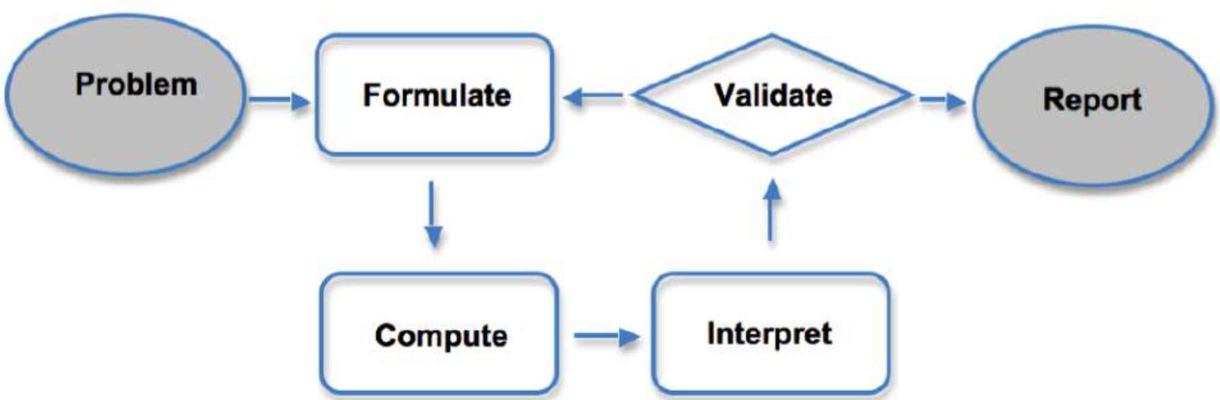
High School: Linear & Exponential Functions
Sample Unit Plan

This instructional unit guide was designed by a team of Delaware educators in order to provide a sample unit guide for teachers to use. This unit guide references some textbook resources used by schools represented on the team. This guide should serve as a complement to district curriculum resources.

Unit Overview

In this unit, students distinguish between, construct, compare, and analyze linear and exponential functions. They differentiate between linear and exponential functions and recognize arithmetic sequences as linear functions and geometric sequences as exponential functions. Students prove that linear functions grow by equal differences over equal intervals (additive rate of change), and that exponential functions grow by equal factors over equal intervals (multiplicative growth factor). Students then interpret these functions given a graphical, numerical, verbal, and symbolic representations. They translate between each of these representations, identify key characteristics, and understand the limitations of linear and exponential functions depending upon the context of the problem. Students also interpret real-world situations in order to determine whether it is can be modeled with a linear or exponential function. Students apply the Modeling Cycle to real-world problems involving linear and exponential data.

The Modeling Cycle



Source: http://www.corestandards.org/Math/Content/HSM/

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The Design Process

The writing team followed the principles of Understanding by Design (Wiggins & McTighe, 2005) to guide the unit development. As the team unpacked the content standards for the unit, they considered the following:

Stage 1: Desired Results
- What long-term transfer goals are targeted?
- What meanings should students make? What essential questions will students explore?
- What knowledge and skills will students acquire?

Stage 2: Assessment Evidence
- What evidence must be collected and assessed, given the desired results defined in stage one?
- What is evidence of understanding (as opposed to recall)?

Stage 3: The Learning Plan
- What activities, experiences, and lessons will lead to achievement of the desired results and success at the assessments?
- How will the learning plan help students Acquisition, Meaning Making, and Transfer?
- How will the unit be sequenced and differentiated to optimize achievement for all learners?

The writing team also incorporated components of the Learning-Focused (LFS) model, including the learning map, and a modified version of the Know-Understand-Do template.

The team also reviewed and evaluated the textbook resources they use in the classroom based on an alignment to the content standard for a given set of lessons. The intention is for a teacher to see what supplements may be needed to support instruction of those content standards. A list of open educational resources (OERs) are also listed with each lesson guide.

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Linear and Exponential Functions
Content and Practice Standards

Transfer Goals (Standards for Mathematical Practice)

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Content Standards

Major Content for this Unit:

Construct and compare linear, quadratic, and exponential models and solve problems.
F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.A.1.a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F.LE.A.1.b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F.LE.A.1.c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model.
F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Build a function that models a relationship between two quantities.

F.BF.A.1 Write a function that describes a relationship between two quantities.*

F.BF.A.1.a Determine an explicit expression, a recursive process, or steps for calculation from a context.

Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

Interpret functions that arise in applications in terms of the context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*

Analyze functions using different representations.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.8.b Use the properties of exponents to interpret expressions for exponential functions.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Build new functions from existing functions.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Summarize, represent, and interpret data on two categorical and quantitative variables

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

S.ID.B.6.a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.B.6.b Informally assess the fit of a function by plotting and analyzing residuals.

S.ID.B.6.c Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.
S.ID.C.9 Distinguish between correlation and causation.

Supporting/Embedded Content for this Unit:

Understand the concept of a function and use function notation.
F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

Create equations that describe numbers or relationships.
A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Represent and solve equations and inequalities graphically.
A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Interpret functions that arise in applications in terms of the context.
F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*
F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Enduring Understandings & Essential Questions

Enduring Understanding	Essential Question(s)
Understanding 1 For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables. Members of a family of functions share the same type of rate of change. This characteristic rate of change determines the kind of real-world phenomena that the function in the family can model.	EQ1a. How can you determine whether a function is a member of the linear or exponential function families? EQ1b. What is rate of change and how can it be found within a table, graph, and equation? How do the rates of change for linear and exponential functions compare? EQ1c. How can we use characteristics of a function family to make decisions about real-world phenomena?
Understanding 2 Linear functions are characterized by a constant rate of change. Reasoning about the similarity of “slope triangles” allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$ for constants m and b. Arithmetic sequences can be thought of as linear functions whose domains are the positive integers.	EQ2a. What characteristics indicate that a real world situation can be modeled with a linear function? EQ2b. How are arithmetic sequences similar to and different from linear functions?
Understanding 3 Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor. Exponential functions connect multiplication to addition through the equation $a^{b+c} = (a^b)(a^c)$. Exponential functions grow and decay by a constant percent rate per unit interval relative to another.	EQ3a. What characteristics indicate that a real world situation can be modeled with an exponential function? EQ3b. How can you use exponential functions to model the increase or decrease of a quantity over time? EQ3c. How are geometric sequences similar to and different from exponential functions?

Geometric sequences can be thought of as exponential functions whose domains are the positive integers.	
Understanding 4 Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function? EQ4b. How can the modeling cycle be applied to real-world situations?

***Enduring understandings and essential questions adapted from NCTM Enduring Understandings**

Source: *Cooney, T.J., Beckmann, S., & Lloyd, G.M. (2010). Developing essential understanding of functions for teaching mathematics in grades 9-12. Reston, VA: The National Council of Teachers of Mathematics, Inc.*

Acquisition

Conceptual Understandings (Know/Understand)	Procedural Fluency (Do)	Application (Apply)
Use rate of change (slope) and *y*-intercept (starting point) to create an equation and graph for a given situation.	Represent linear and exponential relationships graphically.	Apply the modeling cycle to real-world situations.
Distinguish between linear and exponential function families by analyzing rate of change.	Create linear equations in one variable and use them to solve problems. Create exponential equations from situations involving growth and decay.	Apply the modeling cycle to real-world situations.
Identify the similarities and differences between linear and exponential functions (shape, rate of change, etc.)	Graph linear and exponential functions.	Solve real-world problems related to simple versus compound interest.
Understand that a linear function grows by equal differences over equal intervals. (Additive relationship)	Find the slope and y-intercept from a table and write the equation of the line.	Apply the modeling cycle to solve problems.
Use the property of equality, to write an equation (a statement of equality containing one or more variables) and solve it by determining which value(s) of the variables make the equation true.	Write an equation given the slope and one point on the line. Write an equation given two points on the line. Write equations in slope-intercept form. Write equations in standard and point-slope forms. Use order of operations to correctly solve an equation. Find a solution to a linear function for a given variable	Write and solve equations based upon real-world situations.
Reveal key characteristics of Linear Functions (rate of change, *y*-intercept and trend) when converting	Convert a linear function from one representation to another (for example, convert from table to graph or equation to	Analyze and group different representations for the same linear function together (Possible idea:

between tables, graphs, and equations.	graph). Find the slope from each representation (i.e. graph, table, two points, and equation). Find the *x*- and *y*-intercepts from each representation (i.e., graph, table, and equation)	card sort or drag & drop).
Articulate why it is helpful to represent the same linear function in different ways.	Construct, compare and analyze different representations of linear functions	Identify which linear representation would be the best to look at for the answer to a given question and explain why it is the best representation.
Understand that an exponential function grows at a constant rate per unit interval. (Multiplicative growth factor)	Find the growth rate and *y*-intercept from a table and write the equation of the function.	Apply the modeling cycle to real-world situations.
Reveal key characteristics of exponential functions (growth rate, *y*-intercept and trend) when converting between tables, graphs, and equations.	Convert an exponential function from one representation to another (for example, convert from table to graph or equation to graph).	Analyze and group different representations for the same exponential function together (card sort or drag & drop).
Articulate why it is helpful to represent the same exponential function in different ways.	Construct, compare and analyze different representations of exponential functions	Identify which exponential representation would be the best to look at for the answer to a given question and explain why it is the best representation.
Translate key words in a given problem into a written exponential equation using $y = a(1 + r)^t$ for exponential growth and $y = a(1 - r)^t$ for exponential decay.	Understand whether a given situation shows growth or decay. Create an exponential equation from a given situation. Calculate a solution to an exponential function for a given variable (for example, in how many months would your bank account equal zero?)	Apply the modeling cycle to real-world situations.

Reach Back/Reach Ahead Standards

How does this unit relate to the progression of learning? What prior learning do the standards in this unit build upon? How does this unit connect to essential understandings of later content in this course and in future courses? The table below outlines key standards from previous and future courses that connect with this instructional unit of study.

Reach Back Standards	Reach Ahead Standards
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). 8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two	F-BF.B.4 Find inverse functions. A-REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context. F-BF.A.1 Write a function that describes

quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. 8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. 8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*. 8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	a relationship between two quantities F-BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. F-LE.A.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). A-REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. A-REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Common Misunderstandings

- Students may confuse function notation with products of functions. Specifically, students may replace $f(x)$ with f times x.
- Students may struggle with determining whether a function is linear or exponential when given a table, graph or equation.
- Students may compute slope as the change of x over the change of y instead of change of y over change of x.
- Students may confuse the process of finding the slope with the process of plotting a point.
- Students may interchange the initial value with growth rate when writing an exponential function.
- Students may think that the end behavior of all functions depends on the situation rather than understanding the behavior of all exponential functions.
- Students may interchange slope with y-intercept in linear functions when creating an equation. An example would be $y = 3x + 2$ and $y = 2x + 3$.
- Errors made when graphing may include the following:
  - Assuming that if an equation can be graphed, then it is a function.
  - Confusing the x and y distances/directions on a graph.
  - Incorrectly graph an exponential function by having the graph touch the x-axis.
- Students may think that a base raised to the zero power is always zero, when it actually is 1.
- Students may dismiss the domain and range as irrelevant information and possibly switch the values for each.
- Errors made when solving equations may include the following:
  - Forget to maintain balance as they solve the equation.
  - Forget to use order of operations.
  - Identify an incorrect inverse operation.
  - Perform the incorrect order of operations with exponential equations. Students may multiply a and b then raise the product to the exponent.
- Students may confuse the amount inside the parentheses will be greater than 1 for growth and less than 1 for decay when dealing with exponential functions.

SAT Assessment Expectations

Heart of Algebra (HOA)

Heart of Algebra questions ask students to:

HOA.1 Create, solve, or interpret a linear expression or equation in one variable that represents a context. The expression or equation will have rational coefficients, and multiple steps may be required to simplify the expression, simplify the equation, or solve for the variable in the equation.

HOA.3 Build a linear function that models a linear relationship between two quantities. The student will describe a linear relationship that models a context using either an equation in two variables or function notation. The equation or function will have rational coefficients, and multiple steps may be required to build and simplify the equation or function.

HOA.6 Algebraically solve linear equations (or inequalities) in one variable. The equation (or inequality) will have rational coefficients and may require multiple steps to solve for the variable; the equation may yield no solution, one solution, or infinitely many solutions. The student may also be asked to determine the value of a constant or coefficient for an equation with no solution or infinitely many solutions.

HOA.8 Interpret the variables and constants in expressions for linear functions within the context presented. The student will make connections between a context and the linear equation that models the context and will identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.

HOA.9 Understand connections between algebraic and graphical representations. The student will select a graph described by a given linear equation, select a linear equation that describes a given graph, determine the equation of a line given a verbal description of its graph, determine key features of the graph of a linear function from its equation, or determine how a graph may be affected by a change in its equation.

Problem Solving and Data Analysis (PSDA)

Problem Solving and Data Analysis questions ask students to:

PSDA.1 Use ratios, rates, proportional relationships, and scale drawings to solve single- and multistep problems. The student will use a proportional relationship between two variables to solve a multistep problem to determine a ratio or rate; calculate a ratio or rate and then solve a multistep problem; or take a given ratio or rate and solve a multistep problem.

PSDA.4 Given a scatterplot, use linear, quadratic, or exponential models to describe how the variables are related. The student will, given a scatterplot, select the equation of a line or curve of best fit; interpret the line in the context of the situation; or use the line or curve of best fit to make a prediction.

PSDA.5 Use the relationship between two variables to investigate key features of the graph. The student will make connections between the graphical representation of a relationship and properties of the graph by selecting the graph that represents the properties described, or using the graph to identify a value or set of values.

PSDA.6 Compare linear growth with exponential growth. The student will infer the connection between two variables given a context in order to determine what type of model fits best.

Passport to Advanced Math (PAM)

Passport to Advanced Math questions ask students to:

PAM.1 Create a quadratic or exponential function or equation that models a context. The equation will have rational coefficients and may require multiple steps to simplify or solve the equation.

PAM.2 Determine the most suitable form of an expression or equation to reveal a particular trait, given a context.

PAM.4 Create an equivalent form of an algebraic expression by using structure and fluency with operations.

PAM.7 Solve an equation in one variable that contains radicals or contains the variable in the denominator of a fraction. The equation will have rational coefficients, and the student may be required to identify when a resulting solution is extraneous.

PAM.10 Interpret parts of nonlinear expressions in terms of their context. Students will make connections between a context and the nonlinear equation that models the context to identify or describe the real-life meaning of a constant term, a variable, or a feature of the given equation.

PAM.12 Understand a nonlinear relationship between two variables by making connections between their algebraic and graphical representations. The student will select a graph corresponding to a given nonlinear equation; interpret graphs in the context of solving systems of equations; select a nonlinear equation corresponding to a given graph; determine the equation of a curve given a verbal description of a graph; determine key features of the graph of a linear function from its equation; or determine the impact on a graph of a change in the defining equation.

PAM.13 Use function notation, and interpret statements using function notation. The student will use function notation to solve conceptual problems related to transformations and compositions of functions.

PAM.14 Use structure to isolate or identify a quantity of interest in an expression or isolate a quantity of interest in an equation. The student will rearrange an equation or formula to isolate a single variable or a quantity of interest.

Assessment Evidence

What evidence must be collected and assessed, given the desired results defined in stage one? What is evidence of understanding (as opposed to recall)?

Understanding 1:
EQ1a. How can you determine whether a function is a member of the linear or exponential function families?

EQ1b. What is rate of change and how can it be found within a table, graph, and equation? How do the rates of change for linear and exponential functions compare?

EQ1c. How can we use characteristics of a function family to make decisions about real-world phenomena?

Linear or Exponential
https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/629

Laptop Battery Charge 2
https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1559

Example Understanding 1:
A sequence is shown below.

a_n= 7, 10, 13, 16, 19, …

A. Write a formula for a_n in terms of *n*.

B. Explain why the formula you created shows that a_n is a function whose domain is the natural numbers {1, 2, 3, 4, 5, … }

Understanding 2:
EQ2a. What characteristics indicate that a real world situation can be modeled with a linear function?

EQ2b. How are arithmetic sequences similar to and different from linear functions?

Taxi!
https://www.illustrativemathematics.org/content-standards/HSF/LE/B/5/tasks/243

Golf Balls in Water (high school)

http://ccsstoolbox.agilemind.com/parcc/about_highschool_3834.html

Example Understanding 2.1:
The table shows how many people watched two different music videos on the Internet each week for 4 weeks.

Music Video Views

Week	Music Video 1 Number of Views	Music Video 2 Number of Views
0	0	400
1	500	600
2	1000	900
3	1500	1350

If the patterns continue how many views will each video get in week 5? Explain how you got your answers.

Example Understanding 2.2:
Amir is solving the equation below:

$$4(\,3x+5\,) - 8 = 24$$

A. For his first step, Amir added 8 to both sides of the equation. Explain why this step produces an equation that is equivalent to the original equation.

B. Continue Amir's work to solve the equation. Show your work.

C. What properties of operations did you use in part B to solve the equation?

Understanding 3:
EQ3a. What characteristics indicate that a real world situation can be modeled with an exponential function?

EQ3b. How can you use exponential functions to model the increase or decrease of a quantity over time?

EQ3c. How are geometric sequences similar to and different from exponential functions?

Comparing Exponentials
https://www.illustrativemathematics.org/content-standards/HSF/LE/A/tasks/213

Basketball Bounces, Assessment Variation 1
https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/1306

Mathemafish Population
https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/686

Newton's Law of Cooling
https://www.illustrativemathematics.org/content-standards/HSF/LE/B/5/tasks/382

Cellular Growth
http://ccsstoolbox.agilemind.com/parcc/about_highschool_3836.html

Example Understanding 3:

The radioactive substance Iodine-131 is used in various hospitals as a treatment and a diagnostic tool. I-131 can be administered as a single capsule or can be taken orally. It decays according to the equation $y = y_0e^{-.086643397t}$, where y_0 = the initial amount and y = the amount remaining after t years.

A. A local hospital acquires 4 grams of Iodine-131 to use to diagnose possible thyroid conditions. According to the equation, how much Iodine-131 remains after 20 days? Show your work.

B. How long will it take the hospital's supply of Iodine-131 to decay to one-half of the original amount? Show your work.

C. Explain how you found your answer to part B.

Understanding 4:
EQ4a. What do different representations reveal about the behavior of a linear function and/or

exponential function?

EQ4b. How can the modeling cycle be applied to real-world situations?

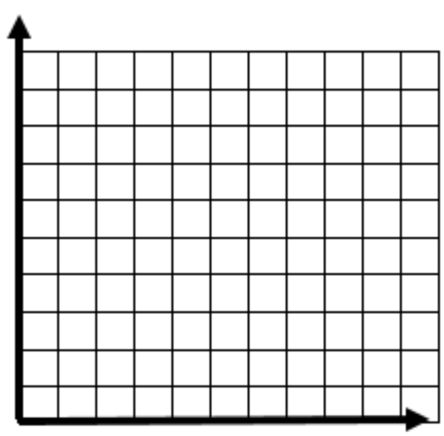
Equations and Formulas
https://www.illustrativemathematics.org/content-standards/HSA/CED/A/4/tasks/393

Example Understanding 4:
The table shows how the number of golf members in two different country clubs is increasing over time.

Country Club	**Description of Increase**
Blue Clay Creek Club	Currently 2,000 members Increasing by 200 members per year
True Manor Country Club	Currently 1,500 members Increasing by 14% per year

A. Write two equations to the describe the golf membership growth at these two clubs. For each equation, let x = the time in years from now, and let y = the number of members.

B. Graph both equations from Part A on this coordinate grid. Show the calculations used to graph the equation.



The Learning Plan: LFS Student Learning Maps

Key Learning 1:
For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables.

Members of a family of functions share the same type of rate of change. This characteristic rate of change determines the kind of real-world phenomena that the function in the family can model.

Unit Essential Questions:

EQ1a. How can you determine whether a function is a member of the linear or exponential function families?

EQ1b. What is rate of change and how can it be found within a table, graph, and equation? How do the rates of change of linear and exponential functions compare?

EQ1c. How can we use characteristics of a function family to make decisions about real-world phenomena?

Concept: Determining Whether a Function is Linear or Exponential	**Concept:** Finding and Comparing Rate of Change	**Concept:** Modeling with Linear and Exponential Functions
LEQ: What key characteristics determine whether a function is linear or exponential?	**LEQ:** How can rate of change be used to describe the covariation between two variables?	**LEQ:** How can we use characteristics of a function family to make decisions about real-world phenomena?
Vocabulary: • Linear function • Exponential function	**Vocabulary:** • Rate of Change • Slope • Growth Rate	**Vocabulary:** • Function Family

Key Learning 2:

Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type f(x) = mx + b for the constants m and b. Arithmetic sequences can be thought of as linear functions whose domains are the positive integers.

Members of a family of functions share the same type of rate of change. This characteristic rate of change determines the kind of real-world phenomena that the function in the family can model.

Unit Essential Questions:

EQ2a. What characteristics indicate that a real-world situation can be modeled with a linear function?

EQ2b. How are arithmetic sequences similar to and different from linear functions?

Concept: Modeling with Linear Functions	**Concept:** Using Arithmetic Sequences	**Concept:** Transformations of Linear Functions
LEQ: How can you model linear relationships given limited information?	**LEQ:** What is an arithmetic sequence and how are arithmetic sequences and linear functions related?	**LEQ:** What are the ways in which you can transform the graph of a linear function?
Vocabulary: • Linear function • Continuous • Discrete • *y*-intercept • Increasing • Decreasing • Slope	**Vocabulary:** • Arithmetic sequences • Common difference • Explicit Rule • Recursive Rule • Term	**Vocabulary:** • Transformation • Translation • Dilation • Reflection • Parallel lines

Key Learning 3:
Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor. Exponential functions connect multiplication to addition through the equation $a^{b+c} = (a^b)(a^c)$.
Exponential functions grow and decay by a constant percent rate per unit interval relative to another. Geometric sequences can be thought of as* exponential functions whose domains are the positive integers.

Unit Essential Questions:

EQ3a. What characteristics indicate that a real-world situation can be modeled with an exponential function?

EQ3b. How can you use exponential functions to model the increase or decrease of a quantity over time?

EQ3c. How are geometric sequences similar to and different from exponential functions?

Concept: Modeling with Exponential Functions	**Concept:** Transforming Exponential Functions	**Concept:** Using Geometric Sequences
LEQ: How can you model exponential relationships given limited information?	**LEQ:** What is the difference between exponential growth and decay? How does the graph of $f(x)=ab^x$ change when *a* and *b* are changed?	**LEQ:** What is a geometric sequence and how are geometric sequences and exponential functions related?
Vocabulary: • Base • Exponents • Exponential function • Asymptote	**Vocabulary:** • Growth • Appreciation • Decay • Depreciation • Simple Interest • Compound Interest • Transformation	**Vocabulary:** • Geometric sequences • Common ratio • Explicit Rule • Recursive Rule • Term

Key Learning 4:

Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics and some may show only part of the function.

Links between algebraic and graphical representations of functions are especially important in studying relationships and change.

Unit Essential Questions:

EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function?

EQ4b. How can you describe the relationship between two variables and use it to make predictions?

EQ4c. How can the modeling cycle be applied to real-world situations?

Concept: Multiple Representations	**Concept:** Analyzing Scatter Plots	**Concept:** Applying the Modeling Cycle
LEQ: How do you convert from one representation to another?	**LEQ:** How do you find a line of best fit and determine if it is a good model?	**LEQ:** How can the modeling cycle be applied to real-world situations?
Vocabulary: • End behavior	**Vocabulary:** • Scatter Plot • Line of Best Fit • Regression • Residuals • Correlation • Causation	**Vocabulary:** • Modeling Cycle • Formulate • Compute • Interpret • Validate • Report

Unit at a Glance

****Note: This is a suggested guideline for pacing of this set of standards. This unit is based on 50-minute lessons. Add additional days for remediation, extra practice or assessment as needed.**

# of days	Topics	Standards
5	Introduction to linear and exponential functions	F.LE.A.1 F.LE.A.1a F.LE.A.1b F.LE.A.1c S.ID.C.7 F.BF.A.1 F.IF.A.2
5	Representing linear and exponential families	F.LE.A.2 F.LE.B.5 F.IF.A.1 F.IF.A.3 F.IF.B.5
10	Representations	F.IF.A.3 F.IF.C.7 F.IF.B.4 F.IF.C.8 F.IF.B.6 F.IF.C.9 A.CED.A.2 A.REI.D.10
2	Transformations of Parent Functions	F.BF.B.3
2	Comparing functions in different forms	F.IF.C.8 F.IF.C.8.b
10	Modeling ● Causation vs. Correlation ● Scatter plots ● Line of best fit ● Residuals ● Nonlinear models	S.ID.B.6 S.ID.B.6.a S.ID.B.6.b S.ID.B.6.c S.ID.C.8 S.ID.C.9
3	Assessment	All standards in this unit should be addressed

<u>Days 1-5</u> - I can identify whether a given situation belongs to the linear or exponential family of functions based upon rate of change. <u>Standard:</u> **F.LE.A.1** **F.LE.A.1a** **F.LE.A.1b** **F.LE.A.1c** **S.ID.C.7** **F.BF.A.1** **F.IF.A.2**	**<u>Days 6-10</u> -** I can find the solution to a problem involving linear and exponential functions. <u>Standard:</u> **F.LE.A.2** **F.LE.B.5** **F.IF.A.1** **F.IF.A.3** **F.IF.B.5**	**<u>Days 11-20</u> -** I can use multiple representations to determine the solutions for linear and exponential functions. <u>Standard:</u> **F.IF.A.3** **F.IF.C.7** **F.IF.B.4** **F.IF.C.8** **F.IF.B.6** **F.IF.C.9** **A.CED.A.2** **A.REI.D.10**	**<u>Days 21-22</u>** I can look at an equation and interpret how it transforms the parent function. <u>Standard:</u> **F.BF.B.3**
<u>Days 23-24</u> - I can manipulate functions into different forms to be able to compare functions. <u>Standards:</u> **F.IF.C.8** **F.IF.C.8.b**	**<u>Days 25-34</u> -** I can use scatterplots to identify correlation and plot a line of best fit to make predictions about the data. <u>Standards:</u> **S.ID.B.6** **S.ID.B.6.a** **S.ID.B.6.b** **S.ID.B.6.c** **S.ID.C.8** **S.ID.C.9**	**<u>Days 35-37</u> -** I can show my understanding of linear and exponential functions. <u>Standards:</u> **Quiz(s) & Unit Test/Assessment(s)**	

Days 1-5: Introduction to Linear and Exponential Functions

Learning Target: I can identify whether a given situation belongs to the linear or exponential family of functions based upon rate of change.

Mathematical Practice Standards
MP.1. Make sense of problems and persevere in solving them
MP.2. Reason abstractly and quantitatively
MP.4. Model with mathematics
MP.8. Look for and express regularity in repeated reasoning

Linked Content Standards:
F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

> F.LE.A.1.a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
> F.LE.A.1.b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
> F.LE.A.1.c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

F.BF.A.1 Write a function that describes a relationship between two quantities.*

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Instructional Notes:

- The focus for these lessons is for students to identify function families as either linear or exponential and use function notation within their appropriate domains.
- Emphasize that members of a family of functions share the same type of rate of change.
- Students need to recognize that linear functions have an additive rate of change and that exponential functions have a multiplicative growth factor.

Reach Back	Reach Ahead
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	Comparing linear and exponential functions to other function families, including quadratic functions (F-LE.A.4, F-BF.A.2, F-IF.B.5, F-IF.C.9, A-CED.A.2)

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 1 For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables. Members of a family of functions share the same type of rate of change. This characteristic rate of change determines the kind of real-world phenomena that the function in the family can model.	EQ1a. How can you determine whether a function is a member of the linear or exponential function families? EQ1b. What is rate of change and how can it be found within a table, graph, and equation? How do the rates of change for linear and exponential functions compare? EQ1c. How can we use characteristics of a function family to make decisions about real-world phenomena?

LEQs:

- What key characteristics determine whether a function is linear or exponential?
- How can rate of change be used to describe the covariation between two variables?
- How can we use characteristics of a function family to make decisions about real-world phenomena?

Text Alignment:

Text	Houghton Mifflin Harcourt Algebra I (2015)	Core Plus Mathematics 1 (2008)	Big Ideas HS Algebra 1: Common Core (2014)
Section(s)	Module 16.4	Unit 1, Lesson 3	Linear: Unit 3 Lesson 1 to 3 Exponential: Unit 6 Lesson 3
Strength of Alignment	Somewhat aligned	Weak Alignment	Weak Alignment

Sample Lesson Activities/Resources:

http://theenlightenedelephant.blogspot.com/2015/03/zombies-and-exponential-functions.html
This is an engaging simulation activity to introduce linear and exponential functions.

https://www.youtube.com/watch?v=GGytywqpGXA
This video introduces exponential growth and decay using the zombie apocalypse.

https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/302
This is a resource that allows students to look at exponential functions with regard to interest. The students will look at two accounts with different starting values and different growth rates.

https://www.illustrativemathematics.org/content-standards/HSF/LE/A/1/tasks/351
This task requires students to understand that the value of an exponential function $f(x) = a * b^x$ increases by a multiplicative factor of *b* when *x* increases by one.

Days 6-10: Function Families

Learning Target: I can find the solution to a problem involving linear and exponential functions.

Mathematical Practice Standards
MP.1. Make sense of problems and persevere in solving them
MP.2. Reason abstractly and quantitatively
MP.5. Use appropriate tools strategically
MP.6. Attend to precision
MP.7. Look for and make use of structure

Linked Content Standard:
F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

F.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.*

Instructional Notes:
- The focus of these lessons is for students to be able to solve linear and exponential functions.
- Emphasize the differences between the domain and range of linear and exponential functions.
- Students need to recognize that arithmetic sequences are represented by linear functions and that geometric sequences are represented by exponential functions.

Reach Back	Reach Ahead
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the	Comparing linear and exponential functions to other function families, including quadratic functions. Representing other function families graphically, with a table of values,

corresponding output. 8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	and with an equation. (F-LE.A.4, F-BF.A.2, F-IF.B.5, F-IF.C.9, A-CED.A.1, A-CED.A.2, A-CED.A.3) Applying understanding of sequences to series (Pre-calculus)

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 2 Linear functions are characterized by a constant rate of change. Reasoning about the similarity of “slope triangles” allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$ for constants m and b. Arithmetic sequences can be thought of as linear functions whose domains are the positive integers.	EQ2a. What characteristics indicate that a real world situation can be modeled with a linear function? EQ2b. How are arithmetic sequences similar to and different from linear functions?
Understanding 3 Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increased	EQ3a. What characteristics indicate that a real world situation can be modeled with an exponential function? EQ3b. How can you use exponential functions to model the increase or decrease

by 1 unit, the output is multiplied by a constant factor. Exponential functions connect multiplication to addition through the equation $a^{b+c} = (a^b)(a^c)$. Exponential functions grow and decay by a constant percent rate per unit interval relative to another. Geometric sequences can be thought of as exponential functions whose domains are the positive integers.	of a quantity over time? EQ3c. How are geometric sequences similar to and different from exponential functions?

LEQs:

- How can you model linear relationships given limited information?
- What is a arithmetic sequence and how are arithmetic sequences and linear functions related?
- How can you model exponential relationships given limited information?
- What is a geometric sequence and how are geometric sequences and exponential functions related?

Text Alignment:

Text	**Houghton Mifflin Harcourt Algebra I (2015)**	**Core Plus Mathematics 1 (2008)**	**Big Ideas HS Algebra 1: Common Core (2014)**
Section(s)	**Module 4 Module 5 Module 15 & Module 16**	**Linear: Unit 3, Lessons 2 Exponential: Unit 5, Lessons 1 & 2**	**Linear: Unit 4 Lesson 6 Exponential: Unit 6 Lesson 6 and 7**
Strength of Alignment	**Somewhat aligned**	**Somewhat aligned**	**Somewhat aligned**

Sample Lesson Activities/Resources:

https://www.illustrativemathematics.org/content-standards/HSF/IF/A/3/tasks/1695
This task requires students view a function through a recursive and an algebraic lens.

https://www.illustrativemathematics.org/content-standards/HSF/IF/A/1/tasks/589 -
This task explores that not all functions have real numbers as domain and range values as well as the process of "restricting the domain".

Days 11-20: Representing Linear and Exponential Functions

Learning Target:
I can use multiple representations to determine the solutions for linear and exponential functions.

Mathematical Practice Standards
MP.2. Reason abstractly and quantitatively
MP.3. Construct viable arguments and critique the reasoning of others
MP.4. Model with mathematics
MP.5. Use appropriate tools strategically
MP.6. Attend to precision
MP.7. Look for and make use of structure

Linked Content Standard:
F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Instructional Notes:

- The focus of these lessons is for students to identify and interpret the key characteristics of a linear and an exponential function.
- Emphasize that key characteristics will be essential in constructing multiple representations (graphs, tables, equations, verbal description) of a linear and exponential function
- Students need to seamlessly transfer between various representations in order to solve a linear or exponential function.

Reach Back	Reach Ahead
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. 8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). 8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional	Comparing linear and exponential functions to other function families, including quadratic functions. Representing other function families graphically, with a table of values, and with an equation. (F-LE.A.4, F-BF.A.2, F-BF.B.4, A-REI.D.11, F-IF.B.5, F-IF.C.9, A-CED.A.1, A-CED.A.2, A-CED.A.3)

relationships represented in different ways.	

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 2 Linear functions are characterized by a constant rate of change. Reasoning about the similarity of “slope triangles” allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$ for constants m and b. Arithmetic sequences can be thought of as linear functions whose domains are the positive integers.	EQ2a. What characteristics indicate that a real world situation can be modeled with a linear function? EQ2b. How are arithmetic sequences similar to and different from linear functions?
Understanding 3 Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor. Exponential functions connect multiplication to addition through the equation $a^{b+c} = (a^b)(a^c)$. Exponential functions grow and decay by a constant percent rate per unit interval relative to another. Geometric sequences can be thought of as exponential functions whose domains are the positive integers.	EQ3a. What characteristics indicate that a real world situation can be modeled with an exponential function? EQ3b. How can you use exponential functions to model the increase or decrease of a quantity over time? EQ3c. How are geometric sequences similar to and different from exponential functions?

LEQ:

- What is the difference between exponential growth and decay?

Text Alignment:

Text	Houghton Mifflin Harcourt Algebra I (2015)	Core Plus Mathematics 1 (2008)	Big Ideas HS Algebra 1: Common Core (2014)
Section(s)	Module 5 & Module 16	Linear: Unit 3, Lessons 1 Exponential: Unit 5, Lessons 1 & 2	Exponential: Unit 6 Lesson 4
Strength of Alignment	Weak Alignment	Somewhat aligned	Weak Alignment

Sample Lesson Activities/Resources:

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2084
This task asks students to use a scenario and different graphs that could describe the relationship of the quantities in the situation.

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2085
In this task, students make sense of the information in several graphs, and then have the opportunity to draw their own graph to share with a partner.

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/2087
In this task, students are asked to develop a story to describe a graph.

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/4/tasks/637
This task is designed for students to correlate symbolic statements about a function using function notation with a graph of the function.

https://www.illustrativemathematics.org/content-standards/HSF/IF/B/6/tasks/1897
In this task, students will interpret, plot, and visualize data in terms of a context.

Days 21-22: Transformations of Parent Functions

Learning Target: I can look at an equation and interpret how it transforms the parent function.

Mathematical Practice Standards
MP.2. Reason abstractly and quantitatively
MP.4. Model with mathematics
MP.5. Use appropriate tools strategically
MP.7. Look for and make use of structure

Linked Content Standard:
F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Instructional Notes:
- The focus for these lessons is understand how to transform parent functions.
- Emphasize function notation and identify the effect on the graph by replacing *f(x)* with *f(x)*+k, k*f(x), f(kx)*, and *f(x+k)* for specific values of *k* (both positive and negative) for transformations of linear and exponential functions.
- Students need interpret representations in a real-world situation.

Reach Back	Reach Ahead
8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described verbally. 8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	Applying transformations to other function families, including quadratic functions (F-BF.B.3, F-IF.C.9, A-SSE.A.3, A-CED.A.2)

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 4 Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function? EQ4b. How can the modeling cycle be applied to real-world situations?

LEQs:

- What are the ways in which you can transform the graph of a linear function?
- How does the graph of $f(x)=ab^x$ change when a and b are changed?

Text Alignment:

Text	**Houghton Mifflin Harcourt Algebra I (2015)**	**Core Plus Mathematics 1 (2008)**	**Big Ideas HS Algebra 1: Common Core (2014)**
Section(s)	**Module 6.4 (Linear)** **Module 15.5 (Expo)**		**Linear:** **Unit 3 Lesson 6** **Unit 4 Lesson 1b & 7**
Strength of Alignment	**Strongly aligned**	**Not aligned**	**Weak Alignment**

Sample Lesson Activities/Resources:
https://teacher.desmos.com/exponential
In this set of learning tasks, students are asked to transform exponential functions, predict the next values in a table, and compare and contrast linear and exponential functions.

Days 23-24: Comparing Functions in Different Forms

Learning Target: I can manipulate functions into different forms to be able to compare functions.

Mathematical Practice Standards
MP.2. Reason abstractly and quantitatively
MP.3. Construct viable arguments and critique the reasoning of others
MP.7. Look for and make use of structure
MP.8. Look for and express regularity in repeated reasoning

Linked Content Standard:
F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.8b Use the properties of exponents to interpret expressions for exponential functions.

Instructional Notes:
- The focus for these lessons is to understand that different representations of a function does not change the function.
- Emphasize the properties of exponents to interpret exponential functions.
- Students need to be able to write a function in different but equivalent forms.

Reach Back	Reach Ahead
8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. 8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$	Writing equations for other function families, including quadratic functions. Converting between factored, standard, and vertex forms for a quadratic function. (F-IF.C.8, A-CED.A.1, A-CED.A.2, A-CED.A.3)

for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*.	

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 4 Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function? EQ4b. How can the modeling cycle be applied to real-world situations?

LEQs:

- How do you convert from one representation to another?
- How can the modeling cycle be applied to real-world situations?

Text Alignment:

Text	**Houghton Mifflin Harcourt Algebra I (2015)**	**Core Plus Mathematics 1 (2008)**	**Big Ideas HS Algebra 1: Common Core (2014)**
Section(s)	**Module 6**		**Linear:** **Unit 4 Lesson 3** **Exponential:** **Unit 6 Lesson 3**
Strength of Alignment	**Somewhat aligned**	**Not aligned**	**Weak Alignment**

Sample Lesson Activities/Resources:

https://www.illustrativemathematics.org/content-standards/HSF/IF/C/8/tasks/758 - After a preserved plant has died, the amount of Carbon 14 is observed over time.

Days 25-34: Modeling Data

Learning Target: I can use scatterplots to identify correlation and plot a line of best fit to make predictions about the data.

Mathematical Practice Standards

MP.1. Make sense of problems and persevere in solving them
MP.2. Reason abstractly and quantitatively
MP.3. Construct viable arguments and critique the reasoning of others
MP.4. Model with mathematics
MP.5. Use appropriate tools strategically
MP.6. Attend to precision
MP.7. Look for and make use of structure
MP.8. Look for and express regularity in repeated reasoning

Linked Content Standard:

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

> S.ID.B.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.
> S.ID.B.6b Informally assess the fit of a function by plotting and analyzing residuals.
> S.ID.B.6c Fit a linear function for a scatter plot that suggests a linear association.

S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.C.9 Distinguish between correlation and causation.

Instructional Notes:

- The focus of these lessons is to use data on a scatter plot to represent linear and exponential models.
- Emphasize the difference between causation and correlation and use technology to find the equation of linear and exponential models.
- Students need to be able to plots coordinate points on a graph and fit the data with its appropriate linear or exponential model.

Reach Back	Reach Ahead
8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x, y*) values, including reading these from a table or from a	Modeling real-world problems with other function families, including quadratic functions. Applying the Modeling Cycle to other functional relationships. (S-ID.B.6, F-IF.B.5, F-IF.C.9, A-CED.A.2, F-BF.B.3)

graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. 8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. 8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	

Linked Essential Understanding(s):	**Linked Unit EQ(s):**
Understanding 4 Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function? EQ4b. How can the modeling cycle be applied to real-world situations?

LEQ:

- How do you find a line of best fit and determine if it is a good model?

Text Alignment:

Text	Houghton Mifflin Harcourt Algebra I (2015)	Core Plus Mathematics 1 (2008)	Big Ideas HS Algebra 1: Common Core (2014)
Section(s)	Module 10.2 (Linear) Module 16.3 (Expo)	Unit 2, Lesson 1 & 2 Unit 3, Lesson 1 (Investigation 3)	Linear: Unit 4 Lesson 1, 4, & 5 Exponential: Unit 4 Lesson 5
Strength of Alignment	Strongly aligned	Weak Alignment	Weak Alignment

Sample Lesson Activities/Resources:
https://www.illustrativemathematics.org/content-standards/HSS/ID/B/6/tasks/941
In this problem, students determine the variables that are quantitative, and use a scatterplot to display a relationship between two quantitative variables.

https://www.illustrativemathematics.org/content-standards/HSS/ID/C/8/tasks/1307
Students learn to interpret the slope of the least-squares line as an estimated increase in *y* per unit change in *x*.

https://www.illustrativemathematics.org/content-standards/HSS/ID/C/9/tasks/1585
Students may find certain data involving correlation and causation confusing, and this task focuses on how to critically think about this issue.

http://algebra2.thinkport.org/module1/index.html
In this lesson, students distinguish between correlation and causation, and identify lurking variables in a given situation.

Days 35-37: Assessing Understanding

Learning Target: I can show my understanding of linear and exponential functions.

Mathematical Practice Standards
MP.1. Make sense of problems and persevere in solving them
MP.3. Construct viable arguments and critique the reasoning of others
MP.4. Model with mathematics
MP.6. Attend to precision
MP.7. Look for and make use of structure

Linked Content Standards:
All major content standards should be represented in the assessment.

Instructional Notes:
- The focus of these lessons is to assess students' level of understanding of linear and exponential functions.

Linked Essential Understanding(s):	Linked Unit EQ(s):
Understanding 1 For functions that map real numbers to real numbers, certain patterns of covariation indicate membership in a particular family of functions and determine the type of formula that the function has. A rate of change describes the covariation between two variables. Members of a family of functions share the same type of rate of change. This characteristic rate of change determines the kind of real-world phenomena that the function in the family can model.	EQ1a. How can you determine whether a function is a member of the linear or exponential function families? EQ1b. What is rate of change and how can it be found within a table, graph, and equation? How do the rates of change for linear and exponential functions compare?
Understanding 2 Linear functions are characterized by a constant rate of change. Reasoning about the similarity of "slope triangles" allows deducing that linear functions have a constant rate of change and a formula of the type $f(x) = mx + b$ for constants m and b.	EQ2a. What characteristics indicate that a real world situation can be modeled with a linear function? EQ2b. How are arithmetic sequences similar to and different from linear functions?

Arithmetic sequences can be thought of as linear functions whose domains are the positive integers.	
Understanding 3 Exponential functions are characterized by a rate of change that is proportional to the value of the function. It is a property of exponential functions that whenever the input is increased by 1 unit, the output is multiplied by a constant factor. Exponential functions connect multiplication to addition through the equation $a^{b+c} = (a^b)(a^c)$. Exponential functions grow and decay by a constant percent rate per unit interval relative to another. Geometric sequences can be thought of as exponential functions whose domains are the positive integers.	EQ3a. What characteristics indicate that a real world situation can be modeled with an exponential function? EQ3b. How can you use exponential functions to model the increase or decrease of a quantity over time? EQ3c. How are geometric sequences similar to and different from exponential functions?
Understanding 4 Changing the way that a function is represented (e.g., algebraically, with a graph, in words, or with a table) does not change the function, although different representations highlight different characteristics, and some may show only part of the function. Links between algebraic and graphical representations of functions are especially important in studying relationships and change.	EQ4a. What do different representations reveal about the behavior of a linear function and/or exponential function? EQ4b. How can the modeling cycle be applied to real-world situations?

Sample Lesson Activities/Resources:
http://www.cfn107.org/uploads/6/1/9/2/6192492/math_sample_performance_assessment.pdf
This a performance task that provides three mathematical scenarios for a student to choose for their dream job salary. The student must use at least two types of representations (table, graph, equation) in order to justify their choice.