and success.

Knowledge

- Conceptual understanding: Students need to understand the why behind the how in mathematics. Concepts build on experience with concrete contexts. Students should access these concepts from a number of perspectives in order to see math as more than a set of disconnected procedures.

- Procedural fluency: We view procedural fluency as solving problems expected by the standards with speed, accuracy, and flexibility.

- Application: Application means applying mathematical or statistical concepts and skills to a novel mathematical or real-world context.

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand and be able to do the mathematics.

Principles for Mathematics Teaching and Learning

Active learning is best: Students learn best and retain what they learn better by solving problems.

- Mathematics instruction is shaped by the belief that if teachers tell students how to solve problems and then students practice, students will learn how to do mathematics.
- students should spend time in math class *doing mathematics*.
- Students should take an active role, both individually and in groups, to see what they can figure out before having things explained to them or being told what to do.
- Teachers play a critical role in mediating student learning, but that role looks different than simply showing, telling, and correcting.
    - The teacher's role is
        1. to ensure students understand the context and what is being asked,
        2. ask questions to advance students' thinking in productive ways,
        3. help students share their work and understand others' work through orchestrating productive discussions, and
        4. synthesize the learning with students at the end of activities and lessons.

How to Use the Materials

Each Lesson and Unit Tells a Story

The story of each grade is told in nine units. Each unit has a narrative that describes the mathematical work that will unfold in that unit. Each lesson in the unit also has a narrative. Lesson Narratives explain:

- The mathematical content of the lesson and its place in the learning sequence.
- The meaning of any new terms introduced in the lesson.
- How the mathematical practices come into play, as appropriate.

Activities within lessons also have narratives, which explain:

- The mathematical purpose of the activity and its place in the learning sequence.
- What students are doing during the activity.
- What teacher needs to look for while students are working on an activity to orchestrate an effective synthesis.
- Connections to the mathematical practices, when appropriate.

Launch - Work - Synthesize

The Launch

- During the launch, the teacher makes sure that students understand the context (if there is one) and *what the problem is asking them to do*. This is not the same as making sure the students know how to do the problem—part of the work that students should be doing for themselves is figuring out how to solve the problem.

Student Work Time

- The launch for an activity frequently includes suggestions for grouping students. This gives students the opportunity to work individually, with a partner, or in small groups.

Activity Synthesis

- During the activity synthesis, the teacher orchestrates some time for students to synthesize what they have learned. This time is used to ensure that all students have an opportunity to understand the mathematical punch line of the activity and situate the new learning within students' previous understanding.

Practice Problems

Each lesson includes an associated set of practice problems. Teachers may decide to assign practice problems for homework or for extra practice in class; they may decide to collect and score it or to provide students with answers ahead of time for self-

assessment. It is up to teachers to decide which problems to assign (including assigning none at all).
The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Distributed practice (revisiting the same content over time) is more effective than massed practice (a large amount of practice on one topic, but all at once).

Are You Ready For More?

Select classroom activities include an opportunity for differentiation for students ready for more of a challenge. We think of them as the “mathematical dessert” to follow the “mathematical entrée” of a classroom activity.
Every extension problem is made available to all students with the heading “Are You Ready for More?” These problems go deeper into grade-level mathematics and often make connections between the topic at hand and other concepts. Some of these problems extend the work of the associated activity, but some of them involve work from prior grades, prior units in the course, or reflect work that is related to the K–12 curriculum but a type of problem not required by the standards. They are not routine or procedural, and they are not just “the same thing again but with harder numbers.”
They are intended to be used on an opt-in basis by students if they finish the main class activity early or want to do more mathematics on their own. It is not expected that an entire class engages in “Are You Ready for More?” problems, and it is not expected that any student works on all of them. “Are You Ready for More?” problems may also be good fodder for a Problem of the Week or similar structure.

Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These lesson plans include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.
Some of the instructional routines, known as Mathematical Language Routines (MLR), were developed by the Stanford University UL/SCALE team. The purpose of each MLR is described here, but you can read more about supports for students with emerging English language proficiency in the Supporting English Language Learners section.

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR3: Critique, Correct, and Clarify
- MLR4: Information Gap Cards
- MLR5: Co-Craft Questions and Problems (old)
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn’t Belong?

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson.
Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word "Let's." They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson.
We do not recommend writing learning targets on the board before class begins, because doing so might spoil the problem-based instruction. (The student-facing learning goals (that start with “Let’s”) are more appropriate for this purpose.)
Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

How to Assess Progress

The materials contain many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment (“Check Your Readiness”) of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.

- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students’ thinking.

- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day’s lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.

- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.

- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.