Unit Title: Thinking with Mathematical Models –Investigation 1: Exploring Data Patterns

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Core Curriculum: Connected Math Project 3 (CMP3)

School: Gateway Lab School

Content Area: Math

Grade Level: 8th

Summary of Unit

In this unit, students will create and analyze patterns in experimental data and number sequences. The unit provides real-world, hands-on experiences with both linear and nonlinear patterns and lays the groundwork for modeling real world patterns with mathematical functions. Students will encounter linear and nonlinear relationships between variables such as bridge length and breaking weight. Using models of real situations will help students to make better supported conjectures about real situations.

Stage 1 – Desired Results
(What students will know, be able to do and understand)

Common Core State Standards

CCSS.Math.Content.8.F.A.3 - Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

CCSS.Math.Content.8.F.B.5 - Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

CCSS.Math.Content.8.SP.A.1 - Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

CCSS.Math.Content.8.SP.A.2 - Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association,

informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Standards for Mathematical Practice

CCSS.Math.Practice.MP1 - Make sense of problems and persevere in solving them.

CCSS.Math.Practice.MP2 - Reason abstractly and quantitatively.

CCSS.Math.Practice.MP3 - Construct viable arguments and critique the reasoning of others.

CCSS.Math.Practice.MP4 - Model with mathematics.

CCSS.Math.Practice.MP5 - Use appropriate tools strategically.

CCSS.Math.Practice.MP6 - Attend to precision.

CCSS.Math.Practice.MP7 - Look for and make use of structure.

CCSS.Math.Practice.MP8 - Look for and express regularity in repeated reasoning.

Big Ideas

- Gathering, organizing, and interpreting data.
- Using graphs and tables to make predictions with linear and non-linear bivariate relationships.
- Recognizing and continuing patterns.
- Representing patterns in tables and graphs.
- Modeling patterns with equations.

Unit Enduring Understandings

- **Grappling / Rigor:** Students will not be given information easily. They will be encouraged to grapple with completing problems. The teacher will not give away any answers until the very end of an activity (and maybe not even then). Students will need to make conjectures, test them, and reach conclusions in a cooperative setting.
- **Constructing Arguments:** Students will be given many opportunities to present their opinions on math topics. An emphasis will be placed on providing evidence for these opinions.
- **Critiquing other students:** Students will be given the opportunity to grade other students' graphs, and evaluate presentations for content and presentation skills. The ability to critique is important for future schooling and career.

- **Self-analysis:** Students will be given opportunities to analyze their performance, which is the cornerstone for self-improvement.

Unit Essential Questions

- How can you describe the relationship between load weight and bridge strength?
- How can you describe the relationship between bridge length and bridge strength?
- How can you predict if a pattern between variables will be linear or nonlinear?

Knowledge and Skills

Students will know...

- Linear and nonlinear situations both real-world, modeled, and patterned.
- Meanings of data found in tables and scatterplots.
- Different types of relationships between two quantities.
- Different types of associations in bivariate data.
- Straight lines are widely used to model relationships between two variables.

Students will be able to...

- Represent patterns using graphs, tables, word descriptions, and algebraic expressions.
- Construct scatterplots to investigate patterns of association.
- Investigate the nature of linear functions in context.
- Compare linear and non-linear situations.
- Measure variation in data, and strength of association in bivariate data.
- Use data to make productions.
- Interpret scatterplots of bivariate data to determine the strength of the linear association between two variables.

Stage 2 – Assessment Evidence
(Evidence that will be collected to determine whether or not Desired Results are achieved)

Suggested Unit Transfer Task(s)

Construction Crane Data Activity (P. 15 #1 a, b, c, d): This homework activity assesses the students' abilities to to apply what they learned in this unit to a situation that is similar but not identical to ones they worked on in the unit. This assesses the students' abilities to:

- Create a scatterplot (In particular to formulate a y-scale that is based on numbers in the thousands [this has not been addressed in the investigation]).
- Interpret a relationship between two quantities.
- Make predictions based on a scatterplot.
- Synthesize knowledge of bivariate relationships by comparing this activity with the bridge experiments from problem 1.1 and 1.2.

Rubric(s) / Scoring Guides

Learning Team Individual Work Rubric:

This rubric is used when students are working cooperatively in learning teams. During an investigation the teacher circulates the room with the following rubric and checks areas for each student (Bu Hao means "no good" in Chinese). The teacher carries one piece of paper per learning team on a clipboard which makes the task of assessing students convenient. When the group work is completed the teacher then averages all the checkmarks to determine a letter grade for the student. The letter grade then indicates how many points out of 20 the student receives for the teamwork part of the investigation grade (as seen in the chart below).

John		Focus on the Task and Participation	Listening, Questioning and Discussing	Attitude
A	Great!	Consistently stays focused on task and what needs to be done. Self-directed. Includes all team members.	Respectfully listens, interacts, discusses and poses questions to all team members during discussions.	Always has a positive attitude about the task(s) and the work of others.
B	Good	Focuses on the task and what needs to be done most of the time.	Respectfully listens, interacts, discusses and poses questions most of the time.	Usually has a positive attitude about the task(s) and the work of others.
C	OK	Focuses on the task and what needs to be done some of the time.	Has difficulty listening and discussing, tends to dominate discussions or rarely participates.	Occasionally is negative and critical of the task(s) or the work of other group members. Or rarely participates.
F	Bu Hao	Rarely focuses on the task and what needs to be done. Lets others do the work.	Has great difficulty listening, argues with teammates, or doesn't participate.	Is often negative and critical of the task(s) or the work of other team members. Or doesn't participate.

	Grade	Percentage
A+	20	100%
A	19	95%
A-	18	90%
B+	17.5	88%
B	17	85%
B-	16	80%
C+	15.5	78%
C	15	75%
C-	14	70%
D+	13.5	68%
D	13	65%
D-	12	60%
F	Anywhere from 0-11 depending on if student had any checks other than an F	

Scatterplot Scoring Guide:

Name (scatterplot creater) _____

Name (grader)_____

1 point maximum for each category

Points	Scatterplot Grading
	Title
	Table
	Proper X and Y axis with Straight Lines
	X label
	X scale
	Y label
	Y scale
	Points
	Point distribution (scale to spread out points)
	Neatness
	Total (10 points maximum)

Lesson 1.1 Question Scoring Guide

Place a 2 (yes), 0 (no), or a 1 (half credit) in each block	Team 1	Team 2	Team 3	Team 4
Question B				
Was the relationship described accurately and totally?				
Was evidence shown in the table?				
Was evidence shown in the graph?				
Question C				
Was the prediction reasonable?				
Was evidence shown for the prediction?				
Question D				
Was the prediction reasonable?				
Was evidence shown for the prediction?				
Question E				
Was the test result explained?				
Was the explanation for non-exact predictions logical?				

Lesson 1.2 Question Scoring Guide

Place a 2 (yes), 0 (no), or a 1 (half credit) in each block	Team 1	Team 2	Team 3	Team 4
Question B				
Was the relationship described accurately and totally?				
Was evidence shown in the table?				
Was evidence shown in the graph?				
Question C				
Were the predictions reasonable?				
Was evidence shown for the predictions?				
Question D				
Was a reasonable similarity(s) accurately described?				
Was evidence shown for the similarity(s)?				
Was a reasonable difference(s) accurately described?				
Was evidence shown for the difference(s)?				

Lesson 1.3 C & D Question Scoring Guide

Place a 2 (yes), 0 (no), or a 1 (half credit) in each block	Team 1	Team 2	Team 3	Team 4
Question C				
Was a reasonable similarity(s) accurately described?				
Was evidence shown for the similarity(s) in the table and graph?				
Was a reasonable difference(s) accurately described?				
Was evidence shown for the difference(s) in the table and graph?				
Question D				
Was a reasonable similarity(s) accurately described?				
Was evidence shown for the similarity(s) in the table and graph?				
Was a reasonable difference(s) accurately described?				
Was evidence shown for the difference(s) in the table and graph?				

Presentation Individual Scoring Rubric

CATEGORY	4	3	2	1
Speaks Clearly	Speaks clearly and distinctly all (100-95%) the time, and mispronounces no words.	Speaks clearly and distinctly all (100-95%) the time, but mispronounces one word.	Speaks clearly and distinctly most (94-85%) of the time. Mispronounces no more than one word.	Often mumbles or can not be understood OR mispronounces more than one word.
Content	Shows a full understanding of the topic.	Shows a good understanding of the topic.	Shows a good understanding of parts of the topic.	Does not seem to understand the topic very well.
Volume	Volume is loud enough to be heard by all audience members throughout the presentation.	Volume is loud enough to be heard by all audience members at least 90% of the time.	Volume is loud enough to be heard by all audience members at least 80% of the time.	Volume often too soft to be heard by all audience members.
Posture and Eye Contact	Stands up straight, looks relaxed and confident. Establishes eye contact with everyone in the room	Stands up straight and establishes eye contact with everyone in the room during the presentation.	Sometimes stands up straight and establishes eye contact.	Slouches and/or does not look at people during the presentation.

Formative Assessment Questions

- What do you need to include in the scatterplot?
- How do you choose a scale for your X and Y axes?
- Is there anything missing from your scatterplot?
- What is missing from your scatterplot?
- What is the evidence for your answer in problem __ ?
- How do you make a prediction based on your graph/table?
- When you tested your prediction were you able to prove that it was correct (or reasonable)?
- How far should the bridge overlap the books on each side?
- Does the amount of pennies the bridge holds determine the number of layers?
- Does the number of layers determine how many pennies the bridge holds?
- How many pennies did you need to collapse a bridge with ___ layers?
- How can you predict a breaking weight for a __ inch bridge using your graph/table?
- What are the similarities/differences in the two graphs you created?
- How can you repeat the pattern on graph paper?
- How can you predict the number of steel rods used in a ___ foot truss?
- How can you predict the number of steel rods used in ___ steps?

- What is the pattern of change?
- What is the evidence for the pattern in your table/graph?

Formative Assessments:

- Weekly warm up sheet
- Vocabulary Check
- Reflection Sheet
- Class discussions
- Group discussions
- Students rephrasing instructions in their own words
- Lesson 1.1, 1.2, and 1.3 Questions
    - Graded with the Question Scoring Guides
- Scatterplots
    - Graded by the teacher using the Scatterplot Scoring Guide
- Scatterplot Scoring Guides - graded by other students.
    - Teacher assesses students' ability to assess other students' scatterplots.
- Individual student work during group activities
    - Graded using the Learning Team Individual Work Rubric and Presentation Individual Scoring Rubric.

Summative Assessments:

- Linear and Non-Linear Relationships Quiz: this quiz assesses individual student's abilities to:
    - Recognize a pattern
    - Continue a pattern
    - Record data from the pattern in the table
    - Create a scatterplot from the data
    - Describing the functional relationship by analyzing a graph
    - Identify a function as linear or nonlinear
    - Explain why the functions is linear or nonlinear
    - Identify a positive linear and negative linear relationship

Pre-assessment:

- Unit Readiness Assessment (See Appendix) - this pre-assessment will be given before students begin the unit, and is intended to help teachers know individual students' levels of understanding and readiness on particular topics. These topics include:
    - Locating integers on a number line

- Solving one-step equations
- Unit rate
- Ratio tables
- Modeling a situation with an equation
- Graphing ordered pairs

Student Self-Assessment and Reflection

- Reflection Sheet
- Scatterplot Grading
- Lesson 1.1, 1.2, and 1.3 Question Scoring Guide

Differentiation Opportunities

- **Group Work**
  - **Learning Team Groupings:** The teacher will analyze data from DCAS testing, MAP testing, easy CBM diagnostic tests, and any other data available. 8th grade students will be ordered according to the ability levels shown in these tests. Teams will be created so that each has at least one student from the high, middle, and low range on the ability level chart.
  - **Group Roles:** Different group roles offer an opportunity for differentiation in team activities.

- **Accommodations (IEP)**
  - **Vision challenges** - a student with a vision accommodation can sit closer to the board or the iPad can be placed on his desk while he is writing vocabulary, doing his warm-up, or anything else that requires reading information projected from the iPad on the whiteboard.
  - **Reading material to students** -while circulating the room the teacher will stop at groups with students who have this accommodation. The teacher will encourage other team members to read the materials out loud. Sometimes the teacher will read.
  - **Extra Time:** students with this accommodation will be given extra time to complete assignments or assessments.
  - **AT Support:** Students with this accommodation will be allowed to use iPads or chrome books to complete their written response questions.
  - **Universal Accommodations:** Some universal accommodations that will be provided for all students at Gateway Lab School to accommodate a large population of students with IEP's are:
    - Simplified instructions

        - Repeat instructions back to the instructor
        - Repetition of information when appropriate
        - Cues to stay on task
        - Writing down tasks and assignments in an agenda
        - Positive reinforcement
    - **Modified materials and assessments:** depending on students' ability levels they will receive different modifications. The modifications are not universal for all students with this accommodation. Some examples of these modifications will be.
        - Modified rubric for presentations (being scored as a 1 or 0 rather than 1-4).
        - Modified grading for extended response questions.
        - Modified grading for team activities.

- **Extra Support:** Students who are working below grade level
    - Graphing Practice from Math-Aids.com (see appendix for example): these worksheets include plotting and connecting points in a sequence to create an object. All students are expected to complete at least seven of the sheets throughout the first three units. The sheets are differentiated; some are very simple and others are very complex. For students working below grade level this is a great activity to sharpen their graphing skills.
    - IXL Exercises: IXL.com has a large database of exercises that students can complete to give extra support for identified deficiencies. Some examples that could be used for this unit are:
        - 7^{th} grade level - X.1 Identify proportional relationships
        - 7^{th} grade level - O.1 Interpret tables
        - 6^{th} grade level - P.1 Write variable expressions to represent word problems

- **Extensions:** These activities will be assigned to accelerated students.
    - Student-Led Discussion: Problem 1.3 A #5: students who correctly analyze and explain the components of the mathematical model will be given the opportunity to present and discuss their findings, and help other students to understand the process.
    - Problem 1.3 B: Accelerated students will be given the opportunity to create or analyze the process of creating a mathematical model for the pattern.
    - pp. 25-27 # 34, 35, 36

- Graphing Practice (see appendix for example): Challenging graphs include characters from The Simpsons and seasonal graphs.
- Khan Academy activities
  - Linear function intercepts
  - Recognizing functions
  - Identifying slope of a line

Stage 3– Learning Plan
(Learning activities need to align with Stage 1 and Stage 2 and show connections to prior learning)

Daily or Periodical Lesson Plan Activities

Warm-up Sheet (Used daily in this unit) - Every day when students arrive in class they complete a brief warm-up. They receive a stamp for quietly working on the warm-up in three minutes or less. At the end of the week warm-up sheets are collected and graded. Students are assessed on their ability to be on time, prepared, working quietly, and their ability to complete the warm-up. The purpose of the warm-up is to set a learning tone for the lesson, and to activate prior knowledge which will be needed for reaching the instructional goals. Students are also required to keep this sheet (which has the reflection sheet on the back) in their three-ring notebooks and bring to class every day. This encourages students to keep an organized notebook, and gives the teacher an assessment of students' organizational abilities.

Reflection Sheet (Used daily in this unit) - At the end of the class students take a few minutes to complete the reflection sheet. They answer two questions: What new concepts did you learn or practice today? How was your understanding? While completing their reflection the teacher stamps the reflection sheet to show that the students complete this at the end of class and not sometime during class. The purpose of this reflection sheet is to give students a chance to briefly recap what they have learned and to assess their own learning. The teacher collects the sheet at the end of the week and assesses students' abilities to summarize their learning and evaluate their learning ability.

Vocabulary (Used every time there is new vocabulary) - New vocabulary and definitions are projected on the white board. A student pulls Popsicle sticks to randomly determine which students will read the vocabulary words. Students read the words and definitions aloud then write them on their vocabulary pages. The teacher then checks that individual students have their vocabulary list and also that they write the new vocabulary. The purpose of this is to pre-

teach vocabulary needed to reach the learning goals, and to serve as a quick notebook check to assess students' organizational abilities. Students need to have their notebooks and need to be able to quickly locate their vocabulary sheets.

Philosophical Chair (used periodically for planned and unplanned discussions) - This is used when students are in a disagreement about any topic. Students typically form three groups around the room: *for* group, *against* group, and *don't know* group. A student in one of the groups briefly presents his/her argument. Students at any time can change groups if they feel that a student's argument is convincing. After the first student presents, a student in another group first summarizes the proceedings student's argument then makes his/her own argument. This continues until a consensus or an impasse is reached (as determined by the teacher).

Problem 1.1 – Bridge Thickness and Strength

Information

CCSS Math Content Standards: 8.F.A.3 , 8.F.B.5 , 8.SP.A.1 , 8.SP.A.2

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7

Essential Question: How can you describe the relationship between load weight and bridge strength?

Real-World Connections: Architecture, specifically bridges and bridge strength.

Objectives:

1. Determine the beam strength of bridges of different thicknesses.
2. Collect and display data in a table and as a graph.
3. Interpret data and use to make predictions.
4. Test predictions.

Vocabulary: Independent Variable, Dependent Variable

Day 1 – Learning Activities

Warm Up: Scatter Plots - reviews basic concepts (activate prior knowledge) – specifically x and y scale.

Launch:

- **Bridge Video** - Students watch a 1 minute animated video of objects being left on a bridge which eventually collapses.
- **Essential Question / Objectives** - students volunteer / or are chosen randomly to read the essential question and objectives aloud.
- **Vocabulary** (Independent Variable, Dependent Variable): Students write vocabulary (as described above). Teacher tells students to write down some situations that could involve independent/dependent variables after writing the vocabulary.
- **Open Discussion** (independent/dependent variables): the teacher leads this discussion and gives examples that are real to students. Students are encouraged to come up with their own relationships. The teacher provides as few examples as is necessary to get students thinking. Teacher elicits responses from students.
- **Demonstration:** Students gather around the teacher in a circle while he performs the bridge penny experiment in problem 1.1. Throughout the demonstration the teacher mentions independent and dependent variables and elicits ideas from students of what is the independent variable - penny weight or bridge thickness.
- **Philosophical Chair** (planned): students will present their viewpoints of whether penny weight or bridge thickness is the independent variable. Students may also have a viewpoint that either one could be the independent variable or that the answer cannot be determined. If all students agree then the activity ends. If students don't agree then the activity ends and they will not be given the answer by the teacher. They will be encouraged to continue considering the variables during the investigation. (This is a planned activity because identifying the independent variable may be difficult without some thought and discussion.)

Movement Break (Dance with light/urgent music): – Students will pair up and portray bridges by extending their arms and touching the tips of fingers. Other students will move from bridge to bridge pushing down and collapsing the bridges.

Problem 1.1 (pp. 8,9 [see appendix]): Students conduct the experiment and record data in tables. This work is done in learning teams of four members.

- **Introduction:** The teacher or student reads the instructions aloud. The teacher has some students rephrase instructions in their own words to check for understanding.
- **Team Roles:** Students write down which role they will fill in their team. The four roles include: paper manager, bridge checker, penny placer, and data recorder. They quickly discuss exactly what they need to do in these roles.
- **Materials:** Students gather the materials and conduct the experiments. Paper in this problem is pre-cut for students with lines marking folds.

- **Assessment**: The teacher circulates the room during the investigation and assesses students:
    - Team Individual Work Rubric (see appendix): the teacher will use this rubric to assess individual student performance during group work.
    - Formative Assessment Questions
        - How far should the bridge overlap the books on each side?
        - Does the amount of pennies the bridge holds determine the number of layers?
        - Does the number of layers determine how many pennies the bridge holds?
        - How many pennies did you need to collapse a bridge with ___ layers?

Reflection: students complete their daily reflection on their warm-up/reflection sheet.

Day 2 – Learning Activities

Warm Up: Scatter Plots – Continue to review basic concept. During this warm-up students will be given the Scatterplot Scoring Guide (shown above) to reference when creating their scatter plots.

Vocabulary (linear, non-linear, positive relationship, negative relationship): Students write vocabulary (as described above). Vocabulary will be briefly discussed. Students will be told that their descriptions of mathematical relationships need to include two parts: mathematics vocabulary, and a description of the real-world relationship.

Problem 1.1 (continued): Students will complete the problem in learning teams. Today they will use their data from day 1 to create scatter plots. They will then answer questions about the problem by analyzing the tables and graphs. The teacher will emphasize the concept of citing evidence from the tables and graphs while explaining their answers. Students will then present their posters and discuss the questions with the class.

- **Introduction:** Instructions for the remainder of the problem will be read aloud in class by students chosen randomly by pulling Popsicle sticks.
- **Discussion:** The teacher leads a discussion on independent/dependent variables in relation to this problem. The issue of which variable is independent (weight or bridge thickness) must be resolved before students begin graphing the data.
- **Scoring:** The teacher reminds students to use their self-scoring guides while creating scatter plots and to provide evidence for their answers after analyzing the graphs and tables.

- **Instructions:** The teacher tells students they will be presenting their posters and that each student in the group will have to discuss one of the questions in problem 1.1 (give students a couple of minutes to discuss this and decide). The teacher also informs students that they will need to refer to their graphs and tables to show evidence for their conclusions.
- **Assessment:** The teacher circulates the room during the investigation and assesses students:
    - Learning Team Individual Work Rubric (see appendix): the teacher will use this rubric to assess individual student performance during group work.
    - Formative Assessment Questions
        - Scatterplot
            - What do you need to include in the scatterplot?
            - How do you choose a scale for your X and Y axes?
        - Problem Questions
            - What is the evidence for your answer in problem __?
            - How do you make a prediction based on your graph/table?
            - When you tested your prediction were you able to prove that it was correct (or reasonable)?

Movement Break (Basic Functions Aerobics [do at an opportune time during problem 1.1]): Four basic functions ($y = x$, $y = -x$, $y = x^2$, $y = -x^2$) are projected onto the whiteboard. These functions have been pre-taught in a prior lesson. The teacher shows how to represent them using arm movement. An example for y = x is that the teacher holds out his/her arms straight to the side and then tilts at a 45° angle.

- The students stand up.
- The teacher says and poses a function.
- The Students copy the movements of the teacher.
- This is repeated a number of times until the students become comfortable.
- Possible alterations
    - A student leads
    - The teacher poses a function; students shout out the function, and then pose the function.
    - Add different y-intercepts. Example: y = x – 1.

Pre-Presentation: Before the presentation the teacher gives students:

- Lesson 1.1 Question Scoring Guide (see above). Teams are instructed to look through the scoring guide and revise their answers making sure that they have included all the information. When complete they should have a "2" in every column for their team.
- Presentation Individual Scoring Rubric (see above). The teacher reviews the scoring rubric which will help students know what is expected of them individually during the presentation.

Reflection: Students complete their daily reflection on their warm-up/reflection sheet.

Homework: Applications p.16 #2 a, b, c

Day 3 – Learning Activities

Warm Up: Students are given two situations and asked to identify the independent and dependent variables.

Scatterplot Assessment: Students use the Scatterplot Scoring Guide to assess other students' scatterplots completed in the homework assignment p.16 #2 a, b, c.

- The teacher collects the homework.
- The teacher redistributes the homework to students so that each student has someone else's homework.
    - For students who didn't do the homework extra examples are available.
- The teacher passes out the Scatterplot Scoring Guide and students staple it to the homework being graded.
- The teacher projects one of the student's scatterplots on the whiteboard from the Elmo.
- The teacher leads the class in scoring their scatterplots.
- Students return the assignments to other students and they check to see if they made any mistakes.
- The teacher later checks the students' scatterplots and also assesses the students ability to assess other students.

Homework Question Discussion: The class briefly discusses the homework questions in a teacher led activity. The main concepts discussed are making predictions and describing relationships. The overlying understanding is to present evidence for the predictions and descriptions.

Movement Break Chair Dancing.

- Dance music is played.
- The teacher sits in an area where all students can see him/her.

- The teacher begins to move his upper body with the music.
- The seated students follow.
- The teacher changes the movements every four measures or so.
- The students continue to follow the teacher.
- If another student wants to, he/she can take the lead.

Presentation: Students present their posters and discuss the answers to questions. Presenting this material should solidify some major concepts: Describing a functional relationship by analyzing a graph, identify a positive linear relationships, and citing evidence from mathematical models to back up predictions and descriptions.

- Students present, while the non-presenting students use the Lesson 1.1 Question Scoring Guide to critique the other teams.
- The teacher uses the Lesson 1.1 Question Scoring Guide to score the student presentations on content.
- The teacher also uses the Presentation Individual Scoring Rubric to grade individual students on their presentations.

Reflection: Students complete their daily reflection on their warm-up/reflection sheet.

Problem 1.2 – Bridge Length and Strength

Information

CCSS Math Content Standards: 8.F.A.3 , 8.F.B.5 , 8.SP.A.1

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Essential Question: How can you describe the relationship between bridge length and bridge strength?

Real-World Connections: Architecture, specifically bridges and bridge strength.

Objectives:

1. Determine the beam strength of bridges of different lengths.
2. Collect and display data in a table and as a graph.
3. Interpret data and use to make predictions.
4. Test predictions.

5. Compare data on bridge length/strength and bridge thickness/strength to find similarities and differences

Day 4 – Learning Activities

Warm Up: How did you make a prediction in the previous investigation? Students write the answer to the question and discuss as a class.

Problem 1.2 (pp. 10,11 [see appendix]): Students conduct the experiment and record data in tables. This work is done in learning teams of four members.

- **Introduction:** As a whole class students read problem 1.2. Students choose other students to read at random by pulling Popsicle sticks. Teacher has some students rephrase instructions as a check for understanding.
- **Discussion:** As a whole class the independent/dependent variables are discussed for the bridge length and bridge strength experiment.
- **Philosophical Chair** (used if students don't agree on the independent/dependent variables): Students will present their viewpoints of whether penny weight or bridge length is the independent variable. Other students will analyze the presented viewpoints and then make decisions to maintain their original views or change views. The teacher will not give the students the answer. The teacher will tell students to continue to consider the independent/dependent variables while performing the experiment.
- **Team Roles**: Students again write down and discuss which role they will fill in their team: paper manager, bridge checker, penny placer, and data recorder.
- **Materials:** The paper managers gather the materials on the list that the teacher projects on the whiteboard: six sheets of paper, one ruler, and one pair of scissors.
- **Cutting the paper:** Students will use the same paper as the first problem, but this time they will be required to cut the bridges to different lengths. The paper needs to be cut in the shape of a rectangle before folded. Students often attempt to create a rectangle by measuring one side (and not the other) then drawing a line by sight to the other side rather than connecting two measured points. This can end up being closer to a trapezoid than a rectangle. This brief activity serves the purpose of helping students understand how to cut rectangles by properly using tools. This emphasizes the standard CCSS.Math.Practice.MP5 (Use appropriate tools strategically). This is an important skill for students to have in the future if they do any type of wood work or home improvement projects.

- **Present Task:** The teacher presents the tasks to the student of cutting the bridges to different lengths. They are cutting rectangles down to shorter rectangles.
- **Instructions** (on white board):
  - Cut the rectangles down to the indicated lengths.
  - Do not fold the paper to measure.
  - Use a ruler, pencil, and scissors only.
- **Think – Group – Share**:
  - Think: Each student holds a piece of paper. Students consider the two following questions (projected on the whiteboard):
    - What are the characteristics of a rectangle?
    - How can you use your ruler to create a small rectangle?
  - Group: Students discuss in their learning teams how to design a method for cutting.
  - Cut: Teams cut one paper to a shorter rectangle.
  - Share: A representative from each team comes up to the Elmo and demonstrates how to cut the paper, and states the evidence for the finished product being a proper rectangle.
  - Assessment/Critique: Other teams evaluate the students methods of cutting the paper and share their opinions on why the methods are successful or unsuccessful.
  - The teacher intervenes only when students are in danger of fully accepting incorrect concepts.

Movement Break (Basic Functions Aerobics): Teacher encourages other students to lead the activity this time.

Investigation Continued:

- **Begin Investigation** Students begin the investigation conducting experiments and recording data.
- **Assessment:** The teacher circulates the room during the investigation and assesses students:
  - Learning Team Individual Work Rubric (see appendix): the teacher again uses this rubric to assess individual student performance during group work.
  - Formative Assessment Questions
    - How far should the bridge overlap the books on each side?
    - What is the independent variable?
    - Does the data show a linear or nonlinear pattern?

- How many pennies did you need to collapse a __ inch bridge?

Reflection: Students complete their daily reflection on their warm-up/reflection sheet.

Day 5 – Learning Activities

Warm Up: Students graph a linear and quadratic function on their TI-83 Plus (or TI-84) calculator then use the table feature to fill in a table for each function. Activates prior knowledge about the functions and calculator use and prepares students for activities that use the TI-83 Plus in the next unit. Students may also begin to make connections between the functions and the graphs they created in the previous lessons.

Problem 1.2 (continued): Students will create scatterplots for their data and answer the reflection questions in learning teams. They will use their data from day 4 to create scatter plots. They will then answer questions about the problem by analyzing the tables and graphs. In addition they will also compare/contrast the new data with the data from the previous problem to synthesize a deeper understanding. The teacher will again emphasize citing evidence from the tables and graphs while explaining their answers. Students will present their posters and discuss the questions with the class.

- **Introduction:** Instructions for the remainder of the problem will be read aloud in class by students chosen randomly by a pulling Popsicle sticks.
- **Discussion:** The teacher leads a discussion on independent/dependent variables in relation to this problem. The issue of which variable is independent (weight or bridge length) must be resolved before students begin graphing the data.
- **Scoring:** The teacher reminds students to again use their self-scoring guides while creating scatter plots and to provide evidence for their answers. Students are given the Lesson 1.2 Question Scoring Guide for self-assessment when answering the questions.
- **Instructions:** The teacher tells students they will be presenting both posters and that each student in the group will have to discuss one of the questions in problem 1.2. The teacher reminds students to refer to their graphs and tables to show evidence for their conclusions.
- **Assessment:** The teacher circulates the room during the investigation and assesses students:
    - Learning Team Individual Work Rubric (see appendix): the teacher will use this rubric to assess individual student performance during group work.
    - Formative Assessment Questions
        - Scatterplot
            - What do you need to include in the scatterplot?
            - How do you choose a scale for your X and Y axes?

- Problem Questions
    - What is the evidence for your answer in problem __?
    - How can you predict a breaking weight for a __ inch bridge using your graph/table?
    - What are the similarities/differences in the two graphs you created?

Movement Break (Mirror Dancing [do at an opportune time during problem 1.2]): Music is played and students pair up and face each other. One student leads and the other follows in each pair. The leader chooses a slow movement and the other student follows. Half way through the activity the students switch roles.

Presentation: Students present their posters and discussed the answers to questions.

- Students present and the non-presenting students use the Lesson 1.2 Question Scoring Guide to critique the other teams.
- The teacher uses the Lesson 1.2 Question Scoring Guide to score the student presentations on content.
- The teacher also uses the Presentation Individual Scoring Rubric to grade individual students on their presentations.

Reflection / exit ticket: Students write down things they understand well and things they don't understand well in this unit so far.

Problem 1.3 – Custom Construction Parts

Information

CCSS Math Content Standards: 8.F.A.3, 8.F.B.5, 8.SP.A.1, 8.SP.A.2

Standards for Mathematical Practice: MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8

Essential Question: How can you predict if a pattern between variables will be linear or nonlinear?

Real-World Connections: Architecture, specifically trusses.

Objectives:

1. Investigate linear and nonlinear sequential patterns.
2. Represent patterns in tables and graphs.
3. Make predictions based on tables and graphs.

4. Show evidence for predictions.
5. Analyze and explain the components of a mathematical model used to describe a real-world pattern.
6. Synthesize information from investigation 1.1 and 1.2 with investigation 1.3

Day 6 – Learning Activities

Warm Up: From memory, students write down the things that they need to include when constructing a scatterplot.

Reflection / exit ticket (from the previous day): The teacher leads a discussion based on the previous day's exit ticket to address any misconceptions or clarify any information about the unit thus far.

Launch:

- **Truss Video:** Students watch a 1 minute animated video that shows a variety of trusses which are structures to support buildings. This video connects prior knowledge of trusses from structures students have seen in photos or real life like the Eiffel Tower to the activities in problem 1.3.
- **Discussion:** the teacher leads a discussion on trusses eliciting ideas from students and real-world connections that they have experienced. Emphasis is given to the concept of repeating geometric patterns.
- **Introduction:** As a whole class students read problem 1.3 introduction and 1.3 A activity. Students choose other students to read at random by pulling Popsicle sticks. Teacher has some students rephrase instructions as a check for understanding.

Problem 1.3 A (p. 10 [see appendix]):

- **Instruction:** The teacher instructs students to work in teams. This time they are expected to create a scatterplot individually on Lab Sheet 1.3-A (see appendix) (not on poster paper as in 1.1 and 1.2). Students are also expected to write down the answers to questions on the back of their sheets individually to be graded by the teacher.
- **Materials:** students will be given the Lab Sheet 1.3-A for this exercise which includes a pre-drawn table and graphing area, as well as space for answering questions. Students will also be given graph paper and will be encouraged to draw continuations of the pattern when needed.
- **Scoring:** The teacher gives students the Scatterplots Scoring Guide to use individually while creating their scatterplots. The teacher will also use this to score students scatterplots.

- **Student-Let Discussion:** A new concept is added in this exploration. Problem 1.3 question five asks students to analyze a mathematical model, and explain the components of the equation and how they were derived from the physical situation. Accelerated students will most likely achieve this before other students, and will be given the opportunity to present their findings to the class, lead a discussion, and help other students to understand the process.
- **Extension Activity** (for all students): The students will input the equation for the pattern into their TI-83/84 calculators and create graphs and check the table. The challenge in this activity is for students to create proper scales for the graphs on the "window" screen. This will be teacher-led and student-led (if possible) with the calculator projected on the white board.
- **Assessment:** The teacher circulates the room during the investigation and assesses students:
    - Class Dojo (see below): while circulating around the room the teacher will carry an iPad and make notes on student performance using the class dojo app. Students will be able to view a constantly updating score of their performance during this activity.
    - Formative Assessment Questions
        - Scatterplot
            - Is there anything missing from your scatterplot?
            - What is missing from your scatterplot?
        - Problem Questions
            - How can you repeat the pattern on graph paper?
            - How can you predict the number of steel rods used in a ___ foot truss?
            - What is the pattern of change?
            - What is the evidence for the pattern in your table/graph?

Movement Break (Dance with light/flowing music [do at an opportune time during problem 1.3A]): Students will team up and portray the repeated pattern found in 1.3 A through movement and creation of geometric figures with arms or legs. Students first briefly discuss how to do this in the teams then they perform their dances. If possible the entire class creates a long truss through dance.

Reflection: Students complete their daily reflection on their warm-up/reflection sheet.

Homework (p. 17 #4 a,b,c): Students will be given graph paper and a copy of page 17 to take home and complete for homework. This assignment is a scaled-down (easier) version of what they will complete tomorrow in class.

Day 7 – Learning Activities

Warm Up: Students take out their homework assignment and note any problems or successes that they had. The homework will then be briefly discussed and collected by the teacher.

Problem 1.3 B:

- **Instruction:** The teacher instructs students to work in teams. They are again expected to create a scatterplot and answer questions individually on their lab sheet to be turned in at the end of the day.
- **Materials:** Students will be given the 1.3 B lab sheet and graph paper to draw continuations of the pattern when needed.
- **Scoring:** Students again reference the Scatterplot Scoring Guide. The teacher uses this to score students scatterplots.
- **Extension:** Accelerated students will be given the opportunity to create a mathematical model for the pattern. This equation is quadratic and students may not have had experience creating this type of model before. If the accelerated students cannot create this they will be given the instructions (see appendix) and will attempt to make sense of the creation of the mathematical model. This will be assessed by the students explaining the model to the teacher.
- **Assessment:** The teacher circulates the room during the investigation and assesses students:
    - Formative Assessment Questions
        - Scatterplot
            - Is there anything missing from your scatterplot?
            - What is missing from your scatterplot?
        - Problem Questions
            - How can you repeat the pattern on graph paper?
            - How can you predict the number of steel rods used in ___ steps?
            - What is the pattern of change?
            - What is the evidence for the pattern in your table/graph?

Movement Break (Mirror Dancing [do after 1.3 B])

Problem 1.3 C, D: the teacher reads the questions from problem 1.3 C and D and gives students the 1.3 C &D Question Scoring Guide. Students are instructed to work in teams to answer these questions which they will need to present the following day. Everyone in the team is to have a role during the presentation. Students are instructed to cite evidence where applicable. These questions synthesize all the information learned in this unit. Students are expected to synthesize these concepts and to present a clear understanding.

Reflection: Students complete their daily reflection on their warm-up/reflection sheet.

Homework (Construction Crane Data Activity [P. 15 #1 a,b,c,d]: As described above in the "Suggested Unit Transfer Task(s)" section.

Day 8 – Learning Activities

Warm Up: What are some questions or uncertainties that you have about the unit? The teacher will address these questions briefly and will emphasize the proper techniques or concepts while going over the previous night's homework assignment.

Homework: The homework will then be briefly discussed as a class and collected by the teacher.

Presentation: Students briefly discuss the answers to problem 1.3 C and D questions.

- Students present, and the non-presenting students use the Presentation Individual Scoring Rubric to critique individual students' presentation performances.
- The teacher uses the Lesson 1.3 C &D Question Scoring Guide to score the student presentations on content.
- The teacher also uses the Presentation Individual Scoring Rubric to grade individual students on their presentations.

Quiz: The students will take the Linear and Non-Linear Relationships Quiz as a summative assessment for their understanding of this unit.

Resources and Teaching Tips

CMP3

- Student text "Thinking with Mathematical Models"
- Teachers Guide "Thinking with Mathematical Models"

- Teacher Resources book
- A Guide to Connected Mathematics 3

On-line

- http://www.ixl.com/
- https://www.khanacademy.org
- https://www.classdojo.com/
- http://www.easycbm.com/
- https://www.nwea.org/assessments/map/
- http://education.ti.com/en/us/home
- http://mypearsontraining.com/tutorials/cmp3_mathxlforschool/index.html
- http://mathdashboard.com/cmp3
- http://www.corestandards.org/other-resources/key-shifts-in-mathematics/
- http://www.corestandards.org/Math/Content/8/introduction/
- http://www.corestandards.org/Math/Practice/
- http://www.math-aids.com/Graphing/

21st Century Learning - http://www.p21.org/

The 21st century learning design principle is embedded throughout this unit. The 21st century learning and innovation skills are emphasized to prepare students for complex life and work environments in the 21st century. These skills include creativity and Innovation, critical thinking and problem solving, and communication and collaboration.

Technology Integration

The ability to responsibly use appropriate technology to communicate, solve problems, and access, manage, integrate, evaluate, and create information

TI-83 Plus/84 Calculator: This will be used at various times throughout the unit to familiarize students with graphing functions and viewing tables.

Khan Academy: This is an online program that has exercises for many different math topics. In this class it will be used mostly for the accelerated students as extension exercises. Students will have the opportunity to learn new concepts that are above and beyond yet related to what they are doing in class.

IXL: This is an online program that has a number of topics where students can answer questions and receive instructions on wrong answers. Students do each exercise until they have mastered the exercise. Students will be given a list of IXL exercises that they need to complete online before the end of the unit. Students can do IXL when they complete other in-class activities early, at home for homework, or any other time they have access to a computer.

Assignments (8th grade level):

- W.1 Identify linear and nonlinear functions - this assignment reviews some prior knowledge the students have as well as teaches them some new concepts which are mirrored in this unit's investigations.
- N.13 Scatter plots - this assignment emphasizes the concepts of positive and negative linear associations and no association. It also allows students to make predictions according to the association.
- V.3 Complete a function table - this assignment reviews some prior knowledge that students have and also teaches students some new concepts of functions. This will be especially important in the next unit.

Class Dojo: This is an app that the teacher uses on an iPad while circulating around the room to make notes on student performance. The app is projected on the white board and students will be able to view a constantly updating score of their performance. Parents can also login at any time to view their student's performance (in real time). Student performance is judged by the pushing of the following buttons that the teacher uses to score the students while circulating the room:

- helping others
- on task
- participating
- persistence
- teamwork
- working hard
- disrespect
- no teamwork
- off task
- talking out of turn

- out of seat/away from team
- unprepared

Connections to Other Areas

Suggestions for integrating instruction with other curricular areas, school support services (health services, counseling, nutrition services, and school climate) families and communities.

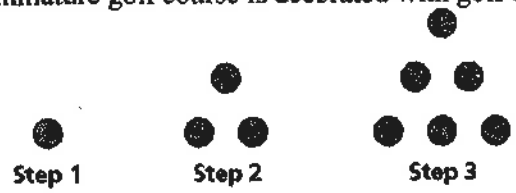
This unit has a strong connection with engineering and physics. The topics include bridge strength and trusses used in architecture. These topics create an understanding in students that math is not an isolated discipline, but integrated into real-world events. This unit also has a strong connection with ELA. Students formalize their opinions and conjectures into writing and then provide written evidence for these opinions. Students also write daily reflections in a log about what they learned that day and current ability level.

Name: _____ Class: _____ Date: _____ ID: A

Linear and Non-Linear Relationships Quiz

Short Answer

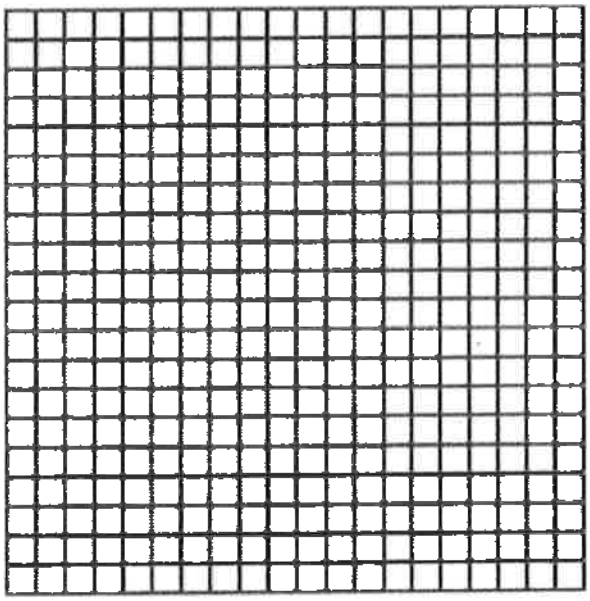
1. The party room at the miniature golf course is decorated with golf ball patterns such as the one below.



a. Complete the table to show the number of golf balls in the next 5 steps of the pattern.

Pattern Step Number	1	2	3	4	5	6	7	8
Number of Balls Used	1	3	6					

b. Create a graph of the step number/balls used data. (below or on graph paper if you prefer ask me)



c. Describe the pattern of change shown in the table and graph.

d. How many balls would be used for pattern step number 11. Show work or explain.

Name: _____ ID: A

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Which is true for a positive linear relationship?

A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

____ 2. Which is true for a negative linear relationship?

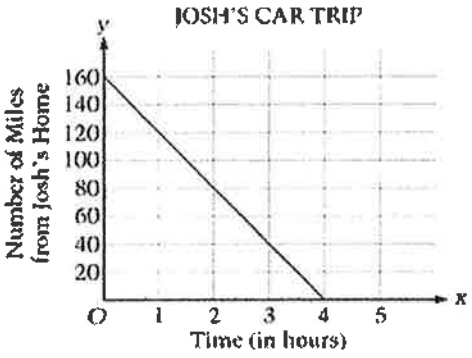
A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

Short answer (Refer to the graphs below to answer the questions)

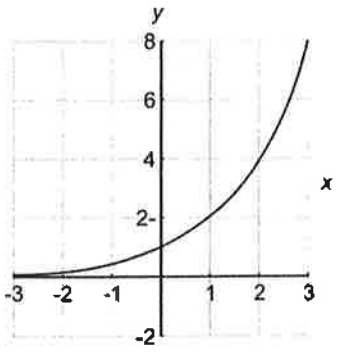
1. Which graphs show linear relationships? Explain how you know.

2. Which graphs show non-linear relationships? Explain how you know.

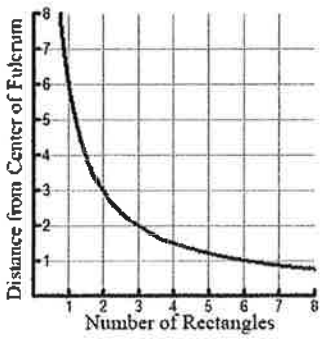
A.



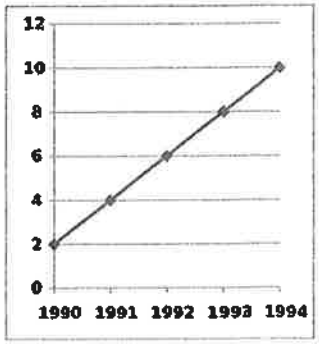
B.



C.



D.



Name: _____ ID: A

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Which is true for a positive linear relationship?

A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

____ 2. Which is true for a negative linear relationship?

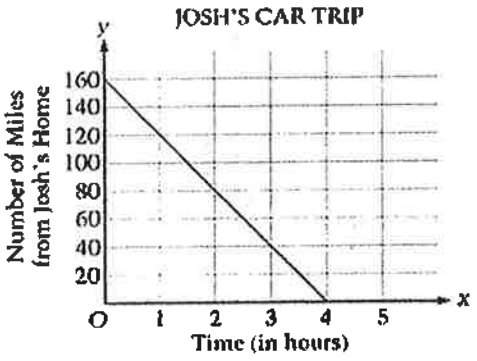
A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

Short answer (Refer to the graphs below to answer the questions)

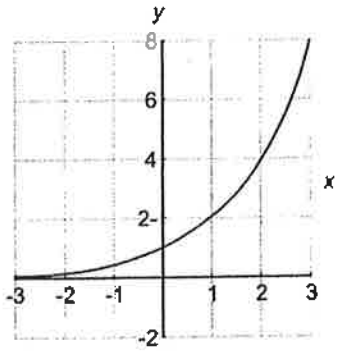
1. Which graphs show linear relationships? Explain how you know.

2. Which graphs show non-linear relationships? Explain how you know.

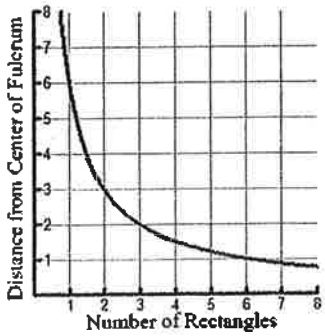
A.



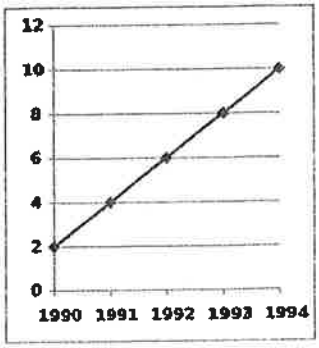
B.



C.



D.



Name: Key Class: _____ Date: _____ ID: A

Linear and Non-Linear Relationships Quiz

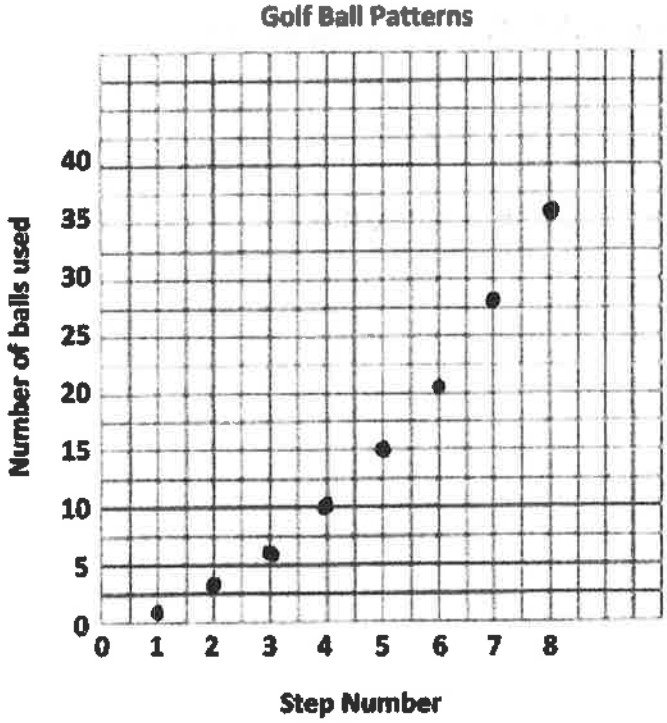
Short Answer

1. The party room at the miniature golf course is decorated with golf ball patterns such as the one below.

a. Complete the table to show the number of golf balls in the next 5 steps of the pattern.

Pattern Step Number	1	2	3	4	5	6	7	8
Number of Balls Used	1	3	6	**10**	**15**	**21**	**28**	**36**

b. Create a graph of the step number-balls used data. (below or on graph paper if you prefer ask me)



c. Describe the pattern of change shown in the table and graph.

The table shows that for every additional pattern step the number of balls added increases by one ball. The graph shows a non-linear increase.

d. How many balls would be used for pattern step number 11. Show work or explain.

Step 8: 36 balls

Step 9: 36 + 9 = 45 balls

Step 10: 45 + 10 = 55 balls

Step 11: 55 + 11 = 66 balls

Name: _____ ID: A

Multiple Choice
Identify the choice that best completes the statement or answers the question.

C 1. Which is true for a positive linear relationship?

A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

A 2. Which is true for a negative linear relationship?

A. x increases as y decreases
B. x and y decrease
C. x increases as y increases
D. x and y are flat

Short answer (Refer to the graphs below to answer the questions)

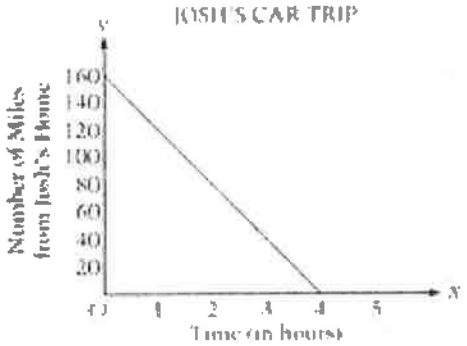
1. Which graphs show linear relationships? Explain how you know.

A and D. Both graphs show a straight line.

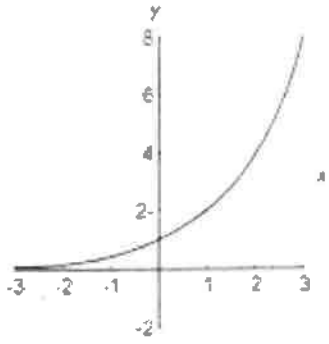
2. Which graphs show non-linear relationships? Explain how you know.

B and C. Both graphs show a curve.

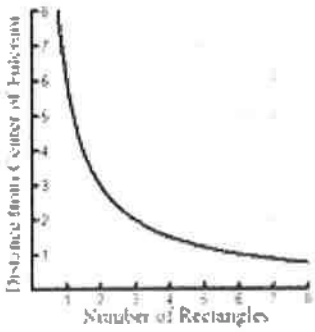
A.



B.



C.



D.

Name: _____ ID: A

Multiple Choice
Identify the choice that best completes the statement or answers the question.

C 1. Which is true for a positive linear relationship?

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B. x and y decrease
C. x increases as y increases
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Short answer (Refer to the graphs below to answer the questions)

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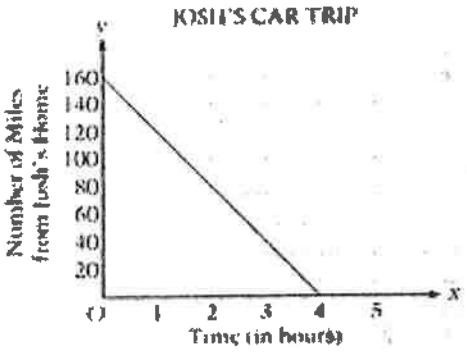
A and D. Both graphs show a straight line.

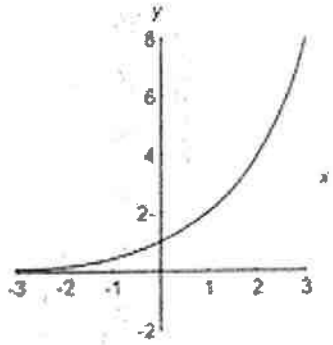
2. Which graphs show non-linear relationships? Explain how you know.

B and C. Both graphs show a curve.

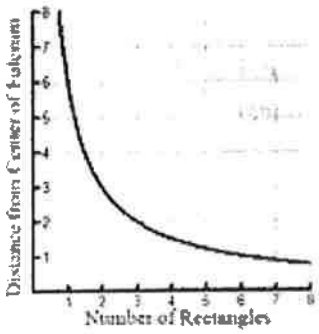
A.



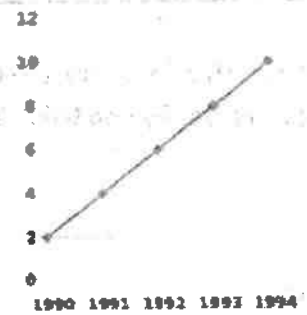
B.



C.



D.



8A Learning Team 1

Jerry		Focus on the Task and Participation	Listening, Questioning and Discussing	Attitude
A	Great!	Consistently stays focused on task and what needs to be done. Self-directed. Includes all team members.	Respectfully listens, interacts, discusses and poses questions to all team members during discussions.	Always has a positive attitude about the task(s) and the work of others.
B	Good	Focuses on the task and what needs to be done most of the time.	Respectfully listens, interacts, discusses and poses questions most of the time.	Usually has a positive attitude about the task(s) and the work of others.
C	OK	Focuses on the task and what needs to be done some of the time.	Has difficulty listening and discussing, tends to dominate discussions or rarely participates.	Occasionally is negative and critical of the task(s) or the work of other group members. Or rarely participates.
F	Bu Hao	Rarely focuses on the task and what needs to be done. Lets others do the work.	Has great difficulty listening, argues with teammates, or doesn't participate.	Is often negative and critical of the task(s) or the work of other team members. Or doesn't participate.

John		Focus on the Task and Participation	Listening, Questioning and Discussing	Attitude
A	Great!	Consistently stays focused on task and what needs to be done. Self-directed. Includes all team members.	Respectfully listens, interacts, discusses and poses questions to all team members during discussions.	Always has a positive attitude about the task(s) and the work of others.
B	Good	Focuses on the task and what needs to be done most of the time.	Respectfully listens, interacts, discusses and poses questions most of the time.	Usually has a positive attitude about the task(s) and the work of others.
C	OK	Focuses on the task and what needs to be done some of the time.	Has difficulty listening and discussing, tends to dominate discussions or rarely participates.	Occasionally is negative and critical of the task(s) or the work of other group members. Or rarely participates.
F	Bu Hao	Rarely focuses on the task and what needs to be done. Lets others do the work.	Has great difficulty listening, argues with teammates, or doesn't participate.	Is often negative and critical of the task(s) or the work of other team members. Or doesn't participate.

Anne		Focus on the Task and Participation	Listening, Questioning and Discussing	Attitude
A	Great!	Consistently stays focused on task and what needs to be done. Self-directed. Includes all team members.	Respectfully listens, interacts, discusses and poses questions to all team members during discussions.	Always has a positive attitude about the task(s) and the work of others.
B	Good	Focuses on the task and what needs to be done most of the time.	Respectfully listens, interacts, discusses and poses questions most of the time.	Usually has a positive attitude about the task(s) and the work of others.
C	OK	Focuses on the task and what needs to be done some of the time.	Has difficulty listening and discussing, tends to dominate discussions or rarely participates.	Occasionally is negative and critical of the task(s) or the work of other group members. Or rarely participates.
F	Bu Hao	Rarely focuses on the task and what needs to be done. Lets others do the work.	Has great difficulty listening, argues with teammates, or doesn't participate.	Is often negative and critical of the task(s) or the work of other team members. Or doesn't participate.

Cedric		Focus on the Task and Participation	Listening, Questioning and Discussing	Attitude
A	Great!	Consistently stays focused on task and what needs to be done. Self-directed. Includes all team members.	Respectfully listens, interacts, discusses and poses questions to all team members during discussions.	Always has a positive attitude about the task(s) and the work of others.
B	Good	Focuses on the task and what needs to be done most of the time.	Respectfully listens, interacts, discusses and poses questions most of the time.	Occasionally is negative and critical of the task(s) or the work of other group members. Or rarely participates.
C	OK	Focuses on the task and what needs to be done some of the time.	Has difficulty listening and discussing, tends to dominate discussions or rarely participates.	Occasionally is critical of the task(s) or the work of other group members. Or rarely participates.
F	Bu Hao	Rarely focuses on the task and what needs to be done. Lets others do the work.	Has great difficulty listening, argues with teammates, or doesn't participate.	Is often negative and critical of the task(s) or the work of other team members. Or doesn't participate.

Investigation 1

Exploring Data Patterns

People in many professions use data and mathematical reasoning to solve problems and make decisions. For example, engineers analyze data from lab tests to determine how much weight a bridge can hold. Market researchers use survey data to predict demand for new products. Stockbrokers use formulas to forecast growth of investments over time.

In several previous *Connected Mathematics* units, you used tables, graphs, and equations to explore and describe relationships between variables. In this Investigation, you will develop your skill in using these tools to organize data from experiments, find patterns, and make predictions.

1.1 Bridge Thickness and Strength

Many bridges are built with frames of steel beams. Steel is very strong, but any beam will bend or break if you put too much weight on it.

Common Core State Standards

8.A.F.3 Interpret the equation $y = mx + b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

8.B.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear) . . .

Also 8.A.F.2, 8.A.SP.1, A-CED.A.1, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-BF.A.1, F-BF.A.1a, F-BF.A.1b

- How do you think the strength of a beam is related to its thickness?
- What other variables might affect the strength of a bridge?

Problem 1.1

Engineers often use scale models to test their designs. You can do your own experiments to discover mathematical patterns involved in building bridges.

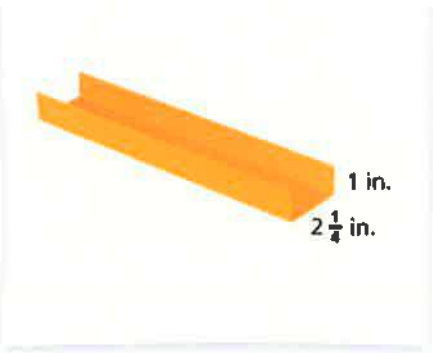
Instructions for a Bridge-Thickness Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- Several 11 inch-by-$4\frac{1}{4}$ inch strips of paper

Instructions:

- Start with one of the paper strips. Make a "bridge" by folding up 1 inch on each long side.



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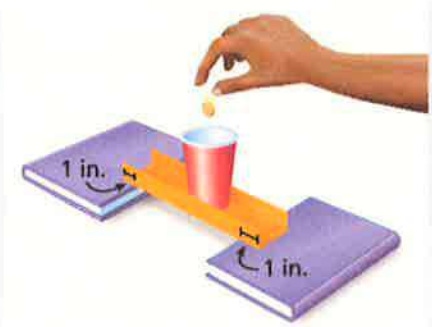
Problem 1.1 continued

- Suspend the bridge between the books. The bridge should overlap each book by 1 inch. Place the cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. This number is the *breaking weight* of the bridge.



- Put two *new* strips of paper together to make a bridge with twice as many layers. Find the breaking weight for this bridge.
- Repeat this experiment to find the breaking weights of bridges made from three, four, and five strips of paper.

A Make a table and a graph of your (bridge layers, breaking weight) data.

B Does the relationship between the number of layers and the breaking weight seem to be linear or nonlinear? How do the graph and the table show this relationship?

C Suppose you could split layers of paper in half. What breaking weight would you predict for a bridge 2.5 layers thick? Explain.

D Predict the breaking weight for a bridge 6 layers thick. Explain your reasoning.

E Test your prediction of strength for the 6-layer bridge. Explain why results from such a test might not exactly match predictions.

ACE Homework starts on page 15.

1.2 Bridge Length and Strength

In the last problem you tested the strength of some paper bridges. You found that bridges with more layers are stronger than bridges with fewer layers.

- How do you think the length and strength of a bridge are related?
- Are longer bridges stronger or weaker than shorter bridges?

Problem 1.2

You can do an experiment to find out how the length and strength of a bridge are related.

Instructions for a Bridge-Length Experiment

Materials:

- Two books of the same thickness
- A small paper cup
- About 50 pennies
- $4\frac{1}{4}$-inch-wide paper strips with lengths 4, 6, 8, 9, and 11 inches

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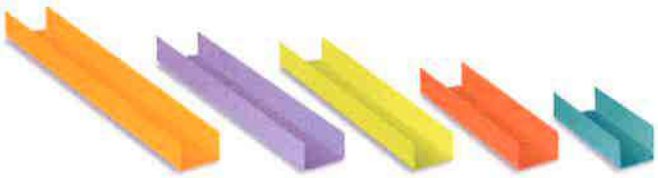
Problem 1.2 continued

Instructions:

- Fold the paper strips to make bridges as shown below.



- Start with the 4-inch bridge. Suspend the bridge between the two books as you did before. The bridge should overlap each book by 1 inch. Place the paper cup in the center of the bridge.
- Put pennies into the cup, one at a time, until the bridge collapses. Record the number of pennies you added to the cup. As in the first experiment, this number is the breaking weight of the bridge.
- Repeat the experiment to find breaking weights for the other bridges.

A Make a graph of your data.

B Describe the relationship between bridge length and breaking weight. How is that relationship shown by patterns in your table and graph?

C Use your data to predict the breaking weights for bridges of lengths 3, 5, 10, and 12 inches. Explain how you made your predictions.

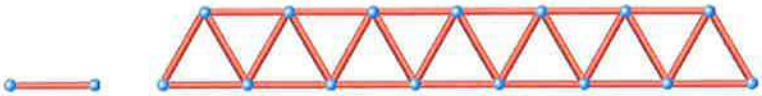
D Compare your data from this experiment to the data from the experiment on bridges with different numbers of layers. How is the relationship between the number of layers in a bridge and its breaking weight similar to the relationship between bridge length and breaking weight? How is it different?

ACE Homework starts on page 15.

1.3 Custom Construction Parts

Finding Patterns

Suppose a company called Custom Steel Products (CSP for short) supplies materials to builders. One common structure that CSP makes is called a *truss*, as shown in the figure below. (You might see a truss holding up the roof of a building.) It is made by fastening together steel rods 1 foot long.



1-foot steel rod

7-foot truss made from 27 rods

This truss has an overall length of 7 feet. The manager at CSP needs to know the number of rods in any length of truss a customer might order.

Problem 1.3

Study the drawing above to see if you can figure out what the manager needs to know. It might help to work out several cases and look for a pattern.

A Copy and complete the table below to show the number of rods in trusses of different overall lengths.

Length of Truss (ft)	2	3	4	5	6	7	8
Number of Rods	7	11				27	

1. Make a graph of the data in your table.
2. Describe the pattern of change in the number of rods used as the truss length increases.

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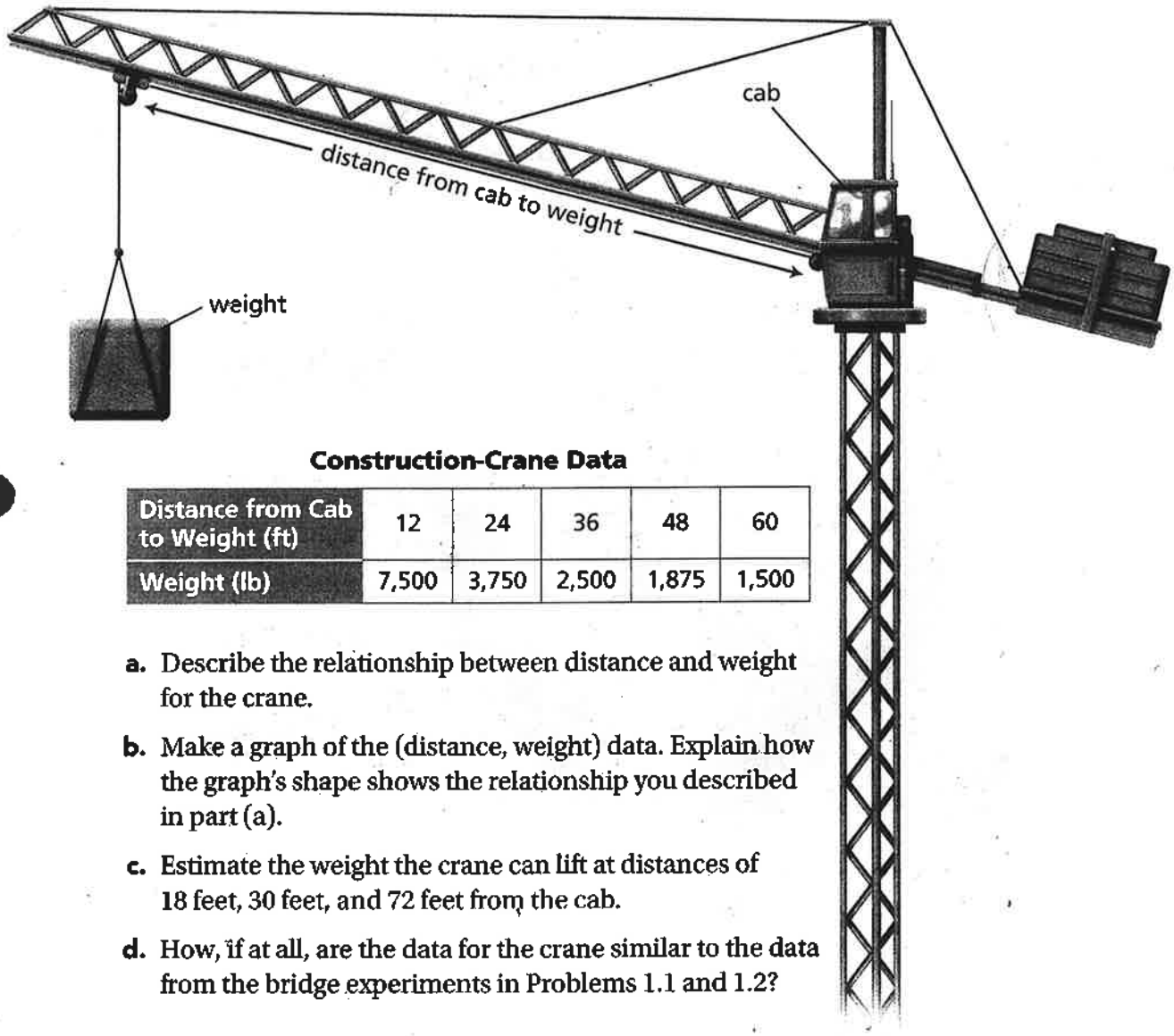
Applications

1. The table shows the maximum weight a crane arm can lift at various distances from its cab.



Construction-Crane Data

Distance from Cab to Weight (ft)	12	24	36	48	60
Weight (lb)	7,500	3,750	2,500	1,875	1,500

a. Describe the relationship between distance and weight for the crane.

b. Make a graph of the (distance, weight) data. Explain how the graph's shape shows the relationship you described in part (a).

c. Estimate the weight the crane can lift at distances of 18 feet, 30 feet, and 72 feet from the cab.

d. How, if at all, are the data for the crane similar to the data from the bridge experiments in Problems 1.1 and 1.2?

2. A group of students conducted the bridge-thickness experiment with construction paper. The table below contains their results.

Bridge-Thickness Experiment

Number of Layers	1	2	3	4	5	6
Breaking Weight (pennies)	12	20	29	42	52	61

a. Make a graph of the (number of layers, breaking weight) data. Describe the relationship between breaking weight and number of layers.

b. Suppose it is possible to use half-layers of construction paper. What breaking weight would you predict for a bridge 3.5 layers thick? Explain.

c. Predict the breaking weight for a construction-paper bridge of 8 layers. Explain how you made your prediction.

4. During the medal ceremonies at a track meet, the top athletes stand on platforms made from stacked wooden boxes. The number of boxes depends on the number of medal winners.

1 medalist
1 box

2 medalists
3 boxes

3 medalists
6 boxes

a. Copy and complete the table below.

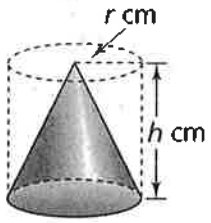
Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	■	■	■	■	■

b. Make a graph of the (number of medalists, number of boxes) data.

c. Describe the pattern of change shown in the table and graph.

33. The volume of the cone in the drawing below is $\frac{1}{3}(28\pi)$ cm^3. Recall that the formula for the volume of a cone is $\frac{1}{3}\pi r^2 h$. What are some possible values of radius and height for the cone?



Extensions

34. Study the patterns in this table. Note that the numbers in the x column may not be consecutive after $x = 6$.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	■	■	■	■
■	■	■	1,024	■
■	■	■	2,048	■
■	■	1,728	■	■
n	■	■	■	■

a. Use the patterns in the first several rows to find the missing values.

b. Are any of the patterns linear? Explain.

35. The table below gives data for a group of middle school students.

Data for Some Middle School Students

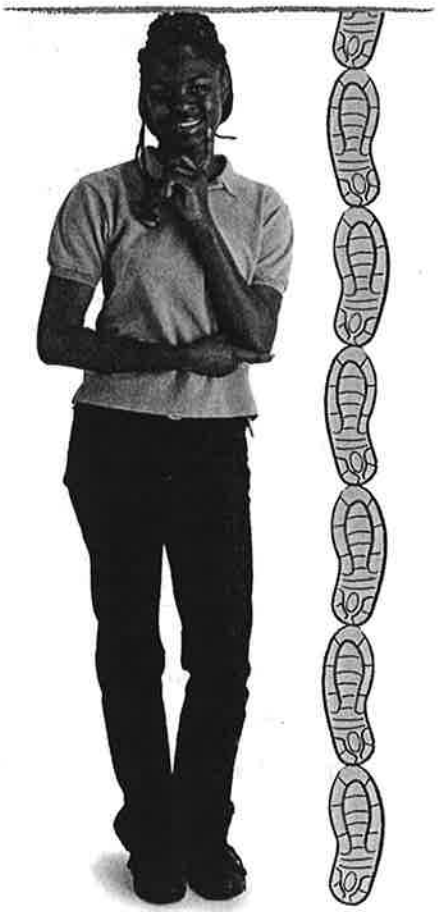
Student	Name Length	Height (cm)	Foot Length (cm)
Thomas Petes	11	126	23
Michelle Hughes	14	117	21
Shoshana White	13	112	17
Deborah Locke	12	127	21
Tonya Stewart	12	172	32
Richard Mudd	11	135	22
Tony Tung	8	130	20
Janice Vick	10	134	21
Bobby King	9	156	29
Kathleen Boylan	14	164	28

a. Make graphs of the (name length, height) data, the (name length, foot length) data, and the (height, foot length) data.

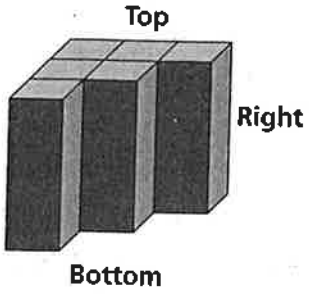
b. Look at the graphs you made in part (a). Which seem to show linear relationships? Explain.

c. Estimate the average height-to-foot-length ratio. How many foot-lengths tall is the typical student in the table?

d. Which student has the greatest height-to-foot-length ratio? Which student has the least height-to-foot-length ratio?



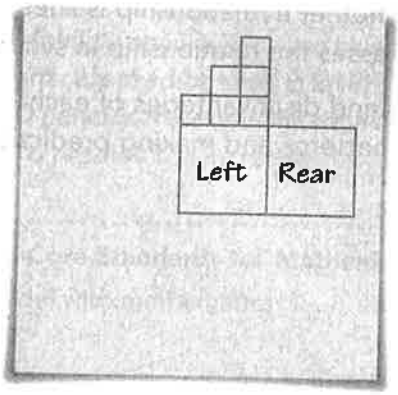
36. A staircase is a type of prism. This is easier to see if the staircase is viewed from a different perspective. In the prism shown here, each of the small squares on the top has an area of 1 square unit.



a. Sketch the base of the prism. What is the area of the base?

b. Rashid tries to draw a flat pattern that will fold up to form the staircase prism. Below is the start of his drawing. Finish Rashid's drawing and give the surface area of the entire staircase.

Hint: You may want to draw your pattern on grid paper and then cut it out and fold it to check.

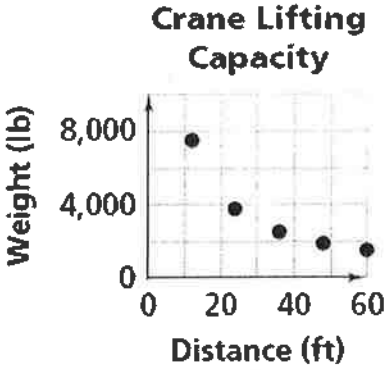


c. Suppose the prism has six stairs instead of three. Assume each stair is the same width as those in the prism above. Is the surface area of this six-stair prism twice that of the three-stair prism? Explain.

P. 15 #1 Answer Key

1. **a.** As distance increases, weight decreases. The decrease is sharper at shorter distances. (The product of distance and weight is always 90,000.)

   b. The graph shows that as distance increases, weight decreases—sharply at first, and then more gradually.

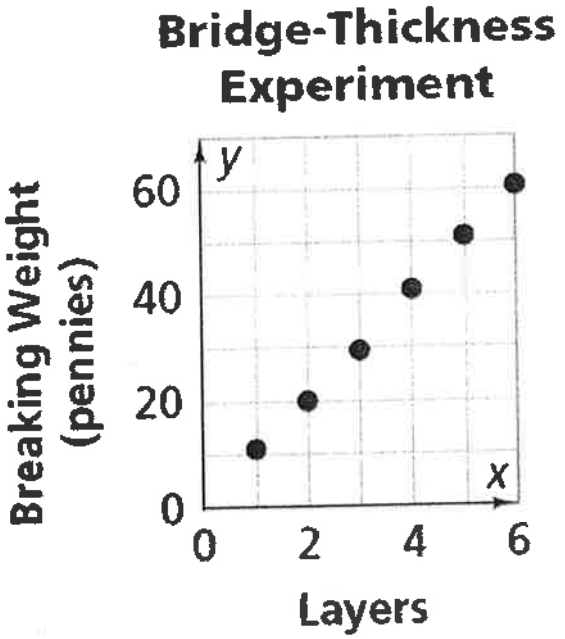


   c. 5,000 lb; ≈ 3,000 lb; ≈ 1,250 lb

   d. The graph's shape is similar to that of the bridge-length experiment because the values of the dependent variable decrease at a decreasing rate.

P. 16 #2 Answer Key

a.



The data are very close to linear. Each time the class adds two layers, the bridge can hold approximately 15 more pennies.

b. 28 pennies. The breaking weight is about 8 pennies per layer. So, for 3.5 layers, the breaking weight would be 28.

c. 80. The breaking weight is about 8 pennies per layer. So, for 10 layers, the breaking weight would be 80.

P. 17 #4 Answer Key

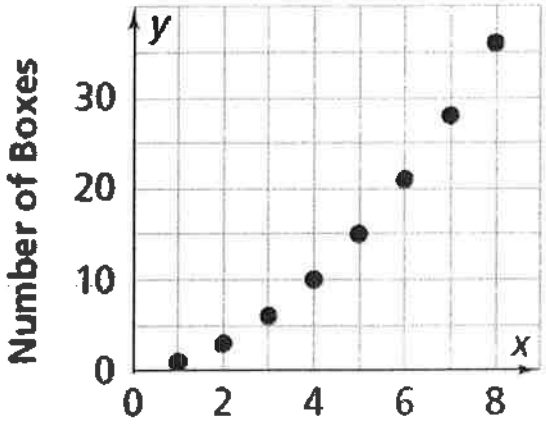
a.

Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	10	15	21	28	36

b.

Medal Platforms



c. This is not a linear relationship. In the table, when you add the second medal winner, you add 2 boxes. When you add a third medal winner, you add 3 more boxes. To add a 29th medal winner, you add 29 boxes to a 28-step platform. The change is increasing at each step. You see this in the graph because the graph rises more and more sharply as you move from left to right along the *x*-axis.

Extensions

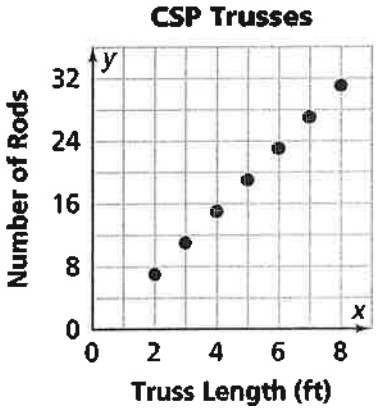
34. a.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	36	216	64	$\frac{1}{6}$
10	100	1,000	1,024	$\frac{1}{10}$
11	121	1,331	2,048	$\frac{1}{11}$
12	144	1,728	4,096	$\frac{1}{12}$
n	n^2	n^3	2^n	$\frac{1}{n}$

b. None of the patterns are linear because a constant change in x does not yield a constant change in y.

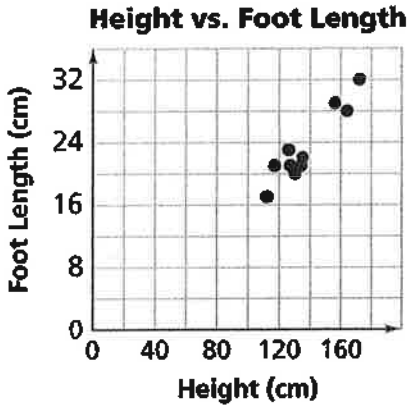
35. Modeling data patterns.

a. The three scatter plots will look like this:



CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	28	40	54	70	88



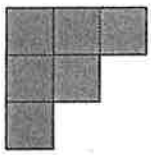
b. Only the (*height, foot length*) graph looks linear.

c. Approximately 6 : 1; The average student is 6 "feet" tall.

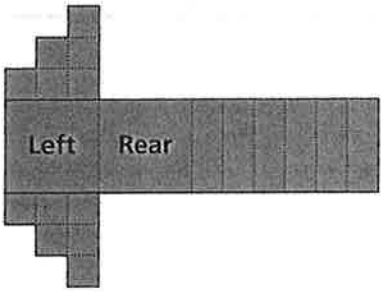
d. Shoshana White; Tonya Stewart

36. Staircase as prism.

a. Orientation of base will vary, but here is one possible sketch from an overhead perspective; area is 6 units^2. This "2" should be superscripted.



b. One possible sketch would be as follows:



Surface Area = top + bottom + left + rear + step + step + step

Surface Area = [6 + 6 + 9 + 9 + (3 + 3) + (3 + 3) + (3 + 3)] units^2. This "2" should be superscripted.

Surface Area = 48 units^2. This "2" should be superscripted.

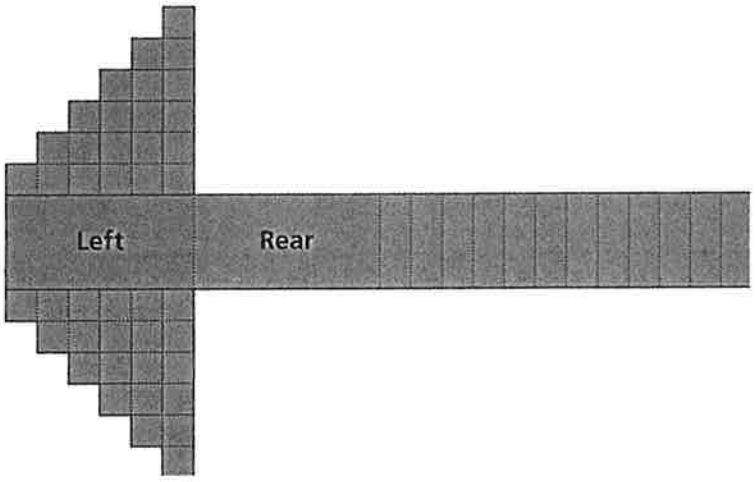
c. New Surface Area = top + bottom + left + rear + step + step + step + step + step + step

New Surface Area = [21 + 21 + 18 + 18 + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3)] units^2. This "2" should be superscripted.

New Surface Area = 114 units^2. This "2" should be superscripted. The top and bottom areas more than doubled. The left and rear areas exactly doubled (but they are no longer squares). The "stair" area doubles. So the total area is more than twice the original. A flat pattern is shown below.

(See Figure 7.)

Figure 7



Investigation 1.3 Equation Model - For Accelerated students

It is not essential that students find an equation for this relationship; in this case the equation is quadratic, and without prior experience with quadratic equations students would not know to look for this kind of relationship.

Students who are having difficulty seeing the pattern may find it useful to expand the calculations as shown below. You can find the number of rods for n steps by adding to the number of rods for $n - 1$ steps.

1 step: 4

2 steps: $4 + (2 \times 3)$

3 steps: $4 + (2 \times 3) + (2 \times 3) - (1 \times 2)$

4 steps: $4 + (2 \times 3) + (2 \times 3) + (1 \times 2) + (2 \cdot 3) - (2 \cdot 2)$

5 steps: $4 + (2 \times 3) + (2 \times 3) + (1 \times 2) + (2 \cdot 3) + (2 \cdot 2) + (2 \times 3) + (3 \cdot 2)$

6 steps: $4 + (2 \times 3) + (2 \times 3) + (1 \times 2) + (2 \cdot 3) + (2 \cdot 2) + (2 \times 3) + (3 \times 2) + (2 \times 3) + (4 \times 2)$

The number of rods for n steps is number for $n - 1$ steps, plus $(2 \times 3) + ((n - 2) \times 2)$.

Some students may see the pattern this way

1 step: $4 = 4$

2 steps: $4 + 6 = 10$

3 steps: $4 + 6 + 8 = 18$

4 steps: $4 + 6 + 8 + 10 = 28$

5 steps: $4 + 6 + 8 + 10 + 12 = 40$

For n steps, the number of rods is the sum of n consecutive even numbers, starting with 4.

Some students might see that $4 = 1 \times 4$, $10 = 2 \times 5$, $18 = 3 \cdot 6$, and so on. So the number of rods is the number of steps times 3 more than the number of steps. This pattern leads to the equation $R = n(n + 3)$. If students see this pattern, challenge them to write the equation in an equivalent form: $R = n^2 + 3n$.

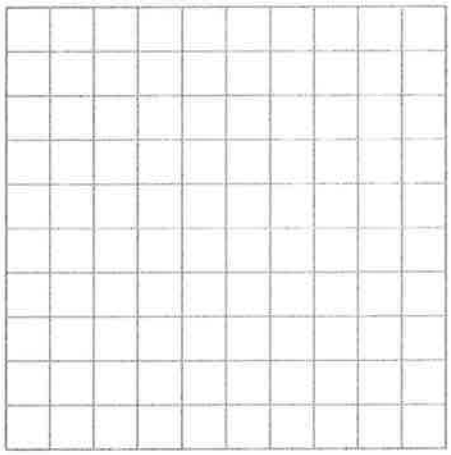
There are many, many ways to see patterns in this problem. Make sure students understand that the relationship is not linear.

Note: The number of rods in an n-step frame is $n^2 + 3n$. Don't expect students to derive this expression, only to see that the relationship is nonlinear.

Name ..... Date ..... Class .....

Labsheet 1.3A Truss

Length of Truss	2	3	4	5	6	7	8
Number of Rods							

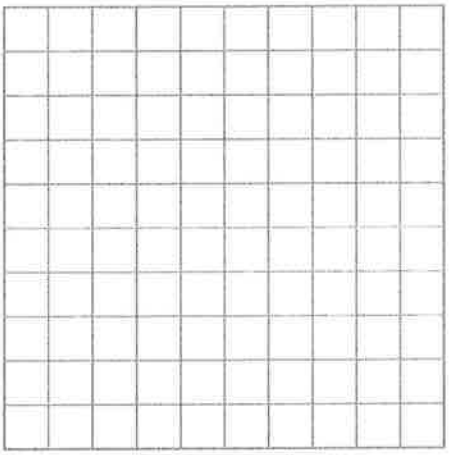


Name ..... Date ..... Class .....

Labsheet 1.3B Steps

CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods								



Name ..... Date ..... Class .....

Unit Readiness Assessment

1. Graph each number on the same number line.

a. 3

b. -1

c. $-(-4)$

d. -6

e. $-(-6)$

2. About $\frac{15}{16}$ of a certain planet's surface is covered with either water or desert. About $\frac{5}{16}$ is covered with water. Solve the equation $d + \frac{5}{16} = \frac{15}{16}$ to find d, the part of the planet's surface covered with desert.

Unit Readiness Assessment (continued)

3. On a certain planet, objects weight about $\frac{7}{12}$ of what they weigh on Earth. An object weighs $13\frac{5}{12}$ pounds on the planet. Solve the equation $\frac{7}{12}w = 13\frac{5}{12}$ for w to find the object's weight on Earth in pounds.

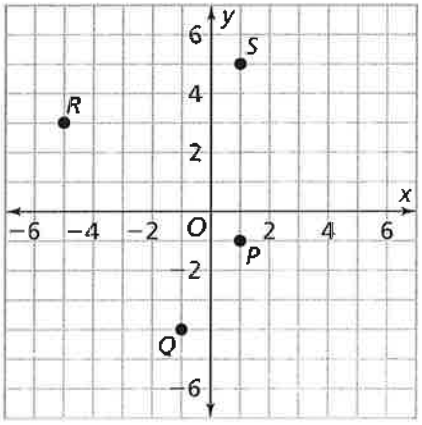
4. In a week, 13 hens laid 65 eggs. What is the unit rate for eggs per hen?

Unit Readiness Assessment (continued)

5. Complete the ratio table. Then graph each x-y pair.

x	y
3	6
6	
9	

6. Name the coordinates of each point on the graph.



Unit Readiness Assessment (continued)

7. You received $112 in all selling paintings. You sold one painting for $27 and the rest for $17 each. Solve the equation below to find x, the number of $17 paintings you sold.

$17x + 27 = 112$

8. A survey of 100 people found that 32% prefer to vacation in the summer rather than the winter. If 25 people were surveyed and the same percentage prefer to vacation in the summer, how many people prefer to vacation in the summer? What if 225 people are surveyed? Complete the table to find 32% of 25 and 32% of 225.

Vacation Preference

People Who Prefer Summer	Total People
	25
32	100
	225

Unit Readiness Assessment (continued)

9. Tim picked up 243 pecans from under one tree and 145 pecans from another tree. Which equation could Tim use to find p, the number of pecans he picked up in all?

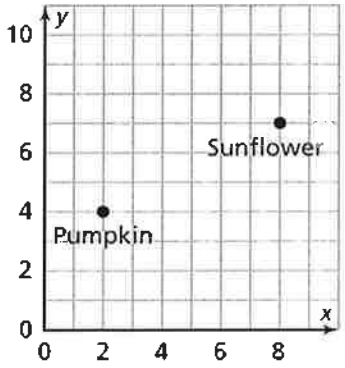
A. $p = 243 + 145$

B. $p = 243 - 145$

C. $p = 243 \times 145$

D. $p = 243 \div 145$

10. Lily made this graph to show the locations of the largest sunflower and the largest pumpkin n her garden. Which ordered pair names the location of the pumpkin?



A. (4, 2)

B. (7 8)

C. (2, 4)

D. (8, 7)

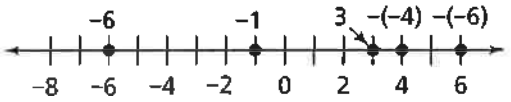
Name ..... Date ..... Class .....

Unit Readiness Assessment Report

Objective	Test Item(s)	Proficient? (Yes or No)
Locate an integer on a number line.	1	
Solve a one-step equation with fractions by adding or subtracting the same number on each side.	2	
Solve a one-step equation with fractions by multiplying each side by the reciprocal.	3	
Understand rate as a ratio and find a unit rate by dividing whole numbers.	4	
Complete a ratio table and graph the points on a coordinate grid.	5	
Name the integer coordinates of given points.	6, 7	
Use a ratio (rate) table to find a percent of a number.	8	
Write an equation with one operation for a situation.	9	
Graph and interpret ordered pairs of whole numbers in the coordinate plane (situational).	10	

Comments

Unit Readiness Assessment Answer Key

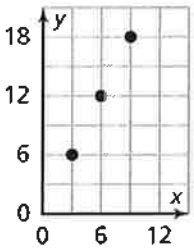
1. 

2. $\frac{5}{8}$

3. 23 pounds

4. 5 eggs per hen

5. 12; 18



6. $P(1, -1)$; $Q(-1, -4)$; $R(-5, 3)$; $S(1, 5)$

7. 5 paintings

8. 8; 72

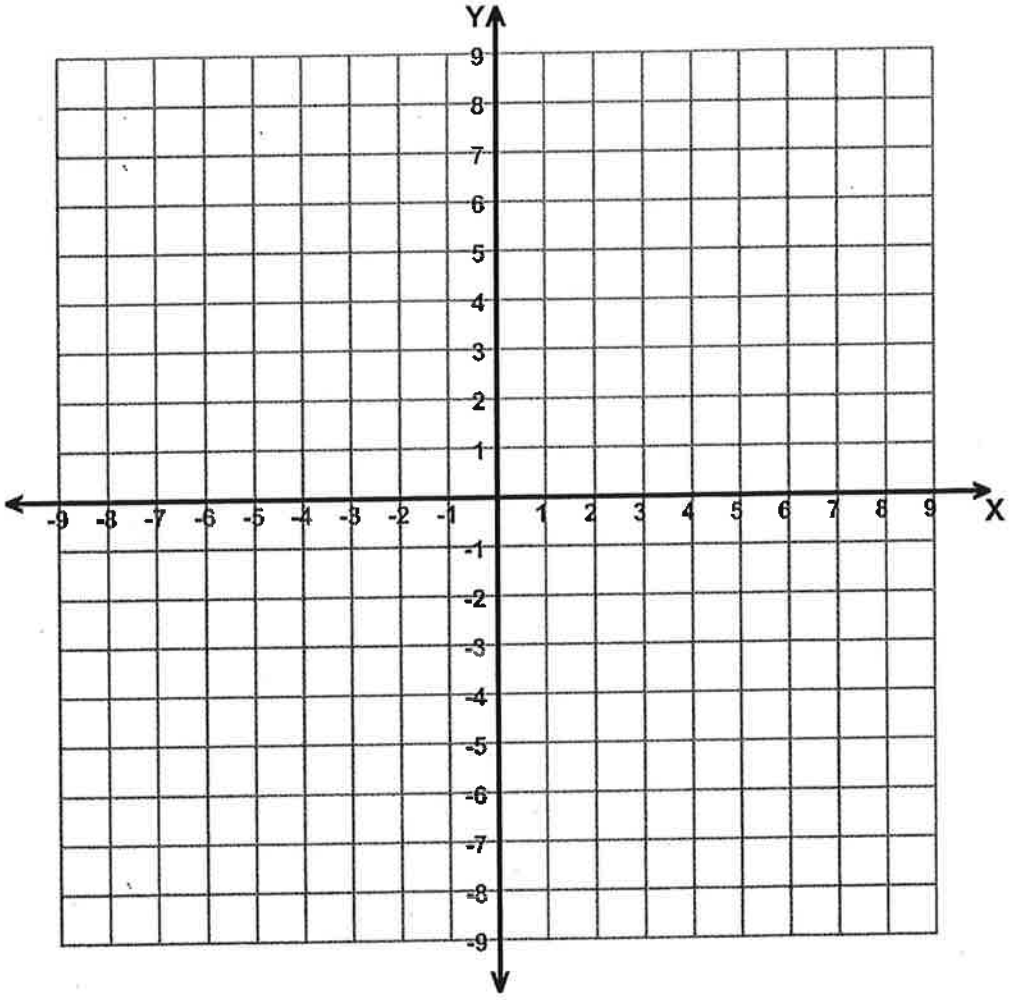
9. A

10. C

Name : _____ Score : _____

Teacher : _____ Date : _____

Four Quadrant Graphing Puzzle



Connect each sequence of points with a line.

(8,7) , (7,3) , (5,1) , (1,-3) , (-1,-5) , (-2.5,-4.5) , (-3.5,-3.5) , (-4,-2) , (-2,0) , (2,4) , (4,6) , (8,7) End of Sequence

(-3.5,-3.5) , (-4,-4) , (-5,-4) , (-5,-5) , (-4,-6) , (-3,-6) , (-3,-5) , (-2.5,-4.5) End of Sequence

(-5,-4) , (-8,-6) , (-6,-6) , (-8,-9) , (-5,-7) , (-5,-9) , (-3,-6) End of Sequence

(4,6) , (5,4) , (7,3) End of Sequence

(2,4) , (3,2) , (5,1) End of Sequence

(0,2) , (1,0) , (3,-1) End of Sequence

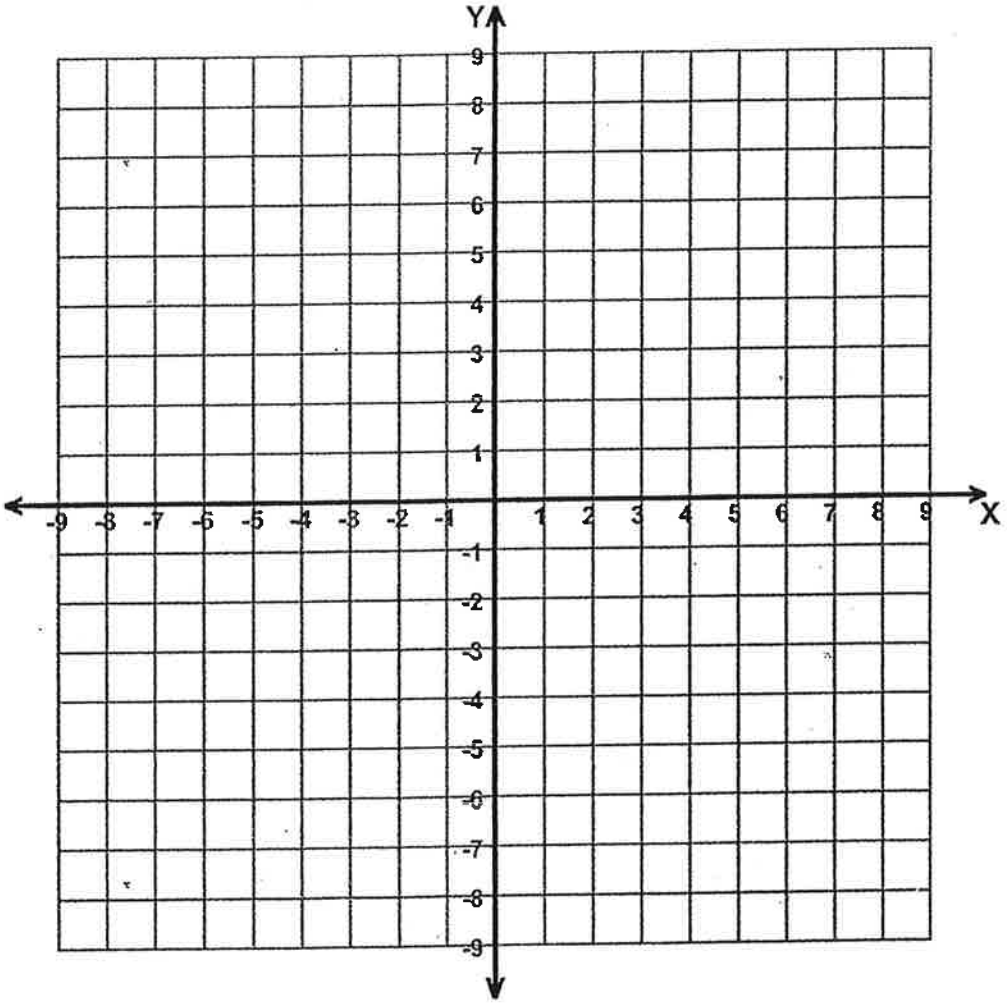
(-2,0) , (-1,-2) , (1,-3) End of Sequence

What is the shape ? _____

Name : _____ Score : _____

Teacher : _____ Date : _____

Four Quadrant Graphing Puzzle



Connect each sequence of points with a line.

(3,7) , (5,7) , (5,3) , (9,3) , (9,1) , (5,1) , (5,-3)

(3,-3) , (3,1) , (-1,1) , (-1,3) , (3,3) , (3,7) End of Sequence

What is the shape ? _____

Name : _____ Score : _____

Teacher : _____ Date : _____

Tweety Bird

For each Shape plot the ordered pairs on the axis and connect them in order.
Do not connect the Shapes to each other.

Shape 1
(1.5,-10) , (1,-10) , (.5,-10) , (0,-10) , (.5,-10) , (.5,-10.5) , (0,-12) , (-.5,-14) , (-2,-15) , (-3,-15.5)
(-5,-16) , (-6,-16) , (-7,-15.5) , (-7.5,-15) , (-7,-14) , (-6,-13.5) , (-5,-13.5)

Shape 2
(-7.5,-15) , (-8,-15) , (-8,-13) , (-7,-12.5) , (-6,-12) , (-4,-12) , (-1,-12) , (-.5,-10.5) , (.5,-10.5)

Shape 3
(1,-10) , (1,-10.5) , (.5,-12.5) , (3,-13.5) , (5,-13.5) , (7,-13) , (8,-12) , (7.5,-11) , (6.5,-10.5) , (5.5,-11)

Shape 4
(8,-12) , (8.5,-11.5) , (8.5,-10.5) , (8,-9.5) , (6,-9.5) , (5,-10) , (3,-11.5) , (1.5,-11.5) , (2,-10.5) , (1,-10.5)

Shape 5
(2,-10.5) , (2.5,-9) , (3,-7) , (3,-6) , (2.5,-4.5) , (1.5,-4) , (2,-3) , (2.5,-2.5) , (1,-2.5) , (0,-2.5)
(1,-4) , (.5,-4) , (-.5,-5.5) , (-.5,-7) , (0,-7.5) , (-.5,-8) , (-1,-8.5) , (-1,-9) , (-.5,-10.5)

Shape 6
(0,-7.5) , (.5,-7) , (.5,-6) , (0,-6.5) , (.5,-6) , (1.5,-5.5)

Shape 7
(-.5,-8) , (-2.5,-7.5) , (-1,-8.5)

Shape 8
(2.5,-2.5) , (3.5,-2.5) , (4.5,-2) , (5,-1) , (4.5,0) , (5,1) , (6,2.5) , (7,4) , (7,6.5) , (6.5,8)
(5.5,9) , (4,10) , (2.5,10.5) , (1.5,10.5) , (0,10.5) , (-1,10.5) , (-2.5,10) , (-4,9) , (-5,8) , (-6,6)
(-6,4) , (-4.5,2) , (-3,0) , (-2.5,.5)

Shape 9
(-3,0) , (-3.5,-1) , (-2.5,-2) , (-1,-2.5) , (0,-2.5)

Shape 10
(-1.5,1) , (-1,1) , (0,1) , (.5,1)

Shape 11
(-1,1) , (-1.5,3) , (-1,5) , (-.5,5.5) , (.5,5.5) , (1,3) , (.5,2) , (0,1)

Shape 12
(-.5,1.5) , (-1,2) , (-1,3) , (-.5,3.5) , (0,3) , (0,2) , (-.5,1.5)

Shape 13
(-1,5) , (-.5,6) , (-1,7.5)

Shape 14
(-.5,5.5) , (0,6) , (.5,7) , (.5,8)

Name : _____ Score : _____

Teacher : _____ Date : _____

Tweety Bird

For each Shape plot the ordered pairs on the axis and connect them in order.
Do not connect the Shapes to each other.

Shape 15
(.5,5) , (1,6) , (1.5,7.5)

Shape 16
(2.5,1) , (3,1) , (4,1) , (4.5,1)

Shape 17
(3,1) , (2.5,3) , (3,4) , (3.5,5) , (4,5) , (4.5,4.5) , (4.5,3.5) , (4.5,2.5) , (4,1)

Shape 18
(3.5,1) , (3,1.5) , (3,2.5) , (3.5,3) , (4,2) , (3.5,1)

Shape 19
(3.5,5) , (3.5,6) , (3,7)

Shape 20
(4,5) , (4,6) , (4,7.5)

Shape 21
(4.5,4.5) , (5,5.5) , (5,6.5)

Shape 22
(2,-.5) , (3,-.5) , (3.5,-.5) , (3,0) , (2,0) , (1.5,-.5) , (2,-1) , (2.5,-1) , (3,-.5)

Shape 23
(0,10.5) , (.5,12.5) , (0,14.5)

Shape 24
(1.5,10.5) , (2,12) , (2.5,13) , (2,15)

Shape 25
(2.5,10.5) , (3.5,11) , (4,12) , (4.5,13) , (5,14)