IM 9–12 MATH v.1

Unit 2
Congruence

GEOMETRY

Lesson 14

Bisect It

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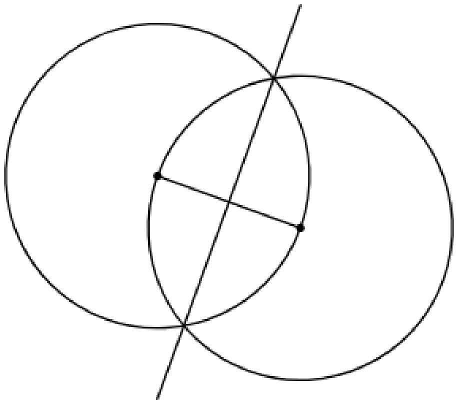
Learning Goal

Let’s prove that some constructions we conjectured about really work.

Geometry

Why Does This Construction Work?

Warm-up



If you are Partner A, explain to your partner what steps were taken to construct the perpendicular bisector in this image.

If you are Partner B, listen to your partner's explanation, and then explain to your partner why these steps produce a line with the properties of a perpendicular bisector.

Then, work together to make sure the main steps in Partner A's explanation have a reason from Partner B's explanation.

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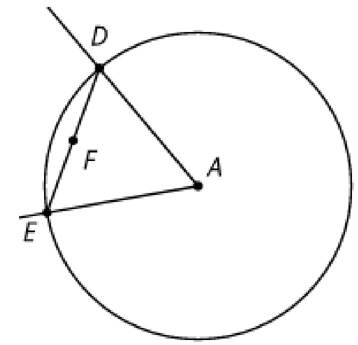
Construction from Definition

Han, Clare, and Andre thought of a way to construct an angle bisector. They used a circle to construct points D and E the same distance from A. Then they connected D and E and found the midpoint of segment DE. They thought that ray AF would be the bisector of angle DAE. Mark the given information on the diagram:



Han's rough-draft justification: F is the midpoint of segment DE. I noticed that F is also on the perpendicular bisector of angle DAE.

Clare's rough-draft justification: Since segment DA is congruent to segment EA, triangle DEA is isosceles. DF has to be congruent to EF because they are the same length. So, AF has to be the angle bisector.

Andre's rough-draft justification: What if you draw a segment from F to A? Segments DF and EF are congruent. Also, angle DAF is congruent to angle EAF. Then both triangles are congruent on either side of the angle bisector line.

1. Each student tried to justify why their construction worked. With your partner, discuss each student's approach.
   - What do you notice that this student understands about the problem?
   - What question would you ask them to help them move forward?
1. Using the ideas you heard and the ways that each student could make their explanation better, write your own explanation for why ray must be an angle bisector.

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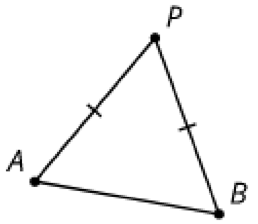
Reflecting on Reflection

1. Here is a diagram of an isosceles triangle APB with segment AP congruent to segment BP. Here is a valid proof that the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.



a. Read the proof and annotate the diagram with each piece of information in the proof.
b. Write a summary of how this proof shows the angle bisector of the vertex angle of an isosceles triangle is a line of symmetry.

- Segment AP is congruent to segment BP because triangle APB is isosceles.
- The angle bisector of APB intersects segment AB. Call that point Q.
- By the definition of angle bisector, angles APQ and BPQ are congruent.
- Segment PQ is congruent to itself.
- By the Side-Angle-Side Triangle Congruence Theorem, triangle APQ must be congruent to triangle BPQ.
- Therefore the corresponding segments AQ and BQ are congruent and corresponding angles AQP and BQP are congruent.
- Since angles AQP and BQP are both congruent and supplementary angles, each angle must be a right angle.
- So PQ must be the perpendicular bisector of segment AB.
- Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the triangle across PQ the vertex P will stay in the same spot and the 2 endpoints of the base, A and B, will switch places.
- Therefore the angle bisector PQ is a line of symmetry for triangle APB.

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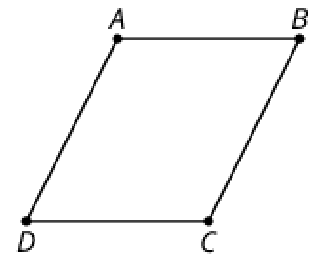
Reflecting on Reflection

2. Here is a diagram of parallelogram *ABCD*. Here is an invalid proof that a diagonal of a parallelogram is a line of symmetry.



a. Read the proof and annotate the diagram with each piece of information in the proof.
b. Find the errors that make this proof invalid. Highlight any lines that have errors or false assumptions.

- The diagonals of a parallelogram intersect. Call that point *M*.
- The diagonals of a parallelogram bisect each other, so *MB* is congruent to *MD*.
- By the definition of parallelogram, the opposite sides *AB* and *CD* are parallel.
- Angles *ABM* and *ADM* are alternate interior angles of parallel lines so they must be congruent.
- Segment *AM* is congruent to itself.
- By the Side-Angle-Side Triangle Congruence Theorem, triangle *ABM* is congruent to triangle *ADM*.
- Therefore the corresponding angles *AMB* and *AMD* are congruent.
- Since angles *AMB* and *AMD* are both congruent and supplementary angles, each angle must be a right angle.
- So *AC* must be the perpendicular bisector of segment *BD*.
- Because reflection across perpendicular bisectors takes segments onto themselves and swaps the endpoints, when we reflect the parallelogram across *AC* the vertices *A* and *C* will stay in the same spot and the 2 endpoints of the other diagonal, *B* and *D*, will switch places.
- Therefore diagonal *AC* is a line of symmetry for parallelogram *ABCD*.

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Bisect It

Lesson Synthesis

- If I told you segment was a line of symmetry of this triangle, what else could you tell me about that line?
- Can you justify your answers using transformations or congruent parts of congruent figures?
- If triangle were not isosceles, would it still have a line of symmetry?
- If triangle were not isosceles, would the angle bisector of and the perpendicular bisector of be the same line?

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- I can critique a proof about constructions.
- I can explain why constructions work.

Learning Targets

Geometry

Going Both Ways Again

Cool-down

Explain the difference between these 2 statements:

Given figure JKL where segment JK is congruent to segment LK, prove that:

1. If ray KP is the angle bisector of angle JKL, and M is the midpoint of segment JL, then ray KP passes through M.
2. If M is the midpoint of segment JL, then ray KM bisects angle JKL.

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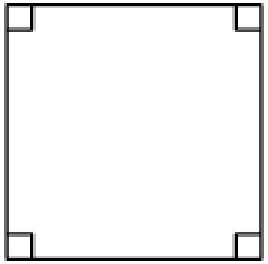
rectangle

A quadrilateral with four right angles.



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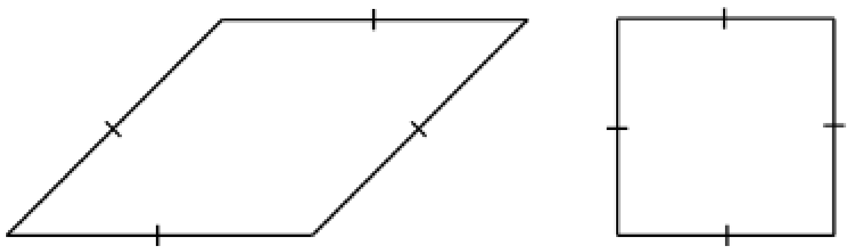
rhombus

A quadrilateral with four congruent sides.



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