Warm Up

For each translation of the point (–2, 5), give the coordinates of the translated point.

1. 6 units down (–2, –1)

2. 3 units right (1, 5)

For each function, evaluate $f(-2)$, $f(0)$, and $f(3)$.

3. $f(x) = x2 + 2x + 6$ 6; 6; 21

4. $f(x) = 2x2 - 5x + 1$ 19; 1; 4

Objectives

Transform quadratic functions.

Describe the effects of changes in the coefficients of $y = a(x - h)2 + k$.

Vocabulary

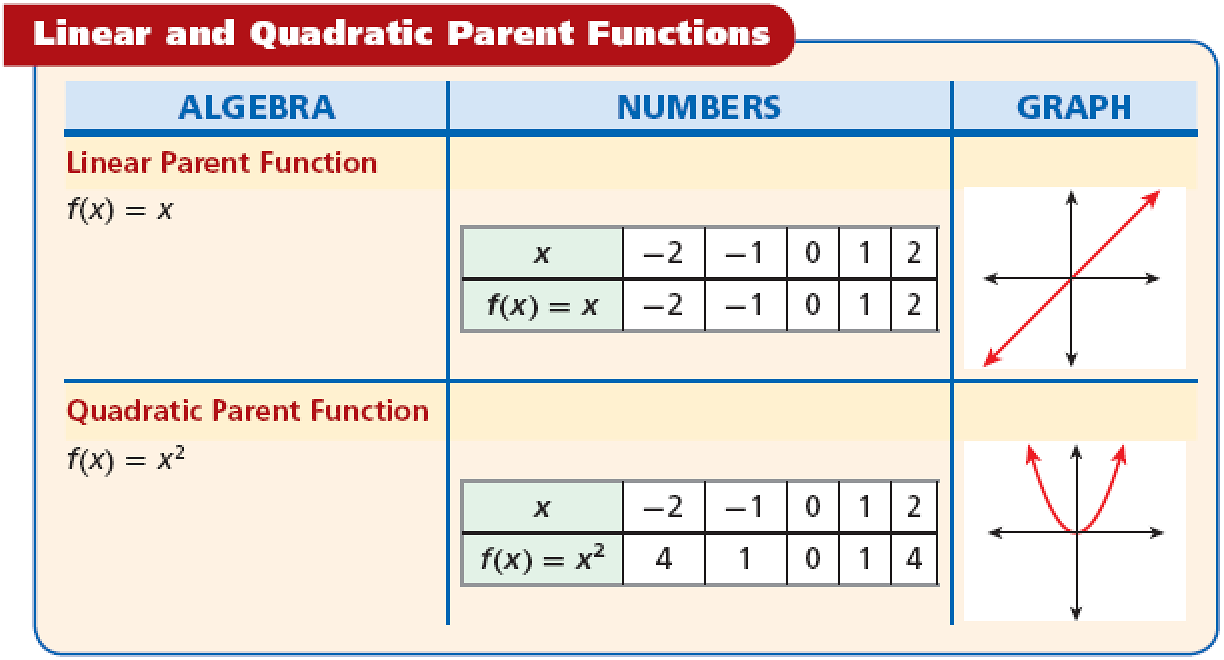
quadratic function

parabola

vertex of a parabola

vertex form

In Chapters 2 and 3, you studied linear functions of the form $f(x) = mx + b$. A **quadratic function** is a function that can be written in the form of $f(x) =$ ~~$a (x – h)2 + k$ (a $\neq$~~ 0). In a quadratic function, the variable is always squared. The table shows the linear and quadratic parent functions.



Linear and Quadratic Parent Functions

ALGEBRA | **NUMBERS** | **GRAPH**

Linear Parent Function

$f(x) = x$

x	-2	-1	0	1	2
$f(x) = x$	-2	-1	0	1	2

Quadratic Parent Function

$f(x) = x^2$

x	-2	-1	0	1	2
$f(x) = x^2$	4	1	0	1	4

Notice that the graph of the parent function $f(x) = x2$ is a U-shaped curve called a **parabola**. As with other functions, you can graph a quadratic function by plotting ~~points wit~~h coordinates that make the equation true.

Example 1: Graphing Quadratic Functions Using a Table

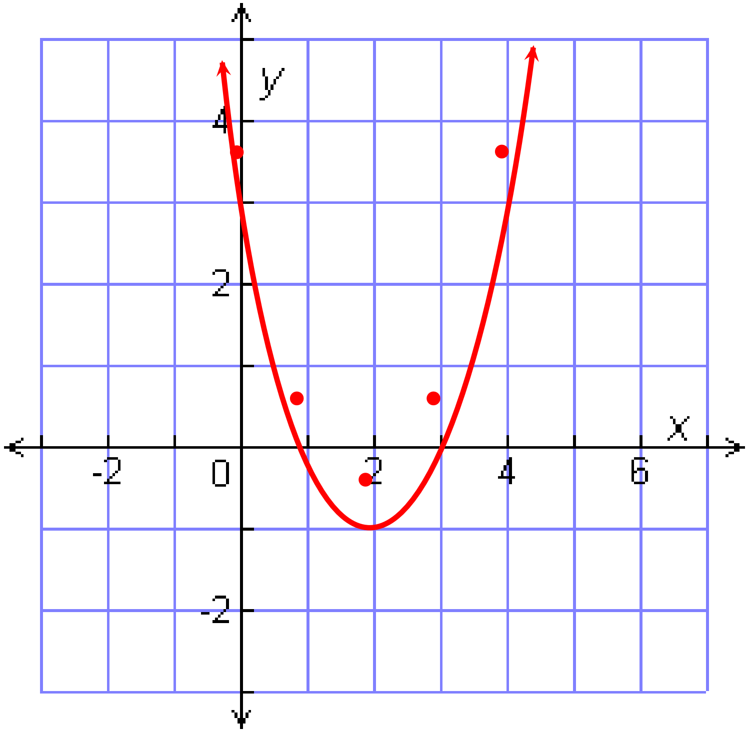
Graph *f*(*x*) = *x*2 – 4*x* + 3 by using a table.

Make a table. Plot enough ordered pairs to see both sides of the curve.

x	$f(x) = x^2 - 4x + 3$	$(x, f(x))$
0	$f(0) = (0)^2 - 4(0) + 3$	(0, 3)
1	$f(1) = (1)^2 - 4(1) + 3$	(1, 0)
2	$f(2) = (2)^2 - 4(2) + 3$	(2,–1)
3	$f(3) = (3)^2 - 4(3) + 3$	(3, 0)
4	$f(4) = (4)^2 - 4(4) + 3$	(4, 3)

Example 1 Continued

$f(x) = x2 – 4x + 3$



Check It Out! Example 1

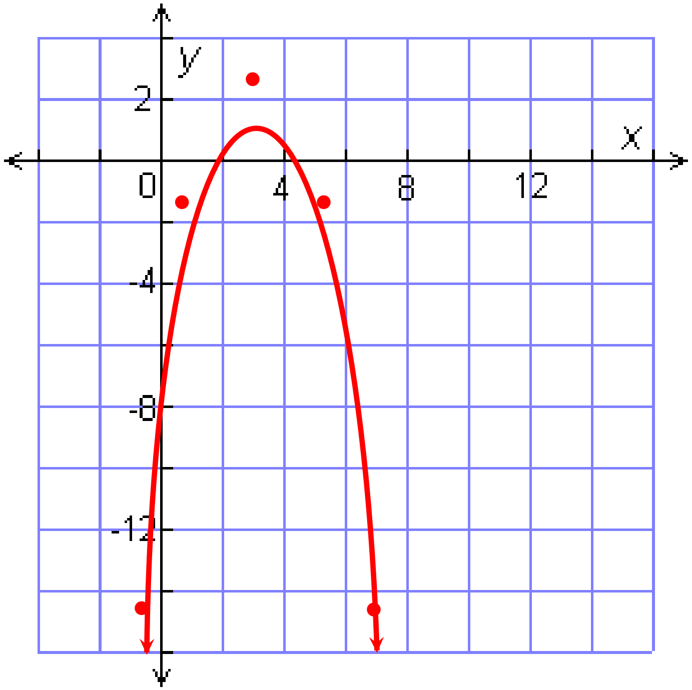
Graph $g(x) = -x2 + 6x - 8$ by using a table.

Make a table. Plot enough ordered pairs to see both sides of the curve.

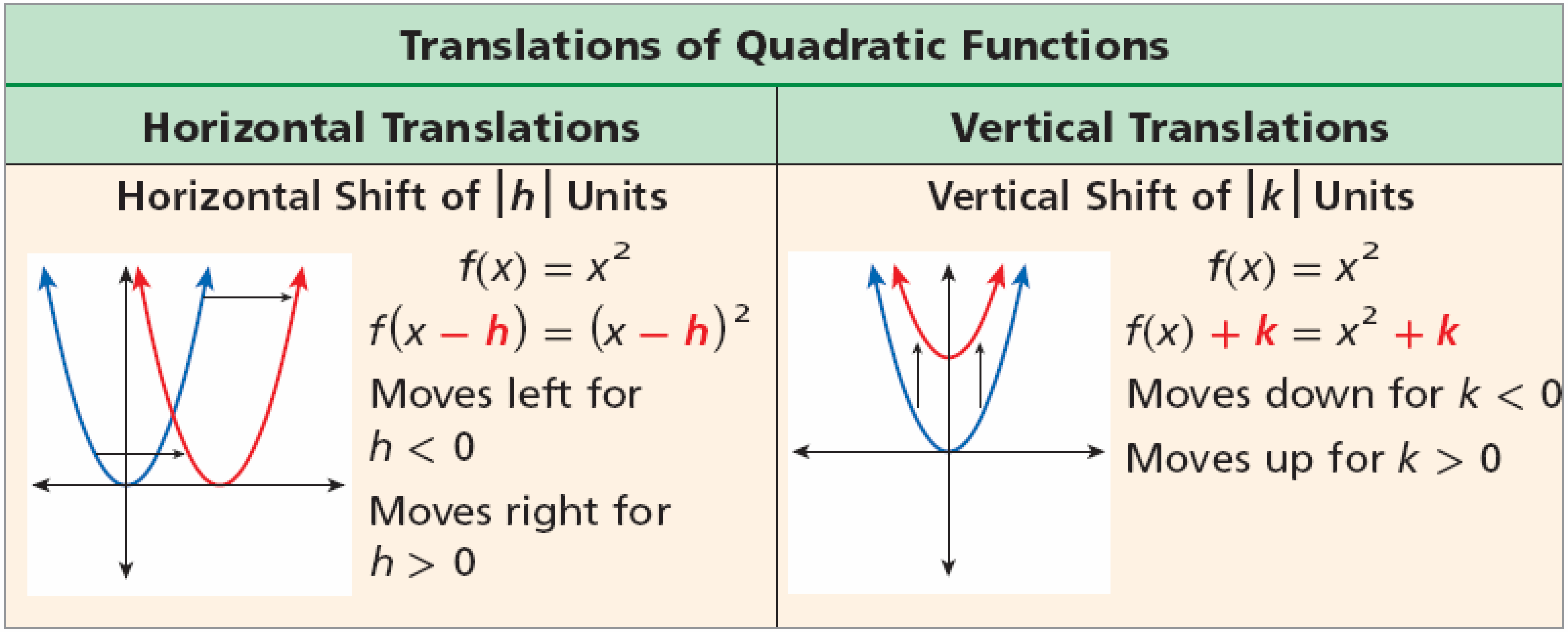
x	$g(x)= -x^2 +6x -8$	$(x, g(x))$
−1	$g(-1)= -(-1)^2 + 6(-1) - 8$	(−1,−15)
1	$g(1)= -(1)^2 + 6(1) - 8$	(1, −3)
3	$g(3)= -(3)^2 + 6(3) - 8$	(3, 1)
5	$g(5)= -(5)^2 + 6(5) - 8$	(5, −3)
7	$g(7)= -(7)^2 + 6(7) - 8$	(7, −15)

Check It Out! Example 1 Continued

$f(x) = -x2 + 6x - 8$



You can also graph quadratic functions by applying transformations to the parent function $f(x) = x^2$. Transforming quadratic functions is similar to transforming linear functions (Lesson 2-6).



Translations of Quadratic Functions

Horizontal Translations	Vertical Translations
Horizontal Shift of $\lvert h \rvert$ Units	**Vertical Shift of $\lvert k \rvert$ Units**
$f(x) = x^2$ $f(x - h) = (x - h)^2$ Moves left for $h < 0$ Moves right for $h > 0$	$f(x) = x^2$ $f(x) + k = x^2 + k$ Moves down for $k < 0$ Moves up for $k > 0$

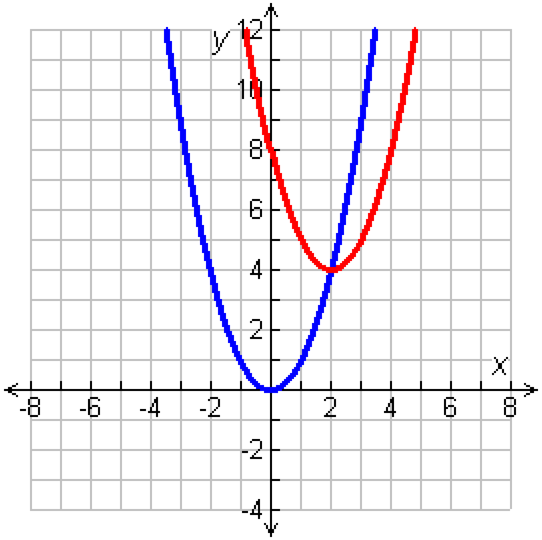
Example 2A: Translating Quadratic Functions

Use the graph of *f*(*x*) = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) = (*x* – 2)2 + 4**

Identify *h* and *k*.

g(*x*) = (*x* – 2)2 + 4



Because *h* = 2, the graph is translated **2 units right**. Because *k* = 4, the graph is translated **4 units up**. Therefore, *g* is *f* translated 2 units right and 4 units up.

Example 2B: Translating Quadratic Functions

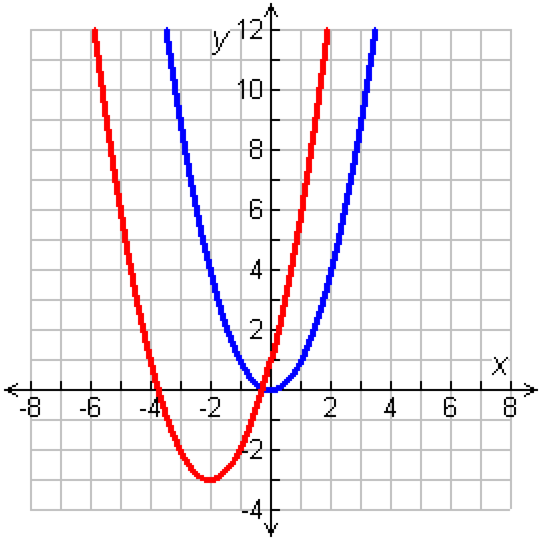
Use the graph of *f*(*x*) = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) = (*x* + 2)2 – 3**

Identify *h* and *k*.

g(*x*) = (*x* – (–2))2 + (–3)

h　　　*k*



Because *h* = –2, the graph is translated **2 units left**. Because *k* = –3, the graph is translated **3 units down**. Therefore, *g* is *f* translated 2 units left and 4 units down.

Check It Out! Example 2a

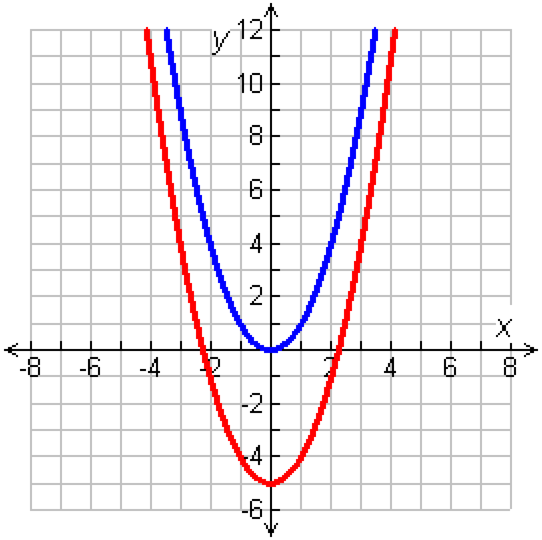
Using the graph of *f*(*x*) = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) = *x*2 – 5**

Identify *h* and *k*.

g(*x*) = *x*2 – 5

k



Because *h* = 0, the graph is not translated horizontally.
Because *k* = –5, the graph is translated **5 units down**. Therefore, *g* is *f* is translated **5 units down**.

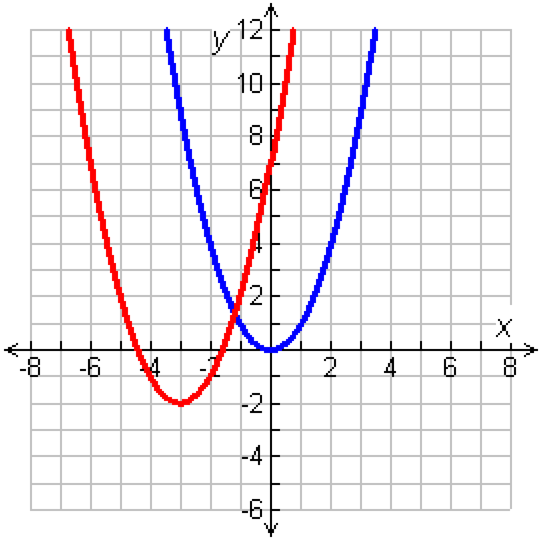
Check It Out! Example 2b

Use the graph of *f*(*x*) =*x*2 as a guide, describe the transformations and then graph each function.

$g(x) = (x + 3)2 - 2$

Identify *h* and *k*.

$g(x) = (x - \underset{\overset{\uparrow}{h}}{(-3)})\ 2 + \underset{\overset{\uparrow}{k}}{(-2)}$

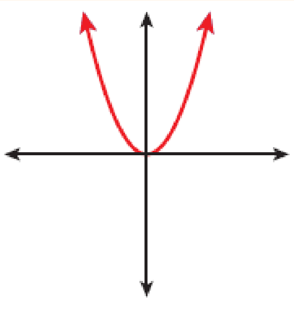


Because *h* = −3, the graph is translated **3 units left**. Because *k* = −2, the graph is translated **2 units down**. Therefore, *g* is *f* translated 3 units left and 2 units down.

Recall that functions can also be reflected, stretched, or compressed.

Reflections

Reflection Across y-axis



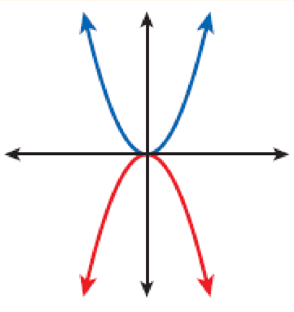
Input values change.

$$f(x) = x^2$$

$$f(-x) = (-x)^2 = x^2$$

The function $f(x) = x^2$ is its own reflection across the y-axis.

Reflection Across x-axis



Output values change.

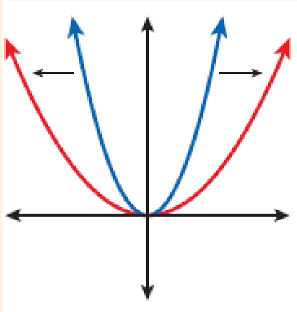
$$f(x) = x^2$$

$$-f(x) = -(x^2) = -x^2$$

The function is flipped across the x-axis.

Stretches and Compressions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

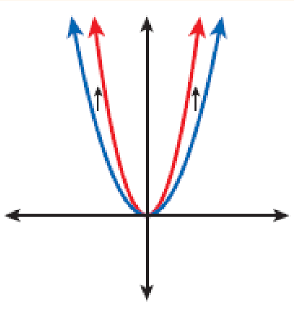
$$f(x) = x^2$$

$$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$$

$|b| > 1$ stretches away from the y-axis.

$0 < |b| < 1$ compresses toward the y-axis.

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$$f(x) = x^2$$

$$a \cdot f(x) = ax^2$$

$|a| > 1$ stretches away from the x-axis.

$0 < |a| < 1$ compresses toward the x-axis.

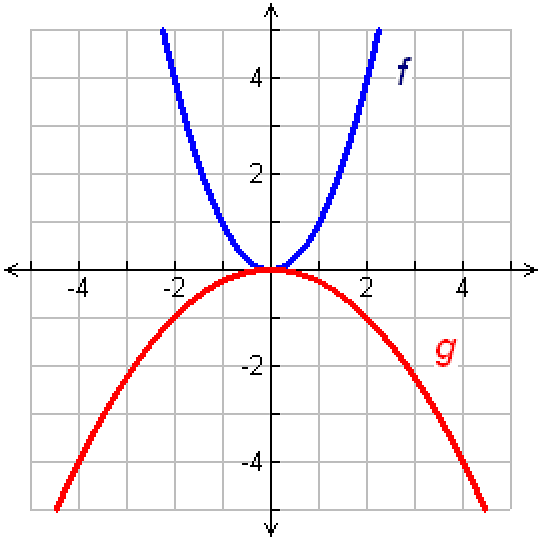
Example 3A: Reflecting, Stretching, and Compressing Quadratic Functions

Using the graph of *f*(*x*) = *x*2 as a guide, describe the transformations and then graph each function.

$$g(x) = -\frac{1}{4}x^2$$

Because *a* is negative, *g* is a reflection of *f* across the *x*-axis.

Because |*a*| = $\frac{1}{4}$, *g* is a vertical compression of *f* by a factor of $\frac{1}{4}$.



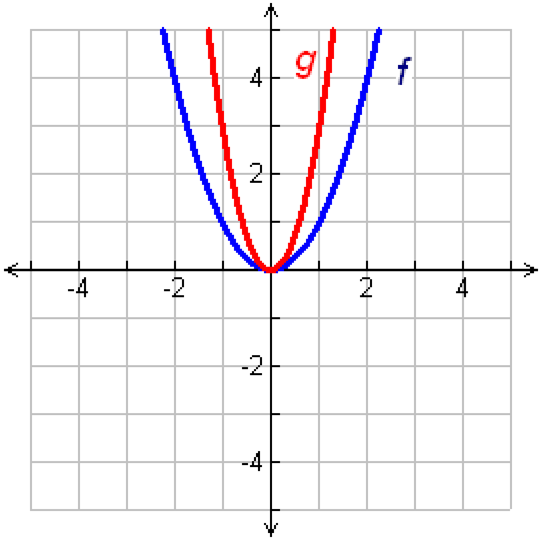
Example 3B: Reflecting, Stretching, and Compressing Quadratic Functions

Using the graph of *f(x)* = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) =(3*x*)2**

Because *b* = , *g* i $\frac{1}{3}$ a horizontal compression of *f* b a factor of .

$\frac{1}{3}$



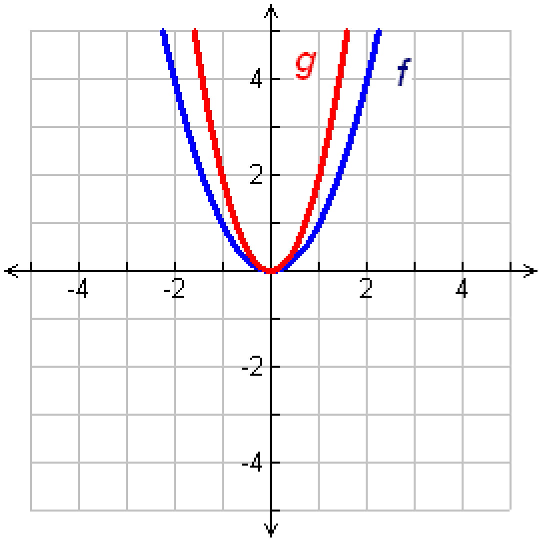
Check It Out! Example 3a

Using the graph of *f(x)* = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) =(2*x*)2**

Because *b* = , *g* i. $\frac{1}{2}$ *a* horizontal compression of *f* by a factor of $\frac{1}{2}$.

$\frac{1}{2}$

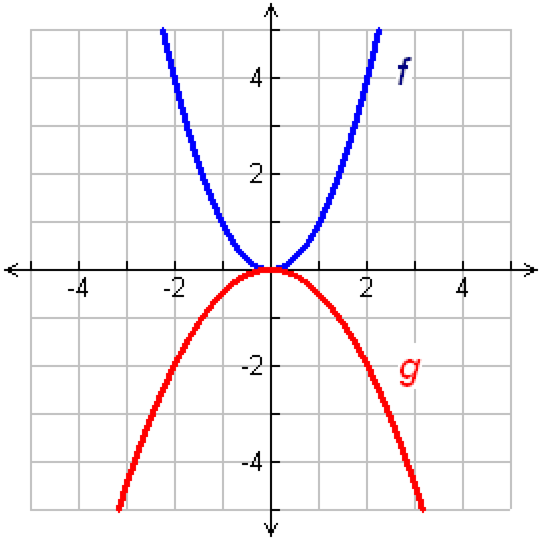


Check It Out! Example 3b

Using the graph of *f*(*x*) = *x*2 as a guide, describe the transformations and then graph each function.

***g*(*x*) = – *x*2** $\frac{1}{2}$

Because *a* is negative, *g* is a reflection of *f* across the *x*-axis.

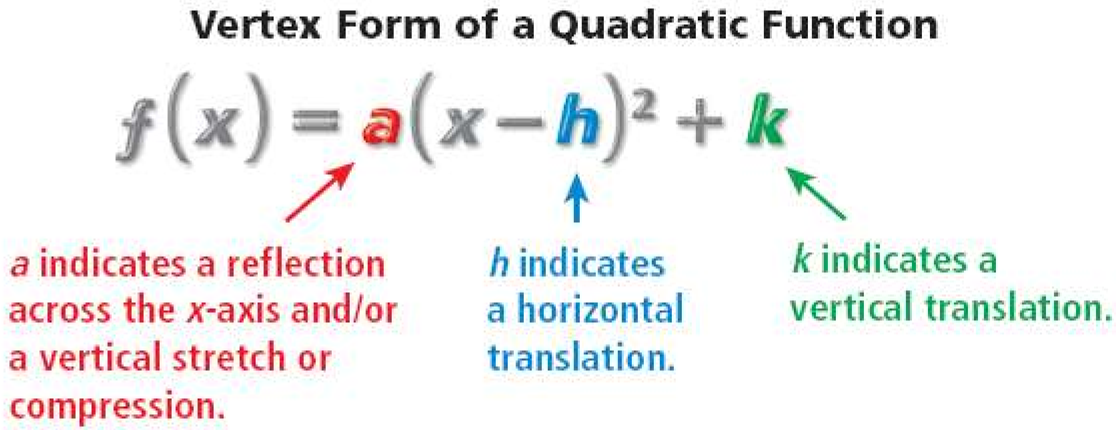


Because |*a*| = $\frac{1}{2}$, *g is* a vertical compression of *f* by a factor of .

$\frac{1}{2}$

If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is the **vertex of the parabola**.

The parent function $f(x) = x2$ has its vertex at the origin. You can identify the vertex of other quadratic functions by analyzing the function in *vertex form.* The **vertex form** of a quadratic function is $f(x) = a(x - h)2 + k$, where ~~a, h, and k ar~~e constants.



Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

a indicates a reflection across the x-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

Because the vertex is translated h horizontal units and k vertical from the origin, the vertex of the parabola is at (h, k).

Helpful Hint

When the quadratic parent function $f(x) = x2$ is written in vertex form, $y = a(x - h)2 + k$,
$a = 1$, $h = 0$, and $k = 0$.

Example 4: Writing Transformed Quadratic Functions

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x2$ is vertically stretched by a factor of and then translated 2 units left and 5 units down to create *g.*

Step 1 Identify how each transformation affects the constant in vertex form.

Vertical stretch by : $\frac{4}{3}$ $a = \frac{4}{3}$

Translation 2 units left: $h = -2$

Translation 5 units down: $k = -5$

Example 4: Writing Transformed Quadratic Functions

Step 2 Write the transformed function.

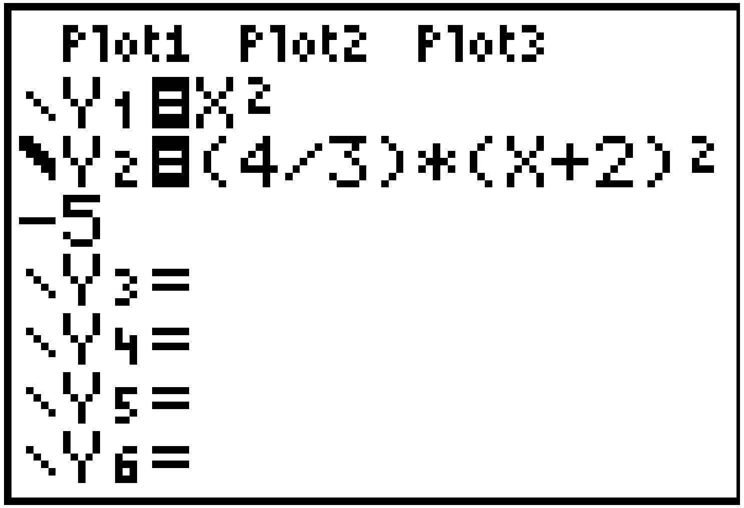
$g(x) = a(x - h)2 + k$ *Vertex form of a quadratic function*

$= \frac{4}{3}(x - (-2))2 + (-5)$ *Substitute $\frac{4}{3}$ for a, –2 for h, and –5 for k.*

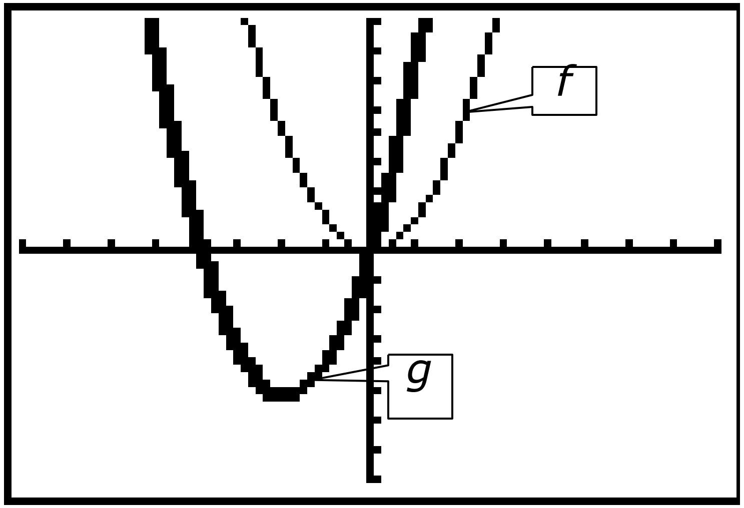
$= \frac{4}{3}(x + 2)2 - 5$ *Simplify.*

$g(x) = \frac{4}{3}(x + 2)2 - 5$

Check Graph both functions on a graphing calculator. Enter *f* as **Y1**, and *g* as **Y2**. The graph indicates the identified transformations.



```
 Plot1  Plot2  Plot3
\Y1=X²
\Y2=(4/3)*(X+2)²
-5
\Y3=
\Y4=
\Y5=
\Y6=
```



Check It Out! Example 4a

Use the description to write the quadratic function in vertex form.

The parent function *f*(*x*) = *x*2 is vertically compressed by a factor of and then translated 2 units right and 4 units down to create $\frac{1}{3}$.

Step 1 Identify how each transformation affects the constant in vertex form.

Vertical compression by : *a* = $\frac{1}{3}$ $\frac{1}{3}$

Translation 2 units right: *h* = 2

Translation 4 units down: *k* = –4

Check It Out! Example 4a Continued

Step 2 Write the transformed function.

$g(x) = a(x - h)2 + k$ — *Vertex form of a quadratic function*

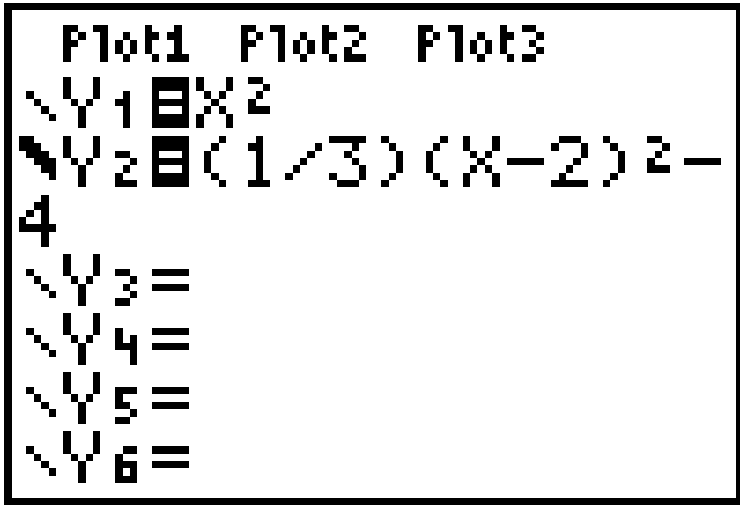
$= \frac{1}{3}x - 2)2 + (-4)$ — *Substitute fo $\frac{1}{3}$, 2 for h, and –4 for k.*

$= \frac{1}{3}x - 2)2 - 4$ — *Simplify.*

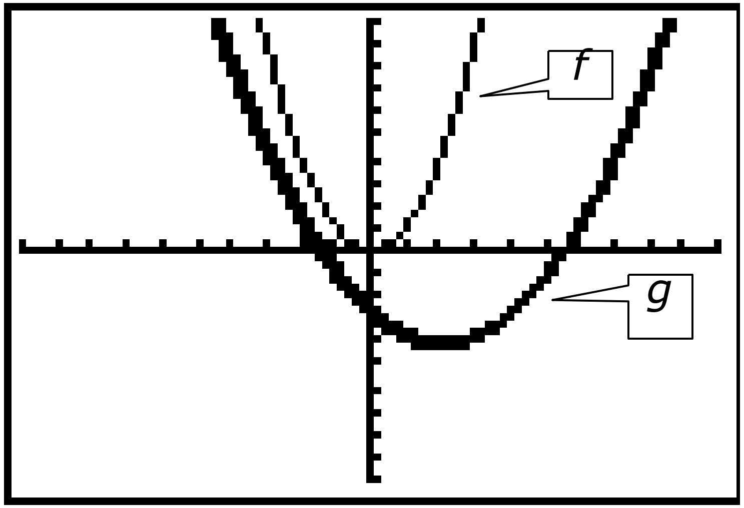
$g(x) = (\frac{1}{3} - 2)2 - 4$

Check It Out! Example 4a Continued

Check Graph both functions on a graphing calculator. Enter *f* as **Y1**, and *g* as **Y2**. The graph indicates the identified transformations.



```
  Plot1  Plot2  Plot3
\Y1=X²
\Y2=(1/3)(X-2)²-
4
\Y3=
\Y4=
\Y5=
\Y6=
```



Check It Out! Example 4b

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x2$ is reflected across the *x*-axis and translated 5 units left and 1 unit up to create *g.*

Step 1 Identify how each transformation affects the constant in vertex form.

Reflected across the *x*-axis: *a* is negative

Translation 5 units left: $h = –5$

Translation 1 unit up: $k = 1$

Check It Out! Example 4b Continued

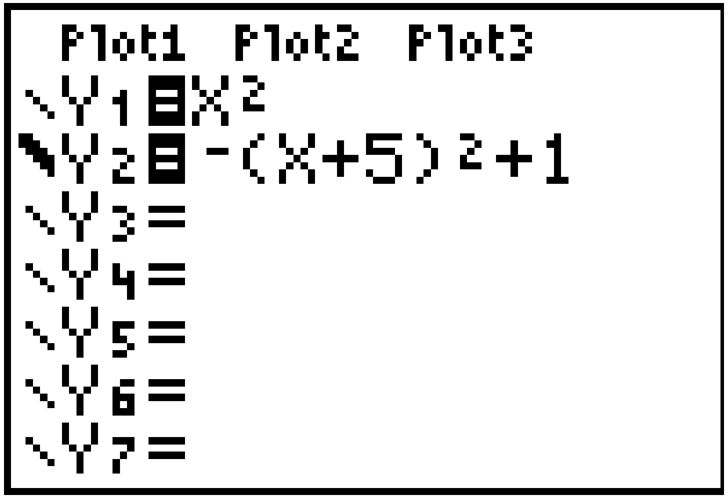
Step 2 Write the transformed function.

$g(x) = a(x - h)2 + k$	*Vertex form of a quadratic function*
$= -(x - (-5)2 + (1)$	*Substitute –1 for a, –5 for h, and 1 for k.*
$= -(x + 5)2 + 1$	*Simplify.*

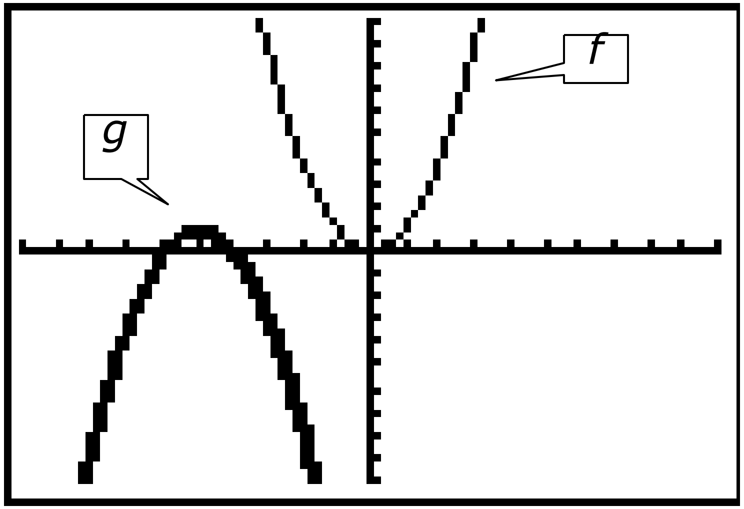
$g(x) = -(x + 5)2 + 1$

Check It Out! Example 4b Continued

Check Graph both functions on a graphing calculator. Enter *f* as **Y1**, and *g* as **Y2**. The graph indicates the identified transformations.



```
 Plot1 Plot2 Plot3
\Y1=X²
\Y2=-(X+5)²+1
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```



Example 5: Scientific Application

On Earth, the distance d in meters that a dropped object falls in t seconds is approximated by $d(t)= 4.9t2$. On the moon, the corresponding function is $dm(t)= 0.8t2$. What kind of transformation describes this change from $d(t)= 4.9t2$, and what does the transformation mean?

Examine both functions in vertex form.

$d(t)= 4.9(t – 0)2 + 0$

$dm(t)= 0.8(t – 0)2 + 0$

Example 5 Continued

The value of *a* has decreased from 4.9 to 0.8. The decrease indicates a vertical compression.

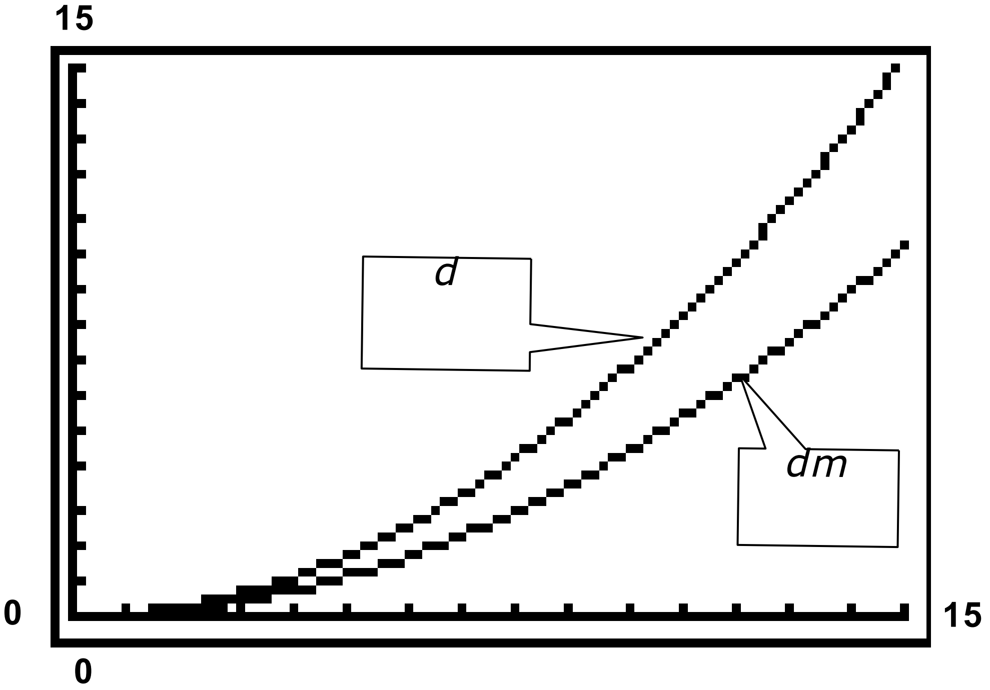
Find the compression factor by comparing the new *a*-value to the old *a*-value.

$$\frac{a \text{ from } dm(t)}{a \text{ from } d(t)} = \frac{0.8}{4.9} \approx 0.16$$

The function *d*m represents a vertical compression of *d* by a factor of approximately 0.16. Because the value of each function approximates the time it takes an object to fall, an object dropped from the moon falls about 0.16 times as fast as an object dropped on Earth.

Example 5 Continued

Check Graph both functions on a graphing calculator. The graph of *dm* appears to be vertically compressed compared with the graph of *d*.



Check It Out! Example 5

The minimum braking distance d in feet for a vehicle on dry concrete is approximated by the function $(v) = 0.045v2$, where v is the vehicle’s speed in miles per hour.

The minimum braking distance dn in feet for a
vehicle with new tires at optimal inflation is $dn(v) = 0.039v2$, where v is the vehicle’s speed in miles per hour. What kind of transformation describes this change from $d(v) = 0.045v2$, and what does
this transformation mean?

Check It Out! **Example 5 Continued**

Examine both functions in vertex form.

$d(v)= 0.045(t - 0)2 + 0$ $dn(t)= 0.039(t - 0)2 + 0$

The value of *a* has decreased from 0.045 to 0.039. The decrease indicates a vertical compression.

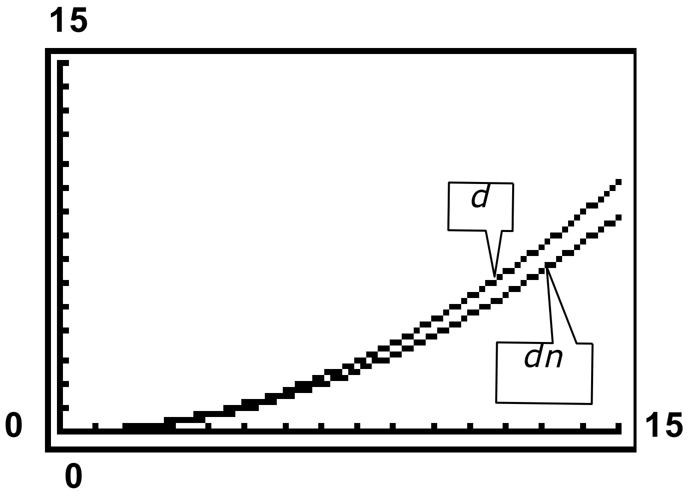
Find the compression factor by comparing the new *a*-value to the old *a*-value.

$$\frac{a \text{ from } dn(t)}{a \text{ from } d(v)} = \frac{0.039}{0.045} = \frac{13}{15}$$

Check It Out! Example 5 Continued

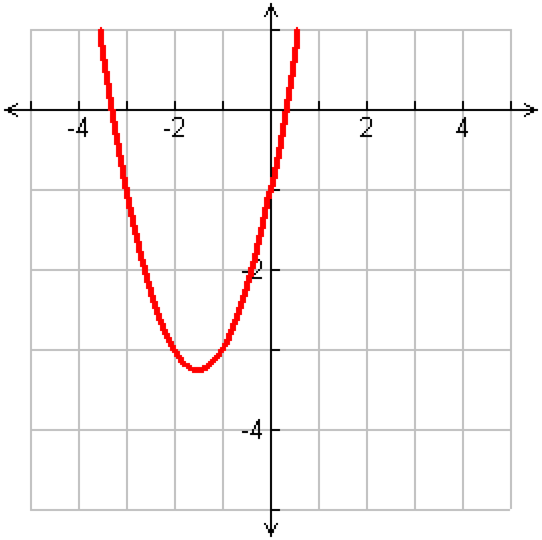
The function *dn* represents a vertical compression of *d* by a factor of . The braking distance will be less with optimally inflated new tires than with tires having more wear. $\frac{13}{15}$

Check Graph both functions on a graphing calculator. The graph of *dn* appears to be vertically compressed compared with the graph of *d*.



Lesson Quiz: Part I

1. Graph $f(x) = x2 + 3x - 1$ by using a table.



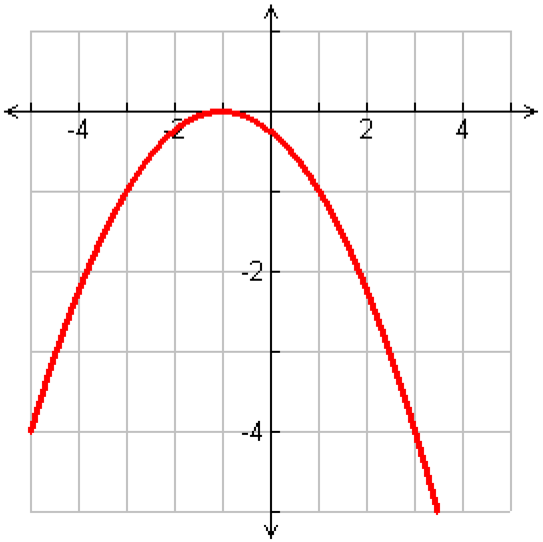
Lesson Quiz: Part II

2. Using the graph of $f(x) = x2$ as a guide, describe the transformations, and then graph $g(x) = \quad (x + 1)2$.

$$-\frac{1}{4}$$

g is *f* reflected across *x*-axis, vertically compressed by a factor of , and translated 1 unit left.

$$\frac{1}{4}$$



Lesson Quiz: Part III

3. The parent function $f(x) = x2$ is vertically stretched by a factor of 3 and translated 4 units right and 2 units up to create *g*. Write *g* in vertex form.

$g(x) = 3(x - 4)2 + 2$