IM 9–12 MATH v.1

Unit 1

Constructions and Rigid Transformations

GEOMETRY

Lesson 21

One Hundred and Eighty

Illustrative Mathematics®

Kendall Hunt

Learning Goal

Let's prove the Triangle Angle Sum Theorem.

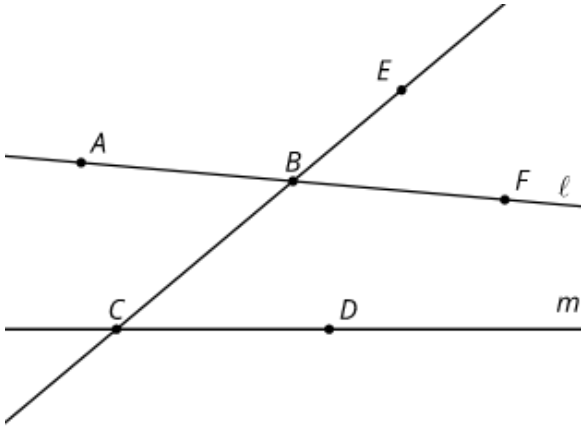
Geometry

What Went Wrong?

Warm-up

Here are 2 lines ℓ and m that are not parallel that have been cut by a transversal.

Tyler thinks angle EBF is congruent to angle BCD because they are corresponding angles and a translation along the directed line segment from B to C would take one angle onto the other. Here are his reasons.

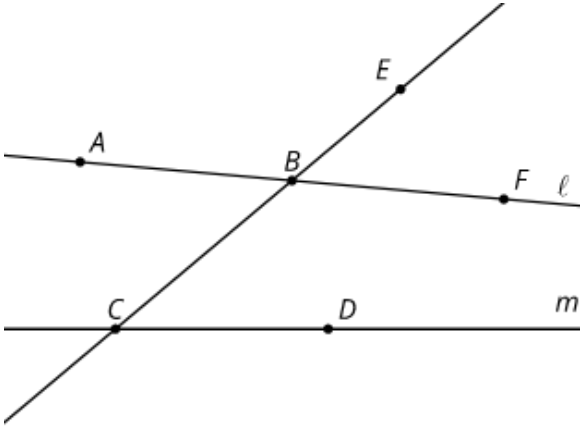


- The translation takes B onto C, so the image of B is C.
- The translation takes E somewhere on ray CB because it would need to be translated by a distance greater than BC to land on the other side of C.
- The image of F has to land somewhere on line because translations take lines to parallel lines and line m is the only line parallel to ℓ that goes through B'.
- The image of F, call it F', has to land on the right side of line BC or else line FF' wouldn't be parallel to the directed line segment from B to C.

What Went Wrong?

Warm-up

Here are 2 lines ℓ and m that are not parallel that have been cut by a transversal.

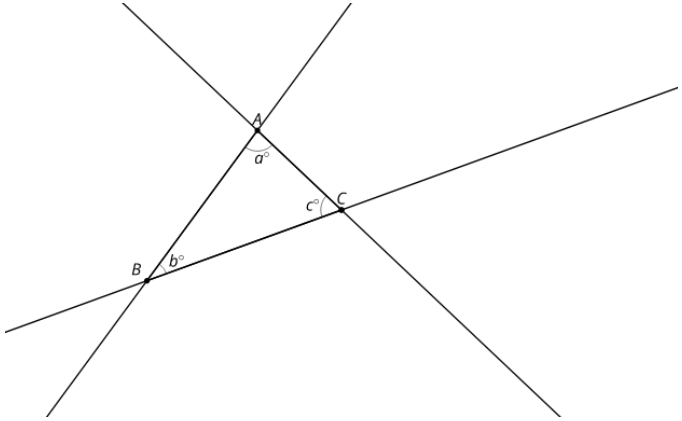


1. Your teacher will assign you one of Tyler's statements to think about. Is the statement true? Explain your reasoning.
2. In what circumstances are corresponding angles congruent? Be prepared to share your reasoning.

Triangle Angle Sum One Way

1. Use technology to create a triangle. Use the Text tool to label the 3 interior angles as *a*, *b*, and *c*.
2. Mark the midpoints of 2 of the sides.
3. Extend the side of the triangle without the midpoint in both directions to make a line.
4. Use what you know about rotations to create a line parallel to the line you made that goes through the opposite vertex.
5. What is the value of $a + b + c$? Explain your reasoning.

Triangle Angle Sum Another Way



1. Translate triangle ABC along the directed line segment from B to C to make triangle $A'B'C'$. Label the measures of the angles in triangle $A'B'C'$.
2. Translate triangle $A'B'C'$ along the directed line segment from A' to C to make triangle $A''B''C''$. Label the measures of the angles in triangle $A''B''C''$.
3. Label the measures of the angles that meet at point C. Explain your reasoning.
4. What is the value of $a + b + c$? Explain your reasoning.

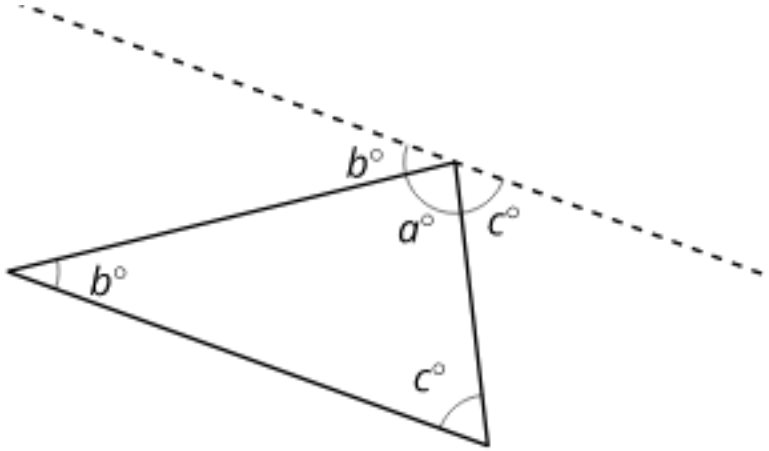
One Hundred and Eighty

Lesson Synthesis

- Look back at the two proofs of the Triangle Angle Sum Theorem. Are there any parts of the argument that depend on the particular measurements of your triangle, or would the same arguments have worked with other kinds of triangles?
- How are the 2 proofs of the Triangle Angle Sum Theorem different? How are they the same?

Triangle Angle Sum Theorem: The three angle measures of any triangle always sum to 180 degrees.

$$a + b + c = 180$$

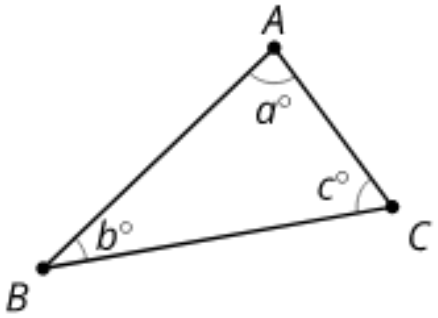


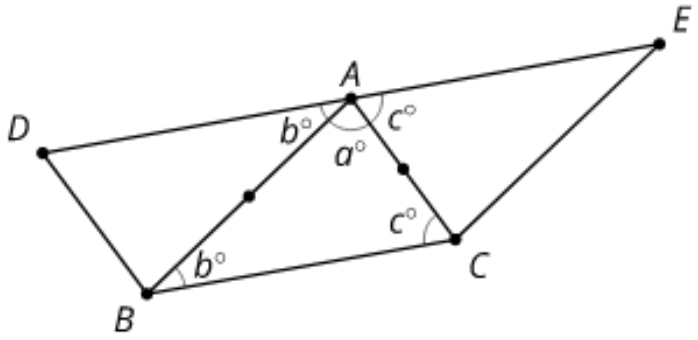
Triangle Angle Sum a Third Way

Cool-down

Elena wants to prove that the measures of the 3 angles in a triangle always sum to 180 degrees. She starts by using rigid motions to rotate triangle *ABC* 180 degrees around 2 midpoints.

Explain how Elena labeled her diagram and finish her proof.





I can prove the angles in a triangle sum to 180 degrees.

Learning Targets

Geometry