



# Georgia Standards of Excellence Frameworks

## Mathematics

GSE Analytic Geometry

Unit 4: Extending the Number System



Richard Woods, Georgia's School Superintendent  
"Educating Georgia's Future"

## Unit 4

### Extending the Number System

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**\*Revised standards indicated in red bold font.**

## OVERVIEW

In this unit students will:

- Operate with polynomials with an emphasis on expressions that simplify to linear or quadratic forms.
- Rewrite expression involving radicals
- Understand that the basic properties of numbers continue to hold with polynomials

During the school-age years, students must repeatedly extend their conception of numbers. From counting numbers to fractions, students are continually updating their use and knowledge of numbers. In Grade 8, students extend this system once more by differentiating between rational and irrational numbers. The basic properties of numbers continue to hold as polynomial simplifications are explored in this unit.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **“Strategies for Teaching and Learning”** in the Comprehensive course Overview and the tasks listed under **“Evidence of Learning”** be reviewed early in the planning process.

## STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

## KEY STANDARDS

### Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

**MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.**  
~~**Perform arithmetic operations with complex numbers.**~~

**Perform arithmetic operations on polynomials**

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

**STANDARDS FOR MATHEMATICAL PRACTICE**

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*SMP = Standards for Mathematical Practice*

**ENDURING UNDERSTANDINGS**

- Develop the structural similarities between the system of polynomials and the system of integers.
- Addition, Subtraction, and Multiplication of polynomials is closed.

**ESSENTIAL QUESTIONS**

- How are polynomial operations related to operations in the real number system?
- How can polynomials be used to express realistic situations?

**CONCEPTS/SKILLS TO MAINTAIN**

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to

be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- number sense
- computation with whole numbers and integers, including application of order of operations
- operations with algebraic expressions
- measuring length and finding perimeter and area of rectangles and squares

## **SELECTED TERMS AND SYMBOLS**

According to Dr. Paul J. Riccomini, Associate Professor at Penn State University,

*“When vocabulary is not made a regular part of math class, we are indirectly saying it **isn’t important!**”* (Riccomini, 2008) Mathematical vocabulary can have significant positive and/or negative impact on students’ mathematical performance.

- ⊙ Require students to use mathematically correct terms.
- ⊙ Teachers must use mathematically correct terms.
- ⊙ Classroom tests must regularly include math vocabulary.
- ⊙ Instructional time must be devoted to mathematical vocabulary.

<http://www.nasd.k12.pa.us/pubs/SpecialED/PDEConference//Handout%20Riccomini%20Enhancing%20Math%20InstructionPP.pdf>

The following terms and symbols are often misunderstood. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers. For help in teaching vocabulary, one technique which could be used is as follows:

*Systematic Vocabulary Instruction; McREL 2008*

Step 1: Present students with a brief explanation or description of the new term or phrase.

Step 2: Present students with a nonlinguistic representation of the new term or phrase.

Step 3: Ask students to generate their own explanation or description of the term or phrase.

Step 4: Ask students to create their own nonlinguistic representations of the term or phrase.

Step 5: Periodically ask students to review the accuracy of their explanations and representation.

<http://www.mcrel.org/topics/products/340/>

For example, the first definition covers the first two steps.

- **Binomial Expression**: An algebraic expression with two unlike terms.
- **Expression**: A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.
- **Monomial Expression**: An algebraic expression with one term.
- **Polynomial function**: A *polynomial function* is defined as a function,  $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-2}x^2 + a_{n-3}x^1 + a_n$ , where the coefficients are real numbers.
- **Rational number**: A number expressible in the form  $a/b$  or  $-a/b$  for some fraction  $a/b$ . The rational numbers include the integers.
- **Standard Form of a Polynomial**: To express a polynomial by putting the terms in descending exponent order.
- **Trinomial**: An algebraic expression with three unlike terms.
- **Whole numbers**. The numbers 0, 1, 2, 3, ....

**The properties of operations**. Here  $a$ ,  $b$  and  $c$  stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

*Associative property of addition*  $(a + b) + c = a + (b + c)$

*Commutative property of addition*  $a + b = b + a$

*Additive identity property of 0*  $a + 0 = 0 + a = a$

*Existence of additive inverses* For every  $a$  there exists  $-a$  so that  $a + (-a) = (-a) + a = 0$ .

*Associative property of multiplication*  $(a \times b) \times c = a \times (b \times c)$

*Commutative property of multiplication*  $a \times b = b \times a$

*Distributive property of multiplication over addition*  $a \times (b + c) = a \times b + a \times c$

This web site has activities to help students more fully understand and retain new vocabulary (i.e. the definition page for *dice* actually generates rolls of the dice and gives students an opportunity to add them).

<http://www.teachers.ash.org.au/jeather/maths/dictionary.html>

Definitions and activities for these and other terms can be found on the InterMath website <http://intermath.coe.uga.edu/dictnary/homepg.asp>

## **TECHNOLOGY RESOURCES**

- <http://brightstorm.com/search/?k=polynomials>
- [http://www.khanacademy.org/search?page\\_search\\_query=polynomials](http://www.khanacademy.org/search?page_search_query=polynomials)
- For mathematical applications
  - <http://www.thefutureschannel.com/>

## **EVIDENCE OF LEARNING**

By the conclusion of this unit, students should be able to

- Results of operations performed between numbers from a particular number set does not always belong to the same set. For example, the sum of two irrational numbers  $(2 + \sqrt{3})$  and  $(2 - \sqrt{3})$  is 4, which is a rational number; however, the sum of a rational number 2 and irrational number  $\sqrt{3}$  is an irrational number  $(2 + \sqrt{3})$

## **FORMATIVE ASSESSMENT LESSONS (FAL)**

**Formative Assessment Lessons** are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

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## TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all Analytic Geometry students. These tasks, or tasks of similar depth and rigor, should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they may also be used for teaching and learning (learning/scaffolding task).

<b>Task Name</b>	<b>Task Type</b> <i>Grouping Strategy</i>	<b>Content Addressed</b>	<b>SMPs Addressed</b>
Polynomial Patterns	Scaffolding Task <i>Individual/Partner Task</i>	Multiplying of Polynomials	2, 7
Modeling	Performance Task <i>Small Group Task</i>	Applying Geometric Representations of Polynomials	2, 7
Visualizing Square Roots	Learning Task <i>Individual/Partner</i>	To build the ideas of square and square root on their geometric interpretation. To justify simplification of radicals using geometric representations.	2,7
Rational & Irrational Numbers #1 (FAL)	Formative Assessment Lesson	Classifying numbers as rational or irrational and moving between different representations of rational and irrational numbers.	3
Rational & Irrational Numbers #2 (FAL)	Formative Assessment Lesson	Finding irrational and rational numbers to exemplify general statements and reasoning with properties of rational and irrational numbers.	3



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The Real Number System	Short Cycle Task	Selecting and applying knowledge from the real number system.	2, 6
Arithmetic with Polynomials and Rational Expressions (FAL)	Formative Assessment Lesson	Determining how to use arithmetic with polynomials and rational expressions	2, 6
Amusement Park	Culminating Task <i>Partner/Small Group Task</i>	Reviewing of Unit Standards	2, 3, 6, 7, 8

## Polynomial Patterns

### Standards Addressed in this Task

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

### Standards of Mathematical Practice

**2. Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.

**7. Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

### Common Student Misconceptions

1. Some students will apply the distributive property inappropriately. Emphasize that it is the distributive property of multiplication over addition. For example, the distributive property can be used to rewrite  $2(x+y)$  as  $2x+ 2y$ , because in this product the second factor is a sum (i.e., involving addition). But in the product  $2(xy)$ , the second factor,  $(xy)$ , is itself a product, not a sum.
2. Some students will still struggle with the arithmetic of negative numbers. Consider the expression  $(-3) \bullet (2 + (-2))$ . On the one hand,  $(-3) \bullet (2 + (-2)) = (-3) \bullet (0) = 0$ . But using the distributive property,  $(-3) \bullet (2 + (-2)) = (-3) \bullet (2) + (-3) \bullet (-2)$ . Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that  $(-3) \bullet (2) = -6$ , then it must follow that  $(-3) \bullet (-2) = 6$ .
3. Students often forget to distribute the subtraction to terms other than the first one. For example, students will write  $(4x+ 3) - (2x+ 1) = 4x+ 3-2x+ 1 = 2x+ 4$  rather than  $4x+ 3 -2x-1 = 2x+ 2$ .
4. Students will change the degree of the variable when adding/subtracting like terms. For example,  $2x+ 3x= 5x^2$  rather than  $5x$ .
5. Students may not distribute the multiplication of polynomials correctly and only multiply like terms. For example, they will write  $(x + 3)(x-2) = x^2 - 6$  rather than  $x^2 -2x+ 3x -6$

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The following activity is a modification from NCTM's Illuminations Polynomial Puzzler <http://illuminations.nctm.org/LessonDetail.aspx?id=L798>

*Comments:*

*When students have completed their puzzles, allow them to share their answers and thinking with the class. Here are some ideas to help you structure this:*

- *Do not simply put up the answer key. Have students write their solutions to the puzzles on the board or fill them in on an overhead copy of the activity sheet. As they fill in the spaces, ask them to explain verbally or in writing how they approached the puzzle.*
- *If students worked in pairs, allow them to present the solutions in pairs.*
- *As students are reflecting, you may wish to ask them questions such as*
  - ❖ *Did you use a traditional method to expand and factor, such as FOIL, or did you develop your own strategies as you worked?*
  - ❖ *Were there certain paths to solving the polynomial puzzles that were easier than others? Why?*

*Assessment can be made by creating tables using the templates below. The question marks can be completed by the instructor, and then, the students will complete the tables.*

	?	
?		?
	?	

		?
?		?
		?

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The following activity is a modification from NCTM's Illuminations Polynomial Puzzler <http://illuminations.nctm.org/LessonDetail.aspx?id=L798>

Can you find the pattern to the number puzzle below?

2	-6	-12
4	1	4
8	-6	-48

Explain the pattern.

Now, use the pattern to complete this table.

3	<i>-5</i>	-15
<i>8</i>	-2	-16
<i>-24</i>	<i>10</i>	-240

HINT: Start with the question marks.

This can be expanded to multiplication with polynomials by solving the following:

1	$x + 3$	<i><math>x + 3</math></i>
$-2x + 5$	2	<i><math>-4x + 10</math></i>
<i><math>-2x + 5</math></i>	<i><math>2x + 6</math></i>	<i><math>-4x^2 - 2x + 30</math></i>

What about this one?

-5	<i><math>-2x + 3</math></i>	$10x - 15$
$3x - 2$	<i>4</i>	<i><math>12x - 8</math></i>
<i><math>-15x + 10</math></i>	$8x + 12$	<i><math>120x^2 - 260x + 120</math></i>

Work the following on your own for 10 minutes, and then complete the tables with a partner.

1	$x + 7$	<i><math>x + 7</math></i>
$-2x + 5$	2	<i><math>-4x + 10</math></i>
<i><math>-2x + 5</math></i>	<i><math>2x + 14</math></i>	<i><math>-4x^2 - 18x + 70</math></i>

<i>-2</i>	$x - 3$	<i><math>-2x + 6</math></i>
3	$-5x + 1$	<i><math>-15x + 3</math></i>
<i>-6</i>	<i><math>-5x^2 + 16x - 3</math></i>	$30x^2 - 96x + 18$

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-4	<i>2</i>	-8
<i><math>x + 3</math></i>	<i><math>x - 3</math></i>	<i><math>x^2 - 9</math></i>
<i><math>-4x - 12</math></i>	$2x - 6$	$-8x^2 + 72$

$x + 3$	<i>3</i>	<i><math>3x + 9</math></i>
2	<i><math>4x</math></i>	8x
<i><math>2x + 6</math></i>	12x	<i><math>24x^2 + 72x</math></i>

<i>2</i>	<i><math>x + 5</math></i>	$2x + 10$
$x + 3$	7	<i><math>7x + 21</math></i>
$2x + 6$	<i><math>7x + 35</math></i>	<i><math>14x^2 + 112x + 210</math></i>

6	<i><math>2x + 2</math></i>	<i><math>12x + 12</math></i>
<i>3</i>	$x + 3$	<i><math>3x + 9</math></i>
18	<i><math>2x^2 + 8x + 6</math></i>	$36x^2 + 144x + 108$

## Polynomial Patterns

### Standards Addressed in this Task

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

### Standards of Mathematical Practice

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Can you find the pattern to the number puzzle below?

2	-6	-12
4	1	4
8	-6	-48

Explain the pattern.

Now, use the pattern to complete this table.

3	?	-15
	-2	?
		-240

HINT: Start with the question marks.

This can be expanded to multiplication with polynomials by solving the following:

1	$x + 3$	
$-2x + 5$	2	

What about this one?

-5		$10x - 15$
$3x - 2$		
	$- 8x + 12$	

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Work the following on your own for 10 minutes, and then complete the tables with a partner.

1	$x + 7$	
$-2x + 5$	2	

	$x - 3$	
3	$-5x + 1$	
		$30x^2 - 96x + 18$

-4		-8
	$2x - 6$	$-8x^2 + 72$

$x + 3$		
2		$8x$
	$12x$	

		$2x + 10$
$x + 3$	7	
$2x + 6$		

6		
	$x + 3$	
18		$36x^2 + 144x + 108$

## Modeling

### Standards Addressed in this Task

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

### Standards of Mathematical Practice

**2. Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.

**7. Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

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### Common Student Misconceptions

1. Some students will apply the distributive property inappropriately. Emphasize that it is the distributive property of multiplication over addition. For example, the distributive property can be used to rewrite  $2(x+y)$  as  $2x+2y$ , because in this product the second factor is a sum (i.e., involving addition). But in the product  $2(xy)$ , the second factor,  $(xy)$ , is itself a product, not a sum.
  2. Some students will still struggle with the arithmetic of negative numbers. Consider the expression  $(-3)\bullet(2 + (-2))$ . On the one hand,  $(-3)\bullet(2 + (-2)) = (-3)\bullet(0) = 0$ . But using the distributive property,  $(-3)\bullet(2 + (-2)) = (-3)\bullet(2) + (-3)\bullet(-2)$ . Because the first calculation gave 0, the two terms on the right in the second calculation must be opposite in sign. Thus, if we agree that  $(-3)\bullet(2) = -6$ , then it must follow that  $(-3)\bullet(-2) = 6$ .
  3. Students often forget to distribute the subtraction to terms other than the first one. For example, students will write  $(4x+3) - (2x+1) = 4x+3-2x+1 = 2x+4$  rather than  $4x+3-2x-1 = 2x+2$ .
  4. Students will change the degree of the variable when adding/subtracting like terms. For example,  $2x+3x = 5x^2$  rather than  $5x$ .
  5. Students may not distribute the multiplication of polynomials correctly and only multiply like terms. For example, they will write  $(x+3)(x-2) = x^2 - 6$  rather than  $x^2 - 2x + 3x - 6$
- 

The problems below will be placed on the walls around the room with large sheets of paper under each. Students will work in teams of four people to travel around the room and write their solutions under the papers. Each team should be given a letter name that corresponds to their starting problem. After each team is given about 3-4 minutes on a problem, the teacher should call time, and the teams move to the next station.

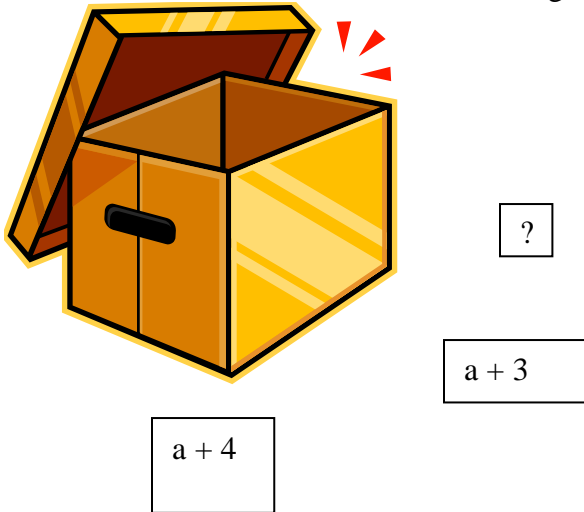
Review and practice: <http://www.wiziq.com/tutorial/111186-7-5-Multiplying-a-Polynomial-by-a-Monomial>

Having each time write in a different color can be beneficial as well as designating roles for each team member such as scribe, director, checker, and presenter. After all teams have rotated



through all the problems, the teams can travel back through to check for differences in answers. This can lead to a discussion on which problems are correct or a discussion on the different methods used to arrive at the same answer.

Problem A: The volume in cubic units of the box is  $a^3 + 8a^2 + 19a + 12$ . Its length is  $a + 4$  units and its width is  $a + 3$  units. What is its height? (**extension problem**)

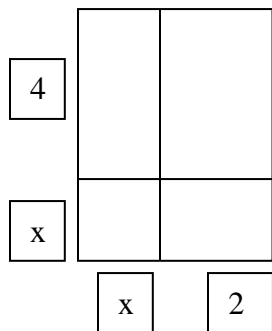


*answer:  $a + 1$*

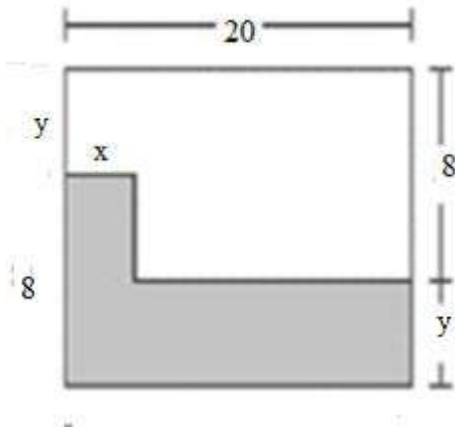
Problem B

What is an illustration of  $(x + 2)(x + 4)$ ?

*Possible answer:*



Problem C: This rectangle shows the floor plan of an office. The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.

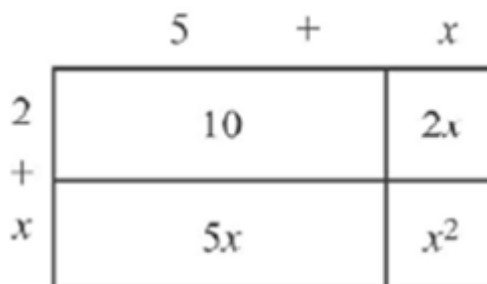


*Possible Answer*

$$8x - xy + 20y$$

Problem D

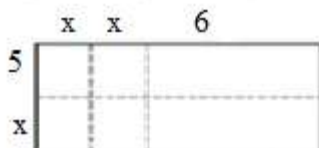
What is the rectangle modeling?



*Answer:  $(x + 5)(x + 2)$*

Problem E

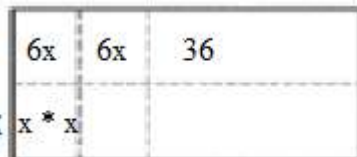
What is the product of the expression represented by the model below?



*Answer:  $2x^2 + 16x + 30$*

Problem F

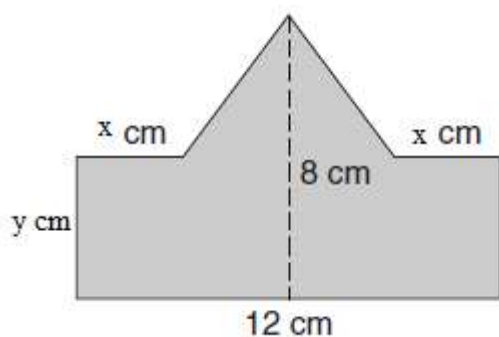
Write the dimensions for the rectangle below.



*Answer:  $(x+6)$  by  $(x+x+6)$  or  $(x+6)$  by  $(2x+6)$*

Problem G

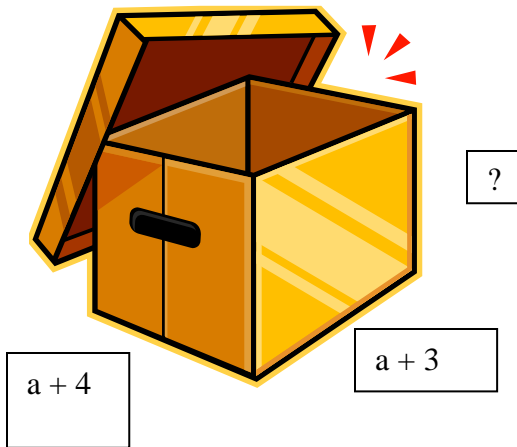
Find the area, including units, of the shape below.



*Answer:  $-8x + xy + 6y + 48$*

## Modeling

Problem A: The volume in cubic units of the box is  $a^3 + 8a^2 + 19a + 12$ . Its length is  $a + 4$  units and its width is  $a + 3$  units. What is its height? (**Extension problem**)

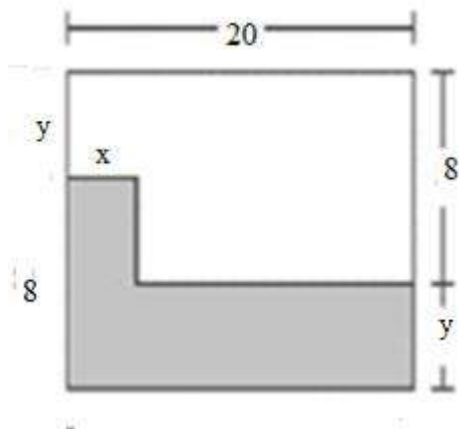


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Problem B

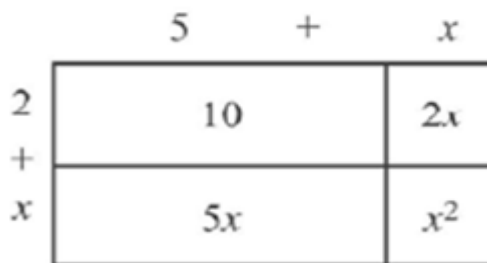
What is an illustration of  $(x + 2)(x + 4)$ ?

Problem C: This rectangle shows the floor plan of an office. The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.



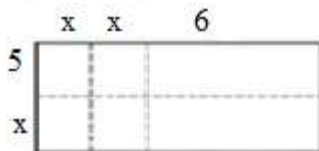
Problem D

What is the rectangle modeling?



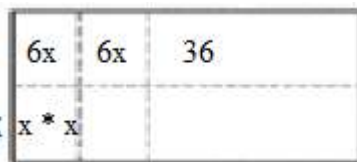
Problem E

What is the product of the expression represented by the model below?



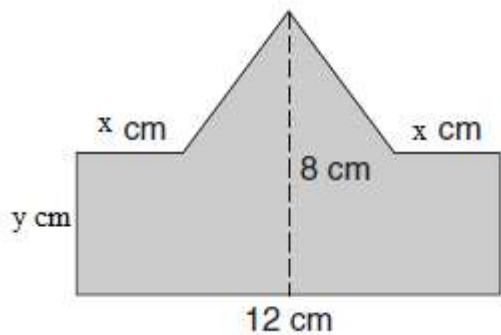
Problem F

Write the dimensions for the rectangle below.



Problem G

Find the area, including units, of the shape below.



## Visualizing Square Roots (Learning Task)

Adapted from <http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf>

### Mathematical Goals:

- To build the ideas of square and square root on their geometric interpretation.
- To justify simplification of radicals using geometric representations.

### Essential Questions:

- How do I represent radicals visually?
- What is the relationship between the radicand and the area of a square?
- How do I justify simplification of radicals using geometric representations?

## GEORGIA STANDARDS OF EXCELLENCE

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

## STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

### Materials Needed:

- Dot Paper , Graph Paper, or Geoboard

### Grouping:

- Individual/Partner

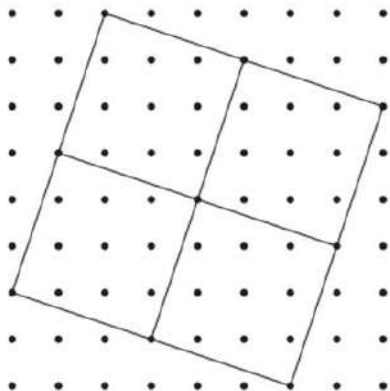
### Time Needed:

- 50-60 minutes



**Visualizing Square Roots (Learning Task)– Teacher Notes**

Adapted from <http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf>



1a. In the above figure, what are the following measures?

- i. The side of one of the small squares  $\sqrt{10}$  units
- ii. The area of one of the small squares  $\sqrt{10} \cdot \sqrt{10} = 10$  units<sup>2</sup>
- iii. The perimeter of one of the small squares  $\sqrt{10} + \sqrt{10} + \sqrt{10} + \sqrt{10} = 4\sqrt{10}$  units.
- iv. The side of the large square  $\sqrt{40}$  units
- v. The area of the large square  $\sqrt{40} \cdot \sqrt{40} = 40$  units<sup>2</sup>

*Please note that the area of the large square could also be found by finding the area the outer square (the circumscribing square) and subtracting the areas of the triangles that would be “cut out” to form the large square (decomposition strategy).*

*The area of the outer square is  $8 \cdot 8 = 64$  square units.*

*The outer square can be decomposed into the large square and four triangles.*

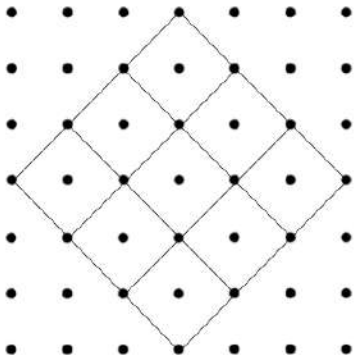
*The area of each triangle is  $\left(\frac{1}{2}\right) 2 \cdot 6 = 6$  units<sup>2</sup>*

*The area of the large square is:  $64 - 4(6) = 40$  units<sup>2</sup> (the area of the outer square minus the areas of four triangles surrounding the large square.)*

- vi. The perimeter of the large square  $\sqrt{40} + \sqrt{40} + \sqrt{40} + \sqrt{40} = 4\sqrt{40}$  units.

1b. Explain, using the answers to Problem 1a, why  $\sqrt{40} = 2\sqrt{10}$ .

Both values represent the length of the side of the large square.

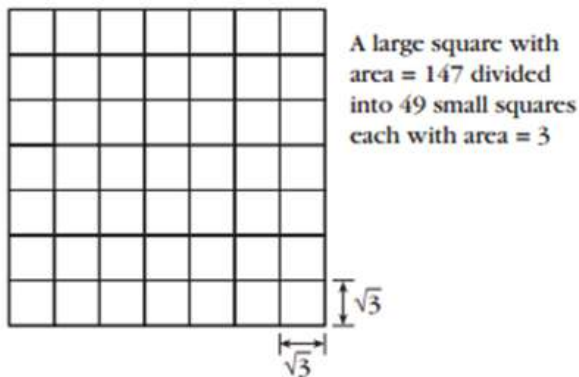


- 2a. In the above figure, what are the following measures?
- i. The area of one of the small squares       $2 \text{ units}^2$
  - ii. The side of one of the small squares       $\sqrt{2} \text{ units}$
  - iii. The area of the large square       $18 \text{ units}^2$
  - iv. The side of the large square       $\sqrt{18} \text{ units}$

2b. Explain, using the answers to problem 2a, why  $\sqrt{18} = 3\sqrt{2}$   
 Both values represent the length of the side of the large square.

3. On dot paper, create a figure to show that  $\sqrt{8} = 2\sqrt{2}$ ,  $\sqrt{18} = 3\sqrt{2}$ ,  $\sqrt{32} = 4\sqrt{2}$ , and  $\sqrt{50} = 5\sqrt{2}$ .
4. On dot paper, create a figure to show that  $\sqrt{20} = 2\sqrt{5}$  and  $\sqrt{45} = 3\sqrt{5}$ .

In the figure on the previous page, and in the figures you made in Problems 3 and 4, a larger square is divided up into a square number of squares. This is the basic idea for writing square roots in *simple radical form*. The figure need not be made on dot paper. For example, consider  $\sqrt{147}$ . Since  $147 = 3 \cdot 49$ , and since 49 is a square number, we can divide a square of area 147 units<sup>2</sup> into 49 squares, each of area 3 units<sup>2</sup>:



You will notice that the side of the larger square is  $\sqrt{147} = 7\sqrt{3}$

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5. Write the following in simple radical form.

i.  $\sqrt{12}$        $2\sqrt{3}$

ii.  $\sqrt{45}$        $3\sqrt{5}$

iii.  $\sqrt{24}$        $2\sqrt{6}$

iv.  $\sqrt{32}$        $4\sqrt{2}$

v.  $\sqrt{75}$        $5\sqrt{3}$

vi.  $\sqrt{98}$        $7\sqrt{2}$

## Visualizing Square Roots (Learning Task)

Name \_\_\_\_\_

Date \_\_\_\_\_

Adapted from <http://www.mathedpage.org/geometry-labs/gl/gl-9.pdf>

### Mathematical Goals:

- To build the ideas of square and square root on their geometric interpretation.
- To justify simplification of radicals using geometric representations.

### Essential Questions:

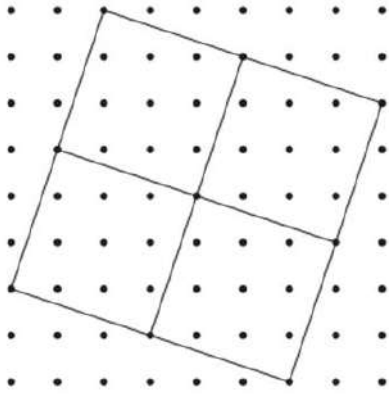
- How do I represent radicals visually?
- What is the relationship between the radicand and the area of a square?
- How do I justify simplification of radicals using geometric representations?

## GEORGIA STANDARDS OF EXCELLENCE

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

## STANDARDS FOR MATHEMATICAL PRACTICE

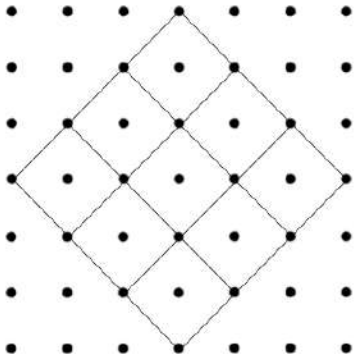
1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure



1a. In the above figure, what are the following measures?

- i. The side of one of the small squares
- ii. The area of one of the small squares
- iii. The perimeter of one of the small squares
- iv. The side of the large square
- v. The area of the large square
- vi. The perimeter of the large square

1b. Explain, using the answers to Problem 1a, why  $\sqrt{40} = 2\sqrt{10}$ .



2a. In the above figure, what are the following measures?

i. The area of one of the small squares

ii. The side of one of the small squares

iii. The area of the large square

iv. The side of the large square

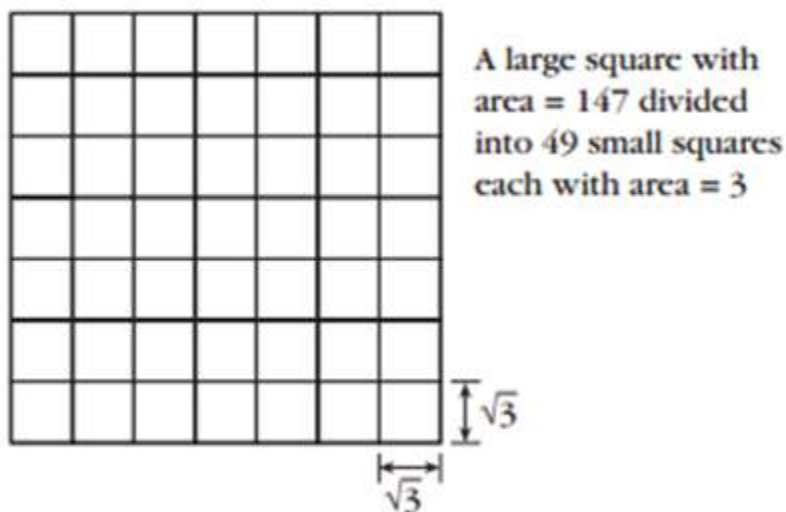
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*GSE Analytic Geometry • Unit 4*

2b. Explain, using the answers to problem 2a, why  $\sqrt{18} = 3\sqrt{2}$

3. On dot paper, create a figure to show that  $\sqrt{8} = 2\sqrt{2}$ ,  $\sqrt{18} = 3\sqrt{2}$ ,  $\sqrt{32} = 4\sqrt{2}$ , and  $\sqrt{50} = 5\sqrt{2}$ .

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You will notice that the side of the larger square is  $\sqrt{147} = 7\sqrt{3}$

5. Write the following in simple radical form.

- i.  $\sqrt{12}$
- ii.  $\sqrt{45}$
- iii.  $\sqrt{24}$
- iv.  $\sqrt{32}$
- v.  $\sqrt{75}$
- vi.  $\sqrt{98}$

## Formative Assessment Lesson: Rational & Irrational Numbers – 1

Source: *Formative Assessment Lesson Materials from Mathematics Assessment Project*  
<http://map.mathshell.org/materials/download.php?fileid=1245>

### ESSENTIAL QUESTIONS:

- How do you classify numbers as rational or irrational?
- How do you move between different representations of rational and irrational numbers?

### TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=formative>

The task, *Rational & Irrational Numbers - 1*, is a Formative Assessment Lesson (FAL) that can be found at the website: <http://map.mathshell.org/materials/lessons.php?taskid=424&subpage=concept>

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:  
<http://map.mathshell.org/materials/download.php?fileid=1245>

### STANDARDS ADDRESSED IN THIS TASK:

#### Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

**MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.**

#### **Standards for Mathematical Practice**



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This lesson uses all of the practices with emphasis on:

**3. Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Formative Assessment Lesson: Rational & Irrational Numbers – 2

Source: *Formative Assessment Lesson Materials from Mathematics Assessment Project*  
<http://map.mathshell.org/materials/download.php?fileid=1267>

### ESSENTIAL QUESTIONS:

- How do you find irrational and rational numbers to exemplify general statements?
- How do you reason with properties of rational and irrational numbers?

### TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=formative>

The task, *Rational & Irrational Numbers - 2*, is a Formative Assessment Lesson (FAL) that can be found at the website:

<http://map.mathshell.org/materials/lessons.php?taskid=434&subpage=concept>

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:

<http://map.mathshell.org/materials/download.php?fileid=1267>

### STANDARDS ADDRESSED IN THIS TASK:

#### Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

**MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.**

**Standards for Mathematical Practice**

This lesson uses all of the practices with emphasis on:

**3. Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## The Real Number System (Short Cycle Task)

Source: *Balanced Assessment Materials from Mathematics Assessment Project*

<http://www.map.mathshell.org/materials/download.php?fileid=830>

### ESSENTIAL QUESTIONS:

- How do you select and apply knowledge from the real number system?

### TASK COMMENTS: (Note: Questions 3 and 4 of this task may be used as Extensions)

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=summative>

The task, *The Real Number System*, is a Mathematics Assessment Project Assessment Task that can be found at the website:

<http://www.map.mathshell.org/materials/tasks.php?taskid=289&subpage=novice>

The PDF version of the task can be found at the link below:

<http://www.map.mathshell.org/materials/download.php?fileid=830>

The scoring rubric can be found at the following link:

<http://www.map.mathshell.org/materials/download.php?fileid=831>

### STANDARDS ADDRESSED IN THIS TASK:

#### Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

#### Standards for Mathematical Practice

This task uses all of the practices with emphasis on:

**2. Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.

**6. Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

## Arithmetic with Polynomials and Rational Expressions (Short Cycle Task)

Source: *Balanced Assessment Materials from Mathematics Assessment Project*

<http://www.map.mathshell.org/materials/download.php?fileid=834>

### ESSENTIAL QUESTIONS:

- How do you use arithmetic with polynomials and rational expressions?

### TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:

<http://www.map.mathshell.org/materials/background.php?subpage=summative>

The task, *Arithmetic with Polynomials and Rational Expressions*, is a Mathematics Assessment Project Assessment Task that can be found at the website:

<http://www.map.mathshell.org/materials/tasks.php?taskid=291&subpage=novice>

The PDF version of the task can be found at the link below:

<http://www.map.mathshell.org/materials/download.php?fileid=834>

The scoring rubric can be found at the following link:

<http://www.map.mathshell.org/materials/download.php?fileid=835>

### STANDARDS ADDRESSED IN THIS TASK:

#### Perform arithmetic operations on polynomials

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

#### **Standards for Mathematical Practice**

This task uses all of the practices with emphasis on:

**2. Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.

**6. Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

## Culminating Task: Amusement Park Problem

Extend the properties of exponents to rational exponents.

### Standards Addressed in this Task

#### Use properties of rational and irrational numbers.

**MGSE9-12.N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents. (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).**

**MGSE9-12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.**

#### Perform arithmetic operations on polynomials

**MGSE9-12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations.**

### Standards for Mathematical Practice

**2. Reason abstractly and quantitatively** by requiring students to make sense of quantities and their relationships to one another in problem situations.

**3. Construct viable arguments and critique the reasoning of others** by engaging students on discussion of why they agree or disagree with responses, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

**6. Attend to precision** by requiring students to calculate efficiently and accurately; and to communicate precisely with others by using clear mathematical language to discuss their reasoning.

**7. Look for and make use of structure** by expecting students to apply rules, look for patterns and analyze structure.

**8. Look for and express regularity in repeated reasoning** by expecting students to understand broader applications and look for structure and general methods in similar situations.

The Radical World of Math is reviewing the master plan of a proposed amusement park coming to your area. Your help is needed with the land space and with park signage.

**First**, the planners need help in designing the land space. The parameters are as follows:

- 15 rows of parking are required
- The rows will be the same length as the park

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- The park size will be square with a length of  $X$  so that expansion is possible.
- “Green Space” for planting, sitting, or picnicking is a must.
- Parking will be adjacent to only two sides of the park

Your task is to choose 3 possible configurations of land use with 15 rows of parking. Find the area of the picnic (green space) for each configuration. There is more than one way to solve the problem. For your maximum picnic space, write an equation for the total AREA of the park.

*As long as all the parameters are met, student designs will be correct. Maximum spaces are the closest to square designs.  $(x + 7)(x + 8)$*

Extension:

The park is expected to be successful and the planners decide to expand the parking lot by adding 11 more rows. Assume the new plan will add not only 11 rows of parking but will also triple the maximum original green space (approximately). Choose 1 of your park configurations (your best) to complete this section and redraw your park configuration. What is the percentage increase in area that was created by expanding to 26 rows of parking?

**Second**, signs have to be designed for the park. For one of the areas called “Radical Happenings”, the signs must show conversions between radical expressions and exponential expressions. There must be at least 10 signs in all that reflect square roots, cube roots, and fourth roots. Create 10 unique signs for use in the park. Would there be appropriate areas for these values to be placed?

*The signs can have values 1 through 10 and be used to designate ten different park areas.*

You must create problems demonstrating each concept and have another team try to “get through” your obstacles.

Extension: Using fractals, create advertising for this park. This link may help: <http://mathworld.wolfram.com/Fractal.html>

## Culminating Task: Amusement Park Problem

Extend the properties of exponents to rational exponents.

### Standards Addressed in this Task

#### Use properties of rational and irrational numbers.

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