Pre-Calculus Summer Assignment NAME: ___________________________

DUE 1st DAY OF SCHOOL

Dear Pre-Calculus student to be,

These are the algebraic and algebraic skills needed for success in Pre-Calculus. If you do not remember how to do some of the problems, search the boldfaced topic on the internet and you will find multiple sites that will help refresh your memory. It is suggested that you work on this worksheet one to two weeks prior to the school year beginning in order to refresh your memory and to get you into "math" mode. For the following exercises, **show all work** and **do not use a calculator**. At Winter Springs High School, the work is more important than the answer. All work must be shown for each problem, neatly and in sequential order. All answers must be **exact** and in **simplest form**.

Enjoy your summer and we look forward to helping you learn Pre-Calculus next year,

Helpful websites: http://www.coolmath.com/algebra/Algebra2/ https://www.khanacademy.org/
http://www.webmath.com/ http://www.math.com/

Summer Review Packet for Students Entering Pre-Calculus. All work must be shown. Due first day.

Radicals:

To simplify means that:

1) No radicand has a perfect square factor and
2) There is no radical in the denominator (rationalize)

Recall – the **Product property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4}\sqrt{6}$ $= 2\sqrt{6}$

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$

If the denominator contains 2 terms – multiply the numerator and the denominator by the conjugate of the denominator. The *conjugate* of $3+\sqrt{2}$ is $3-\sqrt{2}$ (the sign changes between the terms)

Simplify each of the following.

1. $\sqrt{32}$
2. $\sqrt{(2x)^8}$
3. $\sqrt[3]{-64}$
4. $\sqrt{49m^2n^5}$

5. $\sqrt{\frac{11}{9}}$
6. $\sqrt{60} \bullet \sqrt{105}$
7. $(\sqrt{5}-\sqrt{6})(\sqrt{5}+\sqrt{2})$

Rationalize.

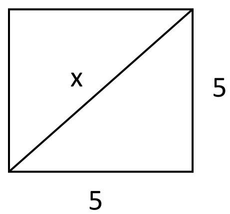
8. $\dfrac{3}{2-\sqrt{5}}$

<u>Geometry:</u>

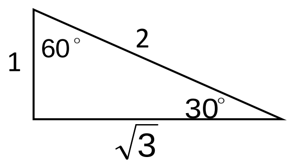
Pythagorean Theorem (right triangles): $a^2+b^2=c^2$

Find the value of x.

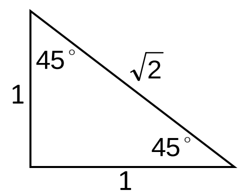
9.



* In 30 – 60 – 90 triangles,
sides are in proportion 1, $\sqrt{3}$, 2.

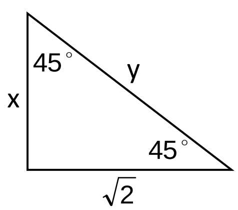


* In 45 – 45 – 90 triangles,
sides are in proportion 1, 1, $\sqrt{2}$



Solve for x and y.

10.



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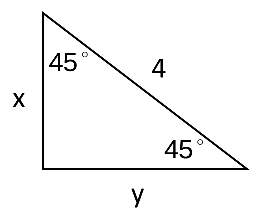

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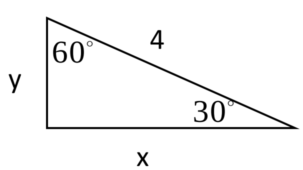

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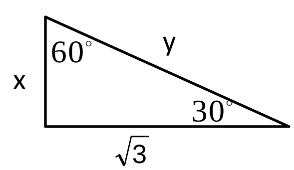

Equations of Lines:

Slope intercept form: y = mx + b	**Vertical line:** x = c (slope is undefined)
Point-slope form: $y - y_1 = m(x - x_1)$	**Horizontal line:** y = c (slope is 0)
Standard Form: Ax + By = C	**Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1}$

14. State the slope and y-intercept of the linear equation: 5x -4y = 8

15. Find the x-intercept and y-intercept of the equation: 2x – y = 5.

Write the equation of the line in the point-slope form with the following conditions:

16. passes through the points (4, 3) and (7, -2)

17. x-intercept (3, 0) and y-intercept (0, 2)

Systems of equations:

$3x + y = 6$
$2x - 2y = 4$

Substitution:
Solve 1 equation for 1 variable.
Rearrange.
Plug into 2nd equation.
Solve for the other variable

Elimination:
Find opposite coefficients for 1 variable.
Multiply equation(s) by constant(s).
Add equations together (lose 1 variable).
Solve for variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$y = 6 - 3x$	solve 1st equation for y
$2x - 2(6-3x) = 4$	plug into 2nd equation
$2x - 12 + 6x = 4$	distribute
$8x = 16$	simplify
$x = 2$	

$6x + 2y = 12$	multiply 1st equation by 2
$2x - 2y = 4$	coefficients of y are opposite
$8x = 16$	add
$x = 2$	simplify

Plug x = 2 back into original:
$3(2) + y = 6$
$6 + y = 6$
$y = 0$

Solve the system of equations by BOTH methods.

18. $\begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \bullet a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\frac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{mn}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \frac{1}{a^n}$ *and* $\frac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

19. $5a^0$ 20. $\frac{3c}{c^{-1}}$ 21. $\frac{2ef^{-1}}{e^{-1}}$ 22. $\frac{(n^3p^{-1})^2}{(np)^{-2}}$

Simplify.

23. $3m^2 \bullet 2m$ 24. $(a^3)^2$ 25. $(-b^3c^4)^5$ 26. $4m(3a^2m)$

Polynomials:

To add / subtract polynomials, combine like terms.		
EX:	$8x - 3y + 6 - (6y + 4x - 9)$	*Distribute the negative through the parentheses*
	$= 8x - 3y + 6 - 6y - 4x + 9$	*Combine terms with similar variables.*
	$= 8x - 4x - 3y - 6y + 6 + 9$	
	$= 4x - 9y + 15$	
To multiply two binomials, use FOIL.		
EX:	$(3x - 2)(x + 4)$	*Multiply the first, outer, inner then last terms.*
	$= 3x^2 + 12x - 2x - 8$	*Combine terms with similar variables.*
	$= 3x^2 + 10x - 8$	

Multiply and simplify.

27. $(3a + 1)(a - 2)$

28. $(c - 5)^2$

29. $(5x + 7y)(5x - 7y)$

Factoring:

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

a) If you have one GCF (other than 1) factor it out front.
b) If you don't have one GCF, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a) Is it difference of two squares?

EX: $x^2 - 25 = (x + 5)(x - 5)$

b) Is it sum or difference of two cubes?

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

EX:
$$m^3 + 64 = (m + 4)(m^2 - 4m + 16)$$
$$p^3 - 125 = (p - 5)(p^2 + 5p + 25)$$

3 Terms

$x^2 + bx + c = (x + \quad)(x + \quad)$

$x^2 - bx + c = (x - \quad)(x - \quad)$

$x^2 + bx - c = (x - \quad)(x + \quad)$

$x^2 - bx - c = (x - \quad)(x + \quad)$

EX:

$x^2 + 7x + 12 = (x + 3)(x + 4)$

$x^2 - 5x + 4 = (x - 1)(x - 4)$

$x^2 + 6x - 16 = (x - 2)(x + 8)$

$x^2 - 2x - 24 = (x - 6)(x + 4)$

4 Terms – Factor by Grouping

a) Pair up first two terms and last two terms
b) Factor out GCF of each pair of numbers.
c) Factor out from the parentheses that the terms have in common.
d) Put leftover terms in parentheses.

Ex:
$$x^3 + 3x^2 + 9x + 27 = (x^3 + 3x^2) + (9x + 27)$$
$$= x^2(x + 3) + 9(x + 3)$$
$$= (x + 3)(x^2 + 9)$$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. if the quadratic does NOT factor, use quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero **FIRST**.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x+3)(x-7) = 0$ *Set each factor equal to zero*

$(x+3) = 0 \quad (x-7) = 0$ *Solve each for x.*

$x = -3 \qquad x = 7$

Solve each equation.

30. $9n^2 = 4$

31. $27z^3 - 8 = 0$

32. $x^2 - 4x = 12$

33. $x^2 + 25 = 10x$

Factor the following:

34. $2mn - 2mt + 2sn - 2st = 0$

Long Division – can be used when dividing any polynomials.
Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

EX: $\dfrac{2x^3+3x^2-6x+10}{x+3}$

Long Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

$$= x+3\overline{)\,2x^3+3x^2-6x+10\,}\quad \text{quotient: } 2x^2-3x+3+\frac{1}{x+3}$$

$(-)\ (2x^3+6x^2)$

$-3x^2-6x$

$(-)\ (-3x^2-9x)$

$3x+10$

$(-)\ (3x+9)$

1

Synthetic Division

$$\frac{2x^3+3x^2-6x+10}{x+3}$$

-3	2	3	-6	10
	↓	-6	9	-9
	2	-3	3	1

$$= 2x-3x+3+\frac{1}{x+3}$$

Divide each polynomial using long division OR synthetic division.

35. $\dfrac{c^3-3c^2+18c-16}{c^2+3c-2}$

36. $\dfrac{x^4-2x^2-x+2}{x+2}$

To evaluate a function for a given value, simply plug the value into the function for x.

Evaluate each function for a given value.

37. $f(x)=x^2-6x+2,\ \ f(3)=$ ______

38. $g(x)=6x-7,\ \ g(x+h)=$ ______

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "f of g of x" "f of g of x" means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x))

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$**,** find f(g(x)).

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Suppose $f(x) = 2x,\ g(x) = 3x - 2,\ and\ \ h(x) = x^2 - 4$. **Find the following:**

39. $f[g(2)] =$

$f[h(3)] =$

40. $f[g(x)] =$

$g[f(x)] =$

To find the inverse of a function simply switch the x and the y and solve for the new "y" value.

Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse, $f^{-1}(x)$, if possible.

41. $f(x)=5x+2$

42. $f(x)=\frac{1}{2}x-\frac{1}{3}$

Addition and subtraction

First, find the least common denominator.

Write each fraction with the LCD

Add / subtract numerators as indicated and leave the denominators as they are.

EX: $\dfrac{3x+1}{x^2+2x}+\dfrac{5x-4}{2x+4}$	*Factor denominator completely.*
$\dfrac{3x+1}{x(x+2)}+\dfrac{5x-4}{2(x+2)}$	*Find LCD.*
$\dfrac{2(3x+1)}{x(x+2)}+\dfrac{x(5x-4)}{2(x+2)}$	*Rewrite each fraction with the LCD as the denominator*
$\dfrac{6x+2+5x^2-4x}{2x(x+2)}$	*Write as one fraction*
$\dfrac{5x^2+2x+2}{2x(x+2)}$	*Combine like terms*

43. $\dfrac{b-a}{a^2b}+\dfrac{a+b}{ab^2}$

44. $\dfrac{2-a^2}{a^2+a}+\dfrac{3a+4}{3a+3}$

<u>Solving Rational Equations:</u>

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

$$\frac{5}{x+2}+\frac{1}{x}=\frac{5}{x}$$

Find LCD first. x(x+2)

$$\left(x\left(x+2\right)\right)\left(\frac{5}{x+2}\right)+\left(x\left(x+2\right)\right)\left(\frac{1}{x}\right)=\left(x\left(x+2\right)\right)\left(\frac{5}{x}\right)$$

Multiply each term by the LCD.

$$5x+1\left(x+2\right)=5\left(x+2\right)$$

Simplify and solve.

$$5x+x+2=5x+10$$

$$6x+2=5x+10$$

Check your answer. Sometimes they do not check!

$$x=8$$

Check:

$$\frac{5}{8+2}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{10}+\frac{1}{8}=\frac{5}{8}$$

$$\frac{5}{8}=\frac{5}{8}$$

Solve each equation. Check your solutions.

45. $\frac{12}{x}+\frac{3}{4}=\frac{3}{2}$

46. $\frac{5}{x-5}=\frac{x}{x-5}-1$