

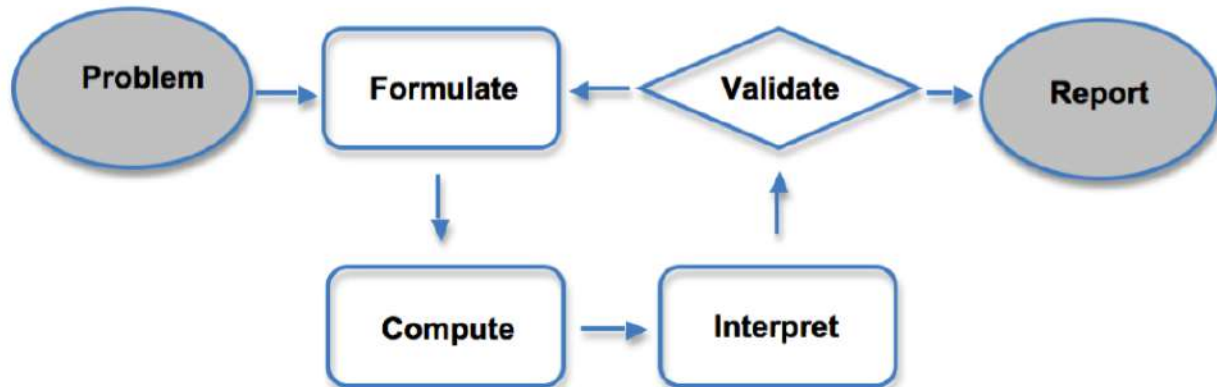
## High School: Quadratic Functions Sample Unit Plan

This instructional unit guide was designed by a team of Delaware educators in order to provide a sample unit guide for teachers to use. This unit guide references some textbook resources used by schools represented on the team. This guide should serve as a complement to district curriculum resources.

### Unit Overview

In this unit, students will generate a quadratic function as a product of two linear equations where they will compare quadratic, linear, and exponential functions. Students will write quadratic equations and inequalities using real world situations and graph them in order to find key features. Using what we know about families of graphs, students will perform transformations on the quadratic functions. In addition, students will apply the modeling cycle to quadratic data and convert between the various forms (vertex, standard, and factored) of quadratic functions. They will then derive the quadratic formula and apply their understanding of quadratics to a variety of real-world situations.

### The Modeling Cycle



Source: <http://www.corestandards.org/Math/Content/HSM/>

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## The Design Process

The writing team followed the principles of Understanding by Design (Wiggins & McTighe, 2005) to guide the unit development. As the team unpacked the content standards for the unit, they considered the following:

### Stage 1: Desired Results

- What long-term transfer goals are targeted?
- What meanings should students make? What essential questions will students explore?
- What knowledge and skills will students acquire?

### Stage 2: Assessment Evidence

- What evidence must be collected and assessed, given the desired results defined in stage one?
- What is evidence of understanding (as opposed to recall)?

### Stage 3: The Learning Plan

- What activities, experiences, and lessons will lead to achievement of the desired results and success at the assessments?
- How will the learning plan help students with Acquisition, Meaning Making, and Transfer?
- How will the unit be sequenced and differentiated to optimize achievement for all learners?

The writing team incorporated components of the Learning-Focused (LFS) model, including the learning map, and a modified version of the Know-Understand-Do template.

The team also reviewed and evaluated the textbook resource they use in the classroom based on an alignment to the content standard for a given set of lessons. The intention is for a teacher to see what supplements may be needed to support instruction of those content standards. A list of open educational resources (OERs) are also listed with each lesson guide.

### A special thanks to the writing team:

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# Quadratic Functions

## Content and Practice Standards

### Transfer Goals (Standards for Mathematical Practice)

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

### Content Standards

#### Major Content for this Unit:

##### **Build a function that models a relationship between two quantities.**

**F.BF.A.1** Write a function that describes a relationship between two quantities.\*

**F.BF.A.1a** Determine an explicit expression, a recursive process, or steps for calculation from a context.

##### **Analyze functions using different representations.**

**F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

**F.IF.C.7a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

##### **Interpret functions that arise in applications in terms of the context.**

**F.IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

##### **Analyze functions using different representations.**

**F.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

**F.IF.C.8a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**F.IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).



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**Build new functions from existing functions.**

**F.BF.B.3** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Summarize, represent, and interpret data on two categorical and quantitative variables**

**S.ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

**S.ID.B.6a** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

**S.ID.B.6b** Informally assess the fit of a function by plotting and analyzing residuals.

**Interpret the structure of expressions.**

**A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it.

**Write expressions in equivalent forms to solve problems.**

**A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\*

**A.SSE.B.3a** Factor a quadratic expression to reveal the zeros of the function it defines.

**A.SSE.B.3b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**Solve equations and inequalities in one variable.**

**A.REI.B.4** Solve quadratic equations in one variable.

**A.REI.B.4a** Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

**A.REI.B.4b** Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

**Supporting/Embedded Content for this Unit:****Understand the concept of a function and use function notation.**

**F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

**F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.



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**Construct and compare linear, quadratic, and exponential models and solve problems.**

**F.LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**Create equations that describe numbers or relationships.**

**A.CED.A.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**Represent and solve equations and inequalities graphically.**

**A.REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**Interpret functions that arise in applications in terms of the context.**

**F.IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.\*

**F.IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

**Interpret the structure of expressions.**

**A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.\*

**A.SSE.A.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**A.SSE.A.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

\*Indicates this standard is considered a Modeling Standard. For more information on the modeling standards, visit <http://www.corestandards.org/Math/Content/HSM/>



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## Enduring Understandings & Essential Questions

Enduring Understanding	Essential Question(s)
<p><b>Understanding 1</b> For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables.</p> <p>Quadratic functions are characterized by a linear rate of change, so the rate of change of the rate of change of a quadratic function is constant.</p> <p>A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model.</p>	<p>EQ1a. What are the characteristics of the rate of change of a quadratic function? How can we determine whether a function can be defined as a quadratic function?</p> <p>EQ1b. How does the rate of change of a quadratic function compare to the rate of change of linear and exponential functions?</p>
<p><b>Understanding 2</b> A quadratic function may be represented as equations in different forms. Each form of the equation for a quadratic reveals different features of the graph, including zeros, relative extrema, and end behavior.</p> <p>Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic function are real, they are symmetric about the x-coordinate of the maximum or minimum point.</p>	<p>EQ2a. What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?</p> <p>EQ2b. How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?</p>
<p><b>Understanding 3</b> Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Some representations of a function may be more useful than others, depending on the context.</p> <p>Links between algebraic and graphical representations of functions are especially</p>	<p>EQ3. What are the key characteristics of each representation of a function and what information does each representation reveal about the graph?</p>



important in studying relationships and change.	
<b>Understanding 4</b> For functions that map the real numbers to the real numbers, composing a function with “shifting” or “scaling” functions changes the formula and graph of the function in readily predictable ways.	EQ4. How do specific transformations impact the graph of the parent function $f(x)=x^2$ ?

**\*Enduring understandings and essential questions adapted from NCTM Enduring Understandings**

Source: Cooney, T.J., Beckmann, S., & Lloyd, G.M. (2010). *Developing essential understanding of functions for teaching mathematics in grades 9-12*. Reston, VA: The National Council of Teachers of Mathematics, Inc.



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## Acquisition

Conceptual Understandings (Know/Understand)	Procedural Fluency (Do)	Application (Apply)
<p>Describe the rate of change for a quadratic function</p> <p>Describe key characteristics of the graph:</p> <ul style="list-style-type: none"> <li>- Symmetry (axis)</li> <li>- Maximum/minimum</li> <li>- Zeros/roots/solutions /x-intercepts</li> <li>- End behavior</li> </ul> <p>Describe the uses for different forms of quadratics:</p> <ul style="list-style-type: none"> <li>- Vertex form</li> <li>- Standard form</li> <li>- Factored form</li> </ul> <p>Compare rates of change of quadratics with linear and exponential functions</p> <p>Understand the relationship between the quadratic formula and the graph of a quadratic function</p> <p>Interpret quadratic inequalities based on the context of a problem</p>	<p>Generate quadratic functions as a product of linear equations</p> <p>Analyze rate of change using first and second differences</p> <p>Determine zeros of a quadratic function, given a table, graph, or equation, and/or using the quadratic formula</p> <p>Find the axis of symmetry based on the vertex</p> <p>Convert between forms of quadratic functions</p> <ul style="list-style-type: none"> <li>- Completing the square</li> <li>- Factoring</li> <li>- Expanding the factored form</li> </ul> <p>Translate, dilate, and reflect quadratic functions</p> <p>Use the appropriate form of the equation to identify the vertex (max/min), y-intercept, and x-intercepts</p> <p>Use the Quadratic Formula to find solutions of a quadratic function or intersections of quadratic and linear functions</p>	<p>Write quadratic functions and inequalities from real-world problems</p> <p>Apply the modeling cycle to real-world situations, including projectile motion &amp; optimization including, speed, height, time, profit, and maximizing the area of an object.</p> <p>Make sense of real world problems/situations using rules, tables, and graphs and make predictions about the situation using these models.</p>



## Reach Back/Reach Ahead Standards

How does this unit relate to the progression of learning? What prior learning do the standards in this unit build upon? How does this unit connect to essential understandings of later content in this course and in future courses? The table below outlines key standards from previous and future courses that connect with this instructional unit of study.

Reach Back Standards	Reach Ahead Standards
<p>8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p>8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p> <p>8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p> <p>F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p>	<p>Apply understanding of functions, key features, and transformations to other function families, including polynomial, radical, rational, logarithmic, and trigonometric functions. (F-IF.5; F-IF.6; F-IF.9; A-CED.2; F-BF.3)</p>



F-LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F-LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

## Common Misunderstandings

- Students may forget to multiply all terms when expanding the factored form.
- Students may interchange  $x$  and  $y$  in a table, graph, or ordered pairs.
- Students may not recognize that  $y$  is equal to  $f(x)$ .
- Students may interpret  $f(x) = 0$  as equivalent to  $f(0)$ .
- Students may represent quadratic functions as a series of line segments rather than connect points using straight lines instead of sketching the curve/parabola.
- When completing the square, students may forget to subtract the number that was added inside the parentheses.
- Students may confuse the effects of different transformations.
- Students may not recognize that completing the square is an algebraic tool that produces the vertex form.
- Recognizing that vertex form is most useful for determining the maximum or minimum value of the function.
- Students may have not connected the concept of a quadratic solution as the zeros ( $x$ -intercepts) of a quadratic graph.
- Students may struggle shading above or below a parabola based on the direction of the inequality.
- Students may struggle with determining when to use dotted or solid lines based on the inequality symbol in the problem.



## SAT Assessment Expectations

### **Problem Solving and Data Analysis (PSDA)**

**Problem Solving and Data Analysis questions ask students to:**

**PSDA.4** Given a scatterplot, use linear, quadratic, or exponential models to describe how the variables are related. The student will, given a scatterplot, select the equation of a line or curve of best fit; interpret the line in the context of the situation; or use the line or curve of best fit to make a prediction.

**PSDA.5** Use the relationship between two variables to investigate key features of the graph. The student will make connections between the graphical representation of a relationship and properties of the graph by selecting the graph that represents the properties described, or using the graph to identify a value or set of values.

**PSDA.10** Evaluate reports to make inferences, justify conclusions, and determine appropriateness of data collection methods. The reports may consist of tables, graphs, or text summaries.

### **Passport to Advanced Math (PAM)**

**Passport to Advanced Math questions ask students to:**

**PAM.1** Create a quadratic or exponential function or equation that models a context. The equation will have rational coefficients and may require multiple steps to simplify or solve the equation.

**PAM.2** Determine the most suitable form of an expression or equation to reveal a particular trait, given a context.

**PAM.4** Create an equivalent form of an algebraic expression by using structure and fluency with operations.

**PAM.5** Solve a quadratic equation having rational coefficients. The equation can be presented in a wide range of forms to reward attending to algebraic structure and can require manipulation in order to solve.

**PAM.6** Add, subtract, and multiply polynomial expressions and simplify the result. The expressions will have rational coefficients.

**PAM.10** Interpret parts of nonlinear expressions in terms of their context. Students will make connections between a context and the nonlinear equation that models the context to identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.

**PAM.11** Understand the relationship between zeros and factors of polynomials, and use that knowledge to sketch graphs. Students will use properties of factorable polynomials to solve conceptual problems relating to zeros, such as determining whether an expression is a factor of a polynomial based on other information provided.

**PAM.12** Understand a nonlinear relationship between two variables by making connections between their algebraic and graphical representations. The student will select a graph corresponding to a given nonlinear equation; interpret graphs in the context of solving systems of equations; select a nonlinear equation corresponding to a given graph; determine the equation of a curve given a verbal description of a graph; determine key features of the graph of a linear function from its equation; or determine the impact on a graph of a change in the defining equation.

**PAM.13** Use function notation, and interpret statements using function notation. The student will use function notation to solve conceptual problems related to transformations.



## Assessment Evidence

### Students will be able to:

Generate a quadratic function by multiplying 2 linear functions.

<https://www.illustrativemathematics.org/content-standards/HSF/BF/A/1/tasks/744>

Compare rate of change of quadratics with linear and exponential functions

Identify key features of a graph of a quadratic function.

Identify the axis of symmetry based on the vertex.

Graph a function on the coordinate plane using transformations.

[http://ccsstoolbox.agilemind.com/parcc/about\\_highschool\\_3828.html](http://ccsstoolbox.agilemind.com/parcc/about_highschool_3828.html)

Create quadratics to model real world situations.

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/1344>

Represent quadratic functions in different forms.

<https://www.illustrativemathematics.org/content-standards/HSA/REI/D/11/tasks/618>

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/2/tasks/87>

<https://www.illustrativemathematics.org/content-standards/HSA/REI/B/4/tasks/1827>

Derive the quadratic formula from standard form.

Apply the quadratic formula to solve quadratic functions.

<https://www.illustrativemathematics.org/content-standards/HSF/IF/C/7/tasks/388>

Use multiple representations to determine the solutions of a quadratic function.

<https://www.illustrativemathematics.org/content-standards/HSA/REI/B/4/tasks/375>

<https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/1279>

<https://www.illustrativemathematics.org/content-standards/tasks/21>

### Additional Resources:

<https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/695>

<https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/741>

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/90>

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/215>

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/A/1/tasks/390>

<https://www.illustrativemathematics.org/content-standards/HSA/SSE/B/3/tasks/434>

<https://www.illustrativemathematics.org/content-standards/HSA/REI/B/4/tasks/618>

<https://www.illustrativemathematics.org/content-standards/HSA/REI/B/4/tasks/1690>



## The Learning Plan: LFS Student Learning Maps

### Key Learning 1:

For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables.

Quadratic functions are characterized by a linear rate of change, so the rate of change of the rate of change of a quadratic function is constant. A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model.

### Unit Essential Questions:

EQ1a. What are the characteristics of the rate of change of a quadratic function? How can we determine whether a function can be defined as a quadratic function?

EQ1b. How does the rate of change of a quadratic function compare to the rate of change of linear and exponential functions?



<b>Concept:</b> Rates of Change for Quadratics Functions	<b>Concept:</b> Finding and Comparing Rates of Change
<b>LEQs:</b> What are the characteristics of the rate of change of a quadratic function?  How can we determine whether a function can be defined as a quadratic function?	<b>LEQ:</b> How does the rate of change of a quadratic function compare to the rates of change of linear and exponential functions?
<b>Vocabulary:</b> <ul style="list-style-type: none"> <li>• Quadratic</li> <li>• Rate of change</li> <li>• Constant</li> <li>• Model</li> <li>• Slope</li> <li>• x-intercept</li> <li>• Variable</li> </ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"> <li>• Function</li> <li>• Linear</li> <li>• Exponential</li> <li>• Slope</li> <li>• y-intercept</li> </ul>





**Key Learning 2:**


A quadratic function may be represented as equations in different forms. Each form of the equation for a quadratic reveals different features of the graph, including zeros, relative extrema, and end behavior.

Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic function are real, they are symmetric about the x-coordinate of the maximum or minimum point.

**Unit Essential Question:**

EQ2a. What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?

EQ2b. How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?



<b>Concept:</b> Graphing Quadratic Equations as a Product of Linear Functions	<b>Concept:</b> Vertex Form and Quadratic Graphs	<b>Concept:</b> Converting Between Different but Equivalent Quadratic Forms
<b>LEQ:</b> What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?	<b>LEQ:</b> How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?	<b>LEQ:</b> How do I algebraically manipulate an equation to represent a function in different but equivalent forms?
<b>Vocabulary:</b> <ul style="list-style-type: none"> <li>• Equivalent</li> <li>• Axis of symmetry</li> <li>• Maximum/minimum</li> <li>• Factors</li> <li>• Standard form</li> <li>• Solutions, Zeros, x-intercepts</li> </ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"> <li>• Completing the square</li> <li>• Vertex</li> <li>• Maximum/minimum</li> <li>• Axis of symmetry</li> <li>• Solutions, Zeros, x-intercepts</li> </ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"> <li>• Vertex</li> <li>• Vertex form</li> <li>• Standard Form</li> <li>• Completing the square Factored form</li> <li>• Solutions, Zeros, x-intercepts</li> </ul>

**Key Learning 3:**

Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Some representations of a function may be more useful than others, depending on the context.

Links between algebraic and graphical representations of functions are especially important in studying relationships and change.

**Unit Essential Questions:**

EQ3. What are the key characteristics of each representation of a function and what information does each representation reveal about the graph?



<b>Concept:</b> Making Connections between Graphical & Algebraic Representations	<b>Concept:</b> Modeling with Quadratics
<b>LEQ:</b> Which characteristics of the quadratic graph relate to which components of the quadratic equation?	<b>LEQs:</b> What are the algebraic and graphical connections in quadratic functions?  What are the key characteristics of each representation of a function and how is it represented algebraically, graphically, in a table and using words based on the context?
<b>Vocabulary:</b> <ul style="list-style-type: none"><li>• Vertex</li><li>• Factors</li><li>• Solutions, zeros, x-intercepts</li><li>• Vertex form</li><li>• Standard form</li><li>• Factored form</li></ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"><li>• Vertex</li><li>• Maximum/minimum</li><li>• Factors</li><li>• Axis of symmetry</li><li>• Solutions, zeros, x-intercepts</li></ul>




**Key Learning 4:**

For functions that map the real numbers to the real numbers, composing a function with "shifting" or "scaling" functions changes the formula and graph of the function in readily predictable ways.

**Unit Essential Question:**

EQ4. How do specific transformations impact the graph of the parent function  $f(x)=x^2$ ?



<b>Concept:</b> Describing the Transformations from Graphs & Equations	<b>Concept:</b> Transforming Quadratic Functions	<b>Concept:</b> Representing Quadratic Functions
<b>LEQ:</b> How can we describe a quadratic function from a graph or an equation?	<b>LEQ:</b> How do we graph quadratic functions using transformations?	<b>LEQ:</b> How can we write an equation from the graph of a transformed parent function?
<b>Vocabulary:</b> <ul style="list-style-type: none"><li>• Parent function</li><li>• Symmetry</li></ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"><li>• Transformations</li><li>• Dilations</li><li>• Translations</li><li>• Reflections</li></ul>	<b>Vocabulary:</b> <ul style="list-style-type: none"><li>• Parent function</li><li>• Transformations</li><li>• Translations</li><li>• Reflections</li></ul>

## Unit at a Glance

**Note: This is a suggested guideline for pacing this set of standards. This unit is based on 50-minute lessons. Add additional days for remediation, extra practice, or assessment as needed.**

# of days	Topics	Standards
4	Generating Quadratic Functions as a product of linear equations Defining quadratic functions Compare quadratic functions to other function families	F.IF.C.7 F.IF.C.9 S.ID.B.6
4	Writing quadratic functions from real-world situations (modeling) Identifying key features of graph Writing quadratic functions in different forms Graphing quadratic functions	F.BF.A.1 F.IF.C.7 F.IF.B.4 F.IF.C.9
3	Transforming Quadratic functions using: <ul style="list-style-type: none"> <li>• Reflections</li> <li>• Stretching/shrinking (dilations)</li> <li>• Translations (shifts)</li> </ul>	F.IF.C.7 F.IF.B.4 F.IF.C.9 F.BF.B.3
3	Modeling with quadratic data with real-world problems	F.BF.A.1 F.IF.C.9 S.ID.B.6
7	Understanding how each form of a quadratic reveals information about the function Converting between forms of a quadratic: <ul style="list-style-type: none"> <li>• Standard</li> <li>• Factored</li> <li>• Vertex</li> </ul>	F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4
4	Solving quadratic equations by completing the square Deriving the quadratic formula	F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4
5	Finding zeros/solutions/roots and intersections between graphs and inequality sets	F.IF.C.7 F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4
3	Review and Unit Test/Assessment(s)	All standards in this unit should be addressed



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<p><b><u>Days 1-4</u></b> - I can generate a quadratic function by multiplying 2 linear functions.</p> <p><u>Standards:</u> F.IF.C.7 F.IF.C.9 S.ID.B.6</p>	<p><b><u>Days 5-8</u></b> - I can identify key features of a graph of a quadratic function.</p> <p><u>Standards:</u> F.BF.A.1 F.IF.C.7 F.IF.B.4 F.IF.C.9</p>	<p><b><u>Day 9-11</u></b> - I can graph a function on the coordinate plane using transformations.</p> <p><u>Standards:</u> F.IF.C.7 F.IF.B.4 F.IF.C.9 F.BF.B.3</p>	<p><b><u>Days 12-14</u></b> - I can create quadratic functions to model real world situations.</p> <p><u>Standards:</u> F.BF.A.1 F.IF.C.9 S.ID.B.6</p>	<p><b><u>Day 15-21</u></b> - I can represent quadratic functions in different forms. I can convert between quadratic forms in order to reach the form that is most beneficial to the problem that I am solving.</p> <p><u>Standards:</u> F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4</p>
<p><b><u>Day 22</u></b> - I can derive the quadratic formula from the standard form.</p> <p><u>Standards:</u> F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4</p>	<p><b><u>Day 23-25</u></b> - I can apply the quadratic formula to solve quadratic functions.</p> <p><u>Standards:</u> F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4</p>	<p><b><u>Day 26-30</u></b> - I can use multiple representations to determine the solutions of a quadratic function.</p> <p><u>Standards:</u> F.IF.C.7 F.IF.C.8 F.IF.C.9 A.SSE.A.2 A.SSE.B.3 A.REI.B.4</p>		



## Days 1-4: Introduction to Quadratic Functions

**Learning Target:** I can generate a quadratic function by multiplying two linear functions.

### Mathematical Practice Standards:

MP.3 Construct viable arguments and critique the reasoning of others

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.7 Look for and make use of structure

MP.8 Look for and express regularity in repeated reasoning

### Linked Content Standards:

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

### Instructional Notes:

- To build an understanding of what a quadratic function is by examining the product of two linear functions. Have students expand those linear products with area models and distribution. See the sample Illuminations lesson listed below for more information.
- Have students compare quadratic functions with linear and exponential functions to define key features of quadratic functions. Have students compare the rates of change and tables in linear and exponential functions with that of quadratic functions. Have students critique each other and add in properties as they compare and contrast.

Reach Back	Reach Ahead
Creating equations, tables of values and graphs for linear and exponential functions. Identifying key features of these function families (A-CED.2, F-IF.B.4)	Comparing quadratic functions with other function families. Extending quadratic functions to other polynomial functions (F-IF.B.5, F-IF.C.9, A-CED.A.2)

Linked Essential Understanding(s):	Linked Unit EQ(s):
Quadratic functions are characterized by a linear rate of change, so the rate of change of the rate of change of a quadratic function is constant. A function's rate of change is one of the main characteristics that determine what kinds of real-world phenomena the function can model.	EQ1a. What are the characteristics of the rate of change of a quadratic function? EQ1b. How does the rate of change of a quadratic function compare to the rate of change of linear and exponential functions?



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**LEQ 1:** How can we determine whether a function can be defined as a quadratic function?

**LEQ 2:** How does the rate of change of a quadratic function compare to the rate of change of linear and exponential functions?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
<b>Section(s)</b>	<b>Chapter 8</b> Pg. 497 Multiplying with Algebra Tiles <b>8-3</b> Multiplying Binomials (pg.498) Apply what you learned (pg. 503) <b>8-4</b> Multiplying Special Cases (pg. 504) <b>9-7</b> Linear, Quadratic, and Exponential Models (pg. 589)	<u>Course 1 Book:</u> Unit 7, Lesson 2, Investigation 1 (7.2.1) pg. 493 #1-4,  <u>Course 2 Book:</u> 5.1.1 5.1.3 All problems 5.2.1 Pg. 345 #2, pg. 360 #1	8-3 Multiplying Polynomials (pg. 478 - using Algebra tiles) pg. 480 (binomials)  8-4 Special Cases (pg. 486) <u>Real World</u> <u>problem:</u> Swimming Pool (pg. 482)  9-1 Algebra Lab: Rate of Change of a Quadratic Function (pg. 554)  9-6 Analyzing Functions with Successive Differences (pg. 590) <u>Real World</u> <u>problem:</u> Book Club (pg. 592)	Chapter 8 - Page 503 Multiplying with Algebra Tiles  8.2 Multiplying Expressions  8-3 Special Cases  Page 509 Application problems - Start exploring  9.8 Compare linear, exponential, and quadratic using tables (Page 628)
<b>Strength of Alignment</b>	<b>Aligned</b>	<b>Aligned**</b>	<b>Aligned</b>	<b>Aligned</b>

**\*\*When using the Core Plus II book in conjunction with the Core Plus I book, the strength of alignment moves to strongly aligned.**



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**Sample Lesson Activities/Resources:**

This lesson focuses on having students make connections among different classes of polynomial functions by exploring the graphs of the functions. The questions in the activity sheets allow students to make connections between the  $x$ -intercepts of the graph of a polynomial and the polynomial's factors. This activity is designed for students who already have a strong understanding of linear functions, some knowledge of quadratic functions, and what is meant by a polynomial function <http://illuminations.nctm.org/Lesson.aspx?id=1091>

These are resources for showing students the different uses of quadratic functions.

<https://plus.maths.org/content/101-uses-quadratic-equation>

<https://plus.maths.org/content/101-uses-quadratic-equation-part-ii>



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## Days 5-8: Modeling Real-World Problems with Quadratics

**Learning Target:** I can identify key features of a graph of a quadratic function.

**Mathematical Practice Standards:**

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.7 Look for and make use of structure

MP.8. Look for and express regularity in repeated reasoning

**Linked Content Standard:**

F.BF.A.1 Write a function that describes a relationship between two quantities.\*

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**Instructional Notes:**

- Graphing quadratic functions should be introduced using multiple representations, including tables and equations. Emphasize using the representations to identify the zeros. Include real-world representations within this section.
- Be sure that students can identify the vertex from an equation and whether it is a maximum or minimum. Have students identify what is happening in the graph regarding end behavior.

Reach Back	Reach Ahead
Comparing linear and nonlinear functions, creating scatter plots for linear data sets (8.F.B.5, 8.SP.A.1)	Modeling real world problems with other function families, including polynomial functions (F-IF.B.5, F-IF.C.9, A-CED.A.2, F-BF.B.3, S-ID.B.6)

Linked Essential Understanding(s):	Linked Unit EQ(s):
Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic function are real, they are symmetric about the x-coordinate of the maximum or minimum point.	EQ2b. How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?



**LEQ 1:** What does each form of a quadratic function reveal about the graph?

**LEQ 2:** How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
<b>Section(s)</b>	<b>Chapter 9</b> <b>9-1</b> Quadratic Graphs and Their Properties pg. 546 <b>9-2</b> Quadratic Functions pg. 553 Concept Byte pg. 559	<u>Course 1 Book:</u> 7.1.3, pg. 476, #5  <u>Course 2 Book:</u> 5.1.1, CYU 5.1.2, pg. 334 #3,4,5, CYU 5.2.2, pg. 366 #6	9-1 Graphing Quadratic Functions (pg. 543) Table comparing linear, exponential, and quadratics (pg. 547) <u>Real World problem:</u> School Spirit (pg. 548)	9.1 Page 570 defines axis of symmetry and vertex. (Not a great resource)  Determine the max/min using a calculator Page 592  ** In Algebra II Holt McDougal Larson - Chapter 1 Section 2 Better aligned
<b>Strength of Alignment</b>	<b>Weak Alignment</b>	<b>Somewhat aligned**</b>	<b>Aligned</b>	<b>Not aligned</b>

**\*\*There are no “axis of symmetry” or “end behavior” problems in either Course books.**

**Sample Lesson Activities/Resources:**

This is an interactive site that students can use to determine the height (vertex) of real world applications, such as the height of a rainbow, or the height of a building. The answers can be found on this site along with definitions.

<http://www.mathwarehouse.com/geometry/parabola/real-world-application.php>

A resource sheet for maximum or minimum of different quadratic functions using tables.

Included is a poster activity for real world application for group work. Have each group of students do a different problem and present to the class on a poster.

<http://www.farragutcareeracademy.org/ourpages/auto/2012/5/14/51834308/Graphing%20Quadratic%20Functions%20Handout.pdf>



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## Days 9-11: Transforming Quadratics

**Learning Target:** I can graph a function on the coordinate plane using transformations.

### Mathematical Practice Standards

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.7 Look for and make use of structure

### Linked Content Standard:

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F.BF.B.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs.

Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

### Instructional Notes:

- Have students describe and write the transformations before they graph the functions.
- When making the equation from a graph, students should recognize the vertical and horizontal shifts and write an equation based on these shifts. Students should recognize the dilations (stretches/shrinks) when describing the graph, but may not be able to generate the exact equation for more complex dilations.

Reach Back	Reach Ahead
Transforming figures in the plane and identifying transformations applied to figures (8.G.A.3, 8.G.A.4)	Applying transformations to other families of functions, including polynomial functions (F-BF.B.3, F-IF.C.9, A-CED.A.2)

Linked Essential Understanding(s):	Linked Unit EQ(s):
For functions that map the real numbers to the real numbers, composing a function with “shifting” or “scaling” functions changes the formula and graph of the function in readily predictable ways.	EQ4. How do specific transformations impact the graph of the parent function $f(x)=x^2$ ?



**LEQ 1:** How can we describe a quadratic function through a graph or an equation?

**LEQ 2:** How do transformations affect the graph of a quadratic function?

**LEQ 3:** How do you write an equation from the graph of a transformed parents function?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
Section(s)		<u>Course 1 Book:</u> 7.1.3 pg. 474, #1-6  <u>Course 2 Book:</u> none	9-3: Graphing Technology Lab Family of Quadratic Functions (pg. 562) 9-3: Transformations of Quadratic Functions (pg. 564)  <u>Real World Problem:</u> Fireworks (pg. 568)	Section 9.1 - Problems will work for assignments, not for instruction  **Algebra II Holt McDougal Larson 1.2 a little better.
Strength of Alignment	Not aligned	Aligned**	Strongly aligned	Not aligned

**\*\*Course 1 & 2 Books never discuss/ the exact transformation of  $b$ .**

**Sample Lesson Activities/Resources:**

These are lesson plans showing the transformations. This is not an activity.

<https://sites.google.com/site/matherudition/technology-and-secondary-school-mathematics/the-transformation-of-the-graph-of-a-quadratic-equation>

This is a Khan Academy video showing the transformations of quadratic functions.

<https://www.khanacademy.org/math/algebra/quadratics/transforming-quadratic-functions/v/shifting-and-scaling-parabolas>

This is a short video showing transformations as a young woman starts her day. It could be used as an introduction to transformations. It does not focus on quadratic functions.

<https://www.youtube.com/watch?v=lappagTQlj8>



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## Days 12-14: Modeling with Quadratic Data

**Learning Target:** I can create quadratics to model real world situations.

### Mathematical Practice Standards

MP.1 Make sense of problems and persevere in solving them

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.8 Look for and express regularity in repeated reasoning

### Linked Content Standard:

F.BF.A.1 Write a function that describes a relationship between two quantities.\*

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

### Instructional Notes:

- Introduce real-world problems that include profit optimization and projectile motion scenarios. Have students find x- and y-intercepts and the vertex in the context of the problem.
- Have students identify types of real-world situations that can be modeled with quadratic functions.
- As students generate a model of best fit, make sure students analyze the residual plot to verify that the model is appropriate.
- Application problems used should be embedded throughout the unit and woven into the instruction of the unit. These problems should NOT be solely introduced completely only during Days 12-14.

Reach Back	Reach Ahead
Creating scatter plots for linear and exponential data. Representing linear and exponential data with an appropriate model. (8.F.A.2, 8.F.B.5 S-ID.B.6)	Extending modeling with data to other function families. Analyzing residuals to determine the best fit for a data set. (S-ID.B.6, A-CED.A.2)

Linked Essential Understanding(s):	Linked Unit EQ(s):
Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ3. What are the key characteristics of each representation of a function and what information does each representation reveal about the graph?



**LEQ 1:** What are the key characteristics of each representation of a function and how is it represented algebraically, graphically, in a table, and using words based on the context?

**LEQ 2:** What are the algebraic and graphical connections in quadratic functions?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
<b>Section(s)</b>	pg. 502-503 #29, 30, 35, 44, 46, 50 Apply What You've Learned (pg. 503) pg. 508-509 #19, 51, 52, 57, 58, 60, pg. 550-552 #26, 27, 38, 46, 49 pg. 557-558 #26, 27, 30, 31, 35, 36 Apply What You've Learned pg. 558 Concept Byte pg. 559	<u>Course 1 Book:</u> 7.1.1 pg. 465, #3 7.1.2 pg. 470, #2 7.1.3 pg. 476, #6  <u>Course 2 Book:</u> 5.1.1 pg. 331 CYU 5.1.3 pg. 338 #7  5.2.1 pg. 360 #1-3 5.2.2 pg. 365 #1-5	<b>Problems stated on other days</b>	Page 603 mixed problem review There are problems throughout chapter 9 in each section.
<b>Strength of Alignment</b>	<b>Somewhat aligned</b>	<b>Strongly aligned</b>	<b>Aligned</b>	<b>Somewhat aligned</b>

**Sample Lesson Activities/Resources:**

This site has a few problems that are real world. It starts with writing quadratic equations from words. You have to scroll down a little before you will see the problems.

<https://www.mathsisfun.com/algebra/quadratic-equation-real-world.html>

Task: Comparing Kicks. This HCPSS Math task provides an opportunity for students to model projectile motion on the moon.

<https://hcpss.instructure.com/courses/99/files/824967/download?verifier=vMzu4Svx4e4eMnc9UIoF2noQJuVL9Kj4GDXWDqkt&wrap=1>



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## Day 15-21: Converting Between Forms

### Learning Targets:

I can represent quadratic functions in different forms.

I can convert between quadratic forms in order to reach the form that is most beneficial to the problem that I am solving.

### Mathematical Practice Standards:

MP.4 Model with mathematics

MP.6 Attend to precision

MP.7 Look for and make use of structure

MP.8 Look for and express regularity in repeated reasoning

### Linked Content Standard:

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.REI.B.4 Solve quadratic equations in one variable.

### Instructional Notes:

- It is important that students make connections between the characteristics of each form and how to gather key information from each form. Students should make connections about which form to use for various situations.
  - Vertex form reveals the maximum or minimum.
  - Standard form reveals the y-intercept.
  - Intercept form reveals the x-intercept(s).
- It may be helpful to have students complete a graphic organizer to compare forms, such as the following:

**What does each form of a quadratic equation reveal?**

<b>Vertex Form</b> $y = a(x - h)^2 + k$	<b>Factored Form</b> $y = a(x - b)(x - c)$	<b>Standard Form</b> $y = ax^2 + bx + c$

- Have students convert between standard/polynomial form, intercept/factor form, and vertex form. Introduce completing the square and factoring using area models and algebra tiles.



Reach Back	Reach Ahead
Write equations for linear functions. Convert between slope-intercept and standard forms (A-CED.A.2)	Write equations for polynomial functions. Convert between factored and standard forms (A-CED.A.2)

Linked Essential Understanding(s):	Linked Unit EQ(s):
<p>A quadratic function may be represented as equations in different forms. Each form of the equation for a quadratic reveals different features of the graph, including zeros, relative extrema, and end behavior.</p> <p>Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Some representations of a function may be more useful than others, depending on the context.</p> <p>Links between algebraic and graphical representations of functions are especially important in studying relationships and change.</p>	<p>EQ2a. What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?</p> <p>EQ3. What are the key characteristics of each representation of a function and what information does each representation reveal about the graph?</p>

**LEQ 1:** What does each form of a quadratic function reveal about the graph?

**LEQ 2:** How do you convert between equivalent quadratic forms?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
<b>Section(s)</b>	<p><b>Chapter 8</b> Using Algebra Tiles to Factor pg. 511</p> <p><b>8-3</b> Multiplying Binomials pg. 498</p> <p><b>8-4</b> Multiplying Special Cases pg. 504</p> <p><b>8-5</b> Factoring</p>	<p><u>Course 1 Book:</u> 7.2.2 pg. 497, #4-7</p> <p><u>Course 2 Book:</u> 5.1.2 pg. 334 #4,5,CYU 5.1.3 pg. 336 #1-11, CYU</p>	<p>8-6 Algebra Lab: Factoring Trinomials (pg. 501)</p> <p>8-6 Solving <math>x^2 + bx + c = 0</math> (pg. 503)</p> <p><u>Real World Problem:</u> Design (pg. 506)</p>	<p>8.5 Factoring - Coefficient of 1</p> <p>8.6 Factoring Coefficient greater than 1</p> <p>8.7 Factoring Special Products</p> <p>Page 524</p>



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	$x^2 + bx + c$ pg. 512 <b>8-6</b> Factoring $ax^2 + bx + c$ pg. 518 <b>8-7</b> Factoring Special Cases pg. 523 <b>Chapter 9</b> <b>9-5</b> Completing the Square pg. 576		8-7 Solving $ax^2 + bx + c = 0$ (pg. 510) <u>Real World Problem:</u> Wildlife (pg. 512)  8-8 Differences of Squares (pg. 516) <u>Real World Problem:</u> (pg. 518 & 519)  8-9 Perfect Squares (pg. 522) <u>Real World Problem:</u> Physical Science (pg. 526)  9-4 Solving Quadratic Equations by Completing the Square (pg. 574) <u>Real World Problem:</u> Jerseys (pg. 576)	Factoring with algebra tiles
<b>Strength of Alignment</b>	<b>Aligned</b>	<b>Aligned</b>	<b>Aligned</b>	<b>Weak Alignment</b>

### Sample Lesson Activities/Resources:

This is an interactive website for students to solve quadratic functions by completing the square. It starts with a lesson of completing the square and then has quiz questions for the students to answer.

<https://www.mathsisfun.com/algebra/completing-square.html>

The videos below show how to teach completing the square and factoring using algebra tiles.

[https://drive.google.com/open?id=0B\\_2\\_-NMZ5KYqck5CMTQxRHgtNEU](https://drive.google.com/open?id=0B_2_-NMZ5KYqck5CMTQxRHgtNEU)

[https://drive.google.com/open?id=0B\\_2\\_-NMZ5KYqVVRpSGJBQIBITDg](https://drive.google.com/open?id=0B_2_-NMZ5KYqVVRpSGJBQIBITDg)



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## Day 22: Deriving the Quadratic Formula

**Learning Target:** I can derive the quadratic formula from standard form.

### Mathematical Practice Standards

MP.1 Make sense of problems and persevere in solving them

MP.2 Reason abstractly and quantitatively

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.8 Look for and express regularity in repeated reasoning

### Linked Content Standard:

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.REI.B.4 Solve quadratic equations in one variable.

### Instructional Notes:

- Begin with having students complete the square to solve quadratics.
- Have students make a connection between completing the square and the graph of the parabola and how that can provide the vertex which is also the axis of symmetry.
- Provide students support during the derivation of the quadratic formula from standard form using completing the square.

Reach Back	Reach Ahead
Write equations for linear functions. Solving equations to find zeros and/or points of intersection. (A-CED.A.1, A-CED.A.2, A-CED.A.3)	Using a variety of algebraic techniques to find solutions to different equations. (A-CED.A.1, A-CED.A.3)

Linked Essential Understanding(s):	Linked Unit EQ(s):
A quadratic function may be represented as equations in different forms. Each form of the equation for a quadratic reveals different features of the graph, including zeros, relative extrema, and end behavior.	EQ2a. What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?





Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic function are real, they are symmetric about the x-coordinate of the maximum or minimum point	
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**LEQ:** How do we derive the Quadratic Formula?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
Section(s)		<b>Course 1 Book:</b> none  <b>Course 2 Book:</b> 5.1.4 pg. 340 #1 (more practice #2-9, CYU)		Page 619 shows the process.
Strength of Alignment	Not aligned	Weak Alignment	Not aligned	Aligned

**Sample Lesson Activities/Resources:**

This site shows how to derive the quadratic formula from standard form. It also includes a video to show a process.

<http://www.wikihow.com/Derive-the-Quadratic-Formula>



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## Day 23-25: Applying the Quadratic Formula

**Learning Target:** I can apply the quadratic formula to solve quadratic functions.

**Mathematical Practice Standards:**

MP.1 Make sense of problems and persevere in solving them

MP.4 Model with mathematics

MP.6 Attend to precision

MP.7 Look for and make use of structure

MP.8 Look for and express regularity in repeated reasoning

**Linked Content Standard:**

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.REI.B.4 Solve quadratic equations in one variable.

**Instructional Notes:**

- Have students solve functions using the quadratic formula, including real-world applications. Emphasize that the quadratic formula produces the zeros of a quadratic function.
- By this time, students should know when and why to apply the quadratic formula instead of factoring. It is imperative that students have to make decisions about what strategy they should use for each problem.

Reach Back	Reach Ahead
Solve equations involving linear and exponential functions. Finding the zeros of a function using a graph. (A-CED.A.1, F-IF.A.2)	Finding intersections between graphs of different types of functions. (F-IF.C.9)

Linked Essential Understanding(s):	Linked Unit EQ(s):
A quadratic function may be represented as equations in different forms. Each form of the equation for a quadratic reveals different features of the graph, including zeros, relative extrema, and end behavior.	EQ2a. What does each form of a quadratic function reveal about the graph? How do you convert between equivalent quadratic forms?  EQ2b. How can we identify the axis of symmetry of a quadratic equation based on the information the vertex provides?



Reasoning about the vertex form of a quadratic allows deducing that the quadratic has a maximum or minimum value and that if the zeros of the quadratic function are real, they are symmetric about the x-coordinate of the maximum or minimum point	
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**LEQ:** How do we use the quadratic formula to discover/reveal the zeroes/solutions?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
Section(s)	Chapter 9 9-6 The Quadratic Formula and the Discriminant pg. 582	Course 1 Book: 7.3.2 pg. 515, #2-4  Course 2 Book: 5.1.1 pg. 347 OYO #8 5.2.2 pg. 365 #4b	9-5 Solving Quadratic Equations by Using the Quadratic Formula (pg. 583)	Chapter 9 Section 6  Page 616 - 2 Error Analysis Problems
Strength of Alignment	Somewhat aligned	Strongly aligned	Aligned	Aligned

**Sample Lesson Activities/Resources:**

This is a great video to show students that may be struggling with applying the quadratic formula with bigger numbers. <https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/quadratic-formula-1>



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## Day 26-30: Finding Solutions for Quadratic Functions and Inequalities

**Learning Target:** I can use multiple representations to determine the solutions of a quadratic function.

### Mathematical Practice Standards

MP.1 Make sense of problems and perseveres in solving them

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision

MP.8 Look for and express regularity in repeated reasoning

### Linked Content Standard:

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.REI.B.4 Solve quadratic equations in one variable.

### Instructional Notes:

- By this point in the unit, students should be finding zeros using any method comfortably and be able to explain why they used that specific method.
- Embed real-world application problems in this portion of the unit.
- Use this opportunity to revisit the quadratic function family and how it compares to other function families.

Reach Back	Reach Ahead
Finding solutions for linear and exponential equations using equations and graphs (8.F.B.5, A-CED.A.1, A-CED.A.2)	Finding solutions for equations and inequalities for other function families (A-CED.A.3, F-IF.6, F-IF.9)

Linked Essential Understanding(s):	Linked Unit EQ(s):
Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the	EQ3. What are the key characteristics of each representation of a function and what information does each representation reveal



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function, although different representations highlight different characteristics, and some may show only part of the function. Some representations of a function may be more useful than others, depending on the context. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	about the graph?
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**LEQ:** What are the algebraic and graphical connections in quadratic functions?

**Text Alignment:**

Text	Pearson Algebra I (2015)	Core Plus Mathematics I (2008)	Glencoe Algebra I (2014)	Algebra I Holt McDougal Larson (2012)
<b>Section(s)</b>	<b>Mix problems from sections:</b> 8-5, 8-6, 8-7, 9-3, 9-4, 9-5, 9-6	<u>Course 1 Book:</u> 7.3.1 pg. 511, #1-4 7.3.2 pg. 515, #1-4 7.3.3 pg. 519, #6  <u>Course 2 Book:</u> 5.1.1 pg. 331 CYU 5.1.2 pg. 333 #2-5, CYU 5.1.3 pg. 336 #1-12, CYU 5.1.4 pg. 342 #5-9, CYU 5.2.2 pg. 365 #3-4, 6	9-4 Solving Quadratic Equations by Completing the Square (pg. 574) <u>Real World Problem:</u> Jerseys (pg. 576)  9-5 Solving Quadratic Equations by Using the Quadratic Formula [Pg. 585] (Different Methods)	Completing the square - Algebra Tiles - Page 604  Chapter 9 section 5 - Solving completing the square  Page 646 Mixed Review Problems
<b>Strength of Alignment</b>	<b>Somewhat aligned</b>	<b>Aligned</b>	<b>Aligned (If you use all of the word problems from earlier days)</b>	<b>Somewhat aligned</b>

**Sample Lesson Activities/Resources:**

This pdf document has examples of application quadratic problems and the answers. It needs to be reformatted but has good selection real world problems.

<https://www.cos.edu/Faculty/jonb/Documents/10.6.pdf>



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