IM 9–12 MATH v.1

Unit 2
Congruence

GEOMETRY

Lesson 3

Congruent Triangles, Part 1

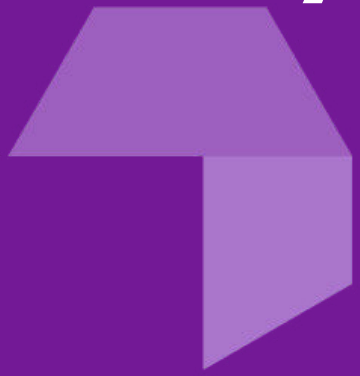
Illustrative Mathematics®

Kendall Hunt

Learning Goal

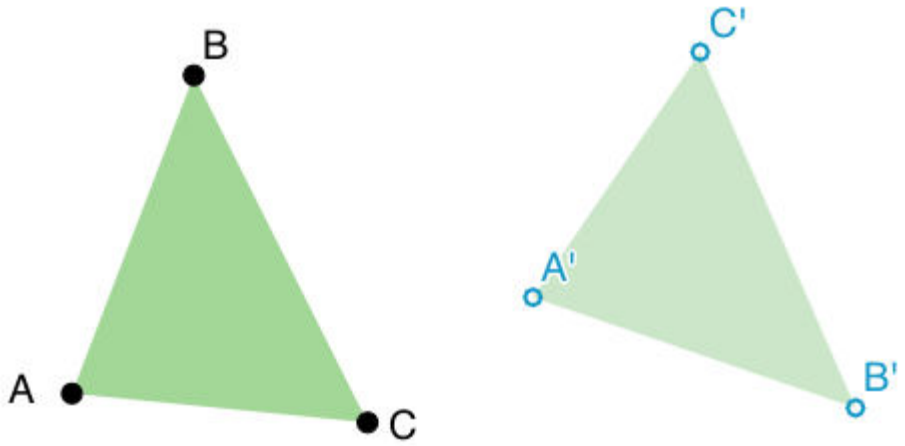
Let's use transformations to be sure that two triangles are congruent.

Geometry



True or . . . Sometimes True?: Triangles

Warm-up

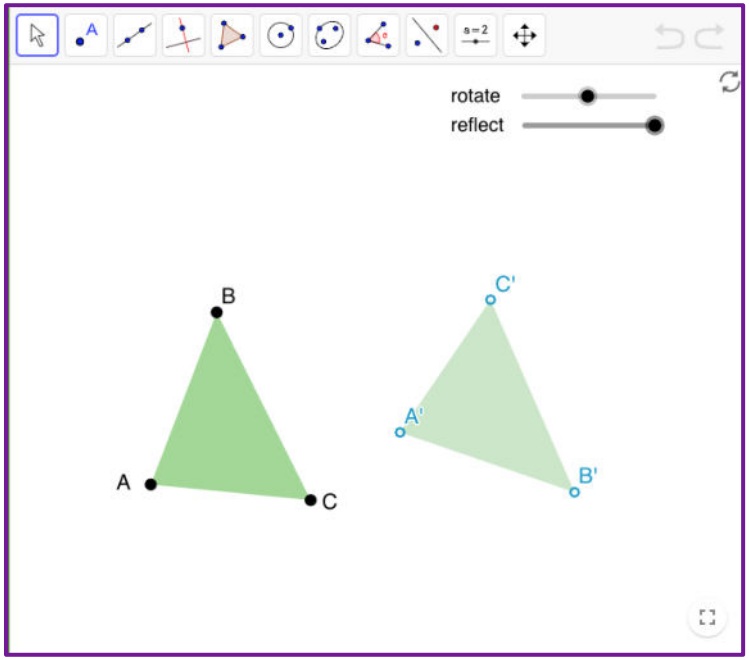


If triangle *ABC* is congruent to triangle *A'B'C'*. . .

1. What must be true?
2. What could possibly be true?
3. What definitely can't be true?

Which Triangles Are Congruent?

Warm-up



Invisible Triangles

Invisible Triangles

Transformer

Listen to hear which parts of the triangles correspond. Then give instructions to take one triangle onto the other.

Possible instructions:

- Translate _____ from _____ to _____.
- Rotate _____ using _____ as the center by angle _____.
- Rotate using _____ as the center so that _____ coincides with _____.
- Reflect _____ across _____.
- Reflect _____ across the perpendicular bisector of _____.

$\overline{JK} \cong \overline{PQ}$, $\overline{JL} \cong \overline{PR}$, $\overline{KL} \cong \overline{QR}$, $\angle J \cong \angle P$, $\angle K \cong \angle Q$, $\angle L \cong \angle R$

Invisible Triangles

Player 1: You are the transformer. Take the transformer card.

Player 2: Select a triangle card. Do not show it to anyone. Study the diagram to figure out which sides and which angles correspond. Tell Player 1 what you have figured out.

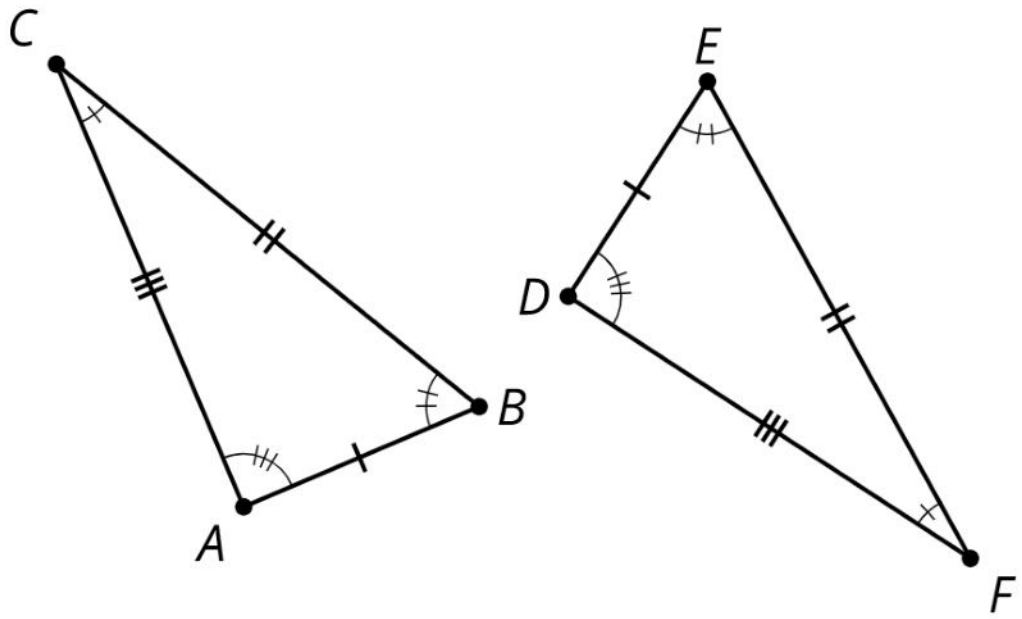
Player 1: Take notes about what they tell you so that you know which parts of their triangles correspond. Think of a sequence of rigid motions you could tell your partner to get them to take one of their triangles onto the other. Be specific in your language. The notes on your card can help with this.

Player 2: Listen to the instructions from the transformer. Use tracing paper to follow their instructions. Draw the image after each step. Let them know when they have lined up 1, 2, or all 3 vertices on your triangles.

Why Do They Coincide?

Noah and Priya were playing Invisible Triangles. For card 3, Priya told Noah that in triangles *ABC* and *DEF*:

- $\overline{AB} \cong \overline{DE}$
- $\overline{AC} \cong \overline{DF}$
- $\overline{BC} \cong \overline{EF}$
- $\angle A \cong \angle D$
- $\angle B \cong \angle E$
- $\angle C \cong \angle F$



Here are the steps Noah had to tell Priya to do before all 3 vertices coincided:

- Translate triangle *ABC* by the directed line segment from *A* to *D*.
- Rotate the image, triangle *A'B'C'*, using *D* as the center, so that rays *A"B"* and *DE* line up.
- Reflect the image, triangle *A"B"C"*, across line *DE*.

Why Do They Coincide?

After those steps, the triangles were lined up perfectly. Now Noah and Priya are working on explaining why their steps worked, and they need some help. Answer their questions.

First, we translate triangle ABC by the directed line segment from A to D. Point A' will coincide with D because we defined our transformation that way. Then, rotate the image, triangle $A'B'C'$, by the angle $B'DE$, so that rays $A''B''$ and DE line up.

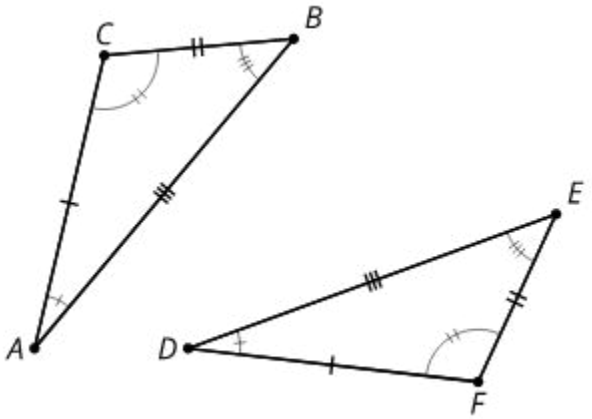
1. We know that rays $A''B''$ and DE line up because we said they had to, but why do points B'' and E have to be in the exact same place?
2. Finally, reflect the image, triangle $A''B''C''$ across DE.
   a. How do we know that now, the image of ray $A''C''$ and ray DF will line up?
   b. How do we know that the image of point C'' and point F will line up exactly?

Congruent Triangles, Part 1

Lesson Synthesis

If all pairs of corresponding sides and all pairs of corresponding angles are congruent, then the triangles must be congruent. (Theorem)

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \overline{AC} \cong \overline{DF}$,
$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$ so
$\triangle ABC \cong \triangle DEF$



Congruent Triangles, Part 1

Lesson Synthesis

Transformations:

- Translate _____ from _____ to _____ .
- Rotate _____ using _____ as the center by angle _____ .
- Rotate _____ using _____ as the center so that _____ coincides with _____ .
- Reflect _____ across _____ .
- Reflect _____ across the perpendicular bisector of _____ .

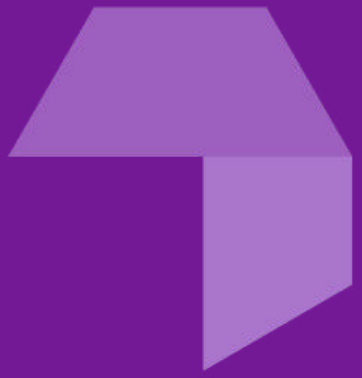
Justifications:

- We know the image of _____ is congruent to _____ because rigid motions preserve measure.
- Points _____ and _____ coincide after translating because we defined our translation that way!
- Since points _____ and _____ are the same distance along the same ray from _____ , they have to be in the same place.
- Rays _____ and _____ coincide after rotating because we defined our rotation that way!
- The image of _____ must be on ray _____ since both _____ and _____ are on the same side of _____ and make the same angle with it at _____ .

I can explain why if all the corresponding sides and angles of two triangles are congruent, then the triangles are congruent.

Learning Targets

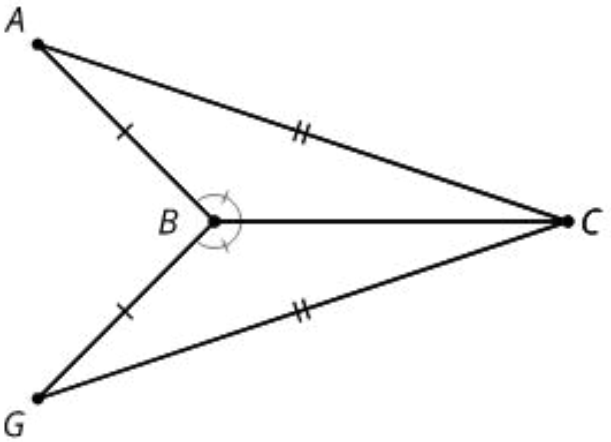
Geometry



Reflecting on Proof

Cool-down

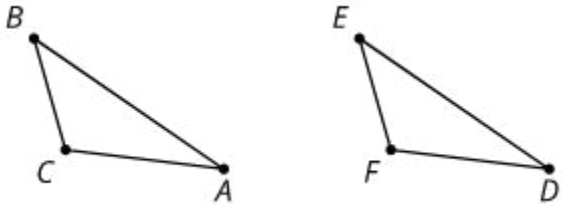
$\overline{AB} \cong \overline{BG}$, $\overline{AC} \cong \overline{GC}$, $\angle ABC \cong \angle GBC$



1. What rigid transformation will take triangle *GBC* onto triangle *ABC*?
2. Explain why *G'* will coincide with *A*.
3. Is triangle *GBC* congruent to triangle *ABC*? Explain your reasoning.

corresponding

For a rigid transformation that takes one figure onto another, a part of the first figure and its image in the second figure are called corresponding parts. We also talk about corresponding parts when we are trying to prove two figures are congruent and set up a correspondence between the parts to see if the parts are congruent.



In the figure, segment *AB* corresponds to segment *DE*, and angle *BCA* corresponds to angle *EFD*.