IM 9–12 MATH v.1

Unit 2
Congruence

GEOMETRY

Lesson 9

Side-Side-Side Triangle Congruence

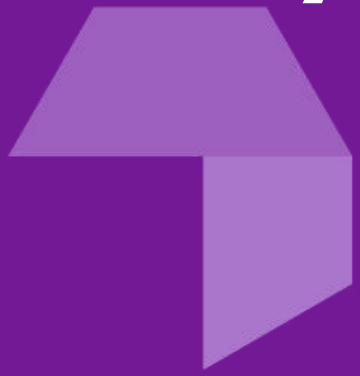
Illustrative Mathematics®

Kendall Hunt

Learning Goal

Let's see if we can prove one more set of conditions that guarantee triangles are congruent, and apply theorems.

Geometry



Dare to Be Different

Warm-up

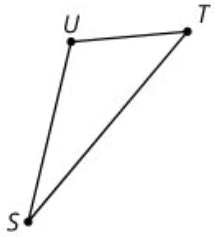
Construct a triangle with the given side lengths using technology.

Side lengths:

- 2 cm
- 1.5 cm
- 2.4 cm

Can you make a triangle that doesn't look like anyone else's?

Proving the Side-Side-Side Triangle Congruence Theorem



Priya was given this task to complete:

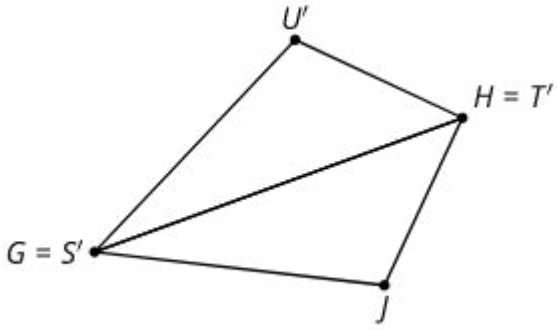
Use a sequence of rigid motions to take STU onto GHJ. Given that segment ST is congruent to segment GH, segment TU is congruent to segment HJ, and segment SU is congruent to segment GJ. For each step, explain how you know that one or more vertices will line up.

Help her finish the missing steps in her proof:

1. ST is the same length as _____ , so they are congruent. Therefore, there is a rigid motion that takes ST to _____ .
2. Apply this rigid motion to triangle STU. The image of T will coincide with _____ , and the image S of will coincide with _____.
3. We cannot be sure that the image of U, which we will call , U' coincides with _____ yet. If it does, then our rigid motion takes STU to GHJ, proving that triangle STU is congruent to triangle GHJ. If it does not, then we continue as follows.

Proving the Side-Side-Side Triangle Congruence Theorem

4. HJ is congruent to the image of _____, because rigid motions preserve distance.
5. Therefore, H is equidistant from U' and _____.
6. A similar argument shows that G is equidistant from U' and _____ .
7. GH is the _____ of the segment connecting U' and J, because the _____ is determined by 2 points that are both equidistant from the endpoints of a segment.
8. Reflection across the _____ of $U'J$, takes _____ to _____.
9. Therefore, after the reflection, all 3 pairs of vertices coincide, proving triangles _____ and _____ are congruent.



Now, help Priya by finishing a few-sentence summary of her proof. "To prove 2 triangles must be congruent if all 3 pairs of corresponding sides are congruent . . . ."

What Else Do We Know For Sure About Parallelograms?

Quadrilateral $ABCD$ is a parallelogram. By definition, that means that segment AB is parallel to segment CD, and segment BC is parallel to segment AD.

Prove that angle B is congruent to angle D.

1. Work on your own to make a diagram and write a rough draft of a proof.
2. With your partner, discuss each other's drafts.
   - What do you notice your partner understands about the problem?
   - What revision would help them move forward?
1. Work together to revise your drafts into a clear proof that everyone in your class could follow and agree with.

Side-Side-Side Triangle Congruence

Lesson Synthesis

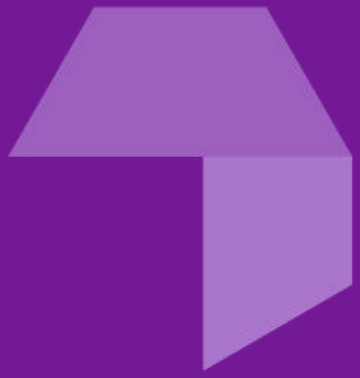
Quadrilateral $ABCD$ is a parallelogram. By definition, that means that segment AB is parallel to segment CD, and segment BC is parallel to segment AD. Prove that angle B is congruent to angle D.

What is always true about parallelograms, based on what we just proved?

- I can explain why the Side-Side-Side Triangle Congruence Theorem works.
- I can use the Side-Side-Side Triangle Congruence Theorem in a proof.

Learning Targets

Geometry



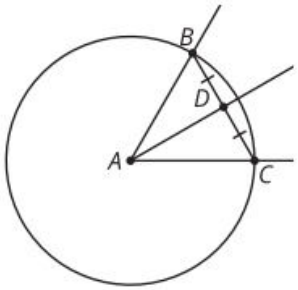
Practice Seeing Shortcuts

Cool-down

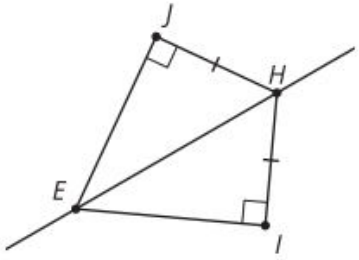
Label each example by whether you could prove the triangles congruent using:

- Side-Side-Side Triangle Congruence Theorem
- Side-Angle-Side Triangle Congruence Theorem
- Angle-Side-Angle Triangle Congruence Theorem
- None of the above

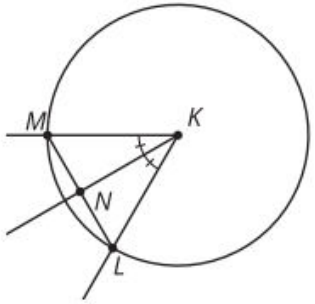
1. Triangle ABD is congruent to triangle ACD.



2. Triangle EJH is congruent to triangle EIH.

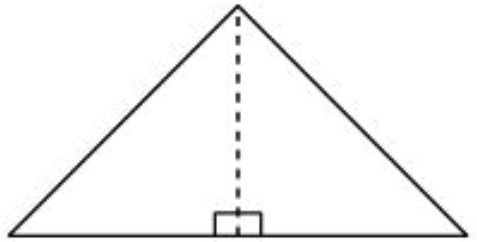


3. Triangle LKN is congruent to triangle MKN.

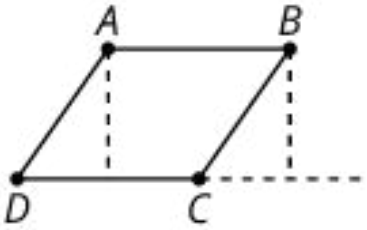


auxiliary line

An extra line drawn in a figure to reveal hidden structure.



For example, the line shown in the isosceles triangle is a line of symmetry, and the lines shown in the parallelogram suggest a way of rearranging it into a rectangle.

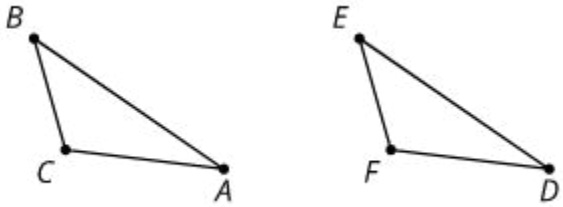


converse

The converse of an if-then statement is the statement that interchanges the hypothesis and the conclusion. For example, the converse of "if it's Tuesday, then this must be Belgium" is "if this is Belgium, then it must be Tuesday."

corresponding

For a rigid transformation that takes one figure onto another, a part of the first figure and its image in the second figure are called corresponding parts. We also talk about corresponding parts when we are trying to prove two figures are congruent and set up a correspondence between the parts to see if the parts are congruent.



In the figure, segment *AB* corresponds to segment *DE*, and angle *BCA* corresponds to angle *EFD*.

parallelogram

A quadrilateral in which pairs of opposite sides are parallel.