Plan for Grade 7 Unit 3: Measuring Circles

Relevant Unit(s) to review: Grade 6 Unit 1: Area and Surface Area

Essential prior concepts to engage with this unit	• Find the perimeter of polygons. • Find the constant of proportionality.
Brief narrative of approach	Begin by reviewing how to find the area of parallelograms (6.1.6) and triangles (6.1.9), then review finding the constant of proportionality. This leads to reasoning that the circumference of a circle is proportional to its diameter, with constant of proportionality π . Finally, students encounter informal derivations of the relationship between area, circumference, and radius.

Lessons to Add	Lessons to Remove or Modify
1. 6.1.6 - Focus on area of parallelograms 2. 6.1.9 - Focus on area of triangles	1. Remove 7.3.5 - lesson is optional 2. Remove 7.3.11 - lesson can be removed or moved to outside of class if time does not allow
Lessons added: 2	Lessons removed: 2

Modified Plan for Grade 7 Unit 3

Day	IM lesson	Notes
	7.3 Check Your Readiness	Note that the Check Your Readiness Assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in 6.1. Questions 1-3 focus on area.
1	6.1.6	Focus on finding areas of parallelograms (Activity 2 and 3).
2	6.1.9	Focus on finding the area of triangles (Activity 1-4).
3	7.3.1	
4	7.3.2	
5	7.3.3	
6	7.3.4	
7	7.3.6	
8	7.3.7	
9	7.3.8	
10	7.3.9	
11	7.3.10	Activity 3 is optional.
12	7.3 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
7.3.1	0	E	The purpose of this lesson is for students to apply what they have learned about proportional relationships to describing geometric figures. The work in this lesson focuses on squares.
7.3.2	+	E	This is the first lesson on circles in a unit that develops and applies methods to find the **circumference** and area of a **circle**. In this lesson, students move from the informal idea of a circle as "a round figure" to the more formal definition that a circle is the set of points that are equally distant from the center, enclosing a circular region.
7.3.3	+	D	In this lesson, students discover that there is a proportional relationship between the **diameter** and **circumference** of a **circle**. They use their knowledge from the previous unit on proportionality to estimate the constant of proportionality. Then they use the constant to compute the diameter given the circumference (and vice versa) for different circles.
7.3.4	+	A	In this lesson, students use the equation $C = \pi d$ to solve problems in a variety of contexts. They compute the **circumference** of **circles** and parts of circles given **diameter** or **radius**, and vice versa. Students develop flexibility using the relationships between **diameter**, **radius**, and **circumference** rather than memorizing a variety of formulas. Understanding the equation $C = 2\pi r$ will help with the transition to the study of area in future lessons.
7.3.5	-	A	*This lesson is optional.* The goal of this lesson is to apply students' understanding of **circumference** to calculate how far a wheel travels when it rolls a certain number of times. This relationship is vital for how odometers and speedometers work in vehicles.
7.3.6	-	E	The purpose of this lesson is for students to practice composing and decomposing irregular regions to calculate their area, in preparation for estimating the **area of circles** in the next lesson.

7.3.7	+	E	This lesson is the first of two lessons that develop the formula for the **area of a circle**. Students start by estimating the area inside different **circles**, deepening their understanding of the concept of area as the number of unit squares that cover a region, and discovering that **area** (unlike **circumference**) is not proportional to **diameter**.
7.3.8	+	D	In the previous lesson, students found that it takes a little more than 3 squares with side lengths equal to the circle's **radius** to completely cover a **circle**. Students may have predicted that the **area of a circle** can be found by multiplying πr^2 .
7.3.9	+	A	In previous lessons, students estimated the **area of circles** on a grid and explored the relationship between the **circumference** and the **area of a circle** to see that $A = \pi r^2$. In this lesson, students apply this formula to solve problems involving the **area of circles** as well as shapes made up of parts of circles and other shapes such as rectangles.
7.3.10	+	A	Students have spent several lessons investigating **circumference**, and then several lessons investigating **area**, separately. In this lesson, both types of problems are mixed together so students have to distinguish which measurement is called for in each problem situation. *Activity 3 is optional.*
7.3.11	0	A	*This culminating lesson is optional.* In this lesson students work on several tasks that combine **circumference** and **area** ideas and computations.

Lesson 6: Area of Parallelograms

6.1: Missing Dots

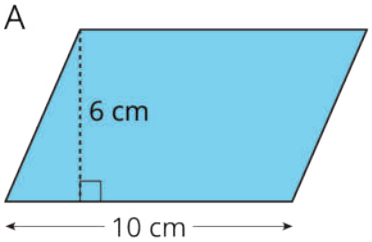
How many dots are in the image?

How do you see them?

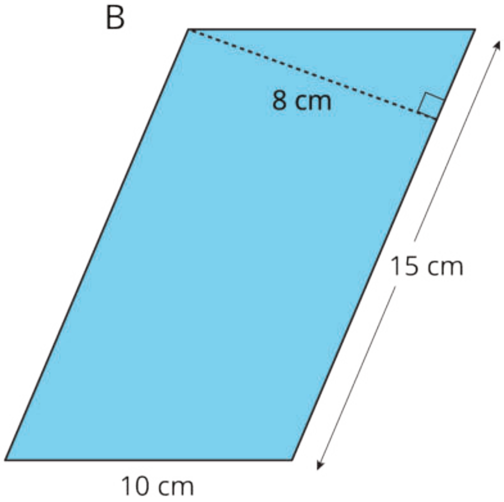
6.2: More Areas of Parallelograms

1. Find the area of each parallelogram. Show your reasoning.

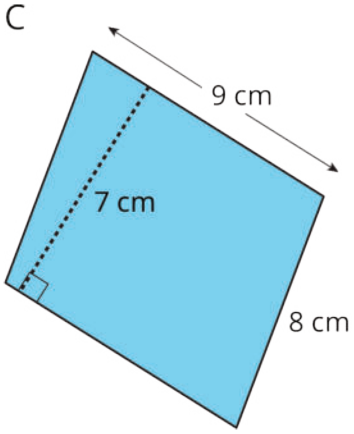
A



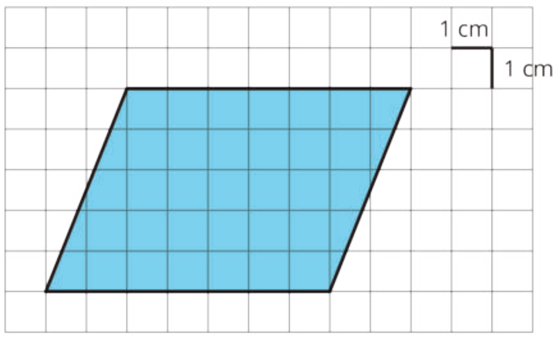
B



C



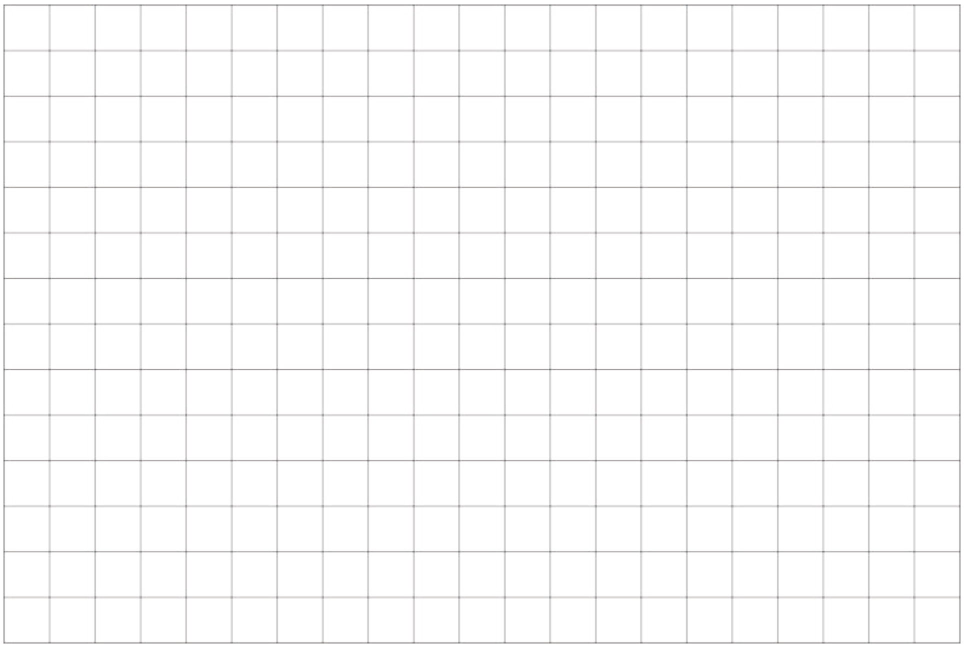
D



2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

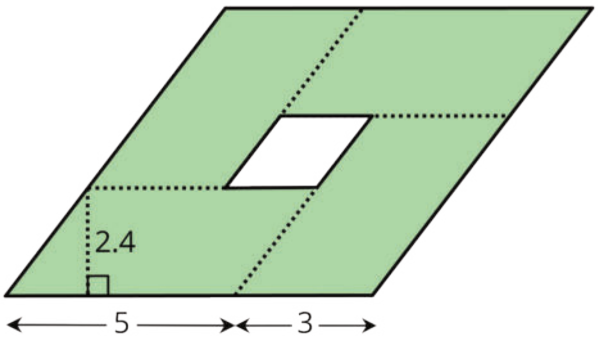
3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

   On the grid, draw two parallelograms that could be P and Q.



Are you ready for more?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

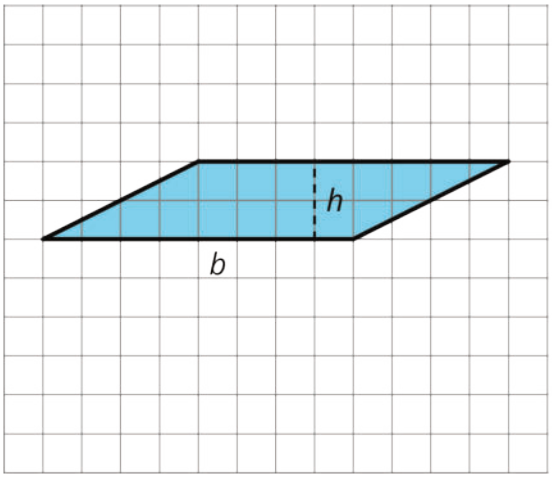


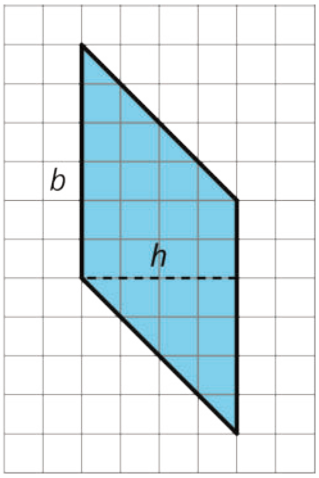
What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

Lesson 6 Summary

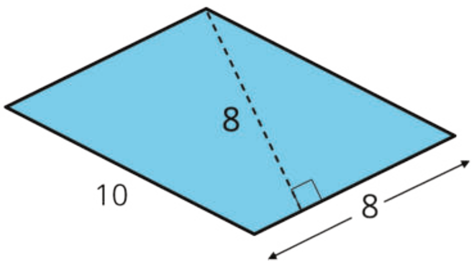
Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.



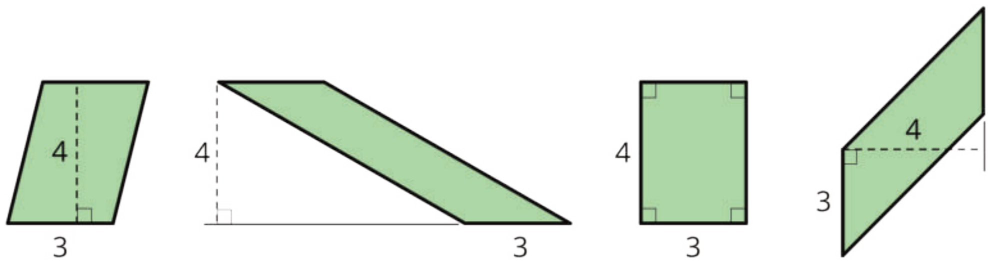


When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



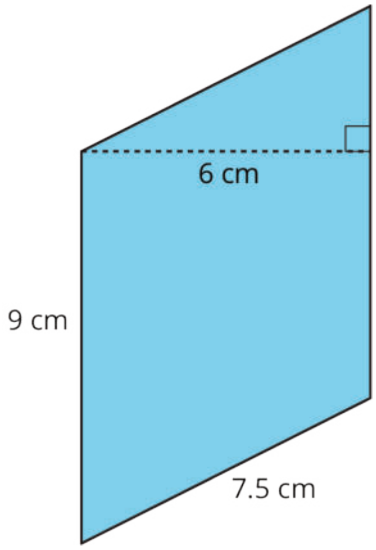
In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.



Lesson 6: Area of Parallelograms

Cool Down: One More Parallelogram

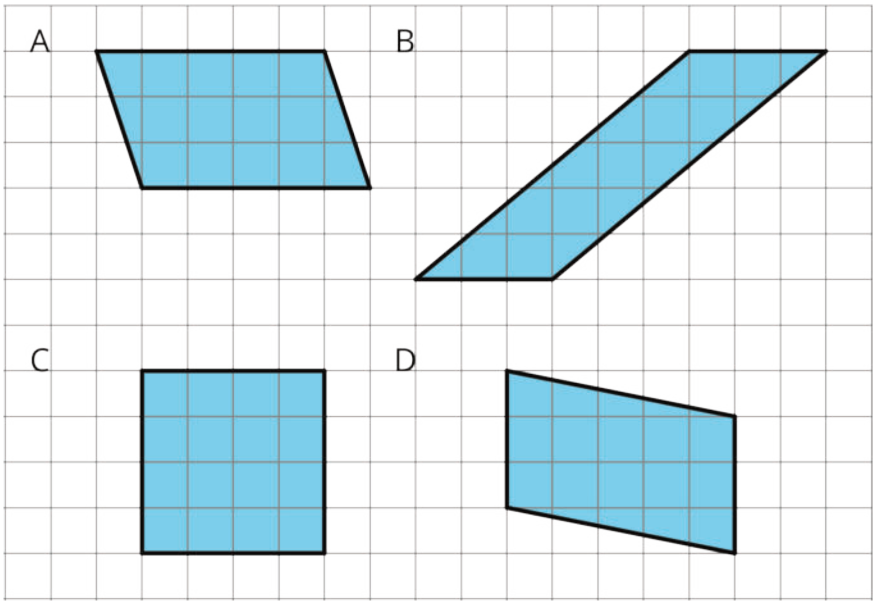


1. Find the area of the parallelogram. Explain or show your reasoning.

2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

Unit 1 Lesson 6 Cumulative Practice Problems

1. Which three of these parallelograms have the same area as each other?



A. A

B. B

C. C

D. D

2. Which pair of base and height produces the greatest area? All measurements are in centimeters.

A. $b = 4$, $h = 3.5$

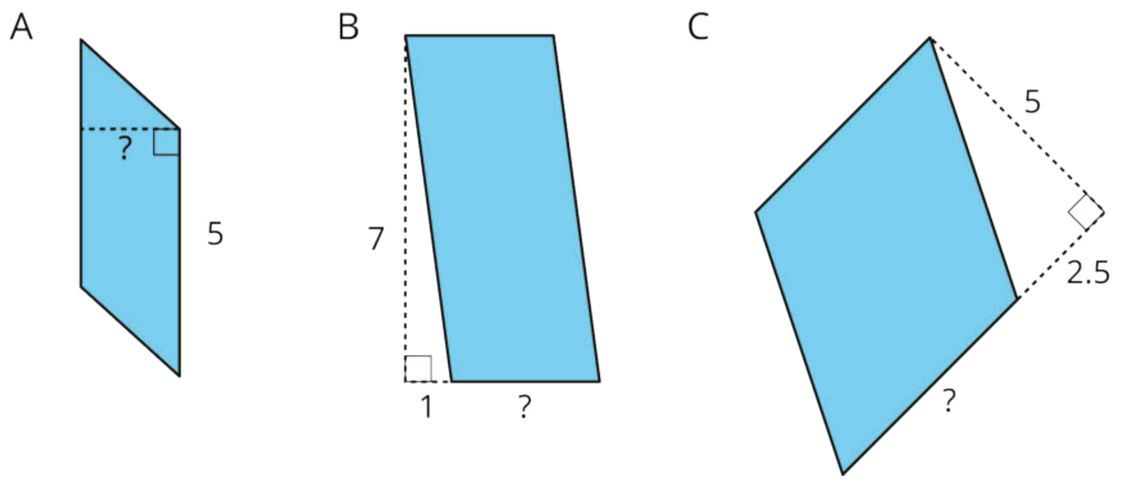
B. $b = 0.8$, $h = 20$

C. $b = 6$, $h = 2.25$

D. $b = 10$, $h = 1.4$

3. Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

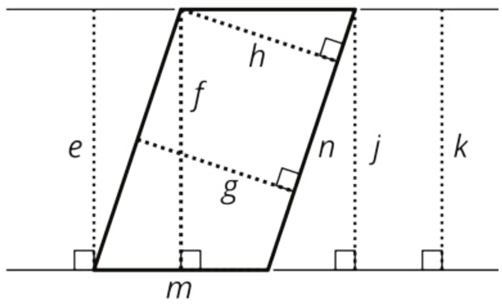
A: 10 square units		B: 21 square units		C: 25 square units



4. The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

5. Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

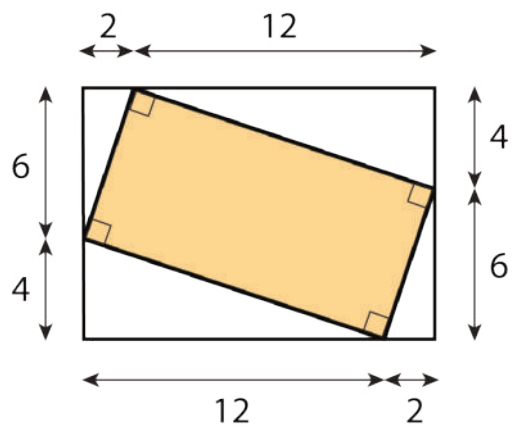
E. j

F. k

G. n

(From Unit 1, Lesson 5.)

6. Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



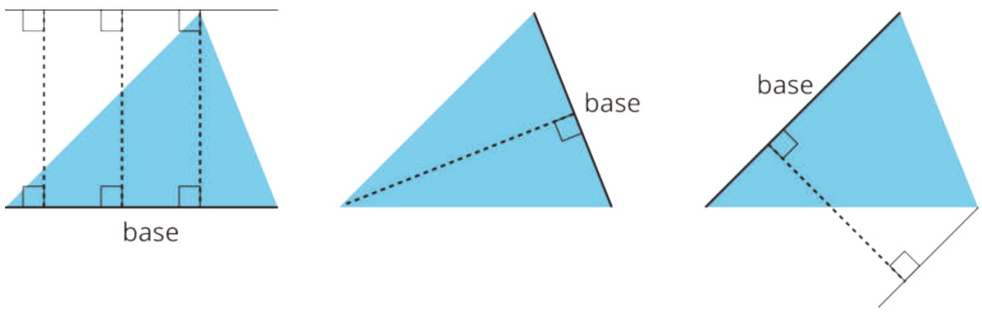
(From Unit 1, Lesson 3.)

Lesson 9: Formula for the Area of a Triangle

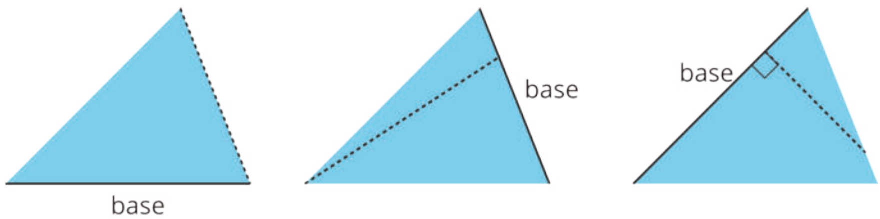
9.1: Bases and Heights of a Triangle

Study the examples and non-examples of **bases** and **heights** in a triangle.

- Examples: These dashed segments represent heights of the triangle.



- Non-examples: These dashed segments do *not* represent heights of the triangle.



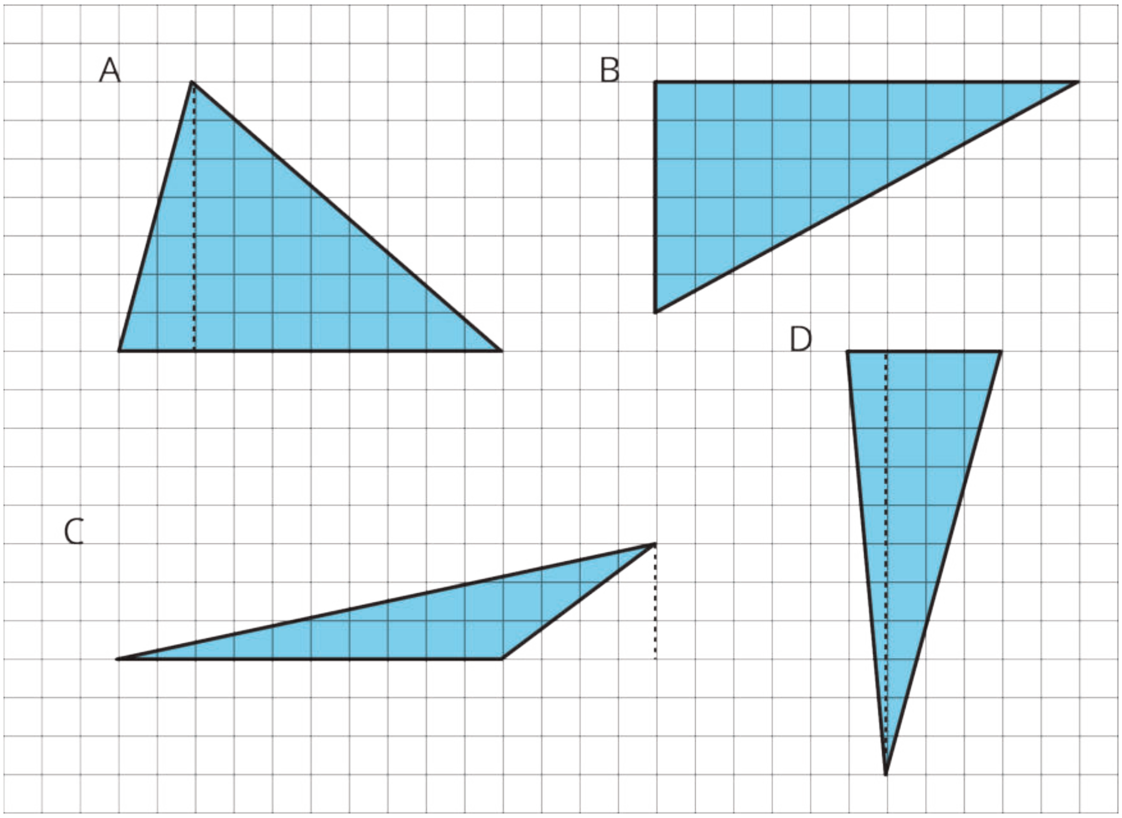
Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.
2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at an acute angle to the base.
5. A height that corresponds to a base must be drawn at a right angle to the base.
6. Once we choose a base, there is only one segment that represents the corresponding height.
7. A segment representing a height must go through a vertex.

9.2: Finding a Formula for Area of a Triangle

For each triangle:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the triangle and record it in the last column of the table.

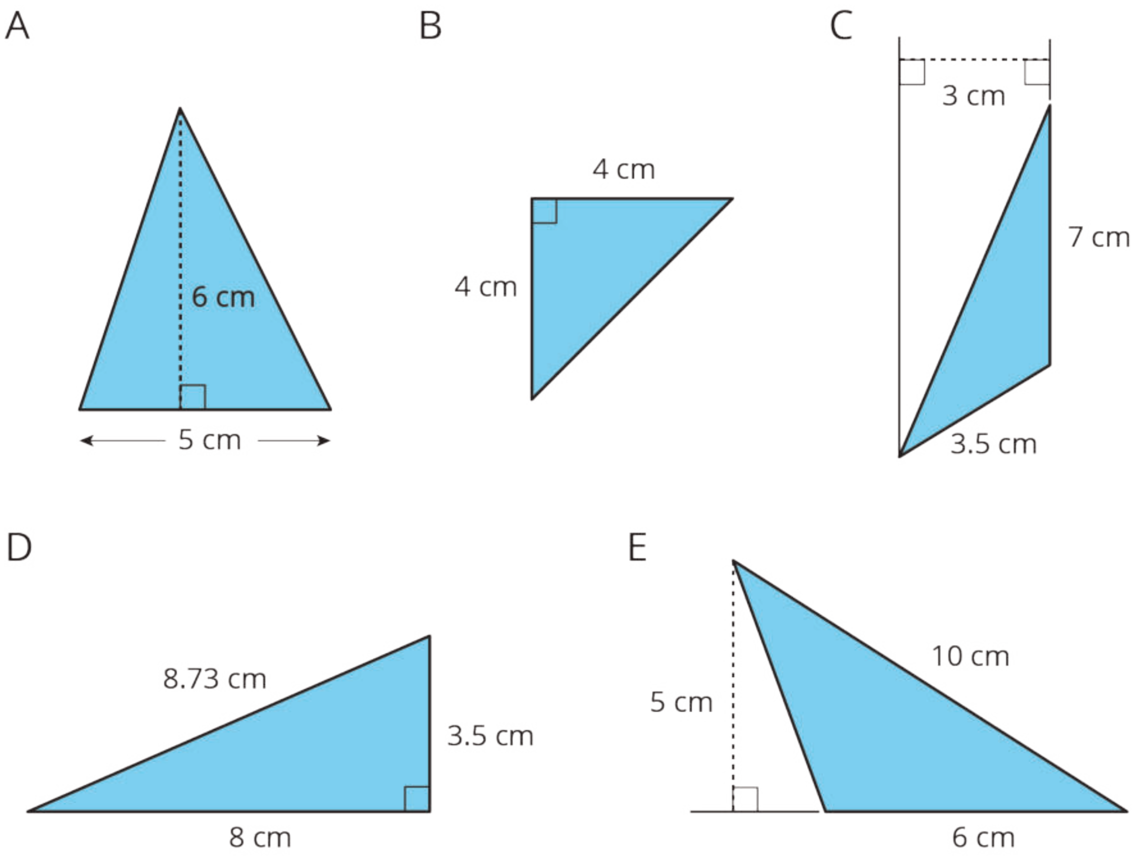


triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

In the last row, write an expression for the area of any triangle, using b and h.

9.3: Applying the Formula for Area of Triangles

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.

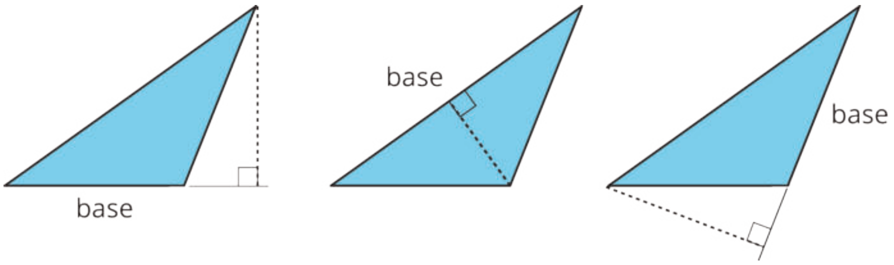

A
6 cm
5 cm
B
4 cm
4 cm
C
3 cm
7 cm
3.5 cm
D
8.73 cm
3.5 cm
8 cm
E
10 cm
5 cm
6 cm

Lesson 9 Summary

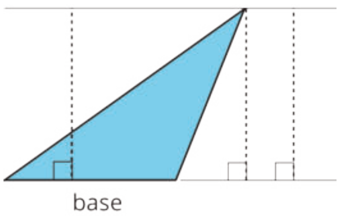
- We can choose any of the three sides of a triangle to call the **base**. The term "base" refers to both the side and its length (the measurement).

- The corresponding **height** is the length of a perpendicular segment from the base to the vertex opposite of it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

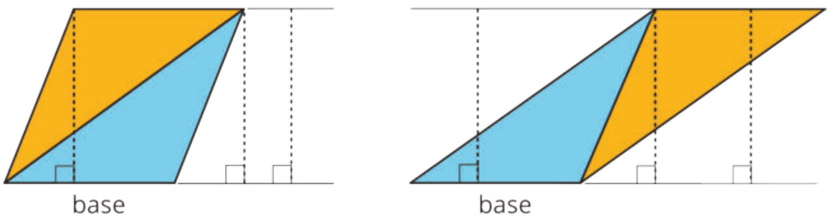
Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.



A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram shares at least one base with the triangle.

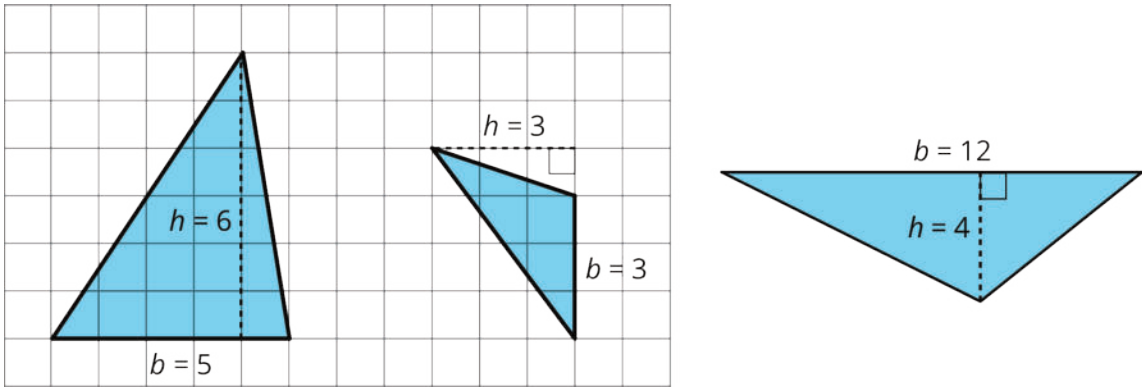


For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area A of a triangle as:

$$A = \frac{1}{2} \cdot b \cdot h$$

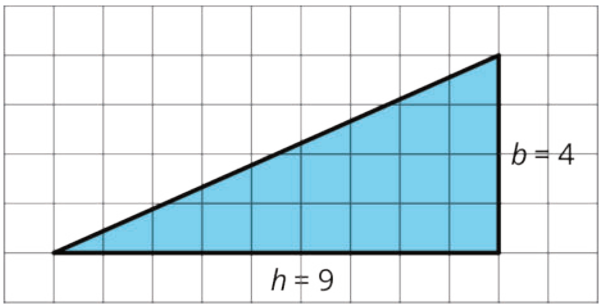


- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.
- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

In right triangles, however, the two sides that are perpendicular can be a base and a height.

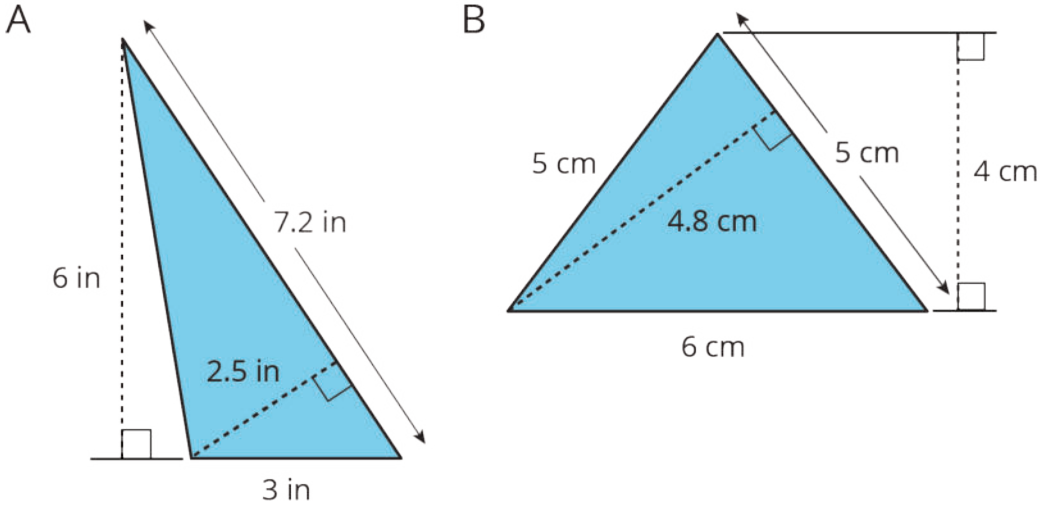
The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.



Lesson 9: Formula for the Area of a Triangle

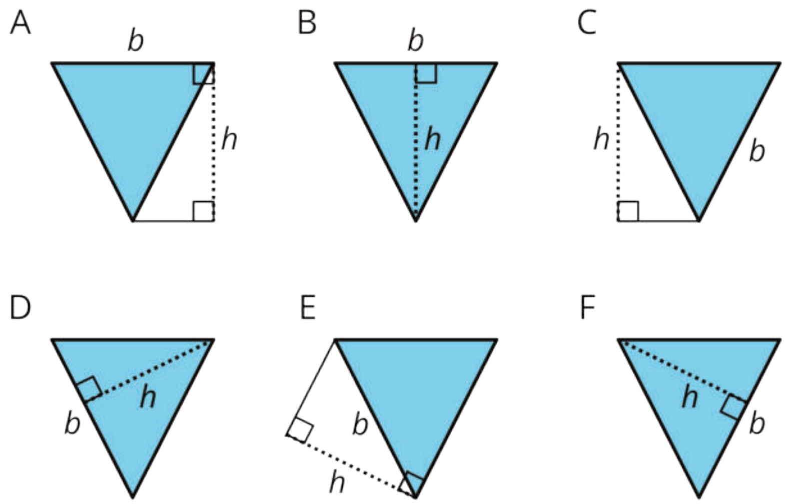
Cool Down: Two More Triangles

For each triangle, identify a base and a corresponding height. Use them to find the area. Show your reasoning.



Unit 1 Lesson 9 Cumulative Practice Problems

1. Select **all** drawings in which a corresponding height h for a given base b is correctly identified.



A. A

B. B

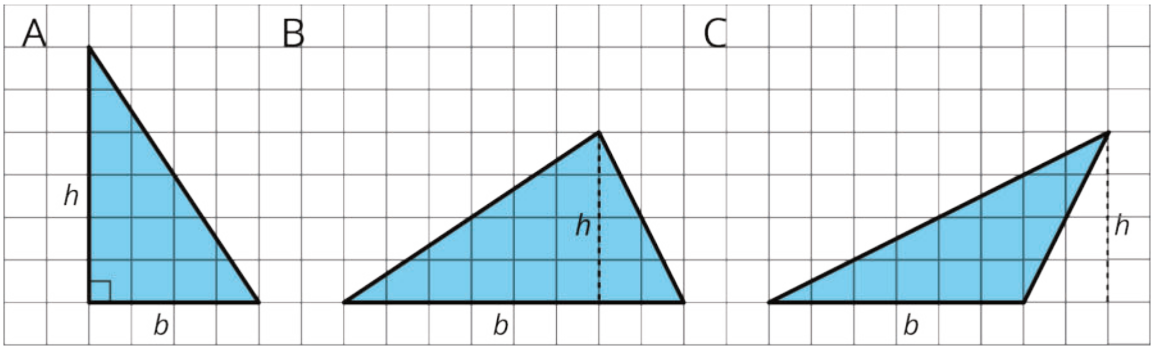
C. C

D. D

E. E

F. F

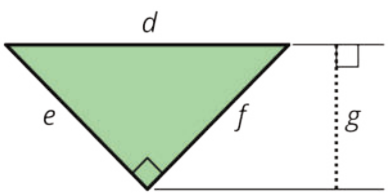
2. For each triangle, a base and its corresponding height are labeled.



a. Find the area of each triangle.

b. How is the area related to the base and its corresponding height?

3. Here is a right triangle. Name a corresponding height for each base.

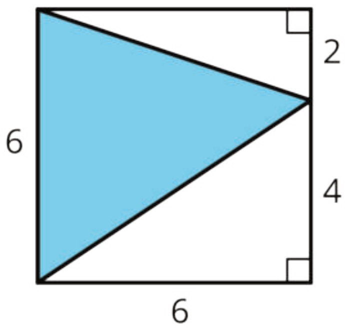


a. Side d

b. Side e

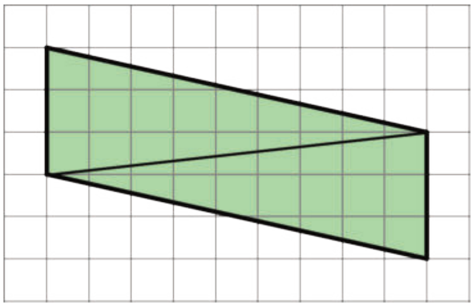
c. Side f

4. Find the area of the shaded triangle. Show your reasoning.



(From Unit 1, Lesson 8.)

5. Andre drew a line connecting two opposite corners of a parallelogram. Select **all** true statements about the triangles created by the line Andre drew.



A. Each triangle has two sides that are 3 units long.

B. Each triangle has a side that is the same length as the diagonal line.

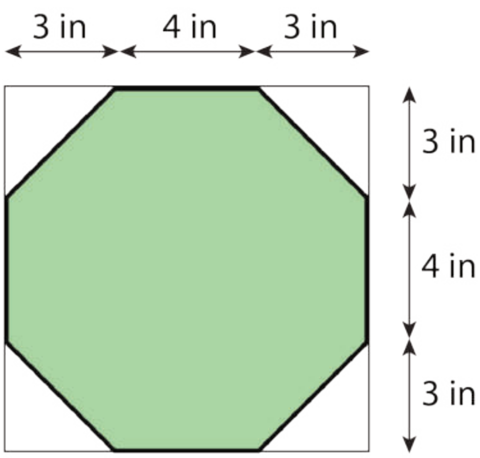
C. Each triangle has one side that is 3 units long.

D. When one triangle is placed on top of the other and their sides are aligned, we will see that one triangle is larger than the other.

E. The two triangles have the same area as each other.

(From Unit 1, Lesson 7.)

6. Here is an octagon. (Note: The diagonal sides of the octagon are *not* 4 inches long.)



a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 square inches. Do you agree? Explain your reasoning.

b. Find the exact area of the octagon. Show your reasoning.

(From Unit 1, Lesson 3.)