11-1 Geometric Sequences

Warm Up

Lesson Presentation

Lesson Quiz

11-1 Geometric Sequences

Warm Up

Find the value of each expression.

1. 2^5 32

2. 2^{-5} $\frac{1}{32}$

3. $7(-4)^2$ 112

4. $15\left(\frac{1}{3}\right)^3$ $\frac{5}{9}$

11-1 Geometric Sequences

Objectives

Recognize and extend geometric sequences.

Find the *n*th term of a geometric sequence.

Vocabulary

geometric sequence
common ratio

11-1 Geometric Sequences

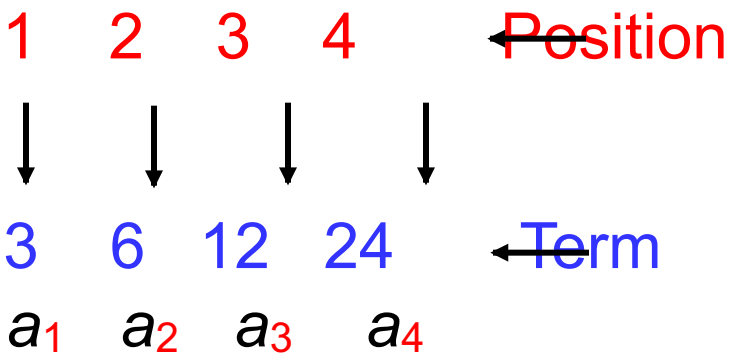
The table shows the heights of a bungee jumper's bounces.

Bounce	1	2	3
Height (ft)	200	80	32

The height of the bounces shown in the table above form a *geometric sequence.* In a **geometric sequence**, the ratio of successive terms is the same number *r*, called the **common ratio**.

11-1 Geometric Sequences

Geometric sequences can be thought of as functions. The term number, or position in the sequence, is the input, and the term itself is the output.



To find a term in a geometric sequence, multiply the previous term by *r*.

To find the common ratio in a geometric sequence, divide any term by the previous term.

11-1 Geometric Sequences

Finding a Term of a Geometric Sequence

The *n*th term of a geometric sequence with common ratio r is

$$a_n = a_{n-1}r$$

11-1 Geometric Sequences

Writing Math

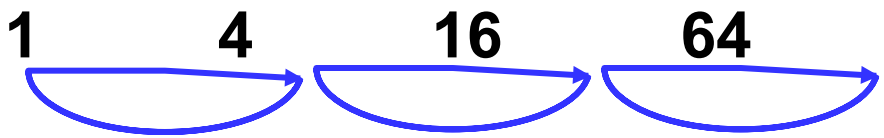
The variable a is often used to represent terms in a sequence. The variable a_4 (read "a sub 4")is the fourth term in a sequence.

Example 1A: Extending Geometric Sequences

Find the next three terms in the geometric sequence.

1, 4, 16, 64,…

Step 1 Find the value of *r* by dividing each term by the one before it.



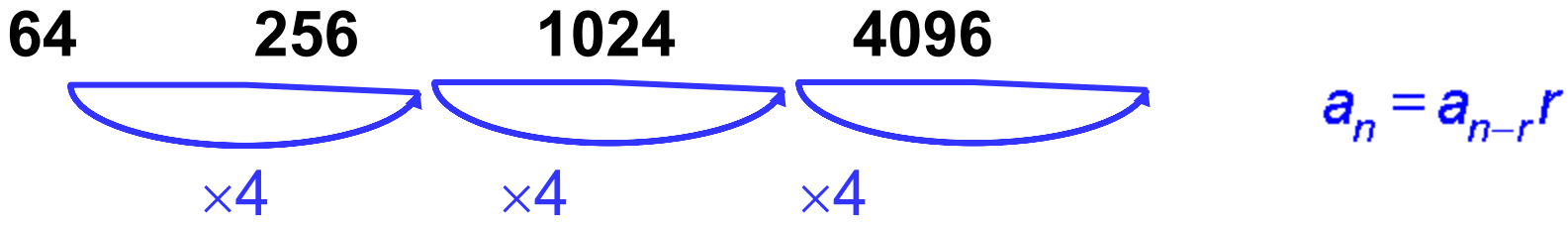
$\frac{4}{1} = 4$ $\frac{16}{4} = 4$ $\frac{64}{16} = 4$ ← The value of *r* is 4.

Example 1A Continued

Find the next three terms in the geometric sequence.

1, 4, 16, 64,…

Step 2 Multiply each term by 4 to find the next three terms.



The next three terms are 256, 1024, and 4096.

Example 1B: Extending Geometric Sequences

Find the next three terms in the geometric sequence.

$-9, 3, -1, \frac{1}{3}, -\frac{1}{9}$

Step 1 Find the value of *r* by dividing each term by the one before it.

$-9 \qquad 3 \qquad -1 \qquad \frac{1}{3} \qquad -\frac{1}{9}$

$\frac{3}{-9} = -\frac{1}{3} \qquad \frac{-1}{3} \qquad \frac{\frac{1}{3}}{\frac{-3}{3}} = -\frac{1}{3} \qquad \frac{-\frac{1}{9}}{\frac{1}{3}} = -\frac{1}{3}$ ← The value of *r* is $-\frac{1}{3}$.

Helpful Hint

When the terms in a geometric sequence alternate between positive and negative, the value of r is negative.

Example 1B Continued

Find the next three terms in the geometric sequence.

$-9, 3, -1, \frac{1}{3}, -\frac{1}{9}$

Step 2 Multiply each term by $-\frac{1}{3}$ to find the next three terms.

$$-\frac{1}{9} \quad \frac{1}{27} \quad -\frac{1}{81} \quad \frac{1}{243}$$

$$\times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right) \quad \times\left(-\frac{1}{3}\right)$$

$a_n = a_{n-r}r$

The next three terms are $\frac{1}{27}, -\frac{1}{81}$ and $\frac{1}{243}$.

Check It Out! Example 1a

Find the next three terms in the geometric sequence.

5, –10, 20,–40,…

Step 1 Find the value of *r* by dividing each term by the one before it.

$\frac{-10}{5} = -2$ $\frac{20}{-10} = -2$ $\frac{-40}{20} = -2$

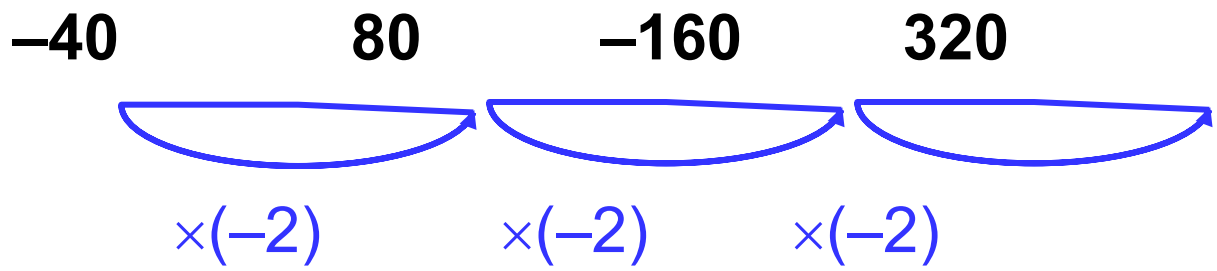
← The value of *r* is –2.

Check It Out! Example 1a Continued

Find the next three terms in the geometric sequence.

5, –10, 20,–40,…

Step 2 Multiply each term by –2 to find the next three terms.



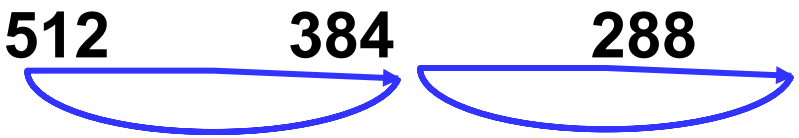
The next three terms are 80, –160, and 320.

Check It Out! Example 1b

Find the next three terms in the geometric sequence.

512, 384, 288,…

Step 1 Find the value of *r* by dividing each term by the one before it.



$\frac{384}{512} = \frac{3}{4}$ $\frac{288}{384} = \frac{3}{4}$

← The value of *r* is 0.75.

Check It Out! Example 1b Continued

Find the next three terms in the geometric sequence.

512, 384, 288,…

Step 2 Multiply each term by 0.75 to find the next three terms.

288 **216** **162** **121.5**

The next three terms are 216, 162, and 121.5.

11-1 Geometric Sequences

To find the output a_n of a geometric sequence when n is a large number, you need an equation, or function rule.

Words	Numbers	Algebra
1^{st} term	3	a_1
2^{nd} term=1^{st} term times r	$3(2^1)$	$a_1(r^1)$
3^{rd} term=1^{st} term times r times r	$3(2^2)$	$a_1(r^2)$
4^{th} term=1^{st} term times r times r times r	$3(2^3)$	$a_1(r^3)$

11-1 Geometric Sequences

Finding the nth term of a Geometric Sequence

If the first term of a geometric sequence is a_1, the nth term is a_n , and the common ratio is r, then

$$a_n = a_1 r^{n-1}$$

nth term

1^{st} term

Common ratio

Example 2A: Finding the *n*th Term of a Geometric Sequence

The first term of a geometric sequence is 500, and the common ratio is 0.2. What is the 7th term of the sequence?

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_7 = 500(0.2)^{7-1}$ *Substitute 500 for a_1, 7 for n, and 0.2 for r.*

$= 500(0.2)^6$ *Simplify the exponent.*

$= 0.032$ *Use a calculator.*

The 7th term of the sequence is 0.032.

Example 2B: Finding the *n*th Term of a Geometric Sequence

For a geometric sequence, $a_1 = 5$, and $r = 2$. Find the 6th term of the sequence.

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_6 = 5(2)^{6-1}$ *Substitute 5 for a_1, 6 for n, and 2 for r.*

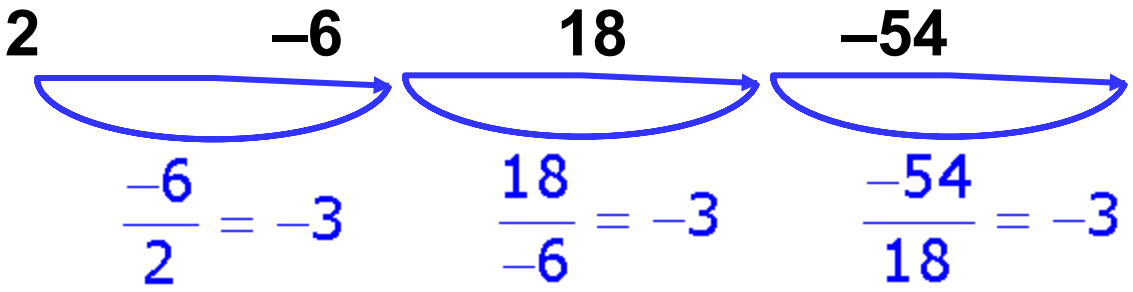
$= 5(2)^5$ *Simplify the exponent.*

$= 160$

The 6th term of the sequence is 160.

Example 2C: Finding the *n*th Term of a Geometric Sequence

What is the 9th term of the geometric sequence 2, –6, 18, –54, …?



The value of r is –3.

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_9 = 2(-3)^{9-1}$ *Substitute 2 for a_1, 9 for n, and –3 for r.*

$= 2(-3)^8$ *Simplify the exponent.*

$= 13{,}122$ *Use a calculator.*

The 9th term of the sequence is 13,122.

Caution

When writing a function rule for a sequence with a negative common ratio, remember to enclose *r* in parentheses. $-2^{12} \neq (-2)^{12}$

Check It Out! Example 2

What is the 8th term of the sequence 1000, 500, 250, 125, …?

1000 500 250 125

$\frac{500}{1000} = \frac{1}{2}$ $\frac{250}{500} = \frac{1}{2}$ $\frac{125}{250} = \frac{1}{2}$

The value of r is $\frac{1}{2}$.

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_8 = 1000\left(\frac{1}{2}\right)^{8-1}$ *Substitute 1000 for a_1, 8 for n, $\frac{1}{2}$ and for r.*

$= 1000\left(\frac{1}{2}\right)^{7}$ *Simplify the exponent.*

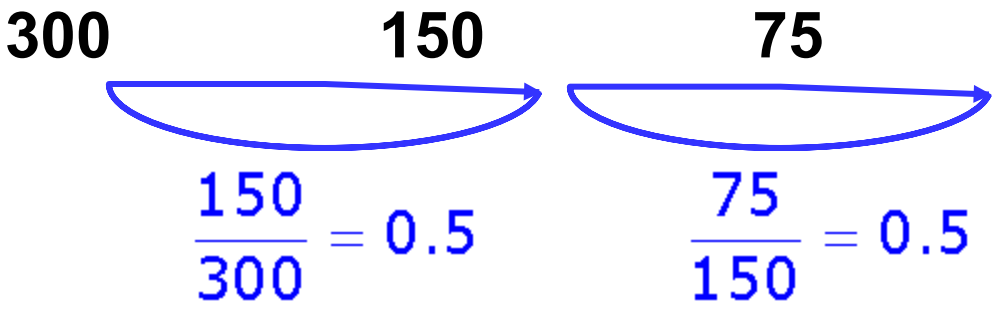
$= 7.8125$ *Use a calculator.*

The 8th term of the sequence is 7.8125.

Example 3: *Application*

A ball is dropped from a tower. The table shows the heights of the balls bounces, which form a geometric sequence. What is the height of the 6th bounce?

Bounce	Height (cm)
1	300
2	150
3	75



The value of r is 0.5.

Example 3 Continued

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_6 = 300(0.5)^{6-1}$ *Substitute 300 for a_1, 6 for n, and 0.5 for r.*

$= 300(0.5)^5$ *Simplify the exponent.*

$= 9.375$ *Use a calculator.*

The height of the 6th bounce is 9.375 cm.

Check It Out! Example 3

The table shows a car's value for 3 years after it is purchased. The values form a geometric sequence. How much will the car be worth in the 10th year?

Year	Value ($)
1	10,000
2	8,000
3	6,400

10,000 → 8,000 → 6,400

$$\frac{8,000}{10,000} = 0.8 \qquad \frac{6,400}{8,000} = 0.8$$

The value of r is 0.8.

Check It Out! Example 3

$a_n = a_1 r^{n-1}$ *Write the formula.*

$a_6 = 10{,}000(0.8)^{10-1}$ *Substitute 10,000 for a_1, 10 for n, and 0.8 for r.*

$= 10{,}000(0.8)^9$ *Simplify the exponent.*

$= 1{,}342.18$ *Use a calculator.*

In the 10th year, the car will be worth $1342.18.

Lesson Quiz: Part I

Find the next three terms in each geometric sequence.

1. 3, 15, 75, 375,… 1875; 9375; 46,875

2. $5, -1, \frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, \ldots$ $-\frac{1}{625}, \frac{1}{3125}, -\frac{1}{15{,}625}$

3. The first term of a geometric sequence is 300 and the common ratio is 0.6. What is the 7th term of the sequence? 13.9968

4. What is the 15th term of the sequence 4, –8, 16, –32, 64…? 65,536

Lesson Quiz: Part II

Find the next three terms in each geometric sequence.

5. The table shows a car's value for three years after it is purchased. The values form a geometric sequence. How much will the car be worth after 8 years?

$5570.39

Year	Value ($)
1	18,000
2	15,300
3	13,005