Plan for Grade 7 Unit 2: Introducing Proportional Relationships

Relevant Unit(s) to review: Grade 6 Unit 3 Unit Rates and Percentages
Grade 6 Unit 6 Expressions and Equations

Essential prior concepts to engage with this unit	• equivalent ratios • derived units: miles per hour; meters per second; dollars per pound; or cents per minute
Brief narrative of approach	Begin by looking at equivalent ratios and their relationship with unit rates (6.3.5–6.3.8), then introduce the concept of proportional relationships by looking at tables of equivalent ratios (7.2.1–7.2.5). Next, use representations to compare rates and consider how each of the representations would change if the independent and dependent variables were switched (6.6.15–6.6.17). This leads to understanding that a proportional relationship can be represented by an equation of the form $y = kx$ where k is the constant of proportionality (7.2.8).

Lessons to Add	**Lessons to Remove or Modify**
1. 6.3.5 - Prioritize activities 2 and 3 2. 6.3.6 3. 6.3.7 - Prioritize activities 2 and 3 4. 6.6.15 5. Combine 6.6.16 and 6.6.17	1. Remove 7.2.6 - additional practice that could be done outside of class 2. Combine 7.2.7 and 7.2.8 - both lessons focus on comparing relationships (7.2.7 - activities 2 and 3, 7.2.8 - activities 2 and 3) 3. Remove 7.2.15 - optional activity
Lessons added: 5	Lessons removed: 3

Modified Plan for Grade 7 Unit 2

Day	IM lesson	Notes
	Diagnostic assessment	7.2 Check Your Readiness Assessment Note that the Check Your Readiness Assessment includes item-by-item guidance to inform just-in-time adjustments to instruction within the lessons in 7.2.
1	6.3.5	If the initial assessment shows that students are not familiar with equivalent ratios, include this activity before continuing with grade-level content.
2	6.3.6	If the initial assessment shows that students are not familiar with unit rates associated with a ratio, include this activity before continuing with grade-level content.
3	6.3.7	If the initial assessment shows that students are not familiar with using tables to examine equivalent ratios, include this activity before continuing with grade-level content.
4	7.2.1	Focus on key ideas in proportional relationships
5	7.2.2	
6	7.2.3	
7	7.2.4	
8	7.2.5	
9	6.6.15	If the initial assessment shows that students are not familiar with evaluating expressions involving exponents, include this activity before continuing with grade-level content.
10	6.6.16 6.6.17	If the initial assessment shows that students are not familiar with independent and dependent variables, include these activities before continuing with grade level content. Consider combining the lessons focusing on activities 2 and 3 in each lesson. Focus on dependent and independent variables.
11	7.2.7 7.2.8	Combine these lessons focusing on activities 2 and 3 in each lesson. Focus on comparing proportional and nonproportional relationships in tables and equations.
12	7.2.9	

13	7.2.10	
14	7.2.11	
15	7.2.12	
16	7.2.13	
17	7.2.14	
18	7.2 End	7.2 End of Unit Assessment

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
7.2.1	+	E	The activities in the lesson are intended to support initial, informal conversations about the key ideas in **proportional relationships** before the next lesson introduces the terms for those ideas.
7.2.2	+	D	The purpose of this lesson is to introduce the concept of a **proportional relationship** by looking at tables of **equivalent ratios**.
7.2.3	+	A	In this lesson, students continue to work with **proportional relationships** represented by tables using contexts familiar from previous grades: unit conversion and constant speed.
7.2.4	0	A	In this lesson, students build on their work with tables and represent **proportional relationships** using equations of the form $y = kx$.
7.2.5	+	E	In this lesson, students write equations for **proportional relationships** two ways, and they see why the two constants of proportionality associated with each way are **reciprocals** of each other.
7.2.6	-	A	In the previous two lessons students learned to represent **proportional relationships** with equations of the form $y = kx$. In this lesson they continue to write equations, and they begin to see situations where using the equation is a more efficient way of solving problems than other methods they have been using, such as tables and **equivalent ratios**.
7.2.7	-	E	In this lesson, students examine tables and explain whether the relationships represented are proportional, not proportional, or possibly proportional.
7.2.8	+	D	This lesson continues the work students did in the previous lesson on comparing **proportional** and **nonproportional relationships**.

7.2.9	+	A	In this lesson students learn to recognize **proportional relationships** from descriptions of the context.
7.2.10	+	E	This lesson introduces an important way of representing a **proportional relationship**: its graph. Students plot points on the graph from tables, and, by the end of the lesson, start to see that the graph of a **proportional relationship** always lies on a line that passes through the **origin**.
7.2.11	+	D	In this lesson, students start to make connections between the graph and the context modeled by the **proportional relationship**, and between the graph and the equation for the **proportional relationship**.
7.2.12	0	A	In this lesson students continue their work with interpreting graphs of **proportional relationships**.
7.2.13	+	E	In this lesson students focus on the relationship between the graph and the equation of a **proportional relationship**. They start with an activity designed to help them see all the different ways in which the graph and the equation are connected.
7.2.14	0	A	In this lesson, students examine tables, equations, and graphs of **proportional relationships**, and use them to reason about relationships that are proportional as well as relationships that are not proportional. Posters can be modified or eliminated for time.
7.2.15	-	A	In this lesson, students use their understanding of **proportional relationships** to explore whether baths or showers use more water. *Optional activity.*

Lesson 5: Comparing Speeds and Prices

5.1: Closest Quotient

Is the value of each expression closer to $\frac{1}{2}$, 1, or $1\frac{1}{2}$?

1. $20 \div 18$
2. $9 \div 20$
3. $7 \div 5$

5.2: More Treadmills

Some students did treadmill workouts, each one running at a constant speed. Answer the questions about their workouts. Explain or show your reasoning.

- Tyler ran 4,200 meters in 30 minutes.
- Kiran ran 6,300 meters in $\frac{1}{2}$ hour.
- Mai ran 6.3 kilometers in 45 minutes.

1. What is the same about the workouts done by:
    a. Tyler and Kiran?

    b. Kiran and Mai?

    c. Mai and Tyler?

2. At what rate did each of them run?

3. How far did Mai run in her first 30 minutes on the treadmill?

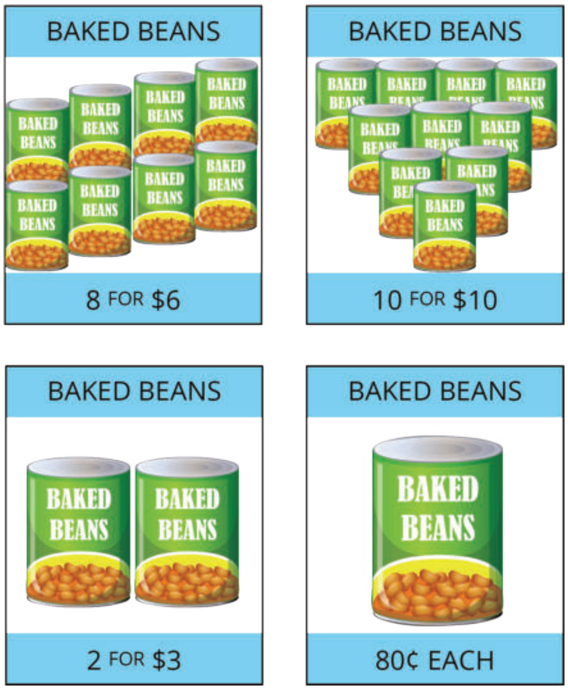
Are you ready for more?

Tyler and Kiran each started running at a constant speed at the same time. Tyler ran 4,200 meters in 30 minutes and Kiran ran 6,300 meters in $\frac{1}{2}$ hour. Eventually, Kiran ran 1 kilometer more than Tyler. How much time did it take for this to happen?

5.3: The Best Deal on Beans

Four different stores posted ads about special sales on 15-oz cans of baked beans.

1. Which store is offering the best deal? Explain your reasoning.



2. The last store listed is also selling 28-oz cans of baked beans for $1.40 each. How does that price compare to the other prices?

Lesson 5 Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

  It is often helpful to compare distances traveled in *1 unit* of time (1 minute, for example), which means finding the speed such as meters per minute.

Let’s compare Diego and Andre’s speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

distance (meters)	time (minutes)
2,550	17
150	1

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity A per 1 unit of quantity B is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *dollars per pound*.

Lesson 5: Comparing Speeds and Prices

Cool Down: A Sale on Sparkling Water

Bottles of sparkling water usually cost $1.69 each. This week they are on sale for 4 bottles for $5. You bought one last week and one this week. Did you pay more or less for the bottle this week? How much more or less?

Unit 3 Lesson 5 Cumulative Practice Problems

1. Mai and Priya were on scooters. Mai traveled 15 meters in 6 seconds. Priya travels 22 meters in 10 seconds. Who was moving faster? Explain your reasoning.

2. Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

   Store X: 4 cans for $2.48 Store Y: 5 cans for $3.00 Store Z: 59 cents per can

3. Costs of homes can be very different in different parts of the United States.

   a. A 450-square-foot apartment in New York City costs $540,000. What is the price per square foot? Explain or show your reasoning.

   b. A 2,100-square-foot home in Cheyenne, Wyoming, costs $110 per square foot. How much does this home cost? Explain or show your reasoning.

4. There are 33.8 fluid ounces in a liter. There are 128 fluid ounces in a gallon. About how many liters are in a gallon?

    a. 2

    b. 3

    c. 4

    d. 5

    Is your estimate larger or smaller than the actual number of liters in a gallon? Explain how you know.

    (From Unit 3, Lesson 4.)

5. Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

    (From Unit 3, Lesson 3.)

6. Name an object that could be about the same length as each measurement.

    a. 4 inches

    b. 6 feet

    c. 1 meter

    d. 5 yards

    a. 6 centimeters

    b. 2 millimeters

    c. 3 kilometers

    (From Unit 3, Lesson 2.)

Lesson 6: Interpreting Rates

6.1: Something per Something

1. Think of two things you have heard described in terms of "something per something."

2. Share your ideas with your group, and listen to everyone else's idea. Make a group list of all unique ideas. Be prepared to share these with the class.

6.2: Cooking Oatmeal

Priya, Han, Lin, and Diego are all on a camping trip with their families. The first morning, Priya and Han make oatmeal for the group. The instructions for a large batch say, "Bring 15 cups of water to a boil, and then add 6 cups of oats."

- Priya says, "The ratio of the cups of oats to the cups of water is $6 : 15$. That's 0.4 cups of oats per cup of water."
- Han says, "The ratio of the cups of water to the cups of oats is $15 : 6$. That's 2.5 cups of water per cup of oats."

1. Who is correct? Explain your reasoning. If you get stuck, consider using the table.

water (cups)	oats (cups)
15	6
1	
	1

2. The next weekend after the camping trip, Lin and Diego each decide to cook a large batch of oatmeal to have breakfasts ready for the whole week.

a. Lin decides to cook 5 cups of oats. How many cups of water should she boil?

b. Diego boils 10 cups of water. How many cups of oats should he add into the water?

3. Did you use Priya's rate (0.4 cups of oats per cup of water) or Han's rate (2.5 cups of water per cup of oats) to help you answer each of the previous two questions? Why?

6.3: Cheesecake, Milk, and Raffle Tickets

For each situation, find the **unit rates**.

1. A cheesecake recipe says, "Mix 12 oz of cream cheese with 15 oz of sugar."
    - How many ounces of cream cheese are there for every ounce of sugar?
    - How many ounces of sugar is that for every ounce of cream cheese?

2. Mai's family drinks a total of 10 gallons of milk every 6 weeks.
    - How many gallons of milk does the family drink per week?
    - How many weeks does it take the family to consume 1 gallon of milk?

3. Tyler paid $16 for 4 raffle tickets.
    - What is the price per ticket?
    - How many tickets is that per dollar?

4. For each problem, decide which unit rate from the previous situations you prefer to use. Next, solve the problem, and show your thinking.
    a. If Lin wants to make extra cheesecake filling, how much cream cheese will she need to mix with 35 ounces of sugar?

    b. How many weeks will it take Mai's family to finish 3 gallons of milk?

    c. How much would all 1,000 raffle tickets cost?

Are you ready for more?

Write a "deal" on tickets for Tyler's raffle that sounds good, but is actually a little worse than just buying tickets at the normal price.

Lesson 6 Summary

Suppose a farm lets us pick 2 pounds of blueberries for 5 dollars. We can say:

blueberries (pounds)	price (dollars)
2	5
1	$\frac{5}{2}$
$\frac{2}{5}$	1

- We get $\frac{2}{5}$ pound of blueberries per dollar.
- The blueberries cost $\frac{5}{2}$ dollars per pound.

The "cost per pound" and the "number of pounds per dollar" are the two *unit rates* for this situation.

A **unit rate** tells us how much of one quantity for 1 of the other quantity. Each of these numbers is useful in the right situation.

If we want to find out how much 8 pounds of blueberries will cost, it helps to know how much 1 pound of blueberries will cost.

blueberries (pounds)	price (dollars)
1	$\frac{5}{2}$
8	$8 \cdot \frac{5}{2}$

If we want to find out how many pounds we can buy for 10 dollars, it helps to know how many pounds we can buy for 1 dollar.

blueberries (pounds)	price (dollars)
$\frac{2}{5}$	1
$10 \cdot \frac{2}{5}$	10

Which unit rate is most useful depends on what question we want to answer, so be ready to find either one!

Lesson 6: Interpreting Rates

Cool Down: Buying Grapes by the Pound

Two pounds of grapes cost $6.

1. Complete the table showing the price of different amounts of grapes at this rate.

grapes (pounds)	price (dollars)
2	6
	1
1	

2. Explain the meaning of each of the numbers you found.

Unit 3 Lesson 6 Cumulative Practice Problems

1. A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint. The table shows different quantities of red and white paint for the same shade of pink. Complete the table.

white paint (cups)	red paint (cups)
4	3
	1
1	
	4
5	

2. A farm lets you pick 3 pints of raspberries for $12.00.

    a. What is the cost per pint?

    b. How many pints do you get per dollar?

    c. At this rate, how many pints can you afford for $20.00?

    d. At this rate, how much will 8 pints of raspberries cost?

3. Han and Tyler are following a polenta recipe that uses 5 cups of water for every 2 cups of cornmeal.

    - Han says, “I am using 3 cups of water. I will need $1\frac{1}{5}$ cups of cornmeal.”
    - Tyler says, “I am using 3 cups of cornmeal. I will need $7\frac{1}{2}$ cups of water.”

    Do you agree with either of them? Explain your reasoning.

4. A large art project requires enough paint to cover 1,750 square feet. Each gallon of paint can cover 350 square feet. Each square foot requires $\frac{1}{350}$ of a gallon of paint.

   Andre thinks he should use the rate $\frac{1}{350}$ gallons of paint per square foot to find how much paint they need. Do you agree with Andre? Explain or show your reasoning.

5. Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Who types faster? Explain your reasoning.

   (From Unit 3, Lesson 5.)

6. A corn vendor at a farmer's market was selling a bag of 8 ears of corn for $2.56. Another vendor was selling a bag of 12 for $4.32. Which bag is the better deal? Explain or show your reasoning.

   (From Unit 3, Lesson 5.)

7. A soccer field is 100 meters long. What could be its length in yards?

   A. 33.3

   B. 91

   C. 100

   D. 109

   (From Unit 3, Lesson 3.)

Lesson 7: Equivalent Ratios Have the Same Unit Rates

7.1: Which One Doesn't Belong: Comparing Speeds

Which one doesn't belong? Be prepared to explain your reasoning.

5 miles in 15 minutes

20 miles per hour

3 minutes per mile

32 kilometers per hour

7.2: Price of Burritos

1. Two burritos cost \$14. Complete the table to show the cost for 4, 5, and 10 burritos at that rate. Next, find the cost for a single burrito in each case.

number of burritos	cost in dollars	unit price (dollars per burrito)
2	14	
4		
5		
10		
b		

2. What do you notice about the values in this table?

3. Noah bought b burritos and paid c dollars. Lin bought twice as many burritos as Noah and paid twice the cost he did. How much did Lin pay per burrito?

	number of burritos	cost in dollars	unit price (dollars per burrito)
Noah	b	c	$\frac{c}{b}$
Lin	$2 \cdot b$	$2 \cdot c$	

4. Explain why, if you can buy b burritos for c dollars, or buy $2 \cdot b$ burritos for $2 \cdot c$ dollars, the cost per item is the same in either case.

7.3: Making Bracelets

1. Complete the table. Then, explain the strategy you used to do so.

time in hours	number of bracelets	speed (bracelets per hour)
2		6
5		6
7		6
	66	6
	100	6

2. Here is a partially filled table from an earlier activity. Use the same strategy you used for the bracelet problem to complete this table.

number of burritos	cost in dollars	unit price (dollars per burrito)
	14	7
	28	7
5		7
10		7

3. Next, compare your results with those in the first table in the previous activity. Do they match? Explain why or why not.

7.4: How Much Applesauce?

It takes 4 pounds of apples to make 6 cups of applesauce.

1. At this rate, how much applesauce can you make with:

    a. 7 pounds of apples?

    b. 10 pounds of apples?

2. How many pounds of apples would you need to make:

    a. 9 cups of applesauce?

    b. 20 cups of applesauce?

pounds of apples	cups of applesauce
4	6
7	
10	
	9
	20

Are you ready for more?

1. Jada eats 2 scoops of ice cream in 5 minutes. Noah eats 3 scoops of ice cream in 5 minutes. How long does it take them to eat 1 scoop of ice cream working together (if they continue eating ice cream at the same rate they do individually)?

2. The garden hose at Andre's house can fill a 5-gallon bucket in 2 minutes. The hose at his next-door neighbor's house can fill a 10-gallon bucket in 8 minutes. If they use both their garden hoses at the same time, and the hoses continue working at the same rate they did when filling a bucket, how long will it take to fill a 750-gallon pool?

Lesson 7 Summary

The table shows different amounts of apples selling at the same rate, which means all of the ratios in the table are equivalent. In each case, we can find the *unit price* in dollars per pound by dividing the price by the number of pounds.

apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

The unit price is always the same. Whether we buy 10 pounds of apples for 4 dollars or 20 pounds of apples for 8 dollars, the apples cost 2.50 dollars per pound.

We can also find the number of pounds of apples we can buy per dollar by dividing the number of pounds by the price.

apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a dollar is the same as well! Whether we buy 10 pounds of apples for 4 dollars or 20 pounds of apples for 8 dollars, we are getting 0.4 pounds per dollar.

This is true in all contexts: when two ratios are equivalent, their unit rates will be equal.

quantity x	quantity y	unit rate 1	unit rate 2
a	b	$\frac{a}{b}$	$\frac{b}{a}$
$s \cdot a$	$s \cdot b$	$\frac{s \cdot a}{s \cdot b} = \frac{a}{b}$	$\frac{s \cdot b}{s \cdot a} = \frac{b}{a}$

Lesson 7: Equivalent Ratios Have the Same Unit Rates

Cool Down: Cheetah Speed

A cheetah can run at its top speed for about 25 seconds. Complete the table to represent a cheetah running at a constant speed. Explain or show your reasoning.

time (seconds)	distance (meters)	speed (meters per second)
4	120	
25		
	270	

Unit 3 Lesson 7 Cumulative Practice Problems

1. A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$		
$1\frac{1}{2}$		
	110	

2. The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

   Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result.

   What is the meaning of the number that Elena has calculated?

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

3. A restaurant is offering 2 specials: 10 burritos for $12, or 6 burritos for $7.50. Noah needs 60 burritos for his party. Should he buy 6 orders of the 10-burrito special or 10 orders of the 6-burrito special? Explain your reasoning.

4. Complete the table so that the cost per banana remains the same.

number of bananas	cost in dollars	unit price (dollars per banana)
4		0.50
6		0.50
7		0.50
10		0.50
	10.00	0.50
	16.50	0.50

5. Two planes travel at a constant speed. Plane A travels 2,800 miles in 5 hours. Plane B travels 3,885 miles in 7 hours. Which plane is faster? Explain your reasoning.

(From Unit 3, Lesson 5.)

6. A car has 15 gallons of gas in its tank. The car travels 35 miles per gallon of gas. It uses $\frac{1}{35}$ of a gallon of gas to go 1 mile.

   - How far can the car travel with 15 gallons? Show your reasoning.
   - How much gas does the car use to go 100 miles? Show your reasoning.

(From Unit 3, Lesson 6.)

7. A box of cereal weighs 600 grams. How much is this weight in pounds? Explain or show your reasoning. (Note: 1 kilogram = 2.2 pounds)

(From Unit 3, Lesson 4.)

Lesson 15: Equivalent Exponential Expressions

15.1: Up or Down?

Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for different values of x. What patterns do you notice?

x	3^x	$\left(\frac{1}{3}\right)^x$
1		
2		
3		
4		

15.2: What's the Value?

Evaluate each expression for the given value of x.

1. $3x^2$ when x is 10
2. $3x^2$ when x is $\frac{1}{9}$
3. $\frac{x^3}{4}$ when x is 4
4. $\frac{x^3}{4}$ when x is $\frac{1}{2}$
5. $9 + x^7$ when x is 1
6. $9 + x^7$ when x is $\frac{1}{2}$

15.3: Exponent Experimentation

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

1. $64 = x^2$
2. $64 = x^3$
3. $2^x = 32$
4. $x = \left(\frac{2}{5}\right)^3$
5. $\frac{16}{9} = x^2$
6. $2 \cdot 2^5 = 2^x$
7. $2x = 2^4$
8. $4^3 = 8^x$

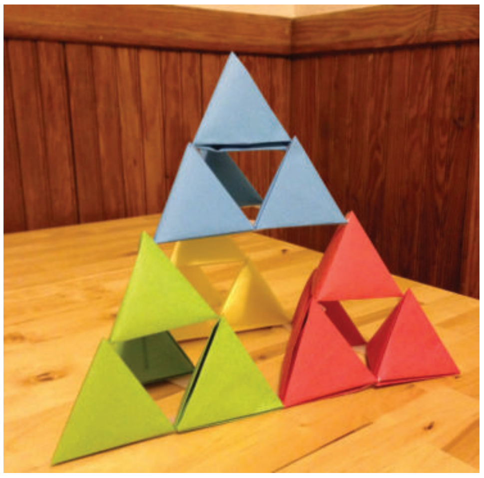
List:

$\frac{8}{125}$ $\frac{6}{15}$ $\frac{5}{8}$ $\frac{8}{9}$ 1 $\frac{4}{3}$ 2 3 4 5 6 8

Are you ready for more?

This fractal is called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has four faces. (The plural of tetrahedron is tetrahedra.)

The small tetrahedra form four medium-sized tetrahedra: blue, red, yellow, and green. The medium-sized tetrahedra form one large tetrahedron.



1. How many small faces does this fractal have? Be sure to include faces you can't see. Try to find a way to figure this out so that you don't have to count every face.

2. How many small tetrahedra are in the bottom layer, touching the table?

3. To make an even bigger version of this fractal, you could take four fractals like the one pictured and put them together. Explain where you would attach the fractals to make a bigger tetrahedron.

4. How many small faces would this bigger fractal have? How many small tetrahedra would be in the bottom layer?

5. What other patterns can you find?

Lesson 15 Summary

In this lesson, we saw expressions that used the letter x as a variable. We evaluated these expressions for different values of x.

- To evaluate the expression $2x^3$ when x is 5, we replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or just 250. So the value of $2x^3$ is 250 when x is 5.
- To evaluate $\frac{x^2}{8}$ when x is 4, we replace the letter x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So $\frac{x^2}{8}$ has a value of 2 when x is 4.

We also saw equations with the variable x and had to decide what value of x would make the equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: $1, 2, 3, 9, 11$. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So 2 is the solution to the equation.

Lesson 15: Equivalent Exponential Expressions

Cool Down: True Statements

Match each equation to a solution.

1. $2^x = 64$
2. $x = \left(\frac{2}{5}\right)^3$
3. $3 \cdot \left(3^4\right) = 3^x$
4. $\frac{16}{25} = x^2$

- $\frac{8}{125}$
- $\frac{4}{5}$
- 5
- 6

Unit 6 Lesson 15 Cumulative Practice Problems

1. Evaluate each expression if $x = 3$.

   a. 2^x

   b. x^2

   c. 1^x

   d. x^1

   e. $\left(\frac{1}{2}\right)^x$

2. Evaluate each expression for the given value of each variable.

   a. $2 + x^3$, x is 3

   b. x^2, x is $\frac{1}{2}$

   c. $3x^2 + y$, x is 5 y is 3

   d. $10y + x^2$, x is 6 y is 4

3. Decide if the expressions have the same value. If not, determine which expression has the larger value.

   a. 2^3 and 3^2

   b. 1^{31} and 31^1

   c. 4^2 and 2^4

   d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$

4. Match each equation to its solution.

A. $7 + x^2 = 16$
B. $5 - x^2 = 1$
C. $2 \cdot 2^3 = 2^x$
D. $\frac{3^4}{3^x} = 27$

1. $x = 1$
2. $x = 2$
3. $x = 3$
4. $x = 4$

5. An adult pass at the amusement park costs 1.6 times as much as a child's pass.

a. How many dollars does an adult pass cost if a child's pass costs:

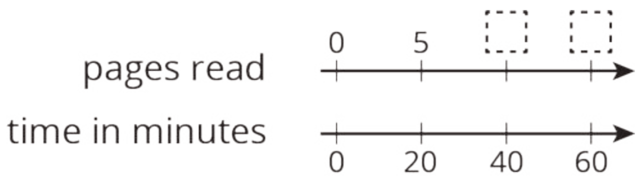
$5?　　$10?　　w dollars?

b. A child's pass costs $15. How many dollars does an adult pass cost?

(From Unit 6, Lesson 6.)

6. Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour?

- Use a double number line to find the answer.



- Use a table to find the answer.

pages read	time in minutes
5	20

Which strategy do you think is better, and why?

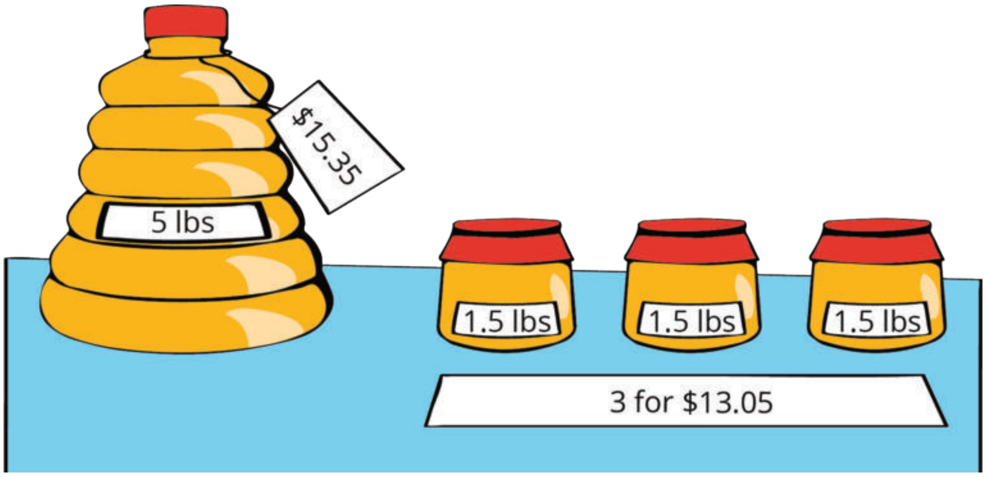
(From Unit 2, Lesson 14.)

Lesson 16: Two Related Quantities, Part 1

16.1: Which One Would You Choose?

Which one would you choose? Be prepared to explain your reasoning.

- A 5-pound jug of honey for $15.35
- Three 1.5-pound jars of honey for $13.05



16.2: Painting the Set

Lin needs to mix a specific shade of orange paint for the set of the school play. The color uses 3 parts yellow for every 2 parts red.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

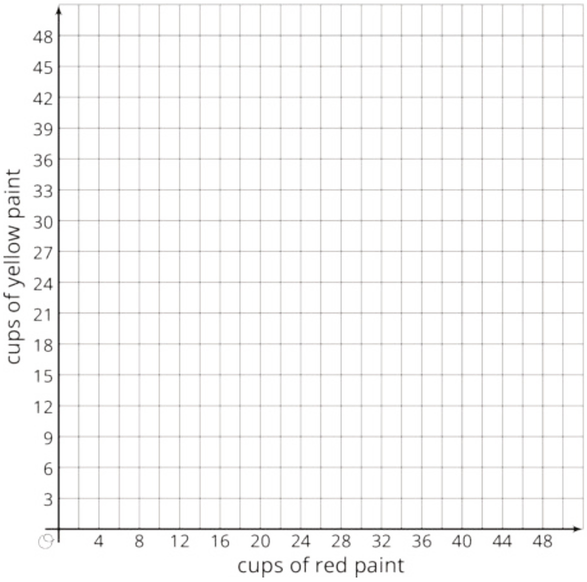
cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	
6		
		20
	18	
14		
16		
		50
	42	

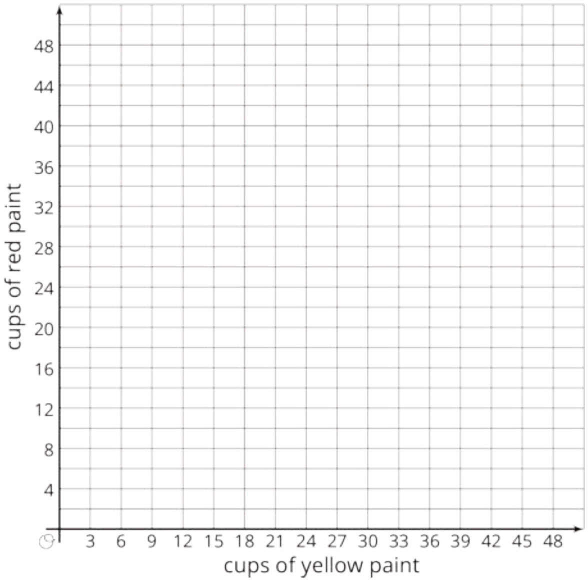
2. Lin notices that the number of cups of red paint is always $\frac{2}{5}$ of the total number of cups. She writes the equation $r = \frac{2}{5}t$ to describe the relationship. Which is the **independent variable**? Which is the **dependent variable**? Explain how you know.

3. Write an equation that describes the relationship between r and y where y is the independent variable.

4. Write an equation that describes the relationship between y and r where r is the independent variable.

5. Use the points in the table to create two graphs that show the relationship between r and y. Match each relationship to one of the equations you wrote.





Are you ready for more?

A fruit stand sells apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of fruit in total. How many of each fruit did they sell?

Lesson 16 Summary

Equations are very useful for describing sets of equivalent ratios. Here is an example.

A pie recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r.

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can write equations to describe the relationship between g and r.

- When we know the number of green apples and want to find the number of red apples, we can write:

  $$r = \frac{5}{3}g$$

  In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

  We can use this equation with any value of g to find r. If 270 green apples are used, then $\frac{5}{3} \cdot (270)$ or 450 red apples are used.

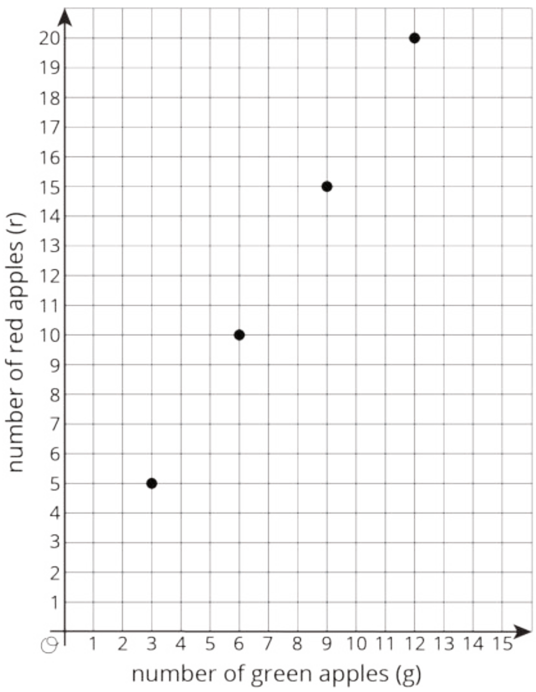
- When we know the number of red apples and want to find the number of green apples, we can write:

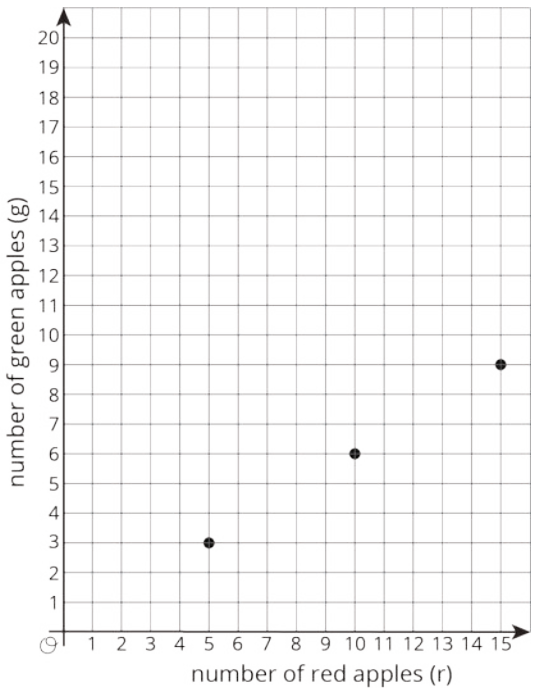
  $$g = \frac{3}{5}r$$

  In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

  We can use this equation with any value of r to find g. If 275 red apples are used, then $\frac{3}{5} \cdot (275)$ or 165 green apples are used.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities.



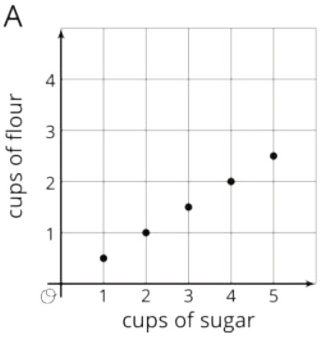


Lesson 16: Two Related Quantities, Part 1

Cool Down: Baking Brownies

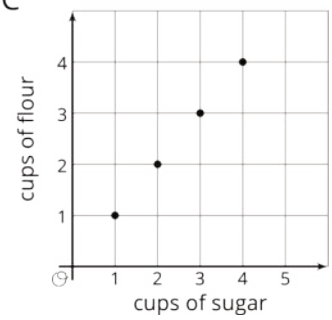
A brownie recipe calls for 1 cup of sugar and $\frac{1}{2}$ cup of flour to make one batch of brownies. To make multiple batches, the equation $f = \frac{1}{2}s$ where f is the number of cups of flour and s is the number of cups of sugar represents the relationship. Which graph also represents the relationship? Explain how you know.

A



B

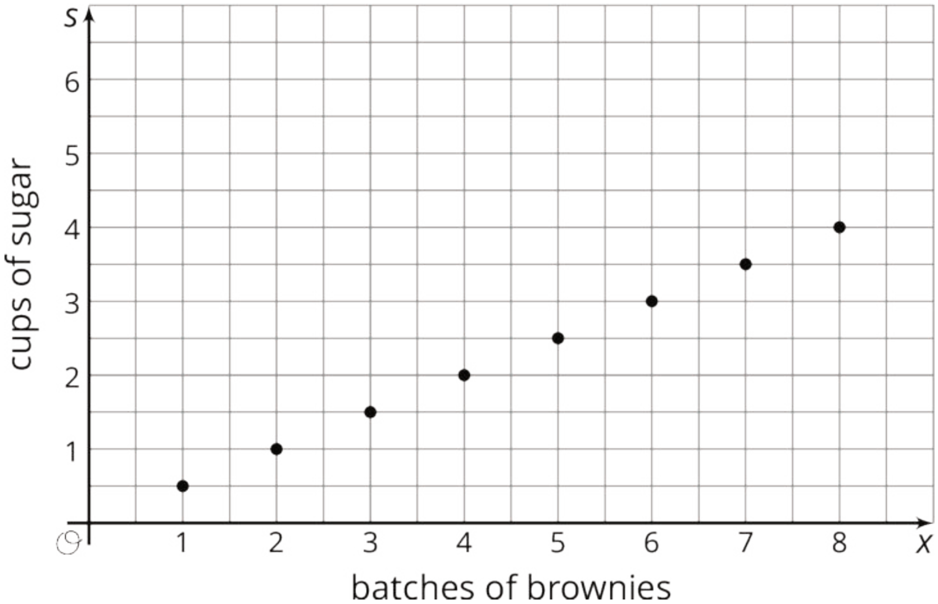
C



Illustrative Mathematics

Unit 6 Lesson 16 Cumulative Practice Problems

1. Here is a graph that shows some values for the number of cups of sugar, s, required to make x batches of brownies.



a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

x	1	2	3	4	5	6	7
s							

b. What does the point $(8, 4)$ mean in terms of the amount of sugar and number of batches of brownies?

c. Write an equation that shows the amount of sugar in terms of the number of batches.

2. Each serving of a certain fruit snack contains 90 calories.

   a. Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories, c, in terms of the number of servings, n.

   b. Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings, n, in terms of the number of calories, c.

3. Kiran shops for books during a 20% off sale.

   a. What percent of the original price of a book does Kiran pay during the sale?

   b. Complete the table to show how much Kiran pays for books during the sale.

   c. Write an equation that relates the sale price, s, to the original price p.

   d. On graph paper, create a graph showing the relationship between the sale price and the original price by plotting the points from the table.

original price in dollars (p)	sale price in dollars (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

4. Evaluate each expression when x is 4 and y is 6.

   a. $(6 - x)^3 + y$

   b. $2 + x^3$

   c. $2^x - 2y$

   d. $\left(\frac{1}{2}\right)^x$

   e. $1^x + 2^x$

   f. $\frac{2^x}{x^2}$

(From Unit 6, Lesson 15.)

5. Find $(12.34) \cdot (0.7)$. Show your reasoning.

(From Unit 5, Lesson 8.)

6. For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

   a. $302.1 \div 0.5$

   b. $12.15 \div 0.02$

   c. $1.375 \div 0.11$

(From Unit 5, Lesson 13.)

Lesson 17: Two Related Quantities, Part 2

17.1: Walking to the Library

Lin and Jada each walk at a steady rate from school to the library. Lin can walk 13 miles in 5 hours, and Jada can walk 25 miles in 10 hours. They each leave school at 3:00 and walk $3\frac{1}{4}$ miles to the library. What time do they each arrive?

17.2: The Walk-a-thon

Diego, Elena, and Andre participated in a walk-a-thon to raise money for cancer research. They each walked at a constant rate, but their rates were different.

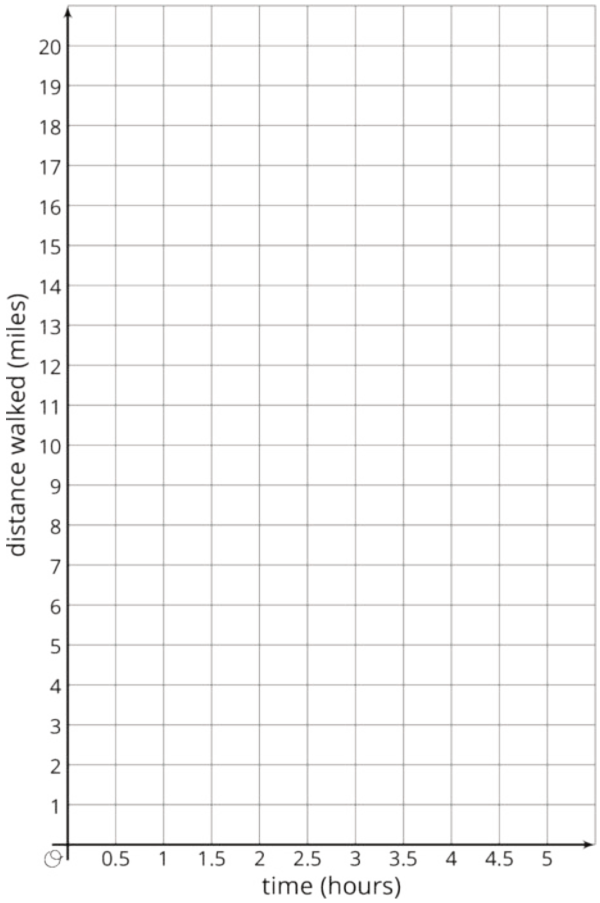
1. Complete the table to show how far each participant walked during the walk-a-thon.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1			
2	6		
	12	11	
5			17.5

2. How fast was each participant walking in miles per hour?

3. How long did it take each participant to walk one mile?

4. Graph the progress of each person in the **coordinate plane**. Use a different color for each participant.



5. Diego says that $d = 3t$ represents his walk, where d is the distance walked in miles and t is the time in hours.

   a. Explain why $d = 3t$ relates the distance Diego walked to the time it took.

   b. Write two equations that relate distance and time: one for Elena and one for Andre.

6. Use the equations you wrote to predict how far each participant would walk, at their same rate, in 8 hours.

7. For Diego's equation and the equations you wrote, which is the dependent variable and which is the independent variable?

Are you ready for more?

1. Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet? One way to start thinking about this problem is to make a table. Add as many rows as you like.

	train A	train B
starting position	0 miles	320 miles
after 1 hour	70 miles	270 miles
after 2 hours		

2. How long will it take a train traveling at 120 miles per hour to go 320 miles?

3. Explain the connection between these two problems.

Lesson 17 Summary

Equations are very useful for solving problems with constant speeds. Here is an example.

A boat is traveling at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?

2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these.

Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When we know the time and want to find the distance, we can write:

$$d = 25t$$

In this equation, if t changes, d is affected by the change, so we t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t. In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.

When we know the distance and want to find the time, we can write:

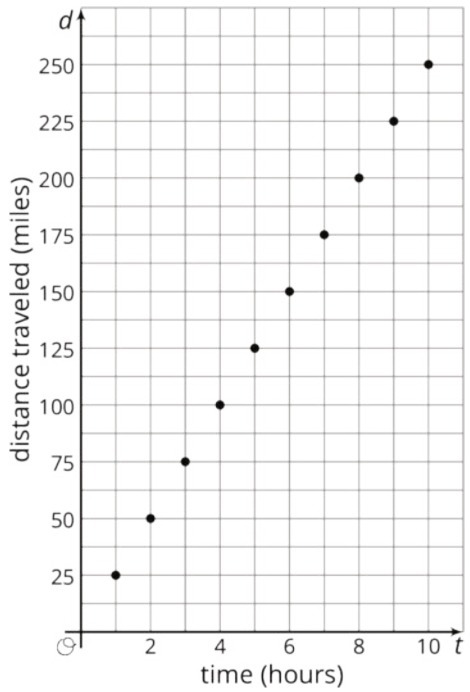
$$t = \frac{d}{25}$$

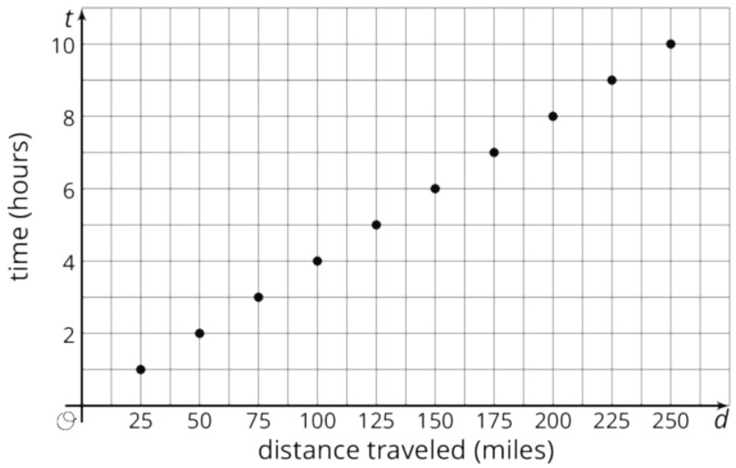
In this equation, if d changes, t is affected by the change, so we d is the independent variable and t is the dependent variable.

This equation can help us find t when for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

These problems can also be solved using important ratio techniques such as a table of equivalent ratios. The equations are particularly valuable in this case because the answers are not round numbers or easy to quickly evaluate.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities:





Lesson 17: Two Related Quantities, Part 2

Cool Down: Interpret the Point

During a walk-a-thon, Noah's time in hours, t, and distance in miles, d, are related by the equation $\frac{1}{3}d = t$. A graph of the equation includes the point $(12, 4)$.

1. Identify the independent variable.

2. What does the point $(12, 4)$ represent in this situation?

3. What point would represent the time it took to walk $7\frac{1}{2}$ miles?

Unit 6 Lesson 17 Cumulative Practice Problems

1. A car is traveling down a road at a constant speed of 50 miles per hour.

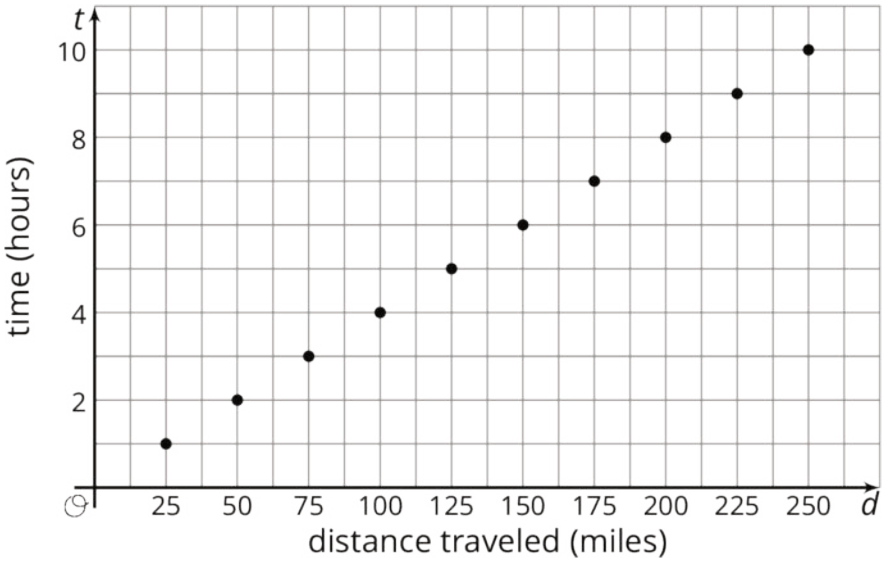
    a. Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

    b. Write an equation that represents the distance traveled by the car, d, for an amount of time, t.

    c. In your equation, which is the dependent variable and which is the independent variable?

time (hours)	distance (miles)
2	
1.5	
t	
	50
	300
	d

2. The graph represents the amount of time in hours it takes a ship to travel various distances in miles.



a. Write the coordinates of one of point on the graph. What does the point represent?

b. What is the speed of the ship in miles per hour?

c. Write an equation that relates the time, t, it takes to travel a given distance, d.

3. Find a solution to each equation in the list that follows (not all numbers will be used):

a. $2^x = 8$

b. $2^x = 2$

c. $x^2 = 100$

d. $x^2 = \frac{1}{100}$

e. $x^1 = 7$

f. $2^x \cdot 2^3 = 2^7$

g. $\frac{2^x}{2^3} = 2^5$

List: $\frac{1}{10}$ $\frac{1}{3}$ 1 2 3 4 5 7 8 10 16

(From Unit 6, Lesson 15.)

4. Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$

B. $(5 + 30 - 15) \cdot x$

C. $x(5 + 30x - 15x)$

D. $5x(1 + 6 - 3)$

E. $5(x + 30x - 15x)$

(From Unit 6, Lesson 11.)

5. Evaluate each expression if x is 1, y is 2, and z is 3.

a. $7x^2 - z$

b. $(x + 4)^3 - y$

c. $y(x + 3^3)$

d. $(7 - y + z)^2$

e. $0.241x + x^3$

(From Unit 6, Lesson 15.)