IM 9–12 MATH v.1

Unit 2
Congruence

GEOMETRY

Lesson 6

Side-Angle-Side Triangle Congruence

Illustrative Mathematics®

Kendall Hunt

Learning Goal

Let's use definitions and theorems to figure out what must be true about shapes, without having to measure all parts of the shapes.

Geometry

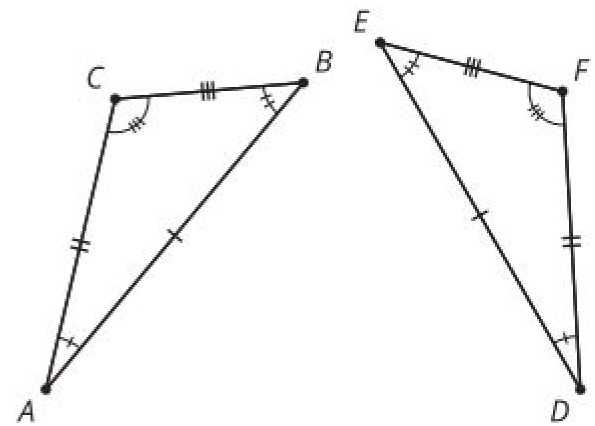
Information Overload?

Warm-up

Highlight each piece of given information that is used in the proof, and each line in the proof where that piece of information is used.

Given:

- $\overline{AB} \cong \overline{DE}$
- $\overline{AC} \cong \overline{DF}$
- $\overline{BC} \cong \overline{EF}$
- $\angle A \cong \angle D$
- $\angle B \cong \angle E$
- $\angle C \cong \angle F$



Proof:

1. Segments *AB* and *DE* are the same length so they are congruent. Therefore, there is a rigid motion that takes *AB* to *DE*.
2. Apply that rigid motion to triangle *ABC*. The image of *A* will coincide with *D*, and the image of *B* will coincide with *E*.

Information Overload?

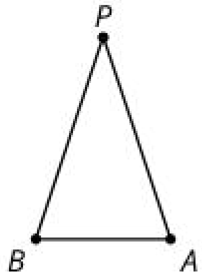
Warm-up

3. We cannot be sure that the image of C coincides with F yet. If necessary, reflect the image of triangle ABC across DE to be sure the image of C, which we will call C', is on the same side of DE as F. (This reflection does not change the image of A or B.)
4. We know the image of angle A is congruent to angle D because rigid motions don't change the size of angles.
5. C' must be on ray DF since both C' and F are on the same side of DE, and make the same angle with it at D.
6. Segment DC' is the image of AC and rigid motions preserve distance, so they must have the same length.
7. We also know AC has the same length as DF. So DC' and DF must be the same length.
8. Since C' and F are the same distance along the same ray from D, they have to be in the same place.
9. We have shown that a rigid motion takes A to D, B to E, and C to F; therefore, triangle ABC is congruent to triangle DEF.

Proving the Side-Angle-Side Triangle Congruence Theorem

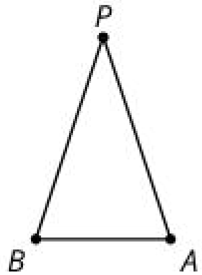
1. Two triangles have 2 pairs of corresponding sides congruent, and the corresponding angles between those sides are congruent. Sketch 2 triangles that fit this description and label them *LMN* and *PQR*, so that:
   - Segment *LM* is congruent to segment *PQ*
   - Segment *LN* is congruent to segment *PR*
   - Angle *L* is congruent to angle *P*
1. Use a sequence of rigid motions to take *LMN* onto *PQR*. For each step, explain how you know that one or more vertices will line up.
2. Look back at the congruent triangle proofs you've read and written. Do you have enough information here to use a proof that is like one you saw earlier? Use one of those proofs to guide you in writing a proof for this situation.

What Do We Know For Sure About Isosceles Triangles?



- Kiran: I’m stumped on this proof.
- Mai: What are you trying to prove?
- Kiran: I’m trying to prove that in an isosceles triangle, the two base angles are congruent. So in this case, that angle is congruent to angle .
- Mai: Let’s think of what geometry ideas we already know are true.
- Kiran: We know if two pairs of corresponding sides and the corresponding angles between the sides are congruent, then the triangles must be congruent.
- Mai: Yes, and we also know that we can use reflections, rotations, and translations to prove congruence and symmetry . . . . The isosceles triangle you’ve drawn makes me think of symmetry. If you draw a line down the middle of it, I wonder if that could help us prove that the angles are the same? [Mai draws the line of symmetry of the triangle and labels the intersection of AB and the line of symmetry Q].

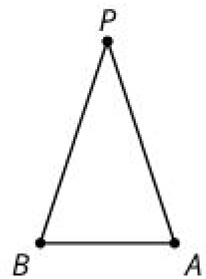
What Do We Know For Sure About Isosceles Triangles?



- Kiran: Wait, when you draw the line, it breaks the triangle into two smaller triangles. I wonder if I could prove those triangles are congruent using Side-Angle-Side Triangle Congruence?
- Mai: It's an isosceles triangle, so we know that one pair of corresponding sides is congruent. [Mai marks the congruent sides].
- Kiran: And this segment in the middle here is part of both triangles, so it has to be the same length for both. Look. [Kiran draws the two halves of the isosceles triangle and marks the shared sides as congruent].
- Mai: So we have two pairs of corresponding sides that are congruent. How do we know the angles between them are congruent?
- Kiran: I'm not sure. Maybe it has to do with how we drew that line of symmetry?

What Do We Know For Sure About Isosceles Triangles?

Mai and Kiran want to prove that in an isosceles triangle, the 2 base angles are congruent. Finish the proof that they started. Draw the auxiliary line and define it so that you can use the Side-Angle-Side Triangle Congruence Theorem to complete each statement in the proof.



Draw ______ .

Segment PA is congruent to segment PB because of the definition of isosceles triangle.

Angle ______ is congruent to angle ______ because ______ .

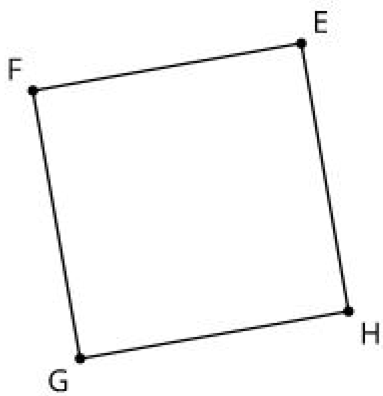
Segment PQ is congruent to itself.

Therefore, triangle APQ is congruent to triangle BPQ by the Side-Angle-Side Triangle Congruence Theorem.

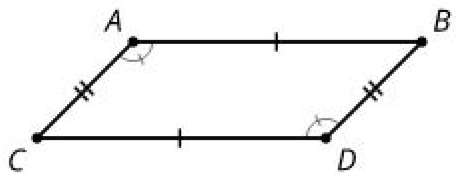
Therefore, ______ .

Side-Angle-Side Triangle Congruence

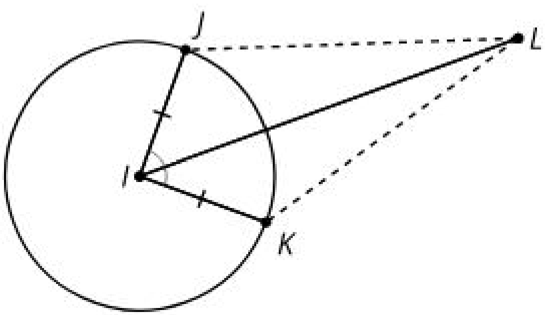
Lesson Synthesis



What auxiliary line would you add to the diagram to help you use the Side-Angle-Side Triangle Congruence Theorem to prove that, in a square, the diagonals form 45º angles with the sides?



What auxiliary line would you add to the diagram to help you use the Side-Angle-Side Triangle Congruence Theorem to prove that, in a quadrilateral with both pairs of opposite sides congruent and one pair of opposite angles congruent, opposite sides are parallel?



This is an example of how not to draw an auxiliary line. Conjecture: Given any two points on a circle and any point outside the circle, the two points on the circle are the same distance to the exterior point. Tyler says, “I know that radii of a circle are congruent, and *LI* is congruent to itself. So then I drew the angle bisector from *I* through *L* and now I can use the Side-Angle-Side Triangle Congruence Theorem to prove that triangles *ILK* and *ILJ* are congruent.” Can that really be true? If not, what went wrong?

- I can explain why the Side-Angle-Side Triangle Congruence Theorem works.
- I can use the Side-Angle-Side Triangle Congruence Theorem in a proof.

Learning Targets

Geometry

Revisiting Perpendicular Bisectors

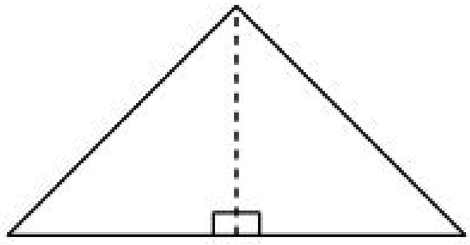
Cool-down

Conjecture: If a point is on the perpendicular bisector of a line segment, then that point must be the same distance from each endpoint of the segment.

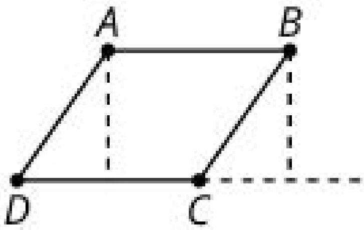
1. Sketch and label a diagram of this situation.
   - Mark any segments or angles you know to be congruent.
   - Mark any other information you know.
1. Find (or add auxiliary lines to find) 2 triangles that appear congruent. Shade in the 2 triangles using different colored pencils.
2. Do you have enough information to prove that the 2 triangles in your diagram are congruent? Explain your reasoning.

auxiliary line

An extra line drawn in a figure to reveal hidden structure.

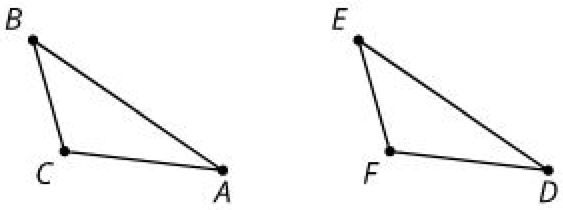


For example, the line shown in the isosceles triangle is a line of symmetry, and the lines shown in the parallelogram suggest a way of rearranging it into a rectangle.



corresponding

For a rigid transformation that takes one figure onto another, a part of the first figure and its image in the second figure are called corresponding parts. We also talk about corresponding parts when we are trying to prove two figures are congruent and set up a correspondence between the parts to see if the parts are congruent.



In the figure, segment AB corresponds to segment DE, and angle BCA corresponds to angle EFD.

Illustrative Mathematics®

Kendall Hunt