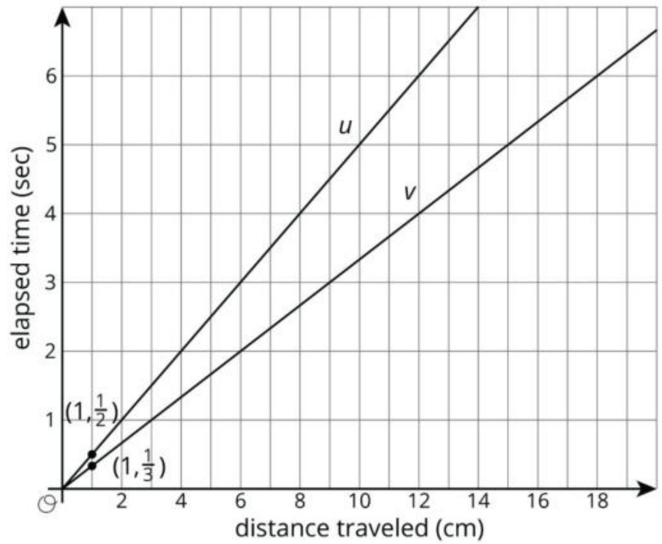
Grade 8 - Unit 3

Lessons 1-4: Proportional Relationships

Lesson 1: Understanding Proportional Relationships

Lesson Synthesis

Display a scaled graph of the two bugs for all to see. Remind students that line u is the ladybug and that line v is the ant.



Ask students:

- "What would the graph of a bug going 3 times faster than the ant look like?" (It would go through the points $(0,0)$, $(1, \frac{1}{9})$, and $(9,1)$.)
- "What would an equation showing the relationship between the bugs' distance and time look like?" (Since it is going 4 times faster and goes through the point $(9,1)$, it has constant of proportionality of $\frac{1}{9}$, which means one equation is $y = \frac{1}{9}x$.)
- "If we wanted to scale the graph so we could see how long it takes the ladybug to travel 50 cm, what numbers could we use on the vertical axis?" (The ladybug travels 50 cm in 25 seconds, so the vertical axis would need to extend to at least that value.)

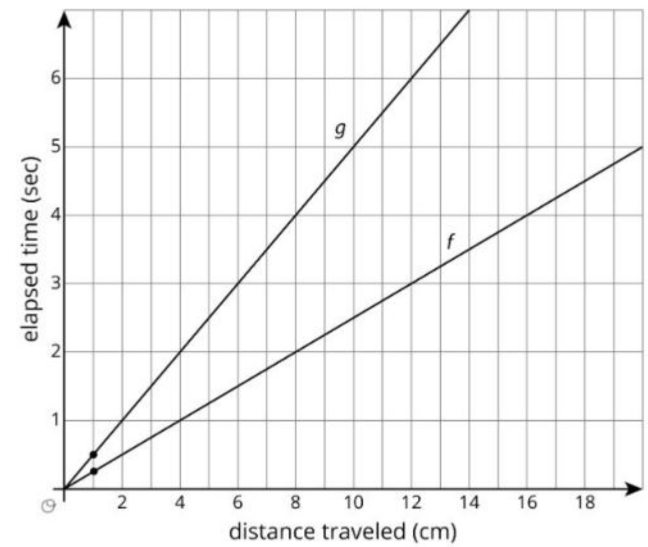
COOL-DOWN: 5 minutes

1.4: Turtle Race

CCSS Standards: Building on	• 7.RP.A.2
CCSS Standards: Building towards	• 8.EE.B.5

Student-Facing Task Statement

This graph represents the positions of two turtles in a race.



1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line g.
2. Explain how your line shows that the turtle is going half as fast.

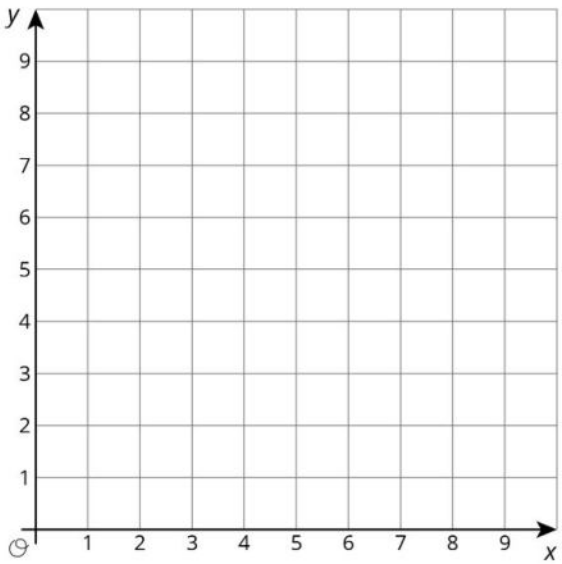
Student Response

1. A line through $(0,0)$, $(1,1)$, $(2,2)$, etc.
2. Looking at the values for 2 seconds, turtle g moves 4 cm and the third turtle moves only 2 cm. This third turtle covers half the distance in the same amount of time.

Lesson 2: Graphs of Proportional Relationships

Lesson Synthesis

Display this blank graph for all to see and provide pairs of students with graph paper.



Ask pairs to draw a copy of the axis and give a signal when they have finished. (You may need to warn students to leave room on their graph paper for a second graph as sometimes students like to draw graphs that fill all the space they are given.) Invite a student to propose a proportional relationship that they consider to have a "steep" line for the class to graph on the axes.

For example, say a student proposes $y = 6x$. After students graph, add the line representing the equation to the graph on display. Then, ask students to make a second graph with the same horizontal scale, but with a vertical scale that makes $y = 6x$ not look as steep when graphed. After students have made the new graph, invite students to share and explain how they decided on their new vertical scale.

Conclude by reminding students that all these graphs of $y = 6x$ are correct since they all show a proportional relationship with a constant of proportionality equal to 6. Ask students, "Can you think of a reason we might want to graph this relationship with such a large vertical scale?" (If we needed to also graph something like $y = 60x$, we would need a pretty big vertical scale in order to see both lines.)

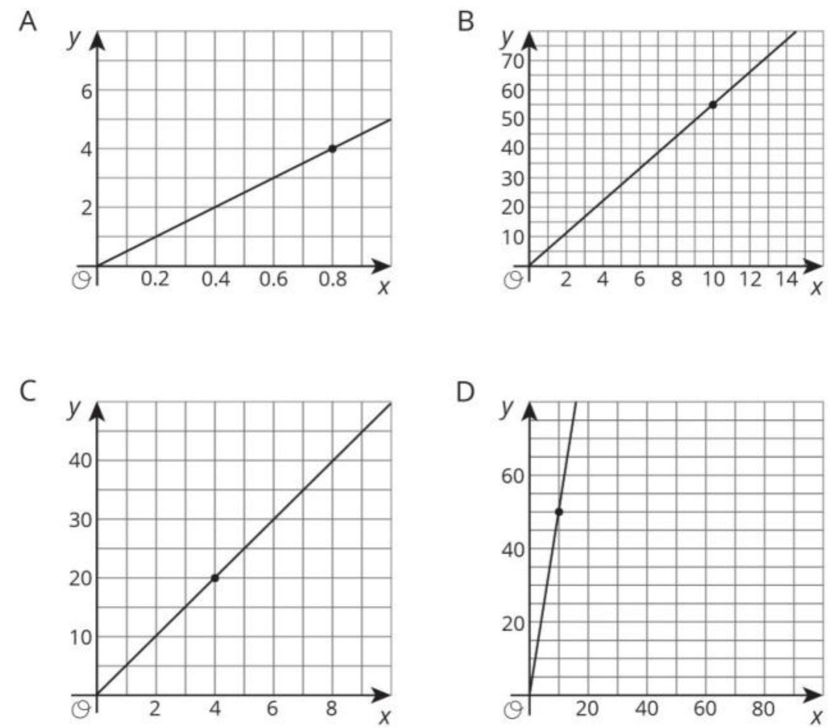
COOL-DOWN: 5 minutes

2.4: Different Axes

CCSS Standards: Addressing	• 8.EE.B
CCSS Standards: Building towards	• 8.EE.B.5

Student-Facing Task Statement

Which one of these relationships is different than the other three? Explain how you know.



Student Response

Answers vary. Sample response: Graphs A, C, and D are all representations of $y = 5x$. Graph B is a representation of $y = 5.5x$.

Lesson 3: Representing Proportional Relationships

Lesson Synthesis

Consider asking some of the following questions.

- "The proportional relationship $y = 5.5x$ includes the point $(18, 99)$ on its graph. How could you choose a scale for a pair of axes with a 10 by 10 grid to show this point?" (Have each grid line represent 10 or 20 units.)
- "What are some things you learned about graphing today that you are going to try to remember for later?"

COOL-DOWN: 5 minutes

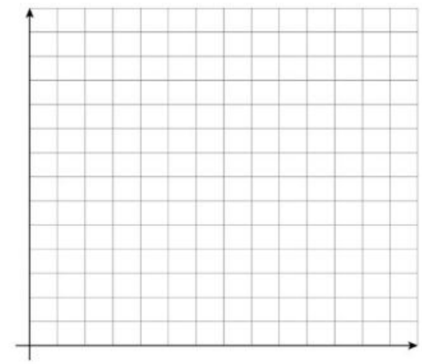
3.4: Graph the Relationship

CCSS Standards: Addressing	• 8.EE.B.5

Student-Facing Task Statement

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

salt (g)	honey (g)	flour (c)
10	14	4
25	35	10



Student Response

Answers vary. Possible graph: Label each axis from 0 to 140. For grams of salt on the horizontal axis and grams of honey on the vertical, the points $(0, 0)$, $(10, 14)$, and $(70, 98)$.

Lesson Synthesis

This lesson asked students to take a single piece of information about a proportional relationship, such as an equation, and use what they know about proportional relationships, rates of change, and representing relationships to compare it with a second proportional relationship in context.

Consider asking some of the following questions. Tell students to use, if possible, examples from today's lesson when responding:

- "What do you need in order to compare two proportional relationships?"
- "What type of wording in a problem statement or description of a situation tells you that you have a rate of change?"
- "How did you decide which representation to use to solve the different types of problems?"

COOL-DOWN: 5 minutes

4.3: Different Salt Mixtures

CCSS Standards: Addressing	• 8.EE.B.5

Student-Facing Task Statement

Here are recipes for two mixtures of salt and water that taste different.

Information about Salt Mixture A is shown in the table.

salt (teaspoons)	water (cups)
4	5
7	$8\frac{3}{4}$
9	$11\frac{1}{4}$

Salt Mixture B is defined by the equation $y = 2.5x$, where x is the number of teaspoons of salt and y is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.
2. Which mixture tastes saltier? Explain how you know.

Student Response

1. Mixture A uses 4 more teaspoons of salt than Mixture B. Mixture A would use 8 teaspoons of salt because I can double the row with 4 and 5 to get 8 and 10. Mixture B would use 4 teaspoons of salt because considering if $10 = 2.5x$, the value of x must be 4.
2. Mixture A tastes saltier because it uses more salt for the same amount of water. Mixture A uses 1.25 cups of water per teaspoon of salt while Mixture B uses 2.5 cups of water per teaspoon of salt.

Grade 8, Unit 3
Lessons 5-8: Representing Linear Relationships

Lesson 5: Introduction to Linear Relationships

Lesson Synthesis
The main focus of this lesson is the transition from proportional relationships to **linear relationships** that are not proportional:

- Understand that there are linear relationships that are not proportional.
- Note that the **rate of change** of the linear relationship is the same value as the **slope** of a line representing the relationship.
- Interpret the rate of change in the context of the situation.

In order to highlight this focus, ask students:

- "How can we tell if a linear relationship is proportional or not? From the graph? From a table? From the context?" (Check that when both variables are 0, this makes sense: on the graph, in the table, or in the context.)
- "What does the rate of change of a linear relationship tell us?" (The slope of the graph.)

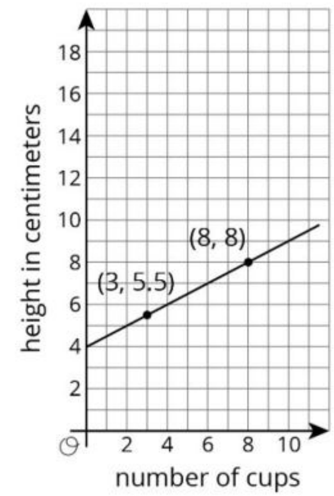
COOL-DOWN: 5 minutes
5.4: Stacking More Cups

CCSS Standards: Addressing	• 8.EE.B

Students apply what they have learned about graphing a non-proportional relationship to another stack of cups that are different in size from those analyzed in the activities.

Student-Facing Task Statement

A shorter style of cup is stacked tall. The graph displays the height of the stack in centimeters for different numbers of cups. How much does each cup after the first add to the height of the stack? Explain how you know.



Student Response

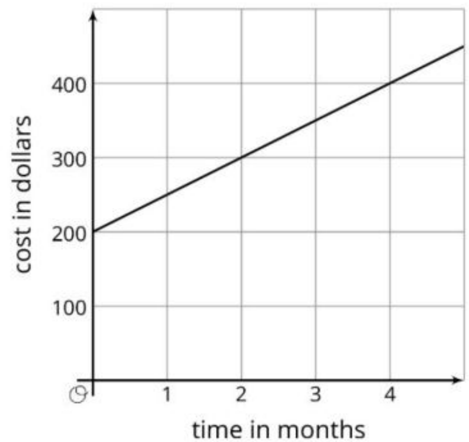
0.5 cm (or equivalent). Since 5 cups add 2.5 cm to the height of the stack, each cup adds 0.5 cm.

Lesson 6: More Linear Relationships

Lesson Synthesis

Lines have a slope and **vertical intercept**. The vertical intercept indicates where the line meets the y-axis. For example, a line represents a proportional relationship when the vertical intercept is 0.

Here is a graph of a line showing the amount of money paid for a new cell phone and monthly plan:



- "What is the vertical intercept for the graph?" ($(0, 200)$)
- "What does it mean?" (There was an initial cost of $200 for the phone.)

The slope of the line is 50 (draw a slope triangle connecting the points such as $(0, 200)$ and $(2, 300)$). This means that the phone service costs $50 per month in addition to the initial $200 for the phone.

COOL-DOWN: 5 minutes

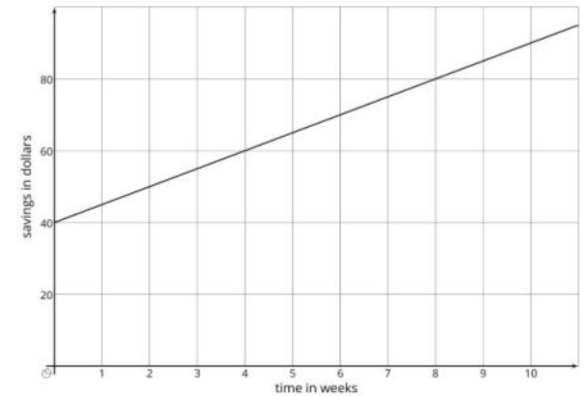
6.4: Savings

CCSS Standards: Addressing	• 8.EE.B.5

Student-Facing Task Statement

The graph shows the savings in Andre's bank account.

1. What is the slope of the line?
2. What is the meaning of the slope in this situation?



Student Response

1. 5
2. Andre saves 5 dollars every week.

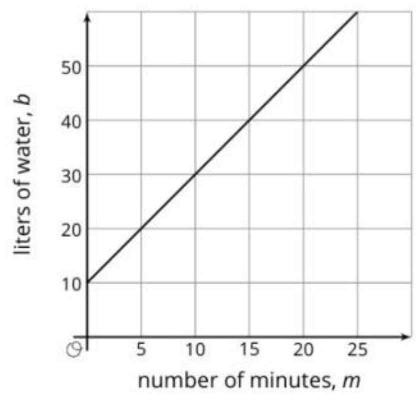
Lesson 7: Representations of Linear Relationships

Lesson Synthesis

Ask students to create and interpret an equation for a situation with linear growth. For example, imagine you have a bucket of water that already contains 10 liters of water and you turn on the water faucet, which adds 2 liters of water every minute. Ask students:

- "Can we use a linear equation to represent this situation?" (Yes.)
- "Why or why not?" (Each minute, 2 more liters of water are added to the bucket; the rate of change is constant.)
- "What is the equation?" ($b = 10 + 2m$ where b is the number of liters of water in the bucket and m is the number of minutes since you turned on the faucet.)

Sketch a graph of the line $b = 10 + 2m$.



Ask students the meaning of 10 from the equation and where they see it on the graph. (It's the y-intercept, and it is how many liters of water were in the bucket at the beginning.) Ask the meaning of 2 from the equation and where they see it on the graph. (It's the slope. A slope triangle with horizontal side length 1 will have vertical side length 2.)

We can find the slope of a line using *any* two points on the line. We use the coordinates of the two points to find the vertical and horizontal side lengths of the slope triangle. The slope is the quotient of the vertical and horizontal lengths. Label two general points (a, b) and (x, y) on the triangle. The vertical side has length $y - b$, and the horizontal side has length $x - a$ so $\frac{y-b}{x-a} = 2$.

COOL-DOWN: 5 minutes

7.4: Graphing a Line

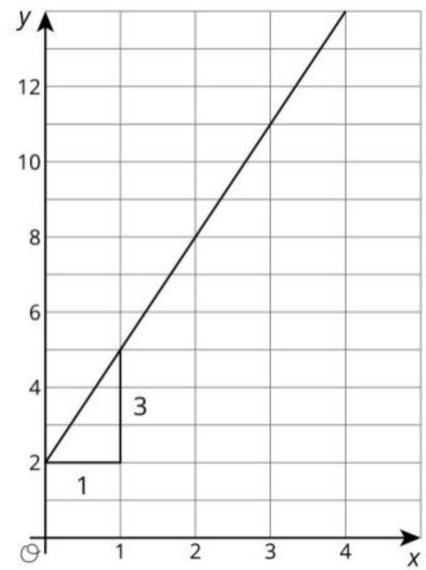
CCSS Standards: Addressing	• 8.EE.B.6

Student-Facing Task Statement

Make a sketch of a linear relationship with slope of 3 that is not a proportional relationship. Show how you know that the slope is 3. Write an equation for the line.

Student Response

Answers vary. Sample response:



The slope triangle has vertical side length 3 and horizontal side length 1, so the slope of the line is 3.

$y = 3x + 2$

Lesson 8: Translating Into $y = mx + b$

Lesson Synthesis

Display a graph of two lines on the same set of axes: one of the form $y = mx$ and the other of the form $y = mx + b$. Discuss:

- "How can we think of one of these lines as a transformation of the other?"
- "What is the equation of the line that goes through the origin?" (Discuss how you need to figure out the slope.)
- "How is the equation of the line that does not go through the origin different?" (Make sure to bring out that the b in $mx + b$ gives the vertical translation to get from the graph of $y = mx$ to the graph of $y = mx + b$; the translation is up when $b > 0$ and down when $b < 0$.)

COOL-DOWN: 5 minutes

8.4: Similarities and Differences in Two Lines

CCSS Standards: Addressing	• 8.EE.B

Student-Facing Task Statement

Describe how the graph of $y = 2x$ is the same and different from the graph of $y = 2x - 7$. Explain or show your reasoning.

Student Response

Answers vary. Students may or may not sketch graphs as part of their solution.

Possible responses to how they are the same:

- They have the same slope.
- They both have a slope of 2.
- They are parallel to each other.

Possible responses to how they are different:

- They are in a different location.
- One is a translation of the other.
- They cross the y-axis (or x-axis) at a different point.
- They don't go through any of the same points.

Grade 8, Unit 3

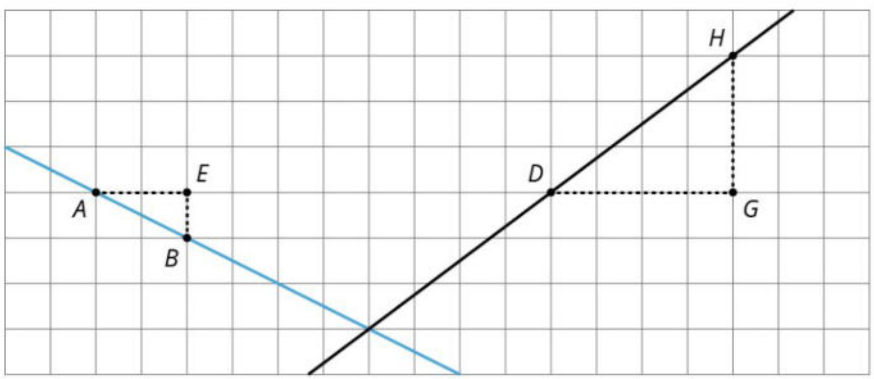
Lessons 9-11: Finding Slopes

<u>Lesson 9: Slopes Don't Have to be Positive</u>

Lesson Synthesis

In this lesson, students learned that the slope of a line can be a negative value or 0. They saw some linear relationships with a negative slope and some with 0 slope. Students learned about cues to identify whether a graphed line has a positive slope, a negative slope, or 0 slope.

Display the graph for all to see. Ask students to pretend that their partner has been absent from class for a few days. Their job is to explain, verbally or in writing, how someone would figure out the slope of one of the graphed lines. Then, switch roles and listen to their partner explain how to figure out the slope of the other line.



COOL-DOWN: 5 minutes

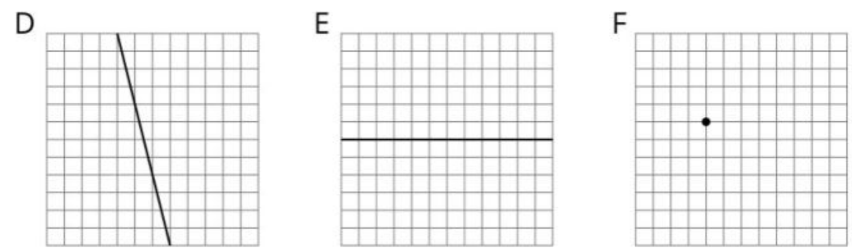
9.5: The Slopes of Graphs

CCSS Standards: Addressing	• 8.EE.B

Student-Facing Task Statement

Each square on a grid represents 1 unit on each side.

1. Calculate the slope of graph D. Explain or show your reasoning.
2. Calculate the slope of graph E. What situation could the graph represent?
3. On the blank grid, draw a line that passes through the indicated point and has slope -2.



Student Response

1. -4. Explanations vary. Sample explanation: for each horizontal change of 1, the vertical change is -4, so the slope of the line is -4.
2. 0. Responses vary. Sample response: The amount of rainfall on a day with no rain.
3. A graph through the indicated point with a slope of -2.

<u>**Lesson 9: Slopes Don't Have to be Positive**</u>

Lesson 10: Calculating Slope

Lesson Synthesis

In this lesson, students explored the interplay between the coordinates of points on a line and the slope of that line, where the slope could be positive or negative.

Ask students, "What information do you need to know exactly where a line is?" Valid responses might be the coordinates of two points on the line or the coordinates of one point and the line's slope. Demonstrate why knowing one point is not enough information (the line goes through it, but could have any slope), and why only knowing the slope is not enough information (you know at what slope to draw the line, but it could be located anywhere—it could be any of a set of parallel lines). It can be helpful to use a yardstick to represent "the line" in this situation as you move it around a coordinate plane on the board.

Ask students, "If you know the coordinates of two points on a line, how can you tell if it has a positive or negative slope?" Responses might include sketching a graph of the line to see if it's "uphill" or "downhill," or an algorithm involving subtraction and division, attending to keeping coordinates in "the same order" and performing operations correctly.

COOL-DOWN: 5 minutes

10.4: Different Slopes

CCSS Standards: Addressing	• 8.EE.B.6

Students calculate the slope of the line through two points. They are only given the coordinates of the points and are specifically directed not to graph the line. By now, students should have internalized an efficient method for finding slope using the coordinates of two points on the line.

Student-Facing Task Statement

Without graphing, find the slope of the line that goes through

1. $(0, 5)$ and $(8, 2)$.
2. $(2, -1)$ and $(6, 1)$.
3. $(-3, -2)$ and $(-1, -5)$.

Student Response

1. $\frac{-3}{8}$
2. $\frac{1}{2}$
3. $\frac{-3}{2}$

Lesson Synthesis

Students have spent considerable time in the 7th and 8th grades solving problems with proportional relationships and non-proportional relationships that can be represented by equations and graphs with positive slopes. Ask students to now consider real-world situations where slopes are not positive.

Ask students, "how can you tell from a real-world situation that the graph of the equation that represents it will be a horizontal line? Be a vertical line? Have a negative slope?"

For horizontal and vertical lines, the key feature is that one of the two variables does *not* vary while the other one can take *any* value. In the x-y plane, when the variable x can take any value, it is a vertical line, and when the variable y can take any value, it is a horizontal line.

COOL-DOWN: 5 minutes

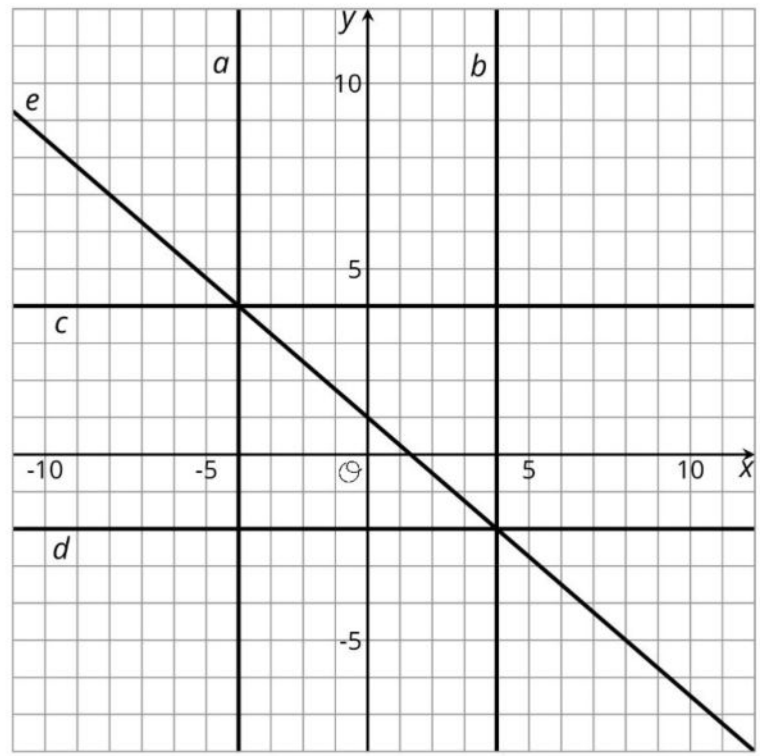
11.4: Line Design

CCSS Standards: Addressing	• 8.EE.B.6

Students write equations for lines that are horizontal, vertical, or have negative slope.

Student-Facing Task Statement

Here are 5 lines on a coordinate grid:



Write equations for lines a, b, c, d, and e.

Student Response

a: $x = -4$ b: $x = 4$ c: $y = 4$ d: $y = -2$ e: $y = \frac{-3}{4}x + 1$ or $\frac{y-1}{x} = \frac{-3}{4}$ (or equivalent equation)

Grade 8, Unit 3

Lessons 12-13: Linear Equations

Lesson 12: Solutions to Linear Equations

Lesson Synthesis

Students explored several big ideas in this lesson:

1. A solution to a linear equation is a pair of values that makes the equation true.
2. Solutions can be found by substituting a value for one of the variables and solving the equation for the other.
3. The set of all the solutions to a linear equation can be shown visually in the coordinate plane and is called the graph of the equation.
4. The graph of a linear equation is a line.
5. Any points in the coordinate plane that do not lie on the line that is on the graph of the linear equation are not solutions to the equation.
6. The number of solutions might be limited in a real-world situation even though the equation has an infinite number of solutions.

Invite students to describe how they found solutions to linear equations and to explain how they knew they had found a solution.

Ask what difference there may be between reading solutions from a graph and calculating them using the equation. Listen for students to identify a possible lack of precision when reading from a graph. If you are using graphing software this will be less of an issue.

Finally, ask if points that are not on the line can be solutions to the equation represented by the line. Listen for students to understand that the line represents *all* pairs that make the equation true so a point not on the line cannot be a solution.

COOL-DOWN: 5 minutes

12.4: Identify the Points

CCSS Standards: Addressing	• 8.EE.C

Students verify whether or not certain points in the x-y plane make a linear equation true.

Student-Facing Task Statement

Which of the following coordinate pairs make the equation $x - 9y = 12$ true?

1. $(12, 0)$
2. $(0, 12)$
3. $(3, -1)$
4. $(0, -\frac{4}{3})$

Student Response

1. Yes.
2. No.
3. Yes.
4. Yes.

Lesson 13: MoreSolutions to Linear Equations

Lesson Synthesis

In order to highlight student thinking about different strategies for finding a solution to a linear equation, ask students:

- "What are different ways to find a solution to the linear equation $3y + x = 12$?" (Substitute in a value for one variable and solve for the other; graph the equation and find points that lie on the line; rearrange the equation so that one variable is written in terms of the other variable.)
- "How do you know when you have found a solution to the equation $3y + x = 12$?" (The coordinates of the point will make the statement true.)
- "What are some easy values to substitute into the equation?" (In this case, a good strategic choice is $x = 0$, which gives $y = 4$, and $y = 0$, which gives $x = 12$. This says that the y-intercept of the graph of the equation is $(0, 4)$. Similarly, the x-intercept of the equation's graph is $(12, 0)$.
- "How can you find the slope of the line?" (Graphing the line shows that the slope is negative, and we can verify this by rewriting the equation as $y = -\frac{1}{3}x + 4$.)

COOL-DOWN

13.4: Intercepted

CCSS Standards: Addressing	• 8.EE.C.8.a

Students show their understanding of solutions to linear equations in two variables and connections to the graph of the equation.

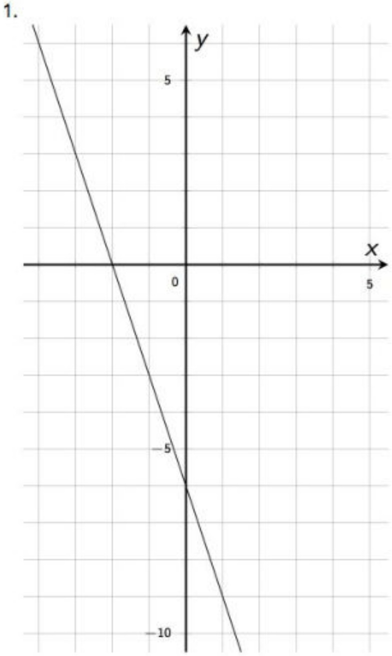
Student-Facing Task Statement

A graph of a linear equation passes through $(-2, 0)$ and $(0, -6)$.

1. Use the two points to sketch the graph of the equation.
2. Is $3x - y = -6$ an equation for this graph? Explain how you know.

Student Response

1.



2. No. Answers vary. Sample response: Test the two given pairs: $3(-2) - 0 = -6 - 0 = -6$ so the coordinates of this point represent a solution to the equation. $3(0) - (-6) = 0 + 6 = 6$, not -6, so the coordinates of this point do not represent a solution to the equation. The graph of a linear equation contains only ordered pairs whose coordinates are solutions to the equation, so the equation is not represented by the line with the two given points.

Grade 8, Unit 3

Lessons 14: Let’s Put it to Work

Lesson 14: Using Linear Relations to Solve Problems

Lesson Synthesis

Ask students to consider the real-world situations described in this lesson. Discuss:

- “Give an example of a solution to an equation that doesn’t make sense in the context it represents.” (Some values might not make sense in the context, like negative values. A length cannot be negative, for example.)
- “If some values make sense in the equation but not in the context, how could this impact the graph?” (We might only draw part of the line or draw some of the points that lie on the line.)

Notes: