Plan for Geometry Unit 6: Coordinate Geometry

Relevant Unit(s) to review: Algebra 1 Unit 6: Introduction to Quadratic Functions [Lessons 8 & 9]

Essential prior concepts to engage with this unit	This unit is accessible due to its visual and kinesthetic approach. Students review, in context, many concepts first introduced in grades 6–8 and in prior units in this course. This includes concepts such as proportional reasoning, scale factor, rigid transformations, and dilations. Access to tracing paper and colored pencils is helpful to support. Two key understandings are important to address as the need arises: ● use of distributive property for binomial pairs ● rewrite trinomials in factored form
Brief narrative of approach	There are a couple of viable options to address unfinished learning due to school closures in spring of 2020. One is outlined here, in which the key prerequisite understandings are introduced and maintain focus on the ideas of this unit. The second option is to introduce the first half (or the entire) of Algebra 1 Unit 6 prior to this unit. We acknowledge that students do complete the square in this unit to transform equations of circles, but there is no need to pre-teach this skill. The lessons in this unit attend to the connection between the distributive property and perfect square trinomials. Familiarity with the use of diagrams of products, first introduced in Algebra 1 Unit 6 will be useful. The greater emphasis is on recognizing the usefulness of manipulating equations to reveal features of the graph. We recommend introducing Lessons 8 and 9 from Algebra 1 Unit 6 prior to Lesson 5 of this unit. Lesson 8 introduces the area model to illustrate the distributive property. We recommend you skip Questions 5, 6, and 8 from practice problems at this time. In Lesson 9, students are introduced to standard and factored forms of quadratic expressions and use the distributive property to move between the equivalent forms.

Lessons to Add	Lessons to Remove or Modify
For this unit, most students will have completed the prerequisite lessons in Grade 8 Unit 8: Pythagorean Theorem and Irrational Numbers. Some may not have completed the prerequisite lessons in Algebra 1 Unit 6. These lessons will give students a ramp into the mathematical concepts in focus in this unit. 1. A1.6.8 2. A1.6.9	1. G.6.1, G.6.2, and G.6.3: move quickly through these if students do well on Question 7 in pre-unit diagnostic assessment 2. G.6.11.2: move quickly through this if students do well on Question 4 in pre-unit diagnostic assessment. 3. Skip G.6.12.2 4. Save G.6.13.2 for Algebra 2 5. Skip G.6.16.3 6. Skip G.6.17.1, G.6.17.3, G.6.17.4, G.6.17.5, and G.6.17.6
Lessons added: 2	Lessons removed: 2

Modified Plan for Geometry Unit 6

Day	IM lesson	Notes
	assessment	G.6 Check Your Readiness Assessment Note that the Check Your Readiness assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in G.6.
1	G.6.1	Can move quickly through the first 3 lessons if students do well in pre-unit diagnostic question 7. Students extend understanding of transformations to figures on a coordinate grid.
2	G.6.2	Students use transformation notation to take points in the plane as inputs and give other points as outputs. This way of thinking is foundational for the rest of the unit as well as in later courses when we study transformations of functions.
3	G.6.3	Students learn to recognize which transformations functions yield figures that are similar, congruent, or neither. Pulls together previous areas of study (Units 1, 2, and 3).

4	G.6.4	Students connect the idea of distance and the definition of a circle to come to understand and be able to write the equation of a circle.
5	A1.6.8	Students extend previous understanding of the distributive property to use diagrams to organize and account for all terms when applying the distributive property. The use of these diagrams will be helpful in upcoming work with transforming equations of circles to reveal key features.
6	A1.6.9	Students apply understandings about the distributive property and use of diagrams to expressions that contain sums and differences. The terms standard form and factored form of quadratic expressions are introduced in this lesson.
7	G.6.5	Students rewrite perfect square trinomials in factored form to find a circle's center. This leads to completing the square in upcoming activities. If students struggled with either question 5 or 6 in the pre-unit diagnostic assessment, plan on amplifying review of distributive property and connection to factored form within this lesson or across the two lessons listed above.
8	G.6.6	Students complete the square to find the center and radius of circles.
9	G.6.7	Students are introduced to the definition of a parabola as the set of points equidistant from a given point, or focus, and a line, directrix. Students practice using distance calculations, which prepares students to write the equation for a parabola in an upcoming lesson.
10	G.6.8	Students write an equation for a parabola and rewrite it in vertex form. If students did not engage in A1 U6, take a moment to highlight the connection between the form of the equation and this feature (vertex) of the graph as seen in Activity 8.2 question 2.
11	G.6.9	Students develop the point-slope form of a linear equation. This will support a focus on the geometric properties of lines, which will play a role in various ways throughout the remainder of the unit. If students struggle on Question 2 from the pre-unit diagnostic assessment, plan to spend more time on slope during the warm-up.
12	G.6.10	Students connect understandings of slope and parallel lines. Students prove the slope of parallel lines are equivalent, then apply this theorem to write equations and prove a quadrilateral is a parallelogram. If students struggle on question 3 from the pre-unit diagnostic assessment, plan to spend some time reviewing how to graph equations of lines in slope-intercept form.

13	G.6.11	If students do well on pre-unit diagnostic question 4, you can move quickly through Activity 2. Students connect an understanding of slope and perpendicular lines.
14	G.6.12	Skip Activity 12.2. This lesson gives students the opportunity to develop proficiency with finding equations of lines parallel or perpendicular to a given line.
15	G.6.13	Activity 13.2 could be saved for Algebra 2. Students have an additional opportunity to attend to ideas of parallel and perpendicular as they investigate finding the intersection points of a line and a circle.
16	G.6.14	Students use coordinates to make conjectures and prove simple geometric theorems algebraically. Another opportunity for students to engage in the use of the distance formula and the Pythagorean Theorem through slopes of parallel and perpendicular lines.
17	G.6.15	Ideas from Unit 3 surface in this lesson, from a different perspective. Students link segment partitioning to the concept of similarity transformations.
18	G.6.16	Skip Activity 16.3. Students apply segment partitioning logic to prove that the medians of a triangle intersect in a single point.
19	G.6.17.2	Skip Activities 17.1, 17.3, 17.4, 17.5, and 17.6. In Activity 17.2 students use the structure of the coordinate plane to examine the observation that the altitudes of a triangle all intersect at a single point.
20	G.6 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

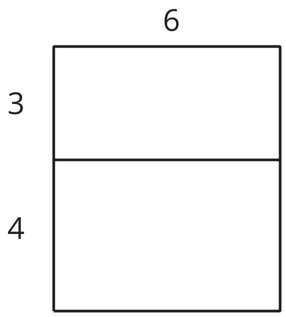
Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
G.6.1	0	E	This lesson connects ideas from several previous units and extends them to the coordinate plane.

G.6.2	+	D	Students use coordinate transformation notation, which supports the understanding of transformations as functions. This is a novel context for students to examine rigid transformations, similarity transformations, and transformations that do not fit any vocabulary students have learned.
G.6.3	+	A	Students describe transformations in the coordinate plane, with a focus on identifying transformations as producing congruent or similar figures (or neither). The focus on congruence via equal distances throughout this lesson prepares students for subsequent activities that use distance to write equations for circles and parabolas.
G.6.4	+	E	Students repeatedly test whether points are on a circle by finding the distance between the points and the circle's center. This lesson requires more algebraic fluency than the previous lessons in this course, as do the next several lessons. Depending on students' familiarity and comfort with the algebra skills involved, it might make sense to spend more than one day on some lessons.
G.6.5	+	D	Work in this lesson leads to completing the square to identify a circle's radius and center in upcoming activities.
G.6.6	+	D	Students complete the square to find the center and radius of circles.
G.6.7	+	D	Students explore the relationship between the positioning of the focus and directrix and the resulting shape of the parabola. One of the activities in this lesson works best when each student has access to devices that can run the Desmos applet because students will benefit from seeing the relationship in a dynamic way.
G.6.8	+	A	Students write an equation for a parabola and rewrite it in vertex form. Then, they match graphs of parabolas to equations.
G.6.9	+	E	Students develop the point-slope form of a linear equation. Students will write equations of lines in the next several lessons, and intercepts will not always be readily available. The point-slope form will require the least algebraic manipulation and allow students to focus on geometric properties.

G.6.10	0 *	D	*Activity G.6.10.3 is a priority Students connect ideas about slopes of lines and parallel lines and prove that non-vertical parallel lines have equal slopes.
G.6.11	+	D	Activity G.6.11.3 is not a priority Students connect ideas about slope and perpendicular lines and prove that non-vertical and non-horizontal perpendicular lines have slopes with opposite reciprocals.
G.6.12	0	A	Students apply concepts of parallel and perpendicular lines to conclude that if two lines are both perpendicular to the same line, they must be parallel.
G.6.13	0*	E	*Activity G.6.13.2 is a priority Students combine concepts of geometry and algebra to find solutions to a system of equations consisting of a linear and a quadratic equation.
G.6.14	0	A	Students use coordinates to make conjectures and prove simple geometric theorems algebraically. One of the activities in this lesson works best when each student has access to devices that can run the Desmos applet because students will benefit from seeing the relationship in a dynamic way.
G.6.15	0	A	Students partition segments in given ratios, they are then introduced to weighted averages and asked to connect weighted averages to segment partitioning.
G.6.16	0	A	The goal of this lesson is to use segment partitioning logic to prove that the medians of a triangle intersect in a single point.
G.6.17	-	A	The goal of this lesson is to hone the algebra skills students encountered during this unit.

Lesson 8: Equivalent Quadratic Expressions

8.1: Diagrams of Products

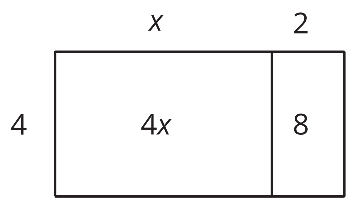


1. Explain why the diagram shows that $6(3+4) = 6 \cdot 3 + 6 \cdot 4$.

2. Draw a diagram to show that $5(x+2) = 5x + 10$.

8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n(2n+5)$ and $2n^2 + 5n$ are equivalent expressions.

2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

   a. $6\left(\frac{1}{3}n+2\right)$

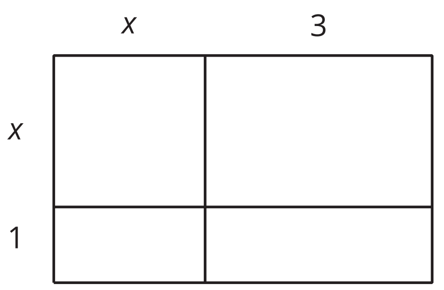
   b. $p(4p+9)$

   c. $5r\left(r+\frac{3}{5}\right)$

   d. $(0.5w+7)w$

8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x + 1$ and $x + 3$. Use this diagram to show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:
    a. $(x + 5)^2$

    b. $2x(x + 4)$

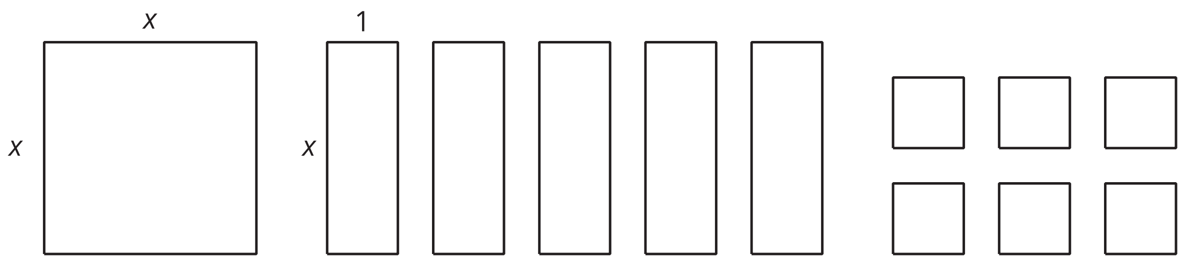
    c. $(2x + 1)(x + 3)$

    d. $(x + m)(x + n)$

3. Write an equivalent expression for each expression without drawing a diagram:
    a. $(x + 2)(x + 6)$

    b. $(x + 5)(2x + 10)$

Are you ready for more?



1. Is it possible to arrange an x by x square, five x by 1 rectangles and six 1 by 1 squares into a single large rectangle? Explain or show your reasoning.

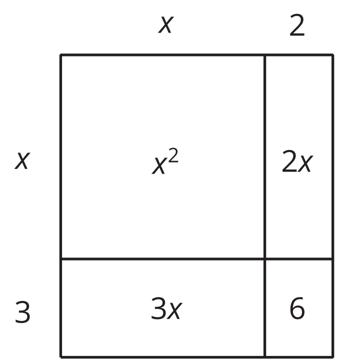
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?

3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed with $x(18-x)$, which can also be written as $18x - x^2$. The former is a product of x and $18-x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x+2)(x+3)$. We can write an equivalent expression by thinking about each factor, the $(x+2)$ and $(x+3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x+2)$ and $(x+3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x+2)(x+3)$ is equivalent to $x^2+2x+3x+6$, or to x^2+5x+6.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x+2$) is multiplied by every term in the other factor (the x and the 3 in $x+3$).

$$(x+2)(x+3)$$
$$= x(x+3) + 2(x+3)$$
$$= x^2 + 3x + 2x + (2)(3)$$
$$= x^2 + (3+2)x + (2)(3)$$

In general, when a quadratic expression is written in the form of $(x+p)(x+q)$, we can apply the distributive property to rewrite it as $x^2+px+qx+pq$ or $x^2+(p+q)x+pq$.

Lesson 8: Equivalent Quadratic Expressions

Cool Down: Writing Equivalent Expressions

1. Use a diagram to show that $(3x + 1)(x + 2)$ is equivalent to $3x^2 + 7x + 2$.

2. Is $(x + 4)^2$ equivalent to $2x^2 + 8x + 8$? Explain or show your reasoning.

Unit 6 Lesson 8 Cumulative Practice Problems

1. Draw a diagram to show that $(2x+5)(x+3)$ is equivalent to $2x^2+11x+15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   A. $(x+2)(x+6)$
   B. $(2x+8)(x+2)$
   C. $(x+8)(x+4)$
   D. $(x+2)(2x+6)$

   1. $x^2+12x+32$
   2. $2x^2+10x+12$
   3. $2x^2+12x+16$
   4. $x^2+8x+12$

3. Select **all** expressions that are equivalent to x^2+4x.

   A. $x(x+4)$
   B. $(x+2)^2$
   C. $(x+x)(x+4)$
   D. $(x+2)^2-4$
   E. $(x+4)x$

4. Tyler drew a diagram to expand $(x+5)(2x+3)$.

   a. Explain Tyler's mistake.

   b. What is the correct expanded form of $(x+5)(2x+3)$?

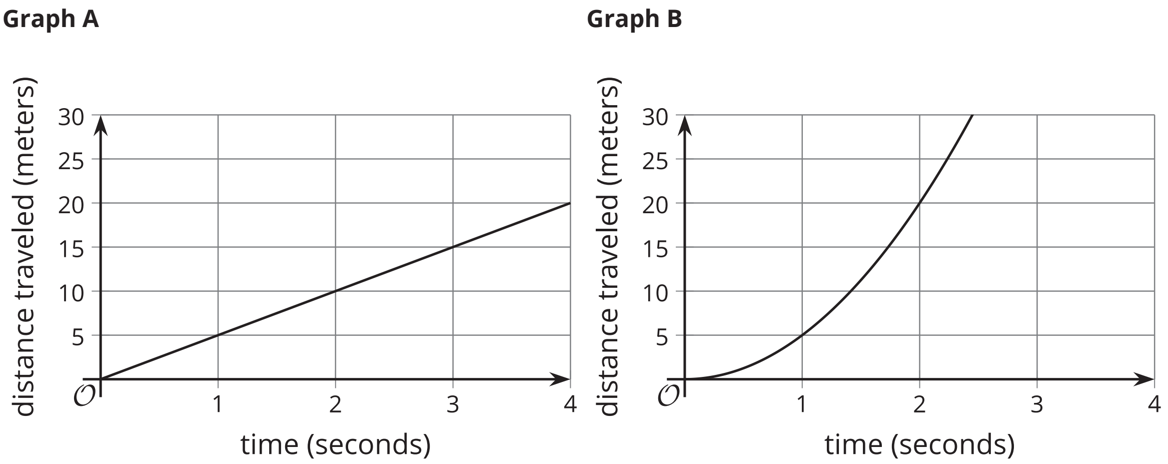
	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

5. Explain why the values of the exponential expression 3^x will eventually overtake the values of the quadratic expression $10x^2$.

(From Unit 6, Lesson 4.)

6. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   Which graph could represent this situation? Explain how you know.



(From Unit 6, Lesson 5.)

7. Consider a function q defined by $q(x) = x^2$. Explain why negative values are not included in the range of q.

(From Unit 4, Lesson 10.)

8. Based on past concerts, a band predicts selling $600 - 10p$ concert tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many concert tickets the band expects to sell and what revenues it expects to receive at the given ticket prices.

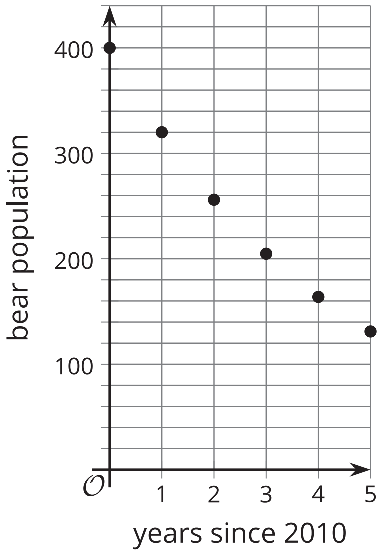
ticket price (dollars)	number of tickets	revenue (dollars)
10		
15		
20		
30		
35		
45		
50		
60		
p		

   b. In this model, at what ticket prices will the band earn no revenue at all?

   c. At what ticket prices should the band sell the tickets if it must earn at least 8,000 dollars in revenue to break even (to not lose money) on a given concert. Explain how you know.

(From Unit 6, Lesson 7.)

9. A population of bears decreases exponentially.



   a. What is the annual factor of decrease for the bear population? Explain how you know.

   b. Using function notation, represent the relationship between the bear population, b, and the number of years since the population was first measured, t. That is, find a function, f, so that $b = f(t)$.

(From Unit 5, Lesson 8.)

10. Equations defining functions a, b, c, d, and f are shown here.

    Select **all** the equations that represent exponential functions.

    A. $a(x) = 2^3 \cdot x$

    B. $b(t) = \left(\frac{2}{3}\right)^t$

    C. $c(m) = \frac{1}{5} \cdot 2^m$

    D. $d(x) = 3x^2$

    E. $f(t) = 3 \cdot 2^t$

(From Unit 5, Lesson 8.)

Lesson 9: Standard Form and Factored Form

9.1: Math Talk: Opposites Attract

Solve each equation mentally.

$40 - 8 = 40 + n$

$25 + -100 = 25 - n$

$3 - \frac{1}{2} = 3 + n$

$72 - n = 72 + 6$

9.2: Finding Products of Differences

1. Show that $(x - 1)(x - 1)$ and $x^2 - 2x + 1$ are equivalent expressions by drawing a diagram or applying the distributive property. Show your reasoning.

2. For each expression, write an equivalent expression. Show your reasoning.
    a. $(x + 1)(x - 1)$

    b. $(x - 2)(x + 3)$

    c. $(x - 2)^2$

9.3: What is the Standard Form? What is the Factored Form?

The quadratic expression $x^2 + 4x + 3$ is written in **standard form**.

Here are some other quadratic expressions. The expressions on the left are written in standard form and the expressions on the right are not.

Written in standard form:	Not written in standard form:
$x^2 - 1$	$(2x + 3)x$
$x^2 + 9x$	$(x + 1)(x - 1)$
$\frac{1}{2}x^2$	$3(x - 2)^2 + 1$
$4x^2 - 2x + 5$	$-4(x^2 + x) + 7$
$-3x^2 - x + 6$	$(x + 8)(-x + 5)$
$1 - x^2$	

1. What are some characteristics of expressions in standard form?

2. $(x + 1)(x - 1)$ and $(2x + 3)x$ in the right column are quadratic expressions written in **factored form**. Why do you think that form is called factored form?

Are you ready for more?

Which quadratic expression can be described as being both standard form and factored form? Explain how you know.

Lesson 9 Summary

A quadratic function can often be represented by many equivalent expressions. For example, a quadratic function f might be defined by $f(x) = x^2 + 3x + 2$. The quadratic expression $x^2 + 3x + 2$ is called the **standard form**, the sum of a multiple of x^2 and a linear expression ($3x + 2$ in this case).

In general, standard form is

$$ax^2 + bx + c$$

We refer to a as the coefficient of the squared term x^2, b as the coefficient of the linear term x, and c as the constant term.

The function f can also be defined by the equivalent expression $(x + 2)(x + 1)$. When the quadratic expression is a product of two factors where each one is a linear expression, this is called the **factored form**.

An expression in factored form can be rewritten in standard form by expanding it, which means multiplying out the factors. In a previous lesson we saw how to use a diagram and to apply the distributive property to multiply two linear expressions, such as $(x + 3)(x + 2)$. We can do the same to expand an expression with a sum and a difference, such as $(x + 5)(x - 2)$, or to expand an expression with two differences, for example, $(x - 4)(x - 1)$.

To represent $(x - 4)(x - 1)$ with a diagram, we can think of subtraction as adding the opposite:

	x	-4
x	x^2	$-4x$
-1	$-x$	4

$$\begin{aligned}
&(x - 4)(x - 1)\\
&= (x + -4)(x + -1)\\
&= x(x + -1) + -4(x + -1)\\
&= x^2 + -1x + -4x + (-4)(-1)\\
&= x^2 + -5x + 4\\
&= x^2 - 5x + 4
\end{aligned}$$

Lesson 9: Standard Form and Factored Form

Cool Down: From One Form to Another

For each expression, write an equivalent expression in standard form. Show your reasoning.

1. $(2x+5)(x+1)$

2. $(x-2)(x+2)$

Unit 6 Lesson 9 Cumulative Practice Problems

1. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x + 4)(x - 1)$

   b. $(2x - 1)(3x - 1)$

2. Consider the expression $8 - 6x + x^2$.

   a. Is the expression in standard form? Explain how you know.

   b. Is the expression equivalent to $(x - 4)(x - 2)$? Explain how you know.

3. Which quadratic expression is written in standard form?

   A. $(x + 3)x$

   B. $(x + 4)^2$

   C. $-x^2 - 5x + 7$

   D. $x^2 + 2(x + 3)$

4. Explain why $3x^2$ can be said to be in both standard form and factored form.

5. Jada dropped her sunglasses from a bridge over a river. Which equation could represent the distance y fallen in feet as a function of time, t, in seconds?

   A. $y = 16t^2$

   B. $y = 48t$

   C. $y = 180 - 16t^2$

   D. $y = 180 - 48t$

   (From Unit 6, Lesson 5.)

6. A football player throws a football. The function h given by $h(t) = 6 + 75t - 16t^2$ describes the football's height in feet t seconds after it is thrown.

   Select **all** the statements that are true about this situation.

   A. The football is thrown from ground level.

   B. The football is thrown from 6 feet off the ground.

   C. In the function, $-16t^2$ represents the effect of gravity.

   D. The outputs of h decrease then increase in value.

   E. The function h has 2 zeros that make sense in this situation.

   F. The vertex of the graph of h gives the maximum height of the football.

   (From Unit 6, Lesson 6.)

7. *Technology required*. Two rocks are launched straight up in the air.

   - The height of Rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$.
   - The height of Rock B is given by function g, where $g(t) = 5 + 20t - 16t^2$.

   In both functions, t is time measured in seconds and height is measured in feet. Use graphing technology to graph both equations.

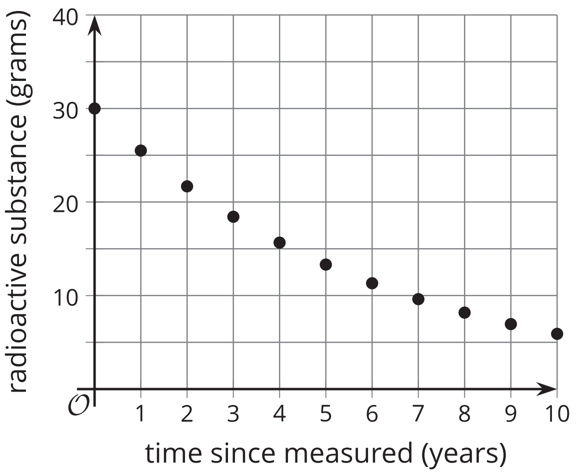
   a. What is the maximum height of each rock?

   b. Which rock reaches its maximum height first? Explain how you know.

   (From Unit 6, Lesson 6.)

8. The graph shows the number of grams of a radioactive substance in a sample at different times after the sample was first analyzed.

   a. What is the average rate of change for the substance during the 10 year period?



   b. Is the average rate of change a good measure for the change in the radioactive substance during these 10 years? Explain how you know.

   (From Unit 5, Lesson 10.)

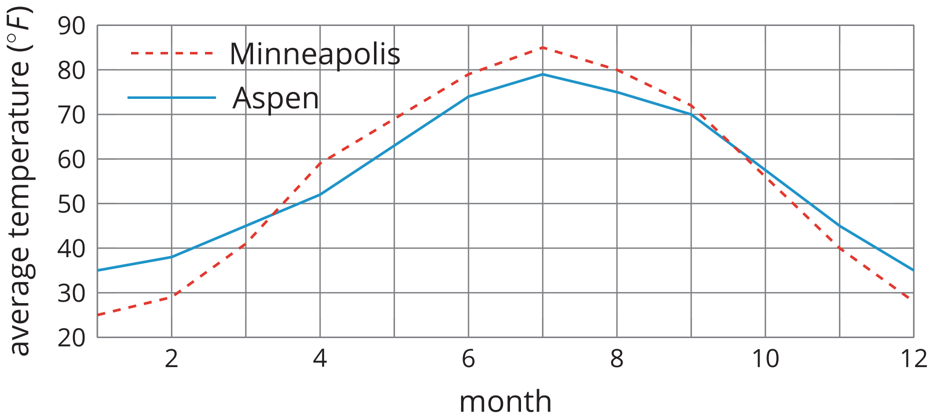
9. Each day after an outbreak of a new strain of the flu virus, a public health scientist receives a report of the number of new cases of the flu reported by area hospitals.

time since outbreak in days	2	3	4	5	6	7
number of new cases of the flu	20	28	38	54	75	105

Would a linear or exponential model be more appropriate for this data? Explain how you know.

(From Unit 5, Lesson 11.)

10. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.



a. What does $A(8)$ mean in this situation? Estimate $A(8)$.

b. Which city had a higher average temperature in February?

c. Were the two cities' average high temperatures ever the same? If so, when?

(From Unit 4, Lesson 9.)