Plan for Unit Grade 7 Unit 4: Proportional Relationships and Percentages

Relevant Unit(s) to review: Grade 6 Unit 3: Unit Rates and Percentages
Grade 6 Unit 5: Arithmetic and Base Ten

Essential prior concepts to engage with this unit	• Ratio, rates, and unit rates • Understanding of rates to include percentages as rates per 100 and reasoning about situations involving whole-number percentages
Brief narrative of approach	Begin by reviewing percentages as rates per 100 by solving percentage problems (6.3.14), then refresh skills of calculating products of decimals (6.5.8) and dividing numbers that result in decimals (6.5.11). This leads to a deeper understanding of ratios, scale factors, unit rates, and proportional relationships, and using them to solve multi-step problems that are set in a wide variety of contexts that involve fractions and percentages.

Lessons to Add	Lessons to Remove or Modify
1. 6.3.14 - focus on strategies to solve problems involving percentages. Problem and Data Card discussion is highly suggested. 2. 6.5.8 - focus on strategies for solving decimal multiplication problems. 3. 6.5.11 - focus on strategies for solving decimal division problems.	4. Combine 7.4.8 and 7.4.9 - remove optional activities to help compact lessons. Optional lessons could be used as additional practice outside of class. 5. Remove 7.4.15 - an optional lesson finding and analyzing intervals of possible error based on maximum possible errors. 6. Move 7.4.16 to outside of class - culminating lesson on percentages, students work at home to collect news clippings that mention percentages and sort them according to whether they are about percent increase or percent decrease, and formulate

	questions about them. Discussion could take place in class.
Lessons added: 3	Lessons removed: 3

Modified Plan for Grade 7 Unit 4

Day	IM lesson	Notes
	assessment	7.4 Check Your Readiness Assessment Note that the Check Your Readiness Assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in 7.4.
1	7.4.1	
2	7.4.2	
3	6.3.14	If the initial assessment shows that students are not familiar with solving problems involving percentages, include these activities before continuing with grade level content.
4	6.5.8	If the initial assessment shows that students are not familiar with solving decimal multiplication problems include these activities before continuing with grade level content.
5	6.5.11	If the initial assessment shows that students are not familiar with solving decimal multiplication problems include these activities before continuing with grade level content.
6	7.4.3	
7	7.4.4	
8	7.4.5	
9	7.4.6	

10	7.4.7	
11	7.4.8 7.4.9	Remove optional activities to help compact the lessons into one day.
12	7.4.10	
13	7.4.11	
14	7.4.12	
15	7.4.13	
16	7.4.14	
17	7.4 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
7.4.1	0	E	The purpose of this lesson is to start the unit with an engaging activity where proportional relationships arise naturally, in the scaling of flags and in questions about what **percentage** of the flag is taken up by a particular part of the design.
7.4.2	+	D	Students start to work with ratios of fractions and fractional percentages. In this lesson they encounter situations where a ratio of fractions arises naturally. They compute scale factors and **unit rates** associated with ratios of fractions.
7.4.3	0	D	In this lesson students move toward solving problems involving proportional relationships by more efficient methods, especially by setting up and reasoning about a two-row table of equivalent ratios. This method encourages them to use the constant of proportionality rather than equivalent ratios.
7.4.4	+	A	In this lesson students see how to use the distributive property to write a compact expression for situations where one quantity is described in relation to another quantity in language such as "half as much again" and "one third more than."
7.4.5	+	A	In this lesson students continue to study situations of fractional increase and decrease. They start to use decimal notation to express the situations. For example, they see that "one quarter less than x" can be expressed as $\frac{3}{4}x$ or as $0.75x$.
7.4.6	0	E	First in a series of 4 lessons about **percent increase and decrease**. The goal of this lesson is to understand what is meant by "20% more than" or "10% less than." Students relate this language to the previous two lessons where they talked about "half as much again" and "one third less than."

7.4.7	0	E	In this second lesson about percent increase and percent decrease, students work with problems where they are given the final amount after a **percent increase or decrease** and must calculate the original amount, or are given the final and original amounts and must calculate the percent increase or decrease. They use **double number lines** to visualize such situations in order to help see clearly which of the two amounts involved, the starting amount or the final amount, is to be regarded as the whole, or 100%.
7.4.8	+	D	In this lesson, students represent situations involving percent increase and percent decrease using equations. Students write equations like $y = 1.06x$ to represent growth of a bank account, and use the equation to answer questions about different starting amounts.
7.4.9	-	A	Until now, students have been working with whole number percentages when they solve **percent increase and percent decrease** problems. As they move towards more complex contexts such as interest rates, taxes, tips and measurement error, they will encounter percentages that are not necessarily whole numbers. In this lesson students consider situations where fractional percentages arise naturally.
7.4.10	+	E	In this lesson students are introduced to contexts involving **sales tax** and **tips**. They can use tape diagrams and double number lines from their grade 6 work, but the lesson provides an opportunity to be more efficient by using an equation of the form $y = kx$.
7.4.11	+	E	In this lesson students are introduced to contexts involving **markups**, **discounts**, and **commissions**, and continue to study contexts involving **tax** and **tips**.
7.4.12	0	D	In this lesson, students consolidate what they have learned over the last few lessons and solve a variety of multi-step **percentage** problems involving **taxes**, **tips**, and **discounts**, including problems involving fractional percentages.
7.4.13	0	D	This is the first of three lessons where students encounter the idea of **percent error**. Unlike situations involving **percent increase** and **percent decrease**, where there is an initial amount and a final amount, situations expressed with **percent error** involve a measured amount and a correct amount.
7.4.14	+	A	In this lesson students get practice using the language of **percent error** in various different situations, and identifying the correct amount, which is the whole, and the incorrect

			amount.
7.4.15	-	A	*This lesson is optional.* In this lesson, students find and analyze intervals of possible error based on maximum possible errors.
7.4.16	-	A	In this culminating lesson on **percentages**, students work in groups to collect news clippings that mention percentages and sort them according to whether they are about percent increase or percent decrease, formulate questions about them, and then share their questions with other groups in a gallery walk.

Lesson 14: Solving Percentage Problems

14.1: Number Talk: Multiplication with Decimals

Find the products mentally.

$6 \cdot (0.8) \cdot 2$

$(4.5) \cdot (0.6) \cdot 4$

14.2: Coupons

Han and Clare go shopping, and they each have a coupon. Answer each question and show your reasoning.

1. Han buys an item with a normal price of $15, and uses a 10% off coupon. How much does he save by using the coupon?

2. Clare buys an item with a normal price of $24, but saves $6 by using a coupon. For what percentage off is this coupon?

Are you ready for more?

Clare paid full price for an item. Han bought the same item for 80% of the full price. Clare said, "I can't believe I paid 125% of what you paid, Han!" Is what she said true? Explain.

14.3: Info Gap: Music Devices

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

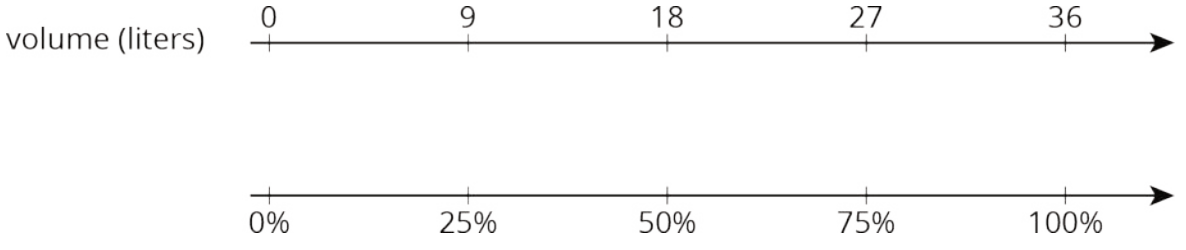
   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Lesson 14 Summary

A pot can hold 36 liters of water. What percentage of the pot is filled when it contains 9 liters of water?

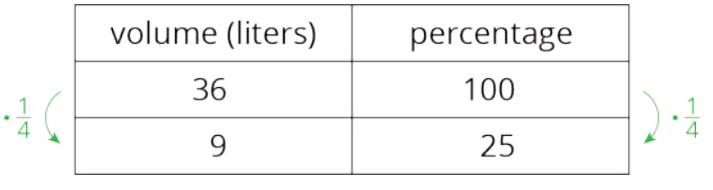
Here are two different ways to solve this problem:

- Using a double number line:



We can divide the distance between 0 and 36 into four equal intervals, so 9 is $\frac{1}{4}$ of 36, or 9 is 25% of 36.

- Using a table:



Info Gap: Music Devices

Problem Card 1

A store sells 3 different music devices:
Device A, Device B, and Device C.

1. Which of the devices can Jada afford?
2. What percentage of the money needed for Device B does she have?

Info Gap: Music Devices

Data Card 1

- Device A costs $15.
- Device B costs $25.
- Device C costs $40.
- Jada has 60% of the money needed to buy Device C.

Info Gap: Music Devices

Problem Card 2

The store starts selling another music device. Jada is interested in Device D, though she does not have enough money to buy it. How much does Device D cost?

Info Gap: Music Devices

Data Card 2

- Jada has $24.
- Jada has 40% of the money needed to buy Device D.

Info Gap: Music Devices

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Lesson 14: Solving Percentage Problems

Cool Down: Walking to School

It takes Jada 20 minutes to walk to school. It takes Andre 80% as long to walk to school.

How long does it take Andre to walk to school?

Unit 3 Lesson 14 Cumulative Practice Problems

1. For each problem, explain or show your reasoning.

    a. 160 is what percentage of 40?

    b. 40 is 160% of what number?

    c. What number is 40% of 160?

2. A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $13, what was the original price of her purchase? Explain or show your reasoning.

3. The original price of a scarf was $16. During a store-closing sale, a shopper saved $12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.

4. Select **all** the expressions whose value is larger than 100.

   A. 120% of 100

   B. 50% of 150

   C. 150% of 50

   D. 20% of 800

   E. 200% of 30

   F. 500% of 400

   G. 1% of 1,000

5. An ant travels at a constant rate of 30 cm every 2 minutes.

   a. At what pace does the ant travel per centimeter?

   b. At what speed does the ant travel per minute?

(From Unit 3, Lesson 8.)

6. Is $3\frac{1}{2}$ cups more or less than 1 liter? Explain or show your reasoning. (Note: 1 cup $\approx$ 236.6 milliliters)

(From Unit 3, Lesson 4.)

7. Name a unit of measurement that is about the same size as each object.

   a. The distance of a doorknob from the floor is about 1 _____.

   b. The thickness of a fingernail is about 1 _____.

   c. The volume of a drop of honey is about 1 _____.

   d. The weight or mass of a pineapple is about 1 _____.

   e. The thickness of a picture book is about 1 _____.

   f. The weight or mass of a buffalo is about 1 _____.

   g. The volume of a flower vase is about 1 _____.

   h. The weight or mass of 20 staples is about 1 _____.

   i. The volume of a melon is about 1 _____.

   j. The length of a piece of printer paper is about 1 _____.

(From Unit 3, Lesson 2.)

Lesson 8: Calculating Products of Decimals

8.1: Number Talk: Twenty Times a Number

Evaluate mentally.

$20 \cdot 5$

$20 \cdot (0.8)$

$20 \cdot (0.04)$

$20 \cdot (5.84)$

8.2: Calculating Products of Decimals

1. A common way to find a product of decimals is to calculate a product of whole numbers, then place the decimal point in the product.

Here is an example for $(2.5) \cdot (1.2)$.

Use what you know about decimals and place value to explain why the decimal point of the product is placed where it is.

$$\begin{array}{rr} & 25 \\ \times & 12 \\ \hline & 50 \\ + & 250 \\ \hline & 300 \end{array}$$

25 • 12 = 300

(2.5) • (1.2) = 3.00

2. Use the method shown in the first question to calculate each product.

   a. $(4.6) \cdot (0.9)$

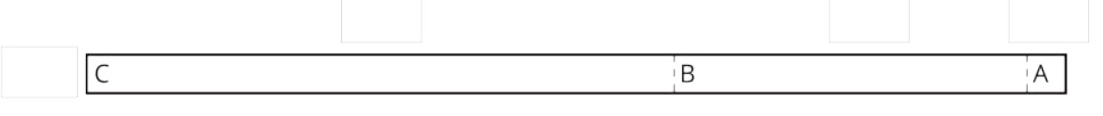
   b. $(16.5) \cdot (0.7)$

3. Use area diagrams to check your earlier calculations. For each problem:

   - Decompose each number into its base-ten units and write them in the boxes on each side of the rectangle.
   - Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.

   a. $(4.6) \cdot (0.9)$

   b. $(16.5) \cdot (0.7)$



4. About how many centimeters are in 6.25 inches if 1 inch is about 2.5 centimeters? Show your reasoning.

8.3: Practicing Multiplication of Decimals

1. Calculate each product. Show your reasoning. If you get stuck, consider drawing an area diagram to help.

   a. $(5.6) \cdot (1.8)$

   b. $(0.008) \cdot (7.2)$

2. A rectangular playground is 18.2 meters by 12.75 meters.

   a. Find its area in square meters. Show your reasoning.

   b. If 1 meter is approximately 3.28 feet, what are the approximate side lengths of the playground in feet? Show your reasoning.

Are you ready for more?

1. Write the following expressions as decimals.

   a. $1 - 0.1$

   b. $1 - 0.1 + 10 - 0.01$

   c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$

2. Describe the decimal that results as this process continues.

3. What would happen to the decimal if all of the addition and subtraction symbols became multiplication symbols? Explain your reasoning.

Lesson 8 Summary

We can use $84 \cdot 43$ and what we know about place value to find $(8.4) \cdot (4.3)$.

Since 8.4 is 84 tenths and 4.3 is 43 tenths, then:

$$(8.4) \cdot (4.3) = \frac{84}{10} \cdot \frac{43}{10} = \frac{84 \cdot 43}{100}$$

That means we can compute $84 \cdot 43$ and then divide by 100 to find $(8.4) \cdot (4.3)$.

$$84 \cdot 43 = 3612$$
$$(8.4) \cdot (4.3) = 36.12$$

Using fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ allows us to find the product of two decimals using the following steps:

- Write each decimal factor as a product of a whole number and a fraction.
- Multiply the whole numbers.
- Multiply the fractions.
- Multiply the products of the whole numbers and fractions.

We know multiplying by fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1{,}000}$ is the same as dividing by 10, 100, and 1,000, respectively. This means we can move the decimal point in the whole-number product to the left the appropriate number of spaces to correctly place the decimal point.

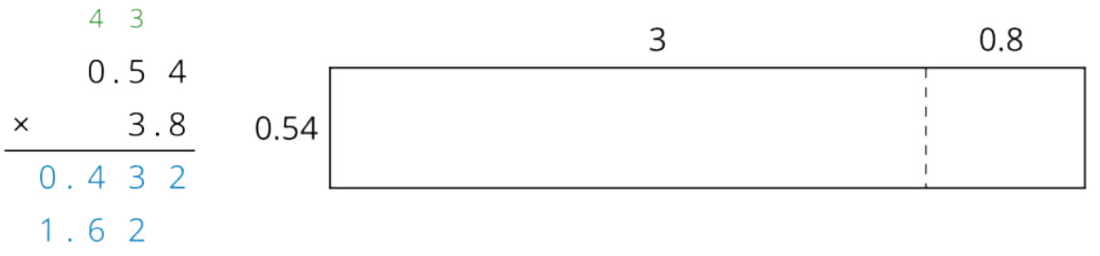
Lesson 8: Calculating Products of Decimals

Cool Down: Calculate This!

Calculate $(1.6) \cdot (0.215)$. Show your reasoning.

Unit 5 Lesson 8 Cumulative Practice Problems

1. Here are an unfinished calculation of $(0.54) \cdot (3.8)$ and a 0.54-by-3.8 rectangle.



a. Which part of the rectangle has an area of 0.432? Which part of the rectangle has an area of 1.62? Show your reasoning.

b. What is $(0.54) \cdot (3.8)$?

2. Explain how the product of 3 and 65 could be used to find $(0.03) \cdot (0.65)$.

3. Use vertical calculation to find each product.

a. $(5.4) \cdot (2.4)$

b. $(1.67) \cdot (3.5)$

4. A pound of blueberries costs $3.98 and a pound of clementines costs $2.49. What is the combined cost of 0.6 pound of blueberries and 1.8 pounds of clementines? Round your answer to the nearest cent.

5. Complete the calculations so that each shows the correct sum or difference.

a.

$$\begin{array}{r} 2.14\square \\ +\ 1.\square 25 \\ \hline \square.865 \end{array}$$

b.

$$\begin{array}{r} 29.\square\square \\ +\ \ 1.42 \\ \hline \square\square.41 \end{array}$$

c.

$$\begin{array}{r} 6.1\square\square \\ -\ 1.009 \\ \hline \square.\square 48 \end{array}$$

(From Unit 5, Lesson 3.)

6. Which has a greater value: $7.4 - 0.0022$ or $7.39 - 0.0012$? Show your reasoning.

(From Unit 5, Lesson 4.)

7. Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings? If you get stuck, consider using the table.

number of saplings	time in minutes
3	30
1	
14	

(From Unit 2, Lesson 12.)

Lesson 11: Dividing Numbers that Result in Decimals

11.1: Number Talk: Evaluating Quotients

Find the quotients mentally.

$400 \div 8$

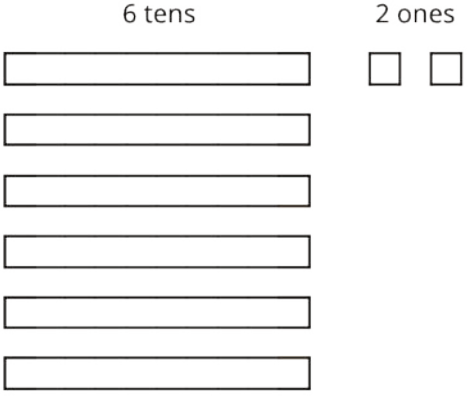
$80 \div 8$

$16 \div 8$

$496 \div 8$

11.2: Keep Dividing

Mai used base-ten diagrams to calculate $62 \div 5$. She started by representing 62.



She then made 5 groups, each with 1 ten. There was 1 ten left. She unbundled it into 10 ones and distributed the ones across the 5 groups.

Here is Mai's diagram for $62 \div 5$.

1. Discuss these questions with a partner and write down your answers:

   a. Mai should have a total of 12 ones, but her diagram shows only 10. Why?

   b. She did not originally have tenths, but in her diagram each group has 4 tenths. Why?

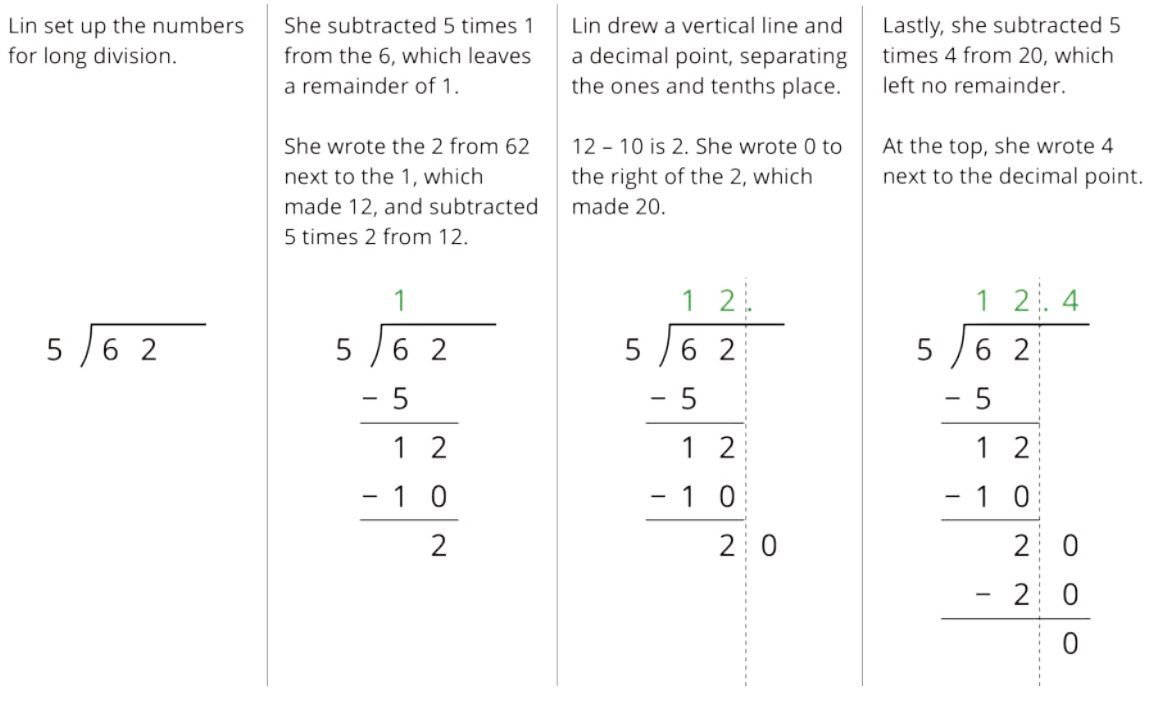
   c. What value has Mai found for $62 \div 5$? Explain your reasoning.

2. Find the quotient of $511 \div 5$ by drawing base-ten diagrams or by using the partial quotients method. Show your reasoning. If you get stuck, work with your partner to find a solution.

3. Four students share a $271 prize from a science competition. How much does each student get if the prize is shared equally? Show your reasoning.

11.3: Using Long Division to Calculate Quotients

Here is how Lin calculated $62 \div 5$.



1. Discuss with your partner:
    - Lin put a 0 after the remainder of 2. Why? Why does this 0 not change the value of the quotient?
    - Lin subtracted 5 groups of 4 from 20. What value does the 4 in the quotient represent?
    - What value did Lin find for $62 \div 5$?

2. Use long division to find the value of each expression. Then pause so your teacher can review your work.

   a. $126 \div 8$

   b. $90 \div 12$

3. Use long division to show that:

   a. $5 \div 4$, or $\frac{5}{4}$, is 1.25.

   b. $4 \div 5$, or $\frac{4}{5}$, is 0.8.

   c. $1 \div 8$, or $\frac{1}{8}$, is 0.125.

   d. $1 \div 25$, or $\frac{1}{25}$, is 0.04.

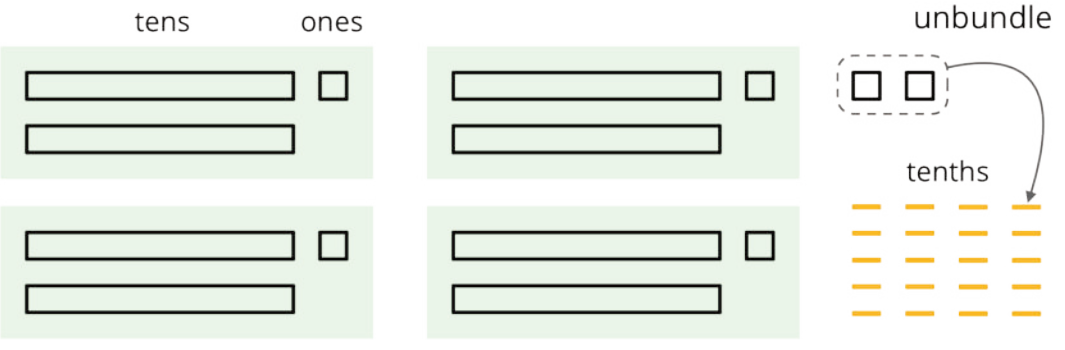
4. Noah said we cannot use long division to calculate $10 \div 3$ because there will always be a remainder.

   a. What do you think Noah meant by "there will always be a remainder"?

   b. Do you agree with him? Explain your reasoning.

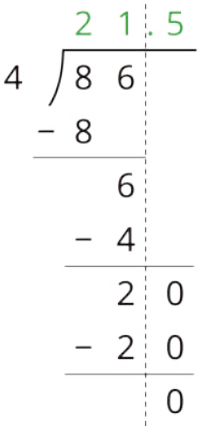
Lesson 11 Summary

Dividing a whole number by another whole number does not always produce a whole-number quotient. Let's look at $86 \div 4$, which we can think of as dividing 86 into 4 equal groups.



We can see in the base-ten diagram that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we can unbundle or decompose the 2 ones into 20 tenths, which enables us to put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.



We can also calculate $86 \div 4$ using long division.

The calculation shows that, after removing 4 groups of 21, there are 2 ones remaining. We can continue dividing by writing a 0 to the right of the 2 and thinking of that remainder as 20 tenths, which can then be divided into 4 groups.

To show that the quotient we are working with now is in the tenth place, we put a decimal point to the right of the 1 (which is in the ones place) at the top. It may also be helpful to draw a vertical line to separate the ones and the tenths.

There are 4 groups of 5 tenths in 20 tenths, so we write 5 in the tenths place at the top. The calculation likewise shows $86 \div 4 = 21.5$.

Lesson 11: Dividing Numbers that Result in Decimals

Cool Down: Calculating Quotients

Use long division to find each quotient. Show your computation and write your answer as a decimal.

1. $22 \div 5$

2. $7 \div 8$

Unit 5 Lesson 11 Cumulative Practice Problems

1. Use long division to show that the fraction and decimal in each pair are equal.

   $\frac{3}{4}$ and 0.75 $\frac{3}{50}$ and 0.06 $\frac{7}{25}$ and 0.28

2. Mai walked $\frac{1}{8}$ of a 30-mile walking trail. How many miles did Mai walk? Explain or show your reasoning.

3. Use long division to find each quotient. Write your answer as a decimal.

   a. $99 \div 12$

   b. $216 \div 5$

   c. $1{,}988 \div 8$

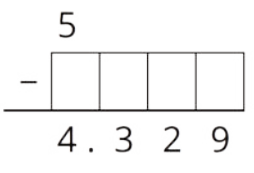
4. Tyler reasoned: "$\frac{9}{25}$ is equivalent to $\frac{18}{50}$ and to $\frac{36}{100}$, so the decimal of $\frac{9}{25}$ is 0.36."

   a. Use long division to show that Tyler is correct.

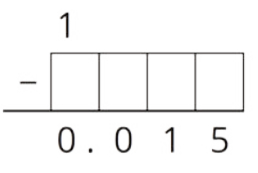
   b. Is the decimal of $\frac{18}{50}$ also 0.36? Use long division to support your answer.

5. Complete the calculations so that each shows the correct difference.

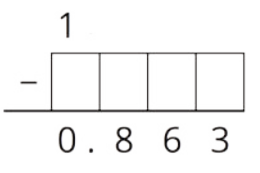
a.



b.



c.



(From Unit 5, Lesson 4.)

6. Use the equation $124 \cdot 15 = 1{,}860$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.24) \cdot (0.15)$.

(From Unit 5, Lesson 6.)