IM 9–12 MATH v.1

Unit 2
Congruence

GEOMETRY

Lesson 8

The Perpendicular Bisector Theorem

Illustrative Mathematics®

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Learning Goal

Let’s convince ourselves that what we’ve conjectured about perpendicular bisectors must be true.

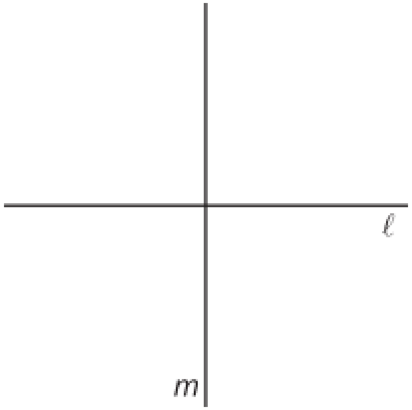
Geometry

Intersecting Lines

Warm-up: Which One Doesn't Belong?

Which one doesn't belong?

A



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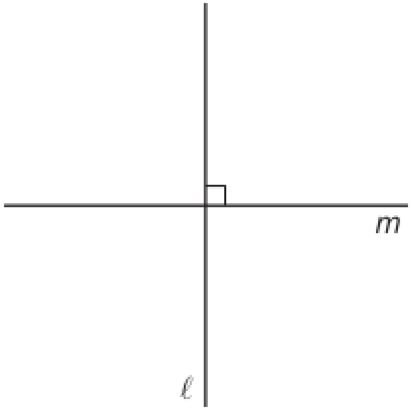

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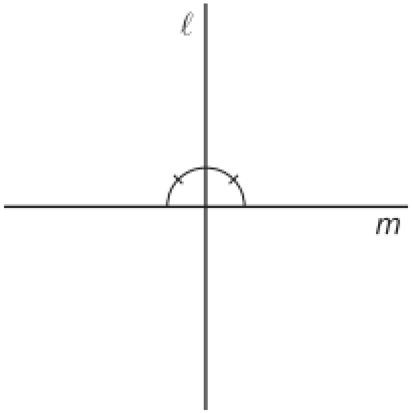
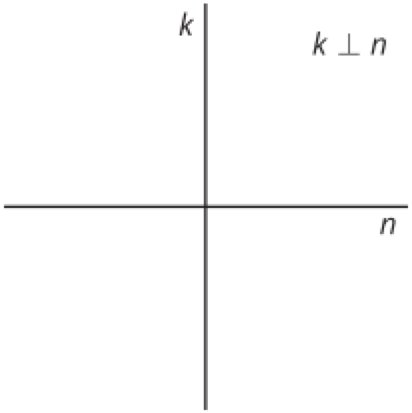
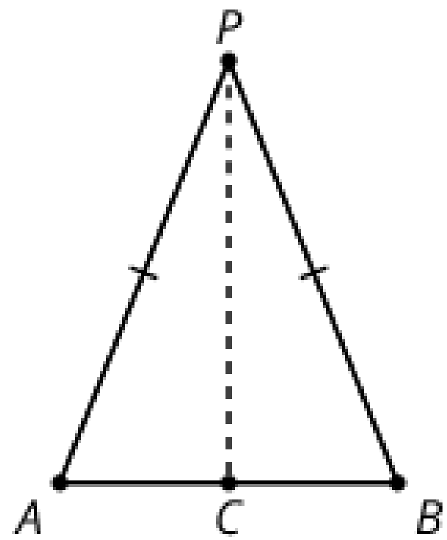


D



Lots of Lines

Notice and Wonder



Lots of Lines

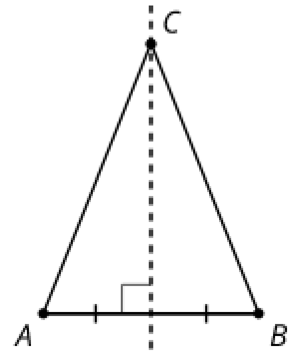
Diego, Jada, and Noah were given the following task:

Prove that if a point C is the same distance from A as it is from B, then C must be on the perpendicular bisector of AB.

At first they were really stuck. Noah asked, “How do you prove a point is on a line?” Their teacher gave them the hint, “Another way to think about it is to draw a line that you know C is on, and prove that line has to be the perpendicular bisector.”

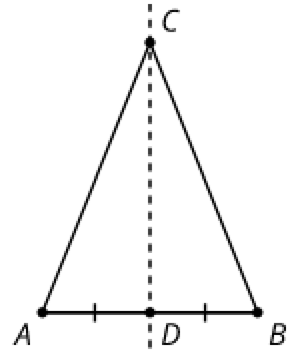
They each drew a line and thought about their pictures. Here are their rough drafts.

Diego’s approach: “I drew a line through C that was perpendicular to AB and through the midpoint of AB. That line is the perpendicular bisector of AB and C is on it, so that proves C is on the perpendicular bisector.”

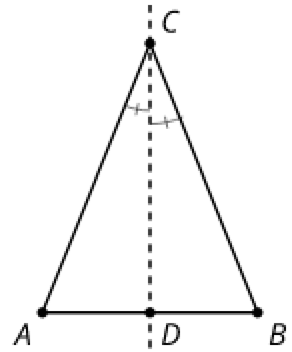


Lots of Lines

Jada's approach: "I thought the line through C would probably go through the midpoint of AB so I drew that and labeled the midpoint D. Triangle ACB is isosceles, so angles A and B are congruent, and AC and BC are congruent. And AD and DB are congruent because D is a midpoint. That made two congruent triangles by the Side-Angle-Side Triangle Congruence Theorem. So I know angle ADC and angle BDC are congruent, but I still don't know if DC is the perpendicular bisector of AB."



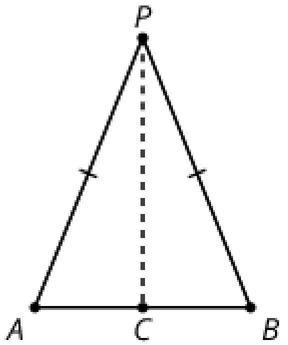
Noah's approach: "In the Isosceles Triangle Theorem proof, Mai and Kiran drew an angle bisector in their isosceles triangle, so I'll try that. I'll draw the angle bisector of angle ACB. The point where the angle bisector hits AB will be D. So triangles ACD and BCD are congruent, which means AD and BD are congruent, so D is a midpoint and CD is the perpendicular bisector."

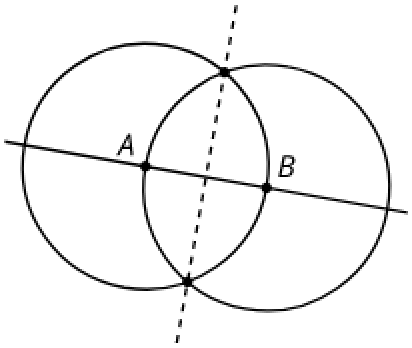


Lots of Lines

1. With your partner, discuss each student's approach.
   - What do you notice that this student understands about the problem?
   - What question would you ask them to help them move forward?
1. Using the ideas you heard and the ways you think each student could make their explanation better, write your own explanation for why C must be on the perpendicular bisector of A and B.

Lots of Lines





Not Too Close, Not Too Far

1. Work on your own to make a diagram and write a rough draft of a proof for the statement:

   If P is a point on the perpendicular bisector of AB, prove that the distance from P to A is the same as the distance from P to B.

1. With your partner, discuss each other's drafts. Record your partner's feedback for your proof.

- What do you notice that your partner understands about the problem?
- What question would you ask them to help them move forward?

The Perpendicular Bisector Theorem

Lesson Synthesis

- If a point C is the same distance from A as it is from B, prove that C must be on the perpendicular bisector of AB.
- If P is a point on the perpendicular bisector of AB, prove that the distance from P to A is the same as the distance from P to B.

- I can critique an explanation of the Perpendicular Bisector Theorem.
- I can explain why the Perpendicular Bisector Theorem is true.

Learning Targets

Geometry

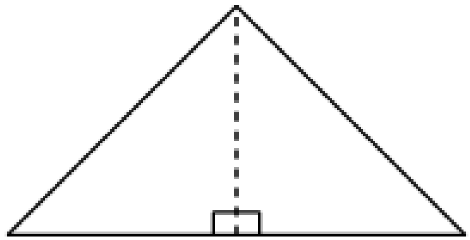
Reflect and Revise

Cool-down

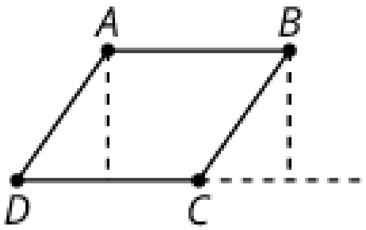
Use your partner's feedback to write a final draft of your proof that if P is a point on the perpendicular bisector of AB, the distance from P to A is the same as the distance from P to B.

auxiliary line

An extra line drawn in a figure to reveal hidden structure.



For example, the line shown in the isosceles triangle is a line of symmetry, and the lines shown in the parallelogram suggest a way of rearranging it into a rectangle.

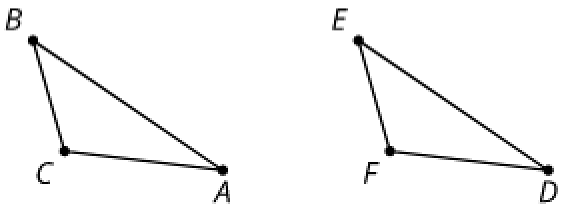


converse

The converse of an if-then statement is the statement that interchanges the hypothesis and the conclusion. For example, the converse of "if it's Tuesday, then this must be Belgium" is "if this is Belgium, then it must be Tuesday."

corresponding

For a rigid transformation that takes one figure onto another, a part of the first figure and its image in the second figure are called corresponding parts. We also talk about corresponding parts when we are trying to prove two figures are congruent and set up a correspondence between the parts to see if the parts are congruent.



In the figure, segment *AB* corresponds to segment *DE*, and angle *BCA* corresponds to angle *EFD*.

parallelogram

A quadrilateral in which pairs of opposite sides are parallel.