Plan for Grade 8 Unit 4: Linear Equations and Linear Systems

Relevant Unit(s) to review: Grade 7 Unit 6: Expressions, Equations, and Inequalities

Essential prior concepts to engage with this unit	In this unit, students build on their grade 6 and 7 work with equivalent expressions and equations with one occurrence of one variable, learning algebraic methods to solve linear equations with multiple occurrences of one variable. Students learn to use algebraic methods to solve systems of linear equations in two variables, building on their grade 7 and 8 work with graphs and equations of linear relationships. Understanding of linear relationships is, in turn, built on the understanding of proportional relationships developed in grade 7 that connected ratios and rates with lines and triangles.
Brief narrative of approach	Because this unit builds on the work of expressions and equations from Grade 7 Unit 6 and that unit may have been compromised, the unit has been revised to integrate just in time review while still meeting the demands of Grade 8 Unit 4 (which features the major work of the grade.) Students learn to identify structure within linear equations in one variable and to assess whether equations have one, infinite, or no solutions. Students then take this understanding of solutions to assess whether graphs of systems of linear equations have no intersection (lines distinct and parallel, no solution), exactly one intersection (lines not parallel, exactly one solution), and the same line (infinitely many solutions). Certain lessons have been removed because the addition of the grade 7 lessons allows the same entry point while going into more depth. Lessons like 8.4.8 have an optional activity that can be skipped as well as a card sort which is hard to replicate virtually. Lessons 8.4.15 and 8.4.16 are synthesis lessons that include some activities such as an Info Gap that are hard to replicate virtually.

Lessons to Add	Lessons to Remove or Modify
1. Combine and add 7.6.7 and 7.6.8 2. 7.6.9 3. 7.6.10 4. 7.6.11	1. Remove 8.4.1: This introductory lesson can be skipped because the inclusion of the lessons from grade 7 addresses the material in more depth. 2. Remove 8.4.8: This lesson contains an optional activity. 3. Remove 8.4.15: This lesson focuses on an Info Gap that is not necessary to understand the concepts of the unit. 4. Remove 8.4.16: This lesson is an application of the concepts from the unit. It can be moved to outside of class.
Lessons added: 4	Lessons removed: 4

Modified Plan for Grade 8 Unit 4

Day	IM lesson	Notes
	assessment	8.4 Check Your Readiness Assessment Note that the Check Your Readiness Assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in 8.4.
1	7.6.7 7.6.8	These lessons integrate hanger diagrams, which are revisited in 8.4.2 and 8.4.3. These lessons are also important for ensuring students understand different approaches to solving equations in the form of $px + q = r$ and $p(x + q) = r$.
2	7.6.9	This lesson combines both understanding the structures of $px + q = r$ and $p(x + q) = r$ and the understanding of rational number operations needed to navigate solving equations with signed

		numbers. This lesson and the next lesson should emphasize approaches of either dividing each side first or applying the distributive property first.
3	7.6.10	The purpose of this lesson is to practice solving equations of the form $p(x + q) = r$, and to notice that one of the two ways of solving may be more efficient depending on the numbers in the equation.
4	7.6.11	Students solve problems that can be represented by equations of the form $px + q = r$ and $p(x + q) = r$. This lesson requires tape diagrams- this lesson can be modified to provide tape diagrams to focus student energy around writing equations and using the equations to solve real world problems.
5	8.4.2	This lesson launches grade 8 lessons using hanger diagrams.
6	8.4.3	This lesson connects hanger diagrams to algebraic methods of solving.
7	8.4.4	In this lesson, students continue to reinforce the connections among three fundamental ideas: a solution to an equation is a number that makes the equation true, performing the same operation on each side of an equation maintains the equality in the equation, and therefore two equations related by such a move have the same solutions.
8	8.4.5	The purpose of this lesson is to move towards a general method for solving linear equations.
9	8.4.6	In previous lessons, students have started to acquire fluency with a general method of solving equations, and have seen that different solution paths are possible. In this lesson, students learn to stop and think ahead strategically before plunging into a solution method.
10	8.4.7	In this lesson, they encounter equations that have no solutions and equations for which every number is a solution.
11	8.4.9	In this lesson, students apply their knowledge of solving equations by considering two real world situations: two tanks where one is filling and the other is emptying and two elevators traveling above and below ground level.
12	8.4.10	This lesson builds upon earlier work with linear equations in two variables in two types of contexts: contexts like distance versus time, in which there are an initial value and a rate of change, and contexts like budgets, in which there is an equation constraining the possible combinations of two quantities. In this lesson, students consider pairs of linear equations in

		each type of context and interpret the meaning of points on the graphs of the equations.
13	8.4.11	The purpose of this lesson is to introduce students to the graphical interpretation of systems of equations. Keeping the graphs in mind will be useful as students navigate algebraic techniques for solving systems in the lessons to come.
14	8.4.12	This lesson formally introduces the concept of a system of equations with different contexts. Students recognize that they have found solutions to systems of equations using graphing in the past few lessons by examining the intersection of graphed lines.
15	8.4.13	In this lesson, students continue to explore systems where the equations are both of the form $y = mx + b$. They connect algebraic and graphical representations of systems, first by matching graphs to systems, then by drawing their own graphs from given systems.
16	8.4.14	In this lesson, students graduate to other types of systems with different structures. They learn that examining structure is a good first step since it is sometimes possible to recognize an efficient method for solving the system through observation.
17	8.4 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
8.4.1	-	E	This activity includes number puzzles as an invitation to the context of the unit.
8.4.2	+	E	This is the first lesson in grade 8 involving hanger diagrams, though this representation should be familiar from grades 6 and 7.
8.4.3	+	D	

8.4.4	+	D	
8.4.5	0	A	
8.4.6	+	A	
8.4.7	+	E	In this lesson, students start to examine what it means to be a solution to an equation and whether there can be infinite or 0 solutions to equations as well.
8.4.8	0	D	
8.4.9	+	A	
8.4.10	+	E	This lesson begins the student work with systems of equations and understanding solutions to systems of equations.
8.4.11	+	E	
8.4.12	+	D	
8.4.13	+	D	
8.4.14	+	A	
8.4.15	-	A	
8.4.16	0	A	

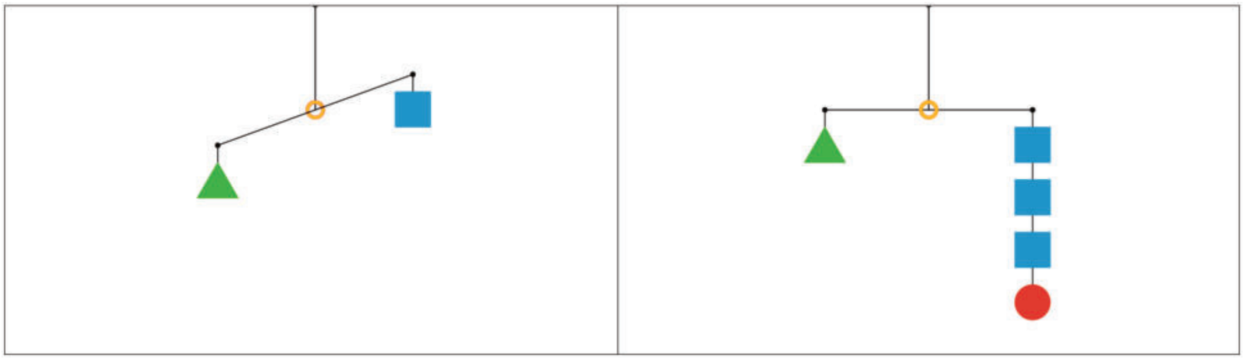
Lesson 7: Reasoning about Solving Equations (Part 1)

7.1: Hanger Diagrams

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true



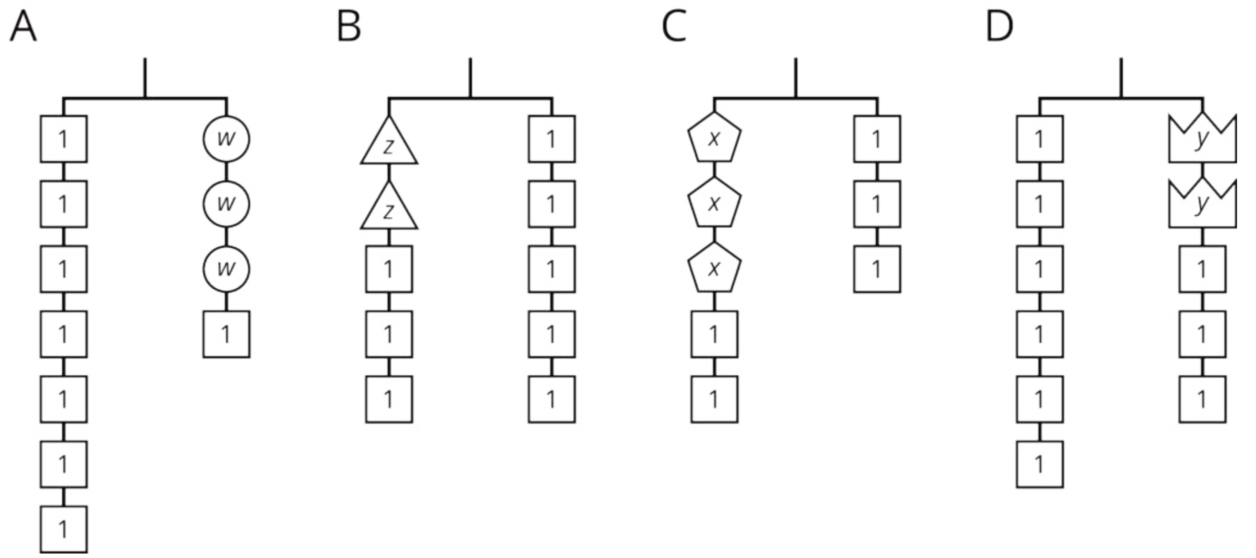
7.2: Hanger and Equation Matching

On each balanced hanger, figures with the same letter have the same weight.

1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   - $2\square + 3 = 5$
   - $3\square + 2 = 3$
   - $6 = 2\square + 3$
   - $7 = 3\square + 1$

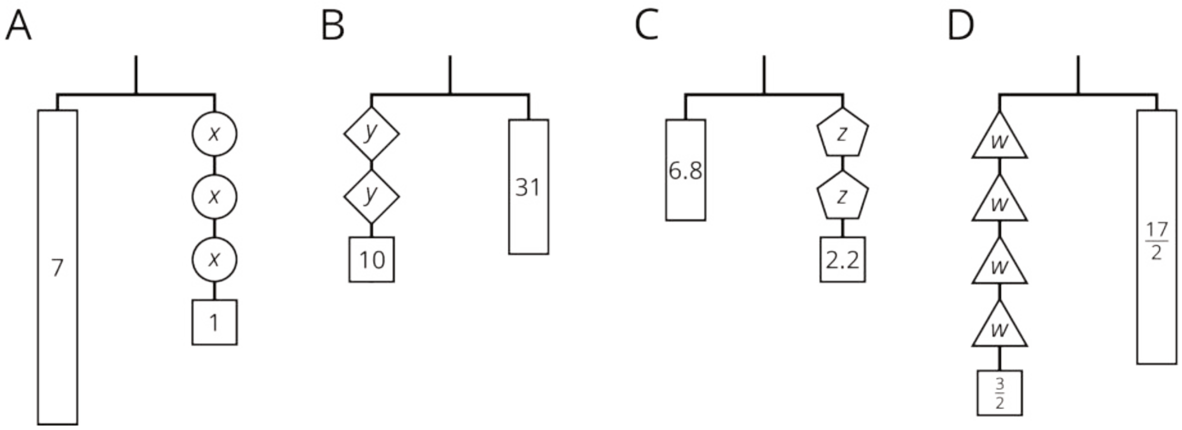
2. Find the solution to each equation. Use the hanger to explain what the solution means.



7.3: Use Hangers to Understand Equation Solving

Here are some balanced hangers where each piece is labeled with its weight. For each diagram:

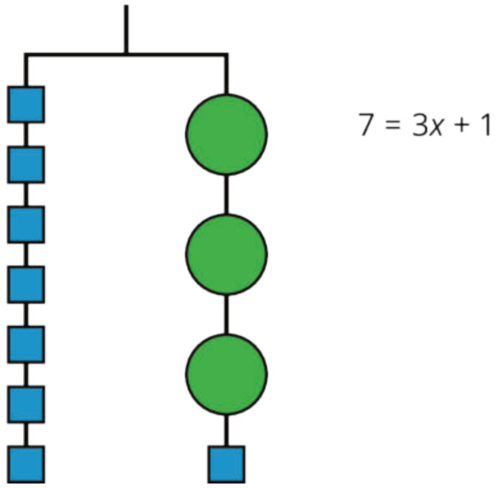
1. Write an equation.
2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.
3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.



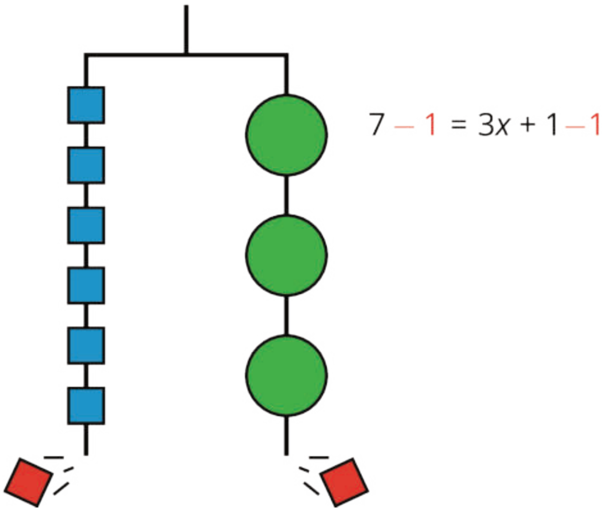
Lesson 7 Summary

In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use a balanced hanger to think about steps to finding an unknown amount in an associated equation.

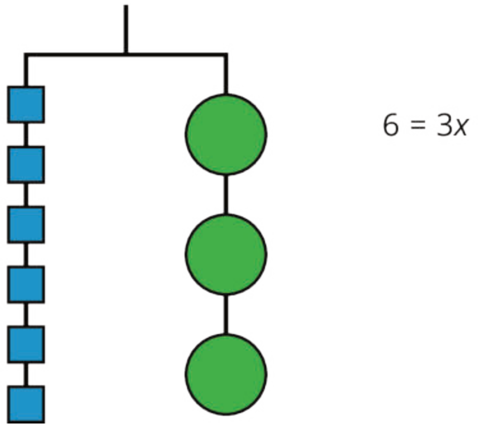
The hanger shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is $7 = 3x + 1$.



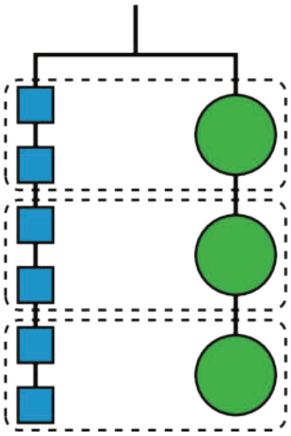
We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.



An equation for the new balanced hanger is $6 = 3x$.

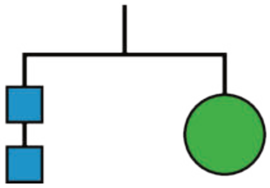


So the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$.



6 = 3*x*

The two sides of the hanger balance with these weights: 6 1-unit weights on one side and 3 weights of unknown size on the other side.



2 = *x*

Here is a concise way to write the steps above:

$7 = 3x + 1$	
$6 = 3x$	after subtracting 1 from each side
$2 = x$	after multiplying each side by $\frac{1}{3}$

Lesson 7: Reasoning about Solving Equations (Part 1)

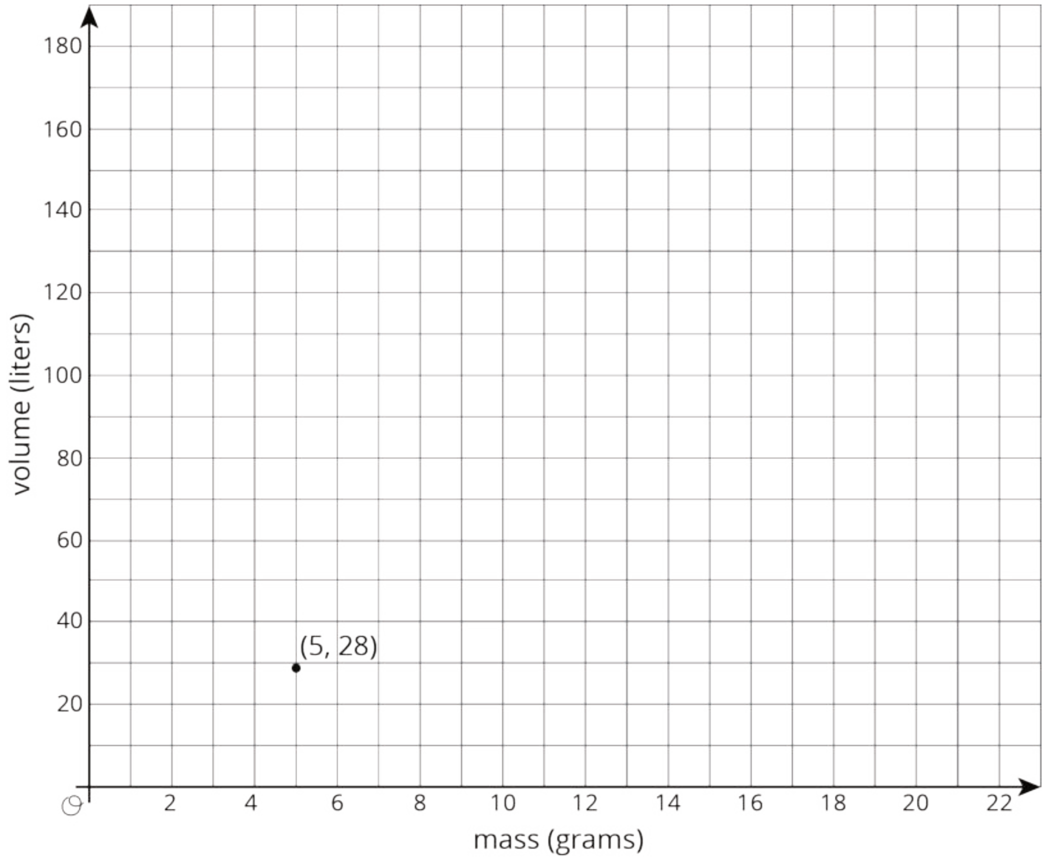
Cool Down: Solve the Equation

Solve the equation. If you get stuck, try using a diagram.

$$5x + \frac{1}{4} = \frac{61}{4}$$

Unit 6 Lesson 7 Cumulative Practice Problems

1. There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters. (5, 28) is shown on the graph below.



   a. What is the constant of proportionality in this relationship?

   b. In this situation, what is the meaning of the number you found in part a?

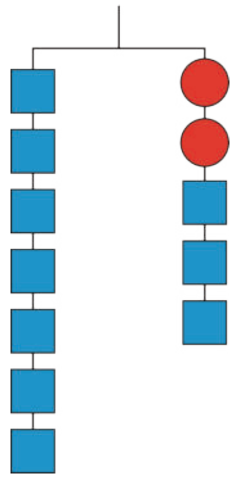
   c. Add at least three more points to the graph above, and label with their coordinates.

   d. Write an equation that shows the relationship between the mass of a sample of helium and its volume. Use m for mass and v for volume.

(From Unit 2, Lesson 11.)

2. Explain how the parts of the balanced hanger compare to the parts of the equation.

$7 = 2x + 3$

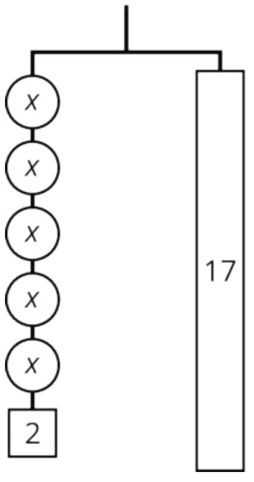


3. For the hanger below:

   a. Write an equation to represent the hanger.

   b. Draw more hangers to show each step you would take to find x. Explain your reasoning.

   c. Write an equation to describe each hanger you drew. Describe how each equation matches its hanger.



Lesson 8: Reasoning about Solving Equations (Part 2)

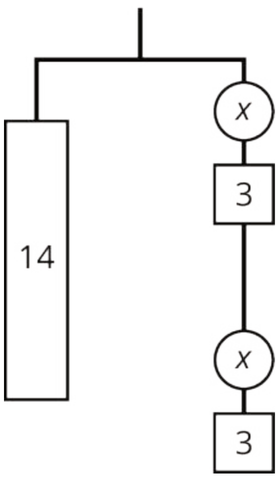
8.1: Equivalent to $2(x + 3)$

Select **all** the expressions equivalent to $2(x + 3)$.

1. $2 \cdot (x + 3)$
2. $(x + 3)2$
3. $2 \cdot x + 2 \cdot 3$
4. $2 \cdot x + 3$
5. $(2 \cdot x) + 3$
6. $(2 + x)3$

8.2: Either Or

1. Explain why either of these equations could represent this hanger:

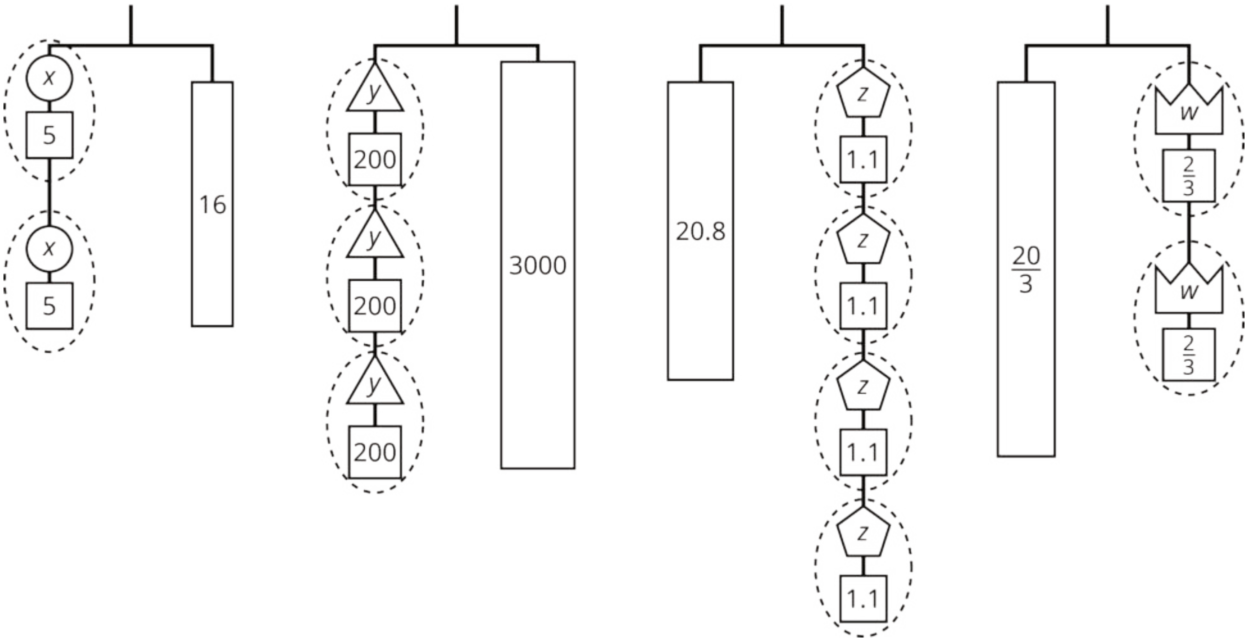


$14 = 2(x + 3)$ or $14 = 2x + 6$

2. Find the weight of one circle. Be prepared to explain your reasoning.

8.3: Use Hangers to Understand Equation Solving, Again

Here are some balanced hangers. Each piece is labeled with its weight.



For each diagram:

1. Assign one of these equations to each hanger:
   $2(x+5)=16$ $\qquad$ $3(y+200)=3{,}000$

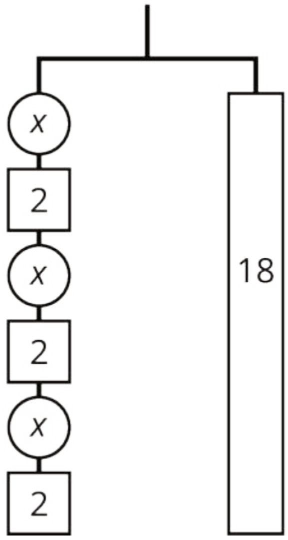
   $20.8=4(z+1.1)$ $\qquad$ $\frac{20}{3}=2\left(w+\frac{2}{3}\right)$

2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.

3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.

Lesson 8 Summary

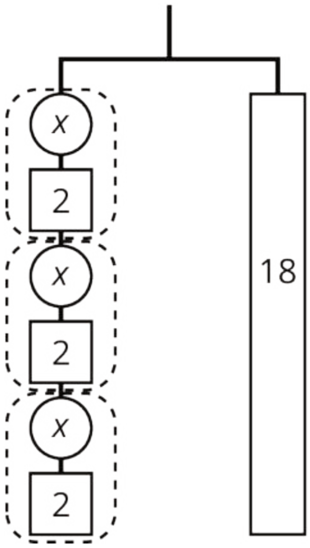
The balanced hanger shows 3 equal, unknown weights and 3 2-unit weights on the left and an 18-unit weight on the right.



There are 3 unknown weights plus 6 units of weight on the left. We could represent this balanced hanger with an equation and solve the equation the same way we did before.

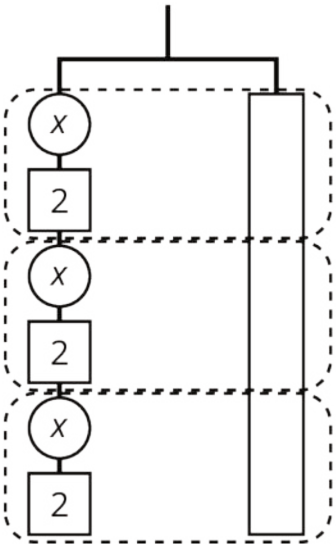
$$3x + 6 = 18$$
$$3x = 12$$
$$x = 4$$

Since there are 3 groups of $x + 2$ on the left, we could represent this hanger with a different equation: $3(x + 2) = 18$.



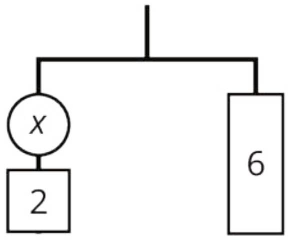
$3(x + 2) = 18$

The two sides of the hanger balance with these weights: 3 groups of $x + 2$ on one side, and 18, or 3 groups of 6, on the other side.



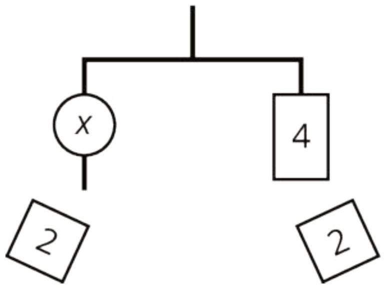
$3(x + 2) = 18$

The two sides of the hanger will balance with $\frac{1}{3}$ of the weight on each side: $\frac{1}{3} \cdot 3(x + 2) = \frac{1}{3} \cdot 18$.



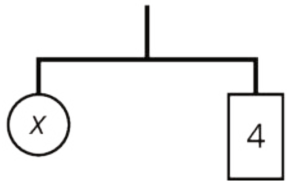
$x + 2 = 6$

We can remove 2 units of weight from each side, and the hanger will stay balanced. This is the same as subtracting 2 from each side of the equation.



$x + 2 = 4 + 2$

An equation for the new balanced hanger is $x = 4$. This gives the solution to the original equation.



$x = 4$

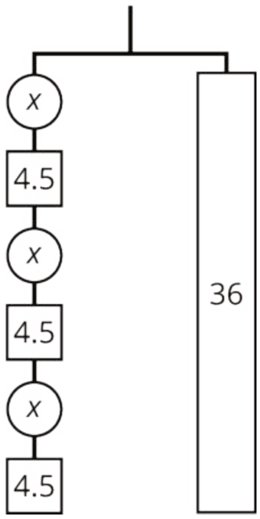
Here is a concise way to write the steps above:

$$\begin{aligned} 3(x+2) &= 18 \\ x + 2 &= 6 && \text{after multiplying each side by } \tfrac{1}{3} \\ x &= 4 && \text{after subtracting 2 from each side} \end{aligned}$$

Lesson 8: Reasoning about Solving Equations (Part 2)

Cool Down: Solve Another Equation

Solve the equation $3(x + 4.5) = 36$. If you get stuck, use the diagram.

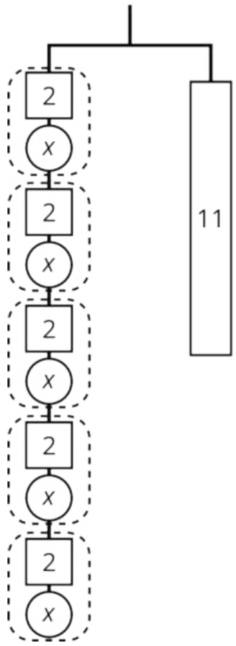


Unit 6 Lesson 8 Cumulative Practice Problems

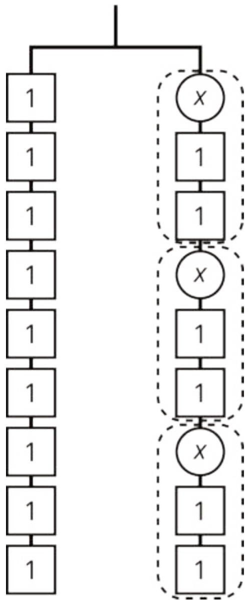
1. Here is a hanger:

   a. Write an equation to represent the hanger.

   b. Solve the equation by reasoning about the equation or the hanger. Explain your reasoning.



2. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.



- x
- 9
- 3
- $x + 2$
- $3(x + 2)$
- the equal sign

3. Select the word from the following list that best describes each situation.

A. You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.

B. For every car sold, a car salesman is paid 6% of the car’s price.

C. Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.

D. An antique furniture store pays $200 for a chair, adds 50% of that amount, and sells the chair for $300.

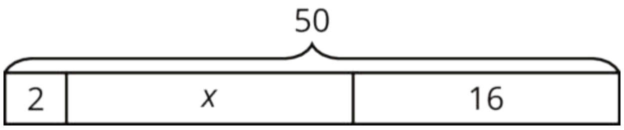
E. The normal price of a mattress is $600, but it is on sale for 10% off.

F. For any item you purchase in Texas, you pay an additional 6.25% of the item's price to the state government.

1. Tax
2. Commission
3. Discount
4. Markup
5. Tip or gratuity
6. Interest

(From Unit 4, Lesson 11.)

4. Clare drew this diagram to match the equation $2x + 16 = 50$, but she got the wrong solution as a result of using this diagram.



a. What value for x can be found using the diagram?

b. Show how to fix Clare's diagram to correctly match the equation.

c. Use the new diagram to find a correct value for x.

d. Explain the mistake Clare made when she drew her diagram.

(From Unit 6, Lesson 3.)

Lesson 9: Dealing with Negative Numbers

9.1: Which One Doesn't Belong: Rational Number Arithmetic

Which equation doesn't belong?

$15 = -5 \cdot -3$

$4 - -2 = 6$

$2 + -5 = -3$

$-3 \cdot -4 = -12$

9.2: Old and New Ways to Solve

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$
2. $x - -4 = -6$
3. $2(x - 1) = -200$
4. $2x + -3 = -23$

9.3: Keeping It True

Here are some equations that all have the same solution.

$$
\begin{aligned}
x &= \text{-}6 \\
x - 3 &= \text{-}9 \\
\text{-}9 &= x - 3 \\
900 &= \text{-}100(x - 3) \\
900 &= (x - 3) \cdot (\text{-}100) \\
900 &= \text{-}100x + 300
\end{aligned}
$$

1. Explain how you know that each equation has the same solution as the previous equation. Pause for discussion before moving to the next question.

2. Keep your work secret from your partner. Start with the equation $\text{-}5 = x$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.

3. See if you can figure out what steps they used to transform $\text{-}5 = x$ into their equation. When you think you know, check with them to see if you are right.

Lesson 9 Summary

When we want to find a solution to an equation, sometimes we just think about what value in place of the variable would make the equation true. Sometimes we perform the same operation on each side (for example, subtract the same amount from each side). The balanced hangers helped us to understand that doing the same to each side of an equation keeps the equation true.

Since negative numbers are just numbers, then doing the same thing to each side of an equation works for negative numbers as well. Here are some examples of equations that have negative numbers and steps you could take to solve them.

Example:

$$\begin{aligned} 2(x-5) &= \text{-}6 \\ \tfrac{1}{2} \cdot 2(x-5) &= \tfrac{1}{2} \cdot (\text{-}6) && \text{multiply each side by } \tfrac{1}{2} \\ x-5 &= \text{-}3 \\ x-5+5 &= \text{-}3+5 && \text{add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} \text{-}2x + \text{-}5 &= 6 \\ \text{-}2x + \text{-}5 - \text{-}5 &= 6 - \text{-}5 && \text{subtract -5 from each side} \\ \text{-}2x &= 11 \\ \text{-}2x \div \text{-}2 &= 11 \div \text{-}2 && \text{divide each side by -2} \\ x &= \text{-}\tfrac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful to solving for the unknown amount. For example, we could take the equation $\text{-}3x+7=\text{-}8$ and add -2 to each side:

$$\begin{aligned} \text{-}3x+7 &= \text{-}8 \\ \text{-}3x+7+\text{-}2 &= \text{-}8+\text{-}2 && \text{add -2 to each side} \\ \text{-}3x+5 &= \text{-}10 \end{aligned}$$

If $\text{-}3x+7=\text{-}8$ is true then $\text{-}3x+5=\text{-}10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x=5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).

Lesson 9: Dealing with Negative Numbers

Cool Down: Solve Two More Equations

Solve each equation. Show your work, or explain your reasoning.

1. $-3x - 5 = 16$
2. $-4(y - 2) = 12$

Unit 6 Lesson 9 Cumulative Practice Problems

1. Solve each equation.

   a. $4x = \text{-}28$

   b. $x - \text{-}6 = \text{-}2$

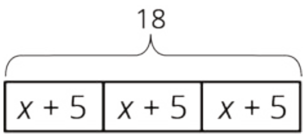
   c. $\text{-}x + 4 = \text{-}9$

   d. $\text{-}3x + 7 = 1$

   e. $25x + \text{-}11 = \text{-}86$

2. Here is an equation $2x + 9 = \text{-}15$. Write three different equations that have the same solution as $2x + 9 = \text{-}15$. Show or explain how you found them.

3. Select **all** the equations that match the diagram.



A. $x + 5 = 18$

B. $18 \div 3 = x + 5$

C. $3(x + 5) = 18$

D. $x + 5 = \frac{1}{3} \cdot 18$

E. $3x + 5 = 18$

(From Unit 6, Lesson 3.)

4. There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

   a. Draw a tape diagram to represent the situation.

   b. What unknown amounts can be found by by using the diagram or reasoning about the situation?

(From Unit 6, Lesson 2.)

5. Match each story to an equation.

A. A stack of nested paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups in the stack adds $\frac{1}{4}$ inch to the height of the stack.

B. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 identical muffins.

C. Elena has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four pieces of equal length.

1. $\frac{1}{4} + 4x = 8$
2. $4 + \frac{1}{4}x = 8$
3. $8x + \frac{1}{4} = 4$

(From Unit 6, Lesson 4.)

Lesson 10: Different Options for Solving One Equation

10.1: Algebra Talk: Solve Each Equation

$100(x-3) = 1{,}000$

$500(x-3) = 5{,}000$

$0.03(x-3) = 0.3$

$0.72(x+2) = 7.2$

10.2: Analyzing Solution Methods

Three students each attempted to solve the equation $2(x-9)=10$, but got different solutions. Here are their methods. Do you agree with any of their methods, and why?

Noah's method:

$$\begin{aligned} 2(x-9) &= 10 \\ 2(x-9)+9 &= 10+9 && \text{add 9 to each side} \\ 2x &= 19 \\ 2x \div 2 &= 19 \div 2 && \text{divide each side by 2} \\ x &= \frac{19}{2} \end{aligned}$$

Elena's method:

$$\begin{aligned} 2(x-9) &= 10 \\ 2x-18 &= 10 && \text{apply the distributive property} \\ 2x-18-18 &= 10-18 && \text{subtract 18 from each side} \\ 2x &= \text{-}8 \\ 2x \div 2 &= \text{-}8 \div 2 && \text{divide each side by 2} \\ x &= \text{-}4 \end{aligned}$$

Andre's method:

$$\begin{aligned} 2(x-9) &= 10 \\ 2x-18 &= 10 && \text{apply the distributive property} \\ 2x-18+18 &= 10+18 && \text{add 18 to each side} \\ 2x &= 28 \\ 2x \div 2 &= 28 \div 2 && \text{divide each side by 2} \\ x &= 14 \end{aligned}$$

10.3: Solution Pathways

For each equation, try to solve the equation using each method (dividing each side first, or applying the distributive property first). Some equations are easier to solve by one method than the other. When that is the case, stop doing the harder method and write down the reason you stopped.

1. $2{,}000(x - 0.03) = 6{,}000$

2. $2(x + 1.25) = 3.5$

3. $\frac{1}{4}(4 + x) = \frac{4}{3}$

4. $-10(x - 1.7) = -3$

5. $5.4 = 0.3(x + 8)$

Lesson 10 Summary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.

Suppose we are trying to solve the equation $\frac{4}{5}(x+27)=16$. Two useful approaches are:

- divide each side by $\frac{4}{5}$
- apply the distributive property

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that $\frac{4}{5} \boldsymbol{\cdot} 27$ will be hard, because 27 isn't divisible by 5. But $16 \div \frac{4}{5}$ gives us $16 \boldsymbol{\cdot} \frac{5}{4}$, and 16 is divisible by 4. Dividing each side by $\frac{4}{5}$ gives:

$$\begin{aligned} \frac{4}{5}(x+27) &= 16 \\ \frac{5}{4} \boldsymbol{\cdot} \frac{4}{5}(x+27) &= 16 \boldsymbol{\cdot} \frac{5}{4} \\ x+27 &= 20 \\ x &= \text{-}7 \end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation $100(x+0.06)=21$. If we first divide each side by 100, we get $\frac{21}{100}$ or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.

$$\begin{aligned} 100(x+0.06) &= 21 \\ 100x+6 &= 21 \\ 100x &= 15 \\ x &= \frac{15}{100} \end{aligned}$$

Lesson 10: Different Options for Solving One Equation

Cool Down: Solve Two Equations

Solve each equation. Show or explain your method.

1. $8.88 = 4.44(x - 7)$
2. $5\left(y + \frac{2}{5}\right) = -13$

Unit 6 Lesson 10 Cumulative Practice Problems

1. Andre wants to buy a backpack. The normal price of the backpack is \$40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

   (From Unit 4, Lesson 11.)

2. On the first math exam, 16 students received an A grade. On the second math exam, 12 students received an A grade. What percentage decrease is that?

   (From Unit 4, Lesson 12.)

3. Solve each equation.

   a. $2(x-3)=14$

   b. $-5(x-1)=40$

   c. $12(x+10)=24$

   d. $\frac{1}{6}(x+6)=11$

   e. $\frac{5}{7}(x-9)=25$

4. Select **all** expressions that represent a correct solution to the equation $6(x+4)=20$.

    A. $(20-4) \div 6$

    B. $\frac{1}{6}(20-4)$

    C. $20-6-4$

    D. $20 \div 6 - 4$

    E. $\frac{1}{6}(20-24)$

    F. $(20-24) \div 6$

5. Lin and Noah are solving the equation $7(x+2)=91$.

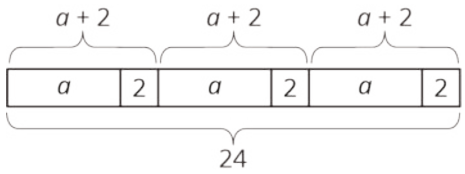
    Lin starts by using the distributive property. Noah starts by dividing each side by 7.

    a. Show what Lin's and Noah's full solution methods might look like.

    b. What is the same and what is different about their methods?

Lesson 11: Using Equations to Solve Problems

11.1: Remember Tape Diagrams

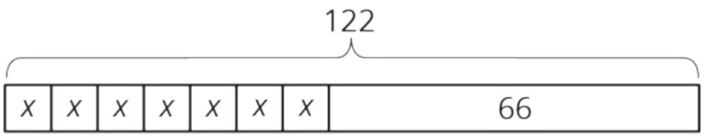


1. Write a story that could be represented by this tape diagram.

2. Write an equation that could be represented by this tape diagram.

11.2: At the Fair

1. Tyler is making invitations to the fair. He has already made some of the invitations, and he wants to finish the rest of them within a week. He is trying to spread out the remaining work, to make the same number of invitations each day. Tyler draws a diagram to represent the situation.



   a. Explain how each part of the situation is represented in Tyler's diagram:

      How many total invitations Tyler is trying to make.

      How many invitations he has made already.

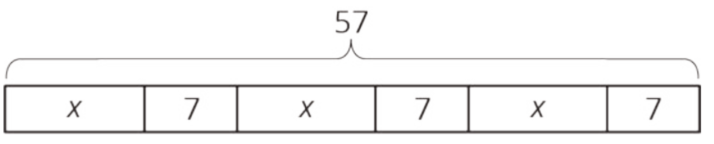
      How many days he has to finish the invitations.

b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.

c. Use Tyler's diagram to write an equation that represents the situation. Explain how each part of the situation is represented in your equation.

d. Show how to solve your equation.

2. Noah and his sister are making prize bags for a game at the fair. Noah is putting 7 pencil erasers in each bag. His sister is putting in some number of stickers. After filling 3 of the bags, they have used a total of 57 items.



a. Explain how the diagram represents the situation.

b. Noah writes the equation $3(x + 7) = 57$ to represent the situation. Do you agree with him? Explain your reasoning.

c. How many stickers is Noah's sister putting in each prize bag? Explain or show your reasoning.

3. A family of 6 is going to the fair. They have a coupon for \$1.50 off each ticket. If they pay \$46.50 for all their tickets, how much does a ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

11.3: Running Around

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.

    a. How many miles did Priya run last week?

    b. Elena wrote the equation $\frac{1}{9}(x-7)=4$ to describe the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?

2. One day last week, 6 teachers joined $\frac{5}{7}$ of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?

3. Priya and Han plan a fundraiser for the running club. They begin with a balance of -80 because of expenses. In the first hour of the fundraiser they collect equal donations from 9 family members, which brings their balance to -44. How much did each parent give?

4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at a rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

Are you ready for more?

A musician performed at three local fairs. At the first he doubled his money and spent \$30. At the second he tripled his money and spent \$54. At the third, he quadrupled his money and spent \$72. In the end he had \$48 left. How much did he have before performing at the fairs?

Lesson 11 Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next six days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where x represents the distance, in miles, she ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

Lesson 11: Using Equations to Solve Problems

Cool Down: The Basketball Game

Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

Unit 6 Lesson 11 Cumulative Practice Problems

1. Find the value of each variable.

   a. $a \cdot 3 = -30$

   b. $-9 \cdot b = 45$

   c. $-89 \cdot 12 = c$

   d. $d \cdot 88 = -88{,}000$

   (From Unit 5, Lesson 9.)

2. Match each equation to its solution and to the story it describes.

   Equations:

   a. $5x - 7 = 3$

   b. $7 = 3(5 + x)$

   c. $3x + 5 = -7$

   d. $\frac{1}{3}(x + 7) = 5$

   Solutions:

   a. -4

   b. $\frac{-8}{3}$

   c. 2

   d. 8

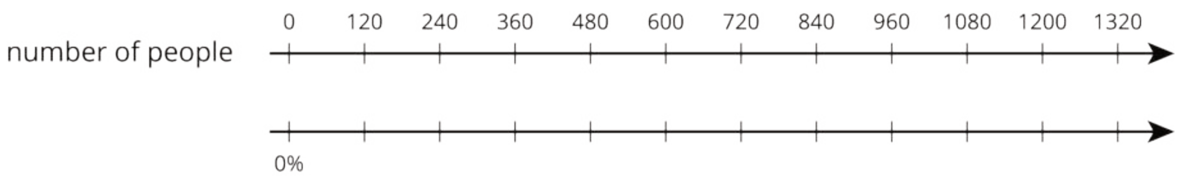
   Stories:

   - The temperature is -7. Since midnight the temperature tripled and then rose 5 degrees. What was temperature at midnight?
   - Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?
   - A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now in 7 hours they can build 3 guitars. By how much did they reduce the time it takes to build each guitar?
   - A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

3. The baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

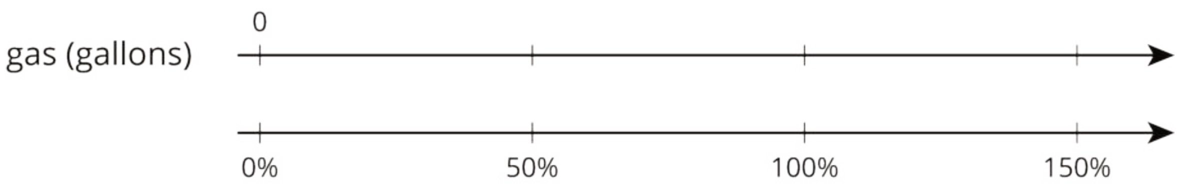
4. Six teams are out on the field playing soccer. The teams all have the same number of players. The head coach asks for 2 players from each team to come help him move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

5. A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?



(From Unit 4, Lesson 7.)

6. The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does it hold?



(From Unit 4, Lesson 7.)