

# Plan for Grade 7 Unit 1: Scale Drawings

*Relevant Unit(s) to review: Grade 6 Unit 2: Introducing Ratios*

<b>Essential prior concepts to engage with this unit</b>	<ul style="list-style-type: none"> <li>• Find the area of polygons.</li> <li>• Generate equivalent ratios and justify why they are equivalent.</li> </ul>
<b>Brief narrative of approach</b>	Begin by looking at finding the area of parallelograms (6.1.5), then review the basics of equivalent ratios. This leads to ratio reasoning with reference to tables and using representations of ratio and rate situations (6.2.11).

Lessons to Add	Lessons to Remove or Modify
<ol style="list-style-type: none"> <li>1. 6.2.5</li> <li>2. 6.2.8</li> <li>3. 6.2.11</li> <li>4. 6.2.14</li> <li>5. Combine 6.1.5 and 6.1.6 - focus on finding area of polygons.</li> </ol>	<ol style="list-style-type: none"> <li>1. Combine lessons 7.1.4 and 7.1.5 skipping optional activities.</li> <li>2. Remove 7.1.6 - optional activity calculating and comparing areas of multiple scaled copies of the same shape.</li> <li>3. Remove 7.1.8 - optional lesson using a scale drawing to estimate the distance an object traveled</li> <li>4. Remove 7.1.11 - activity can be moved to at home if additional time is needed</li> <li>5. Remove 7.8.13 - lesson can be removed as it is additional practice with creating scale drawings</li> </ol>
Lessons added: 5	Lessons removed: 5

## Modified Plan for Grade 7 Unit 1

Day	IM lesson	Notes
	<a href="#">6.1 Check Your Readiness Assessment</a> <a href="#">6.2 End Assessment</a> <a href="#">7.1 Check Your Readiness</a>	6.1.CYR.2 - Area of a polygon 6.2.EOU.2 - Find equivalent ratios 6.2.EOU.4 - Equivalent ratios in a table 7.1.CYR.4 - Areas of polygons 7.1.CYR.6 - Equivalent ratios in a table 7.1.CYR.7 - Scaling polygons  Use questions from the 6.2 End Assessment or the 7.1 Check Your Readiness Assessment, or a combination of items to determine student needs for incorporating below level content.
1	<a href="#">7.1.1</a>	
2	<a href="#">7.1.2</a>	
3	<a href="#">7.1.3</a>	
4	<a href="#">6.2.5</a>	If the initial assessment shows that students are not familiar with equivalent ratios, consider including this activity before continuing with grade level content.
5	<a href="#">6.2.8</a>	If the initial assessment shows that students are not familiar with equivalent ratios, consider including this activity before continuing with grade level content.
6	<a href="#">6.2.11</a>	If the initial assessment shows that students are not familiar with equivalent ratios, consider including this activity before continuing with grade level content.
7	<a href="#">6.2.14</a>	If the initial assessment shows that students are not familiar with equivalent ratios, consider including this activity before continuing with grade level content.
8	<a href="#">6.1.5</a> <a href="#">6.1.6</a>	If the initial assessment shows that students are not familiar with areas of polygons, consider including a combination of 6.1.5 and 6.1.6, focusing on finding the areas of polygons.
9	<a href="#">7.1.4</a> <a href="#">7.1.5</a>	Combine Lessons 4 and 5, skipping optional activities.

10	<a href="#">7.1.7</a>	
11	<a href="#">7.1.9</a>	
12	<a href="#">7.1.10</a>	
13	<a href="#">7.1.11</a>	
14	<a href="#">7.1.12</a>	
15	<a href="#">7.8 End Assessment</a>	

## Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

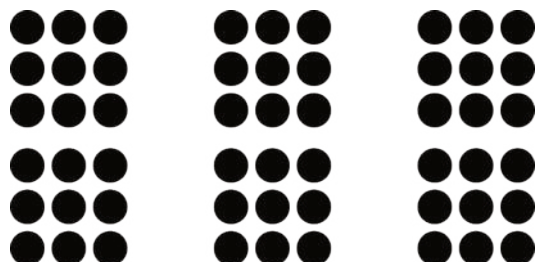
Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
7.1.1	+	E	This lesson introduces students to the idea of a <b>scaled copy</b> of a picture or a figure. Students learn to distinguish scaled copies from those that are not—first informally, and later, with increasing precision.
7.1.2	+	D	This lesson develops the vocabulary for talking about <b>scaling</b> and <b>scaled copies</b> more precisely (MP6), and identifying the structures in common between two figures (MP7). Specifically, students learn to use the term <b>corresponding</b> to refer to a pair of points, segments, or angles in two figures that are scaled copies. Students also begin to describe the numerical relationship between the corresponding lengths in two figures using a scale factor.
7.1.3	+	A	In the previous lesson, students learned that we can use scale factors to describe the relationship between corresponding lengths in scaled figures. Here they apply this idea to draw scaled copies of simple shapes on and off a grid. They also strengthen their understanding that the relationship between scaled copies is multiplicative, not additive.
7.1.4	0	D	In previous lessons, students looked at the relationship between a figure and a scaled copy by finding the scale factor that relates the side lengths and by using tracing paper to compare the angles. This lesson takes both of these comparisons a step further and identifies <b>corresponding distances</b> .
7.1.5	0	D	In this lesson, students deepen their understanding of scale factors in two ways: classifying scale factors by size noticing how each class of factors affects the scaled copies, and seeing that the scale factor that takes an original figure to its copy and the one that takes the copy to the original are reciprocals.
7.1.6	-	E	<i>This lesson is optional.</i> In this lesson, students are introduced to how the area of a scaled copy relates to the area of the original shape. Students build on their grade 6 work with exponents to recognize that the area increases by the square of the scale factor by which

			the sides increased.
7.1.7	+	E	In this lesson, students begin to look at <b>scale drawings</b> , or scaled two-dimensional representations of actual objects or places.
7.1.8	-	D	<i>This lesson is optional.</i> In this lesson, students apply what they have learned about scale drawings to solve problems involving constant speed.
7.1.9	0	A	This is the first lesson where students use the actual distance to calculate the scaled distance and create their own scale drawings.
7.1.10	+	A	In this lesson, students are given a scale drawing and asked to recreate it at a different scale.
7.1.11	-	A	In this lesson, students learn that a scale can be expressed without units.
7.1.12	+	A	In this lesson, students analyze various scales and find that sometimes it is helpful to rewrite scales with units as scales without units in order to compare them.
7.1.13	-	A	<i>This culminating lesson is optional.</i> Students use what they have learned in this unit to create a scale floor plan of their classroom.

# Lesson 5: Defining Equivalent Ratios

## 5.1: Dots and Half Dots

Dot Pattern 1:



Dot Pattern 2:



## 5.2: Tuna Casserole

Here is a recipe for tuna casserole.

---

### Ingredients

- 3 cups cooked elbow-shaped pasta
- 6 ounce can tuna, drained
- 10 ounce can cream of chicken soup
- 1 cup shredded cheddar cheese
- $1\frac{1}{2}$  cups French fried onions



### Instructions

Combine the pasta, tuna, soup, and half of the cheese. Transfer into a 9 inch by 18 inch baking dish. Put the remaining cheese on top. Bake 30 minutes at 350 degrees. During the last 5 minutes, add the French fried onions. Let sit for 10 minutes before serving.

- 
1. What is the ratio of the ounces of soup to the cups of shredded cheese to the cups of pasta in one batch of casserole?
  2. How much of each of these 3 ingredients would be needed to make:
    - a. twice the amount of casserole?
    - b. half the amount of casserole?
    - c. five times the amount of casserole?
    - d. one-fifth the amount of casserole?
  3. What is the ratio of cups of pasta to ounces of tuna in one batch of casserole?
  4. How many batches of casserole would you make if you used the following amounts of ingredients?
    - a. 9 cups of pasta and 18 ounces of tuna?
    - b. 36 ounces of tuna and 18 cups of pasta?

c. 1 cup of pasta and 2 ounces of tuna?

### Are you ready for more?

The recipe says to use a 9 inch by 18 inch baking dish. Determine the length and width of a baking dish with the same height that could hold:

1. Twice the amount of casserole
2. Half the amount of casserole
3. Five times the amount of casserole
4. One-fifth the amount of casserole

## 5.3: What Are Equivalent Ratios?

The ratios  $5 : 3$  and  $10 : 6$  are **equivalent ratios**.

1. Is the ratio  $15 : 12$  equivalent to these? Explain your reasoning.
  
2. Is the ratio  $30 : 18$  equivalent to these? Explain your reasoning.
  
3. Give two more examples of ratios that are equivalent to  $5 : 3$ .
4. How do you know when ratios are equivalent and when they are *not* equivalent?
  
5. Write a definition of *equivalent ratios*.



Pause here so your teacher can review your work and assign you a ratio to use for your visual display.

6. Create a visual display that includes:

- the title "Equivalent Ratios"
- your best definition of *equivalent ratios*
- the ratio your teacher assigned to you
- at least two examples of ratios that are equivalent to your assigned ratio
- an explanation of how you know these examples are equivalent
- at least one example of a ratio that is *not* equivalent to your assigned ratio
- an explanation of how you know this example is *not* equivalent

Be prepared to share your display with the class.

### Lesson 5 Summary

All ratios that are **equivalent** to  $a : b$  can be made by multiplying both  $a$  and  $b$  by the same number.

For example, the ratio  $18 : 12$  is equivalent to  $9 : 6$  because both 9 and 6 are multiplied by the same number: 2.

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot 2 & \downarrow \downarrow & \cdot 2 \\ & 18 : 12 & \end{array}$$

$3 : 2$  is also equivalent to  $9 : 6$ , because both 9 and 6 are multiplied by the same number:  $\frac{1}{3}$ .

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot \frac{1}{3} & \downarrow \downarrow & \cdot \frac{1}{3} \\ & 3 : 2 & \end{array}$$

Is  $18 : 15$  equivalent to  $9 : 6$ ?

No, because 18 is  $9 \cdot 2$ , but 15 is *not*  $6 \cdot 2$ .

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot 2 & \downarrow \downarrow & \text{Nope.} \\ & 18 : 15 & \end{array}$$

# Lesson 5: Defining Equivalent Ratios

## Cool Down: Why Are They Equivalent?

1. Write another ratio that is equivalent to the ratio 4 : 6.
  
2. How do you know that your new ratio is equivalent to 4 : 6? Explain or show your reasoning.

## Unit 2 Lesson 5 Cumulative Practice Problems

1. Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a diagram that shows why they are equivalent ratios.

a.  $4 : 5$  and  $8 : 10$

a.  $2 : 7$  and  $10,000 : 35,000$

b.  $18 : 3$  and  $6 : 1$

2. Explain why  $6 : 4$  and  $18 : 8$  are not equivalent ratios.

3. Are the ratios  $3 : 6$  and  $6 : 3$  equivalent? Why or why not?

4. This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.

white paint (cups)



yellow paint (cups)



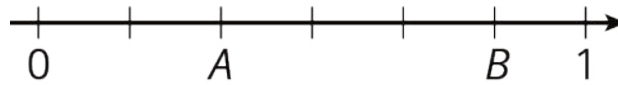
(From Unit 2, Lesson 4.)

5. In the fruit bowl there are 6 bananas, 4 apples, and 3 oranges.

- For every 4 \_\_\_\_\_, there are 3 \_\_\_\_\_.
- The ratio of \_\_\_\_\_ to \_\_\_\_\_ is 6 : 3.
- The ratio of \_\_\_\_\_ to \_\_\_\_\_ is 4 to 6.
- For every 1 orange, there are \_\_\_\_\_ bananas.

(From Unit 2, Lesson 1.)

6. Write fractions for points *A* and *B* on the number line.



(From Unit 2, Lesson 1.)

## Lesson 8: How Much for One?

### 8.1: Number Talk: Remainders in Division

Find the quotient mentally.

$$246 \div 12$$

### 8.2: Grocery Shopping

Answer each question and explain or show your reasoning. If you get stuck, consider drawing a double number line diagram.

1. Eight avocados cost \$4.
  - a. How much do 16 avocados cost?
  - b. How much do 20 avocados cost?
  - c. How much do 9 avocados cost?



2. Twelve large bottles of water cost \$9.
  - a. How many bottles can you buy for \$3?
  - b. What is the cost per bottle of water?
  - c. How much would 7 bottles of water cost?



3. A 10-pound sack of flour costs \$8.
  - a. How much does 40 pounds of flour cost?
  - b. What is the cost per pound of flour?

### Are you ready for more?

It is commonly thought that buying larger packages or containers, sometimes called *buying in bulk*, is a great way to save money. For example, a 6-pack of soda might cost \$3 while a 12-pack of the same brand costs \$5.

Find 3 different cases where it is not true that buying in bulk saves money. You may use the internet or go to a local grocery store and take photographs of the cases you find. Make sure the products are the same brand. For each example that you find, give the quantity or size of each, and describe how you know that the larger size is not a better deal.

## 8.3: More Shopping

- Four bags of chips cost \$6.
  - What is the cost per bag?
  - At this rate, how much will 7 bags of chips cost?
- At a used book sale, 5 books cost \$15.
  - What is the cost per book?
  - At this rate, how many books can you buy for \$21?
- Neon bracelets cost \$1 for 4.
  - What is the cost per bracelet?
  - At this rate, how much will 11 neon bracelets cost?

Pause here so you teacher can review your work.

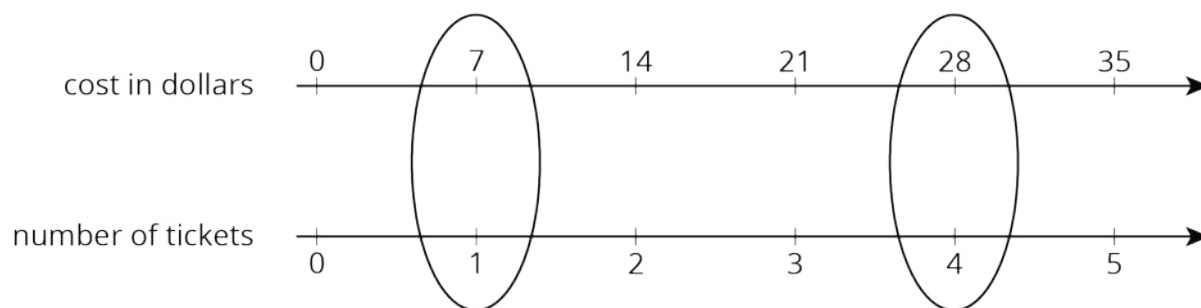


- Your teacher will assign you one of the problems. Create a visual display that shows your solution to the problem. Be prepared to share your solution with the class.

## Lesson 8 Summary

The **unit price** is the price of 1 thing—for example, the price of 1 ticket, 1 slice of pizza, or 1 kilogram of peaches.

If 4 movie tickets cost \$28, then the unit price would be the cost *per* ticket. We can create a double number line to find the unit price.

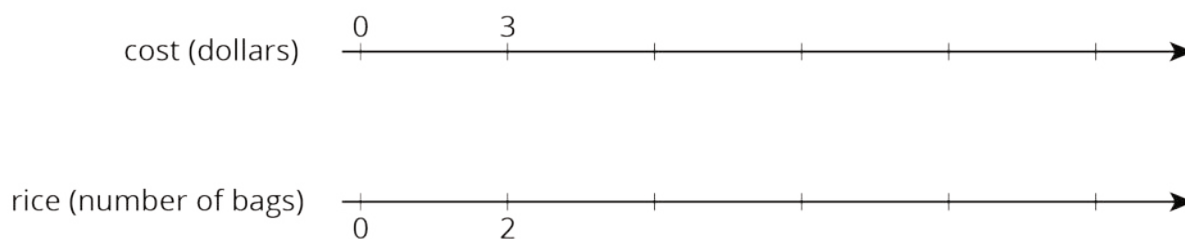


This double number line shows that the cost for 1 ticket is \$7. We can also find the unit price by dividing,  $28 \div 4 = 7$ , or by multiplying,  $28 \cdot \frac{1}{4} = 7$ .

# Lesson 8: How Much for One?

## Cool Down: Unit Price of Rice

Here is a double number line showing that it costs \$3 to buy 2 bags of rice:

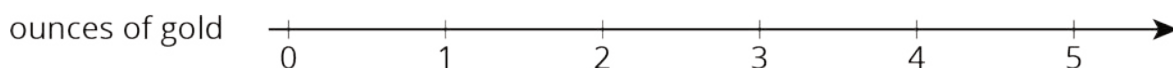
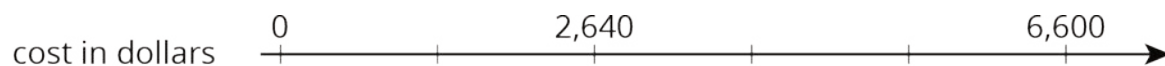


1. At this rate, how many bags of rice can you buy with \$12?
  
2. Find the cost per bag.
  
3. How much do 20 bags of rice cost?

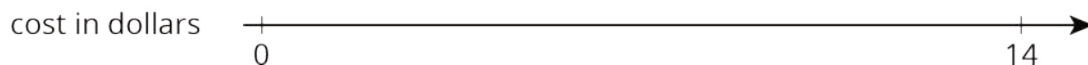
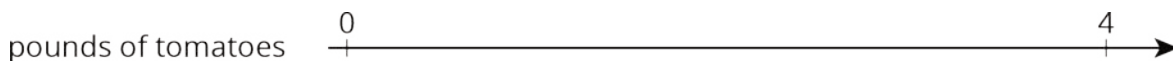


## Unit 2 Lesson 8 Cumulative Practice Problems

1. In 2016, the cost of 2 ounces of pure gold was \$2,640. Complete the double number line to show the cost for 1, 3, and 4 ounces of gold.



2. The double number line shows that 4 pounds of tomatoes cost \$14. Draw tick marks and write labels to show the prices of 1, 2, and 3 pounds of tomatoes.



3. 4 movie tickets cost \$48. At this rate, what is the cost of:

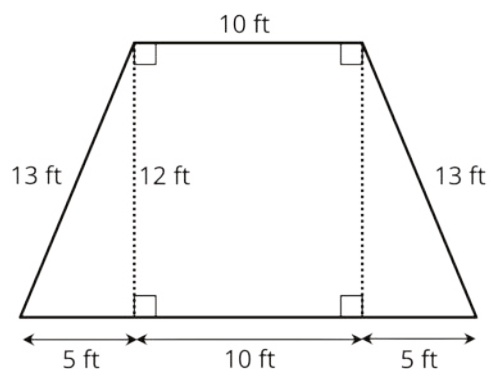
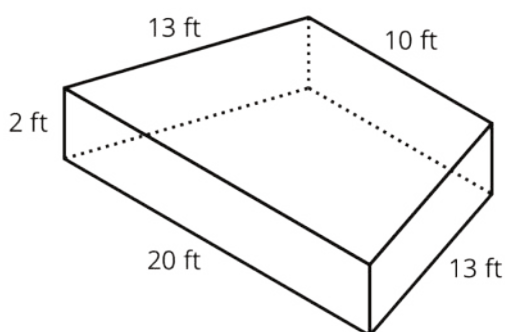
- a. 5 movie tickets?
  
- b. 11 movie tickets?

4. Priya bought these items at the grocery store. Find each unit price.
- 12 eggs for \$3. How much is the cost per egg?
  - 3 pounds of peanuts for \$7.50. How much is the cost per pound?
  - 4 rolls of toilet paper for \$2. How much is the cost per roll?
  - 10 apples for \$3.50. How much is the cost per apple?

5. Clare made a smoothie with 1 cup of yogurt, 3 tablespoons of peanut butter, 2 teaspoons of chocolate syrup, and 2 cups of crushed ice.
- Kiran tried to double this recipe. He used 2 cups of yogurt, 6 tablespoons of peanut butter, 5 teaspoons of chocolate syrup, and 4 cups of crushed ice. He didn't think it tasted right. Describe how the flavor of Kiran's recipe compares to Clare's recipe.
  - How should Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

(From Unit 2, Lesson 3.)

6. A drama club is building a wooden stage in the shape of a trapezoidal prism. The height of the stage is 2 feet. Some measurements of the stage are shown here.



What is the area of all the faces of the stage, excluding the bottom? Show your reasoning. If you get stuck, consider drawing a net of the prism.

(From Unit 1, Lesson 15.)

# Lesson 11: Representing Ratios with Tables

## 11.1: How Is It Growing?

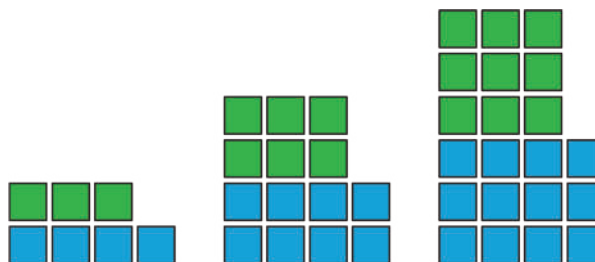
Look for a pattern in the figures.

1. How many total tiles will be in:

a. the 4th figure?

b. the 5th figure?

c. the 10th figure?



2. How do you see it growing?

## 11.2: A Huge Amount of Sparkling Orange Juice

Noah's recipe for one batch of sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

1. Use the double number line to show how many liters of each ingredient to use for different-sized batches of sparkling orange juice.

orange juice (liters) 

soda water (liters) 

2. If someone mixes 36 liters of orange juice and 45 liters of soda water, how many batches would they make?
3. If someone uses 400 liters of orange juice, how much soda water would they need?
4. If someone uses 455 liters of soda water, how much orange juice would they need?
5. Explain the trouble with using a double number line diagram to answer the last two questions.

## 11.3: Batches of Trail Mix

A recipe for trail mix says: “Mix 7 ounces of almonds with 5 ounces of raisins.” Here is a **table** that has been started to show how many ounces of almonds and raisins would be in different-sized batches of this trail mix.

almonds (oz)	raisins (oz)
7	5
28	
	10
3.5	
	250
56	

1. Complete the table so that ratios represented by each row are equivalent.
2. What methods did you use to fill in the table?
3. How do you know that each row shows a ratio that is equivalent to  $7 : 5$ ? Explain your reasoning.

### Are you ready for more?

You have created a best-selling recipe for chocolate chip cookies. The ratio of sugar to flour is  $2 : 5$ .

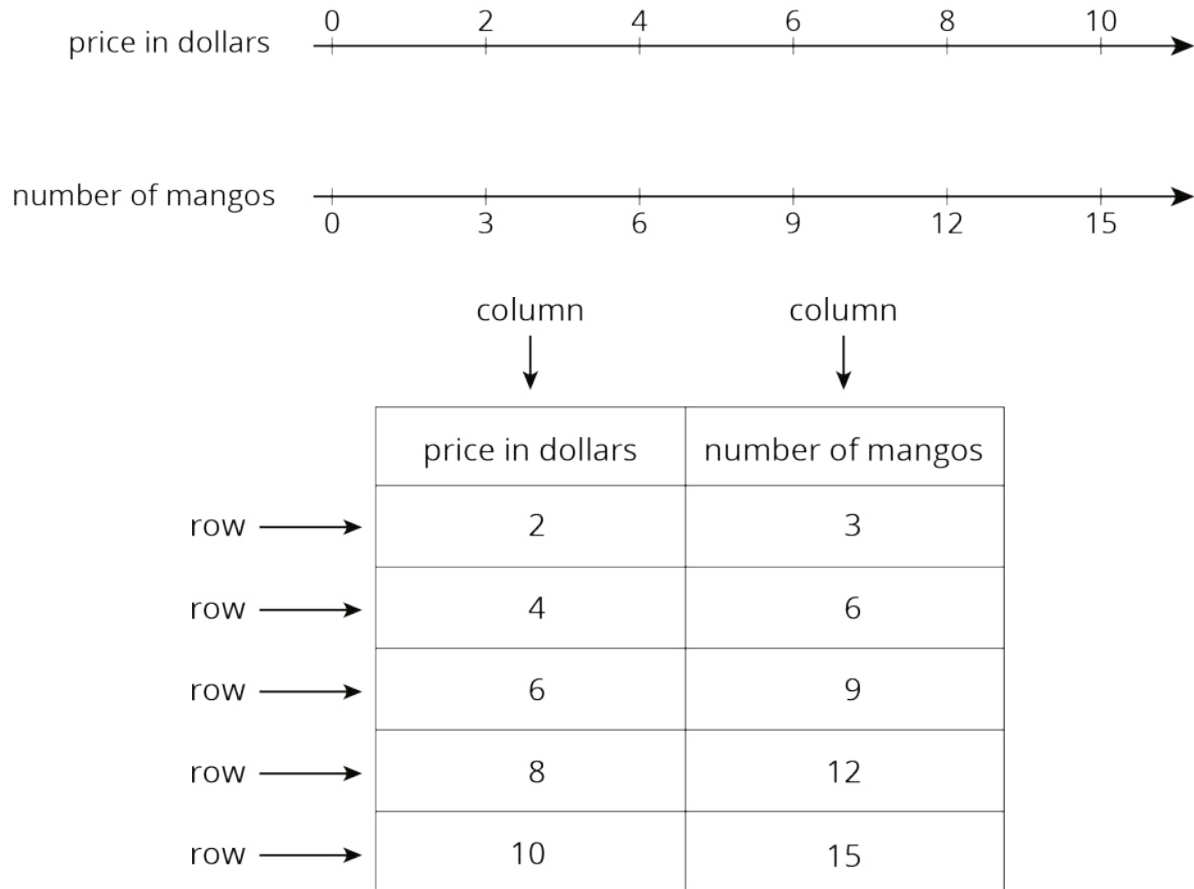
Create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amounts using more than 500 units of flour.

## Lesson 11 Summary

A **table** is a way to organize information. Each horizontal set of entries is called a *row*, and each vertical set of entries is called a *column*. (The table shown has 2 columns and 5 rows.) A table can be used to represent a collection of equivalent ratios.

Here is a double number line diagram and a table that both represent the situation: “The price is \$2 for every 3 mangos.”



# Lesson 11: Representing Ratios with Tables

## Cool Down: Batches of Cookies in a Table

In previous lessons, we worked with a diagram and a double number line that represent this cookie recipe. Here is a table that represents the same situation.

flour (cups)	vanilla (teaspoons)
5	2
15	6
$2\frac{1}{2}$	1

1. Write a sentence that describes a ratio shown in the table.
  
2. What does the second row of numbers represent?
  
3. Complete the last row for a different batch size that hasn't been used so far in the table. Explain or show your reasoning.



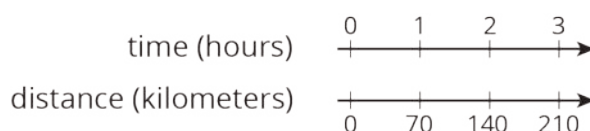
## Unit 2 Lesson 11 Cumulative Practice Problems

1. Complete the table to show the amounts of yellow and red paint needed for different-sized batches of the same shade of orange paint.

yellow paint (quarts)	red paint (quarts)
5	6

Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange as the mixture in the first row of the table.

2. A car travels at a constant speed, as shown on the double number line.

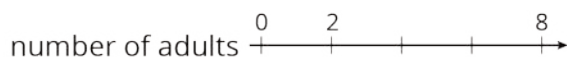


How far does the car travel in 14 hours?  
Explain or show your reasoning.

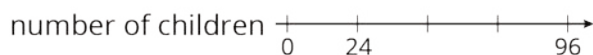
3. The olive trees in an orchard produce 3,000 pounds of olives a year. It takes 20 pounds of olives to make 3 liters of olive oil. How many liters of olive oil can this orchard produce in a year? If you get stuck, consider using the table.

olives (pounds)	olive oil (liters)
20	3
100	
3,000	

4. At a school recess, there needs to be a ratio of 2 adults for every 24 children on the playground. The double number line represents the number of adults and children on the playground at recess.



- a. Label each remaining tick mark with its value.



- b. How many adults are needed if there are 72 children? Circle your answer on the double number line.

(From Unit 2, Lesson 6.)

5. While playing basketball, Jada's heart rate goes up to 160 beats per minute. While jogging, her heart beats 25 times in 10 seconds. Assuming her heart beats at a constant rate while jogging, which of these activities resulted in a higher heart rate? Explain your reasoning.

(From Unit 2, Lesson 10.)

6. A shopper bought the following items at the farmer's market:

a. 6 ears of corn for \$1.80. What was the cost per ear?

b. 12 apples for \$2.88. What was the cost per apple?

c. 5 tomatoes for \$3.10. What was the cost per tomato?

(From Unit 2, Lesson 8.)

# Lesson 14: Solving Equivalent Ratio Problems

## 14.1: What Do You Want to Know?

Consider the problem: A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?

What information would you need to be able to solve the problem?

## 14.2: Info Gap: Hot Chocolate and Potatoes

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

### 14.3: Comparing Reading Rates

- Lin read the first 54 pages from a 270-page book in the last 3 days.
- Diego read the first 100 pages from a 320-page book in the last 4 days.
- Elena read the first 160 pages from a 480-page book in the last 5 days.

If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

#### Are you ready for more?

The ratio of cats to dogs in a room is  $2 : 3$ . Five more cats enter the room, and then the ratio of cats to dogs is  $9 : 11$ . How many cats and dogs were in the room to begin with?

## Lesson 14 Summary

To solve problems about something happening at the same rate, we often need:

- Two pieces of information that allow us to write a ratio that describes the situation.
- A third piece of information that gives us one number of an equivalent ratio. Solving the problem often involves finding the other number in the equivalent ratio.

Suppose we are making a large batch of fizzy juice and the recipe says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” We know that the ratio of cranberry juice to soda water is  $5 : 2$ , and that we need 2.5 cups of cranberry juice per cup of soda water.

We still need to know something about the size of the large batch. If we use 16 cups of soda water, what number goes with 16 to make a ratio that is equivalent to  $5 : 2$ ?

To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

cranberry juice (cups)	soda water (cups)
5	2
2.5	1
40	16

# Lesson 14: Solving Equivalent Ratio Problems

## Cool Down: Water Faucet

Jada wants to know how fast the water comes out of her faucet. What information would she need to know to be able to determine that?



## Unit 2 Lesson 14 Cumulative Practice Problems

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for \$3.00 and allows customers to buy any amount of vinegar. Decide whether each of the following ratios correctly represents the price of vinegar.

- a. 4 gallons to \$3.00
- b. 1 gallon to \$1.50
- c. 30 gallons to \$45.00
- d. \$2.00 to 30 gallons
- e. \$1.00 to  $\frac{2}{3}$  gallon

2. A caterer needs to buy 21 pounds of pasta to cater a wedding. At a local store, 8 pounds of pasta cost \$12. How much will the caterer pay for the pasta there?

- a. Write a ratio for the given information about the cost of pasta.
- b. Would it be more helpful to write an equivalent ratio with 1 pound of pasta as one of the numbers, or with \$1 as one of the numbers? Explain your reasoning, and then write that equivalent ratio.
- c. Find the answer and explain or show your reasoning.

3. Lin is reading a 47-page book. She read the first 20 pages in 35 minutes.
- If she continues to read at the same rate, will she be able to complete this book in under 1 hour?
  - If so, how much time will she have left? If not, how much more time is needed? Explain or show your reasoning.

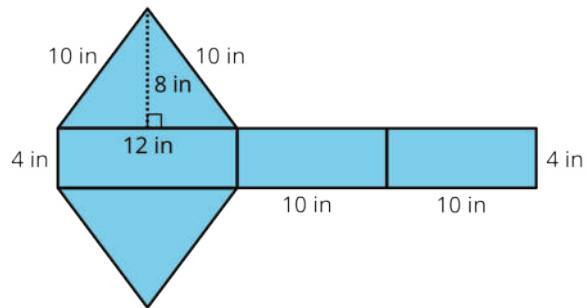
4. Diego can type 140 words in 4 minutes.
- At this rate, how long will it take him to type 385 words?
  - How many words can he type in 15 minutes?

If you get stuck, consider creating a table.

5. A train that travels 30 miles in  $\frac{1}{3}$  hour at a constant speed is going faster than a train that travels 20 miles in  $\frac{1}{2}$  hour at a constant speed. Explain or show why.

(From Unit 2, Lesson 10.)

6. Find the surface area of the polyhedron that can be assembled from this net. Show your reasoning.

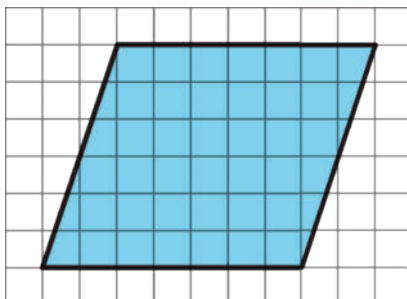


(From Unit 1, Lesson 14.)

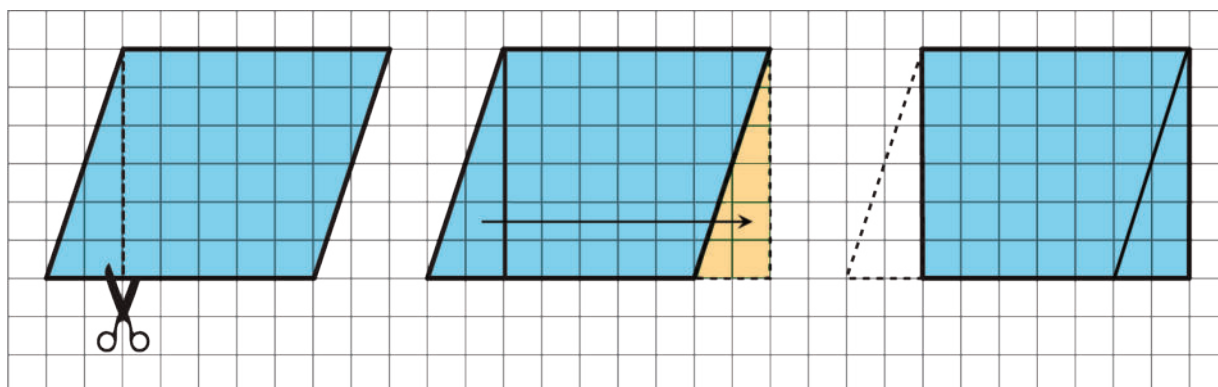
# Lesson 5: Bases and Heights of Parallelograms

## 5.1: A Parallelogram and Its Rectangles

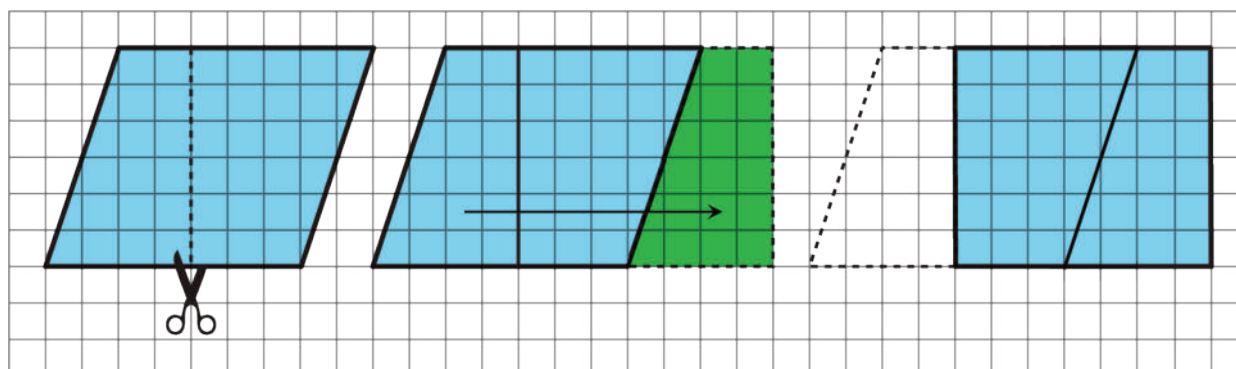
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:

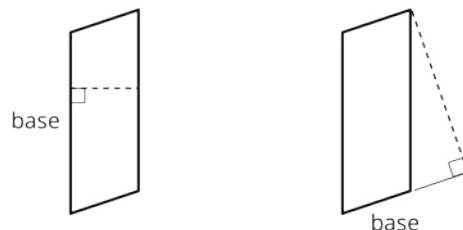
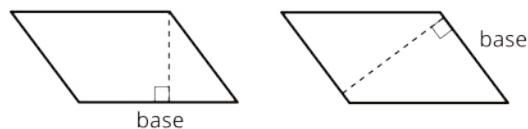


How are the two strategies for finding the area of a parallelogram the same? How they are different?

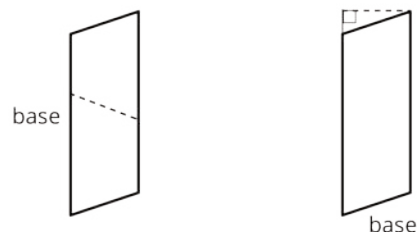
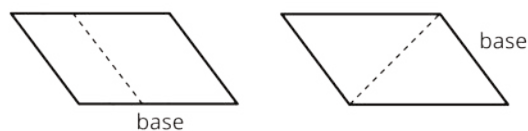
## 5.2: The Right Height?

Study the examples and non-examples of **bases** and **heights** of parallelograms.

- Examples: The dashed segments in these drawings represent the corresponding height for the given base.

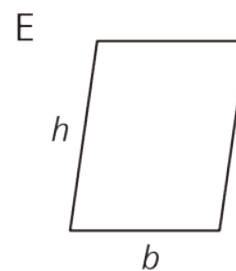
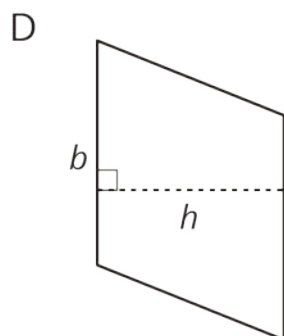
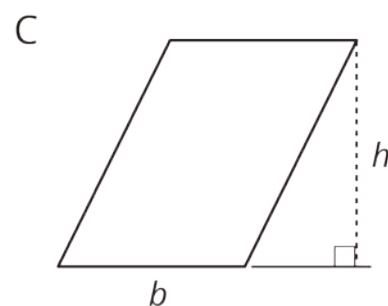
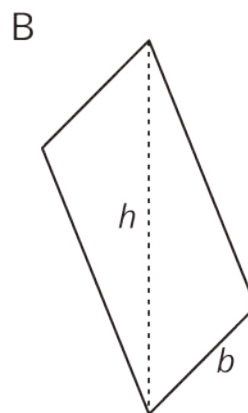
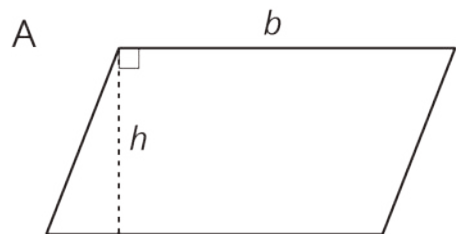


- Non-examples: The dashed segments in these drawings do *not* represent the corresponding height for the given base.



1. Select **all** the statements that are true about bases and heights in a parallelogram.
  - a. Only a horizontal side of a parallelogram can be a base.
  - b. Any side of a parallelogram can be a base.
  - c. A height can be drawn at any angle to the side chosen as the base.
  - d. A base and its corresponding height must be perpendicular to each other.
  - e. A height can only be drawn inside a parallelogram.
  - f. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.
  - g. A base cannot be extended to meet a height.

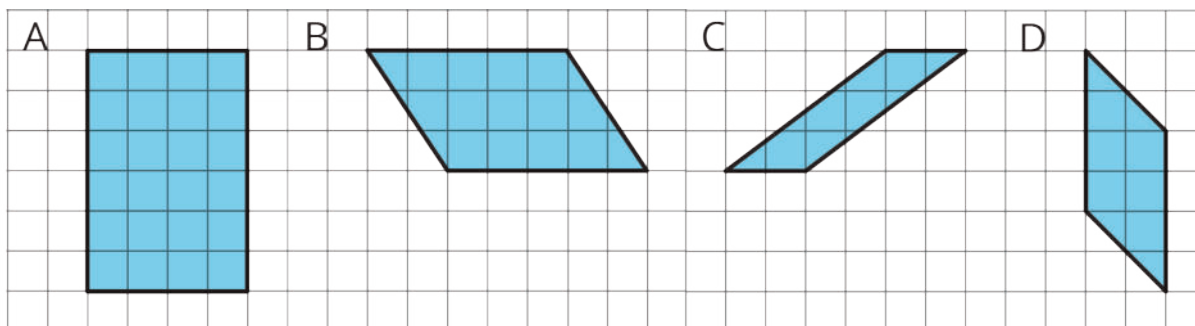
2. Five students labeled a base  $b$  and a corresponding height  $h$  for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.



### 5.3: Finding the Formula for Area of Parallelograms

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	$b$	$h$	

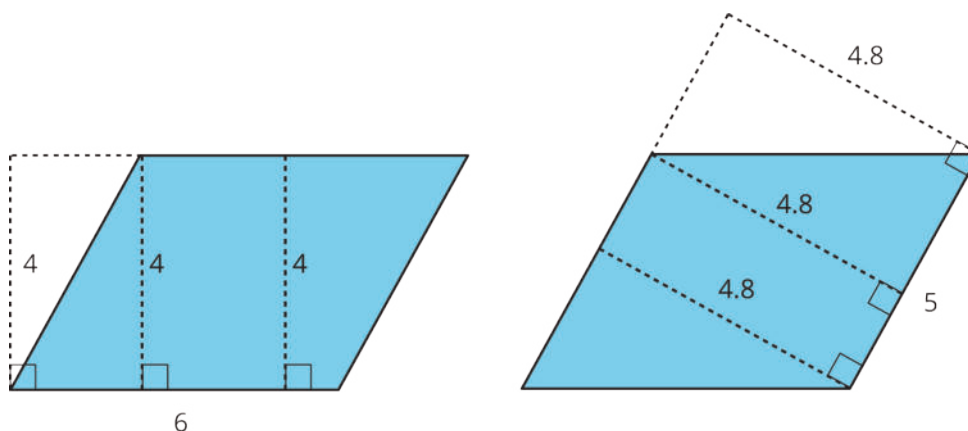
In the last row, write an expression for the area of any parallelogram, using  $b$  and  $h$ .

#### Are you ready for more?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

## Lesson 5 Summary

- We can choose any of the four sides of a parallelogram as the **base**. Both the side (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!



Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

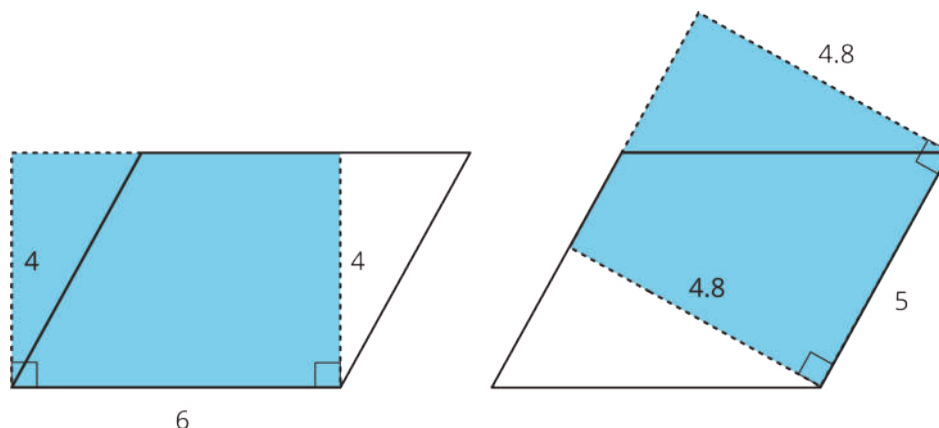
No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$$4 \times 6 = 24$$

and

$$4.8 \times 5 = 24$$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.





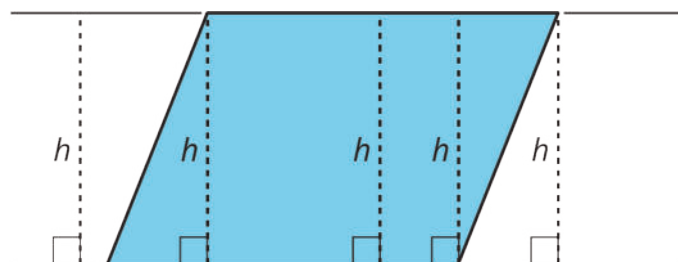
Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as the parallelogram.

We often use letters to stand for numbers. If  $b$  is base of a parallelogram (in units), and  $h$  is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a  $\times$  symbol. This is so that we don't get confused about whether  $\times$  means multiply, or whether the letter  $x$  is standing in for a number.

In high school, you will be able to prove that a perpendicular segment from a point on one side of a parallelogram to the opposite side will always have the same length.

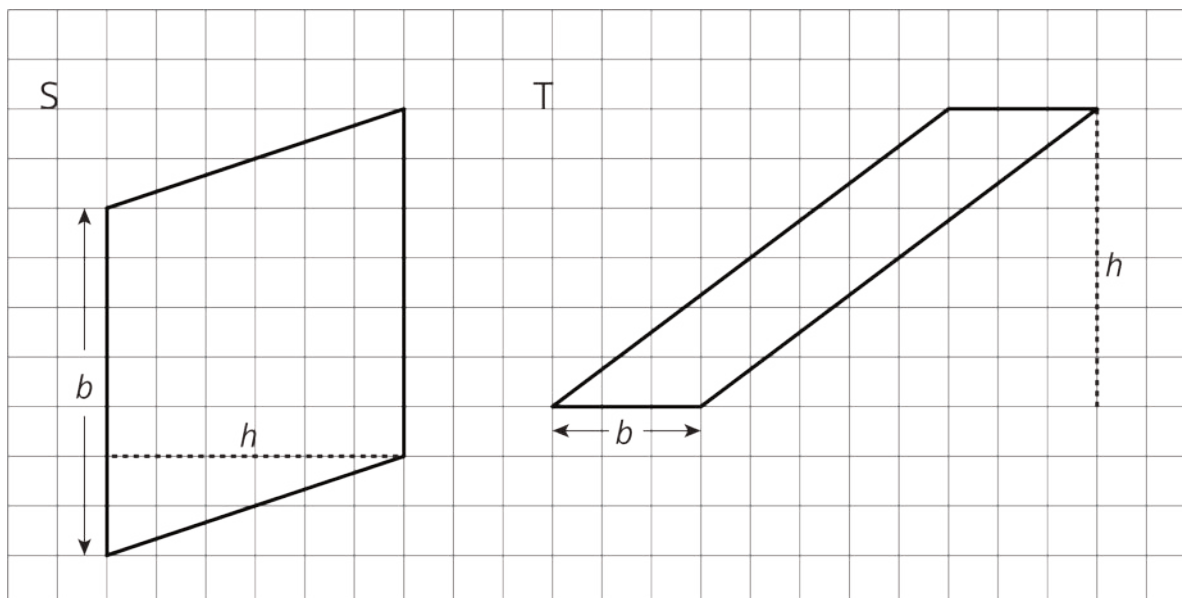


You can see this most easily when you draw a parallelogram on graph paper. For now, we will just use this as a fact.

# Lesson 5: Bases and Heights of Parallelograms

## Cool Down: Parallelograms S and T

Parallelograms S and T are each labeled with a base and a corresponding height.



1. What are the values of  $b$  and  $h$  for each parallelogram?

○ Parallelogram S:  $b = \underline{\hspace{2cm}}$ ,  $h = \underline{\hspace{2cm}}$

○ Parallelogram T:  $b = \underline{\hspace{2cm}}$ ,  $h = \underline{\hspace{2cm}}$

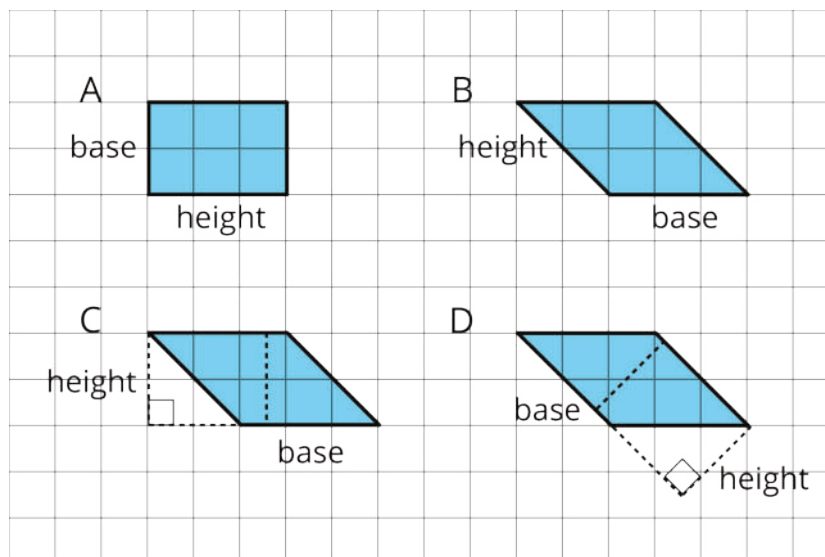
2. Use the values of  $b$  and  $h$  to find the area of each parallelogram.

○ Area of Parallelogram S:

○ Area of Parallelogram T:

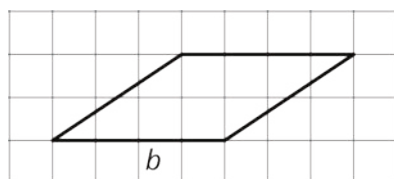
## Unit 1 Lesson 5 Cumulative Practice Problems

1. Select **all** parallelograms that have a correct height labeled for the given base.



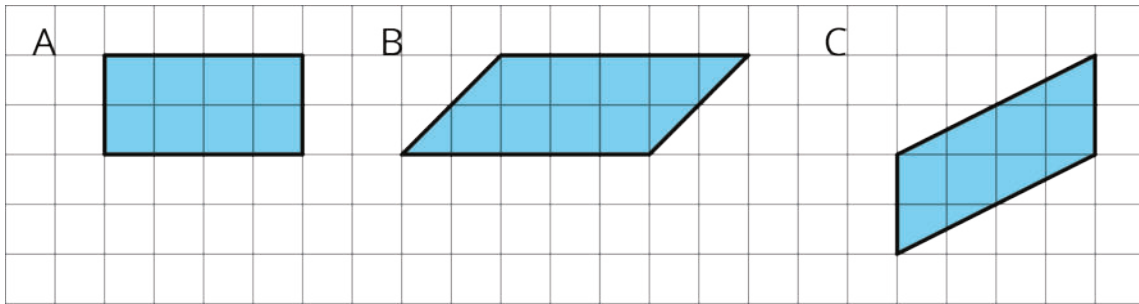
- A. A
- B. B
- C. C
- D. D

2. The side labeled  $b$  has been chosen as the base for this parallelogram.

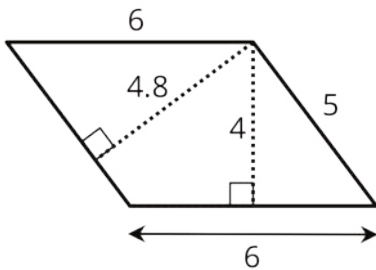


Draw a segment showing the height corresponding to that base.

3. Find the area of each parallelogram.

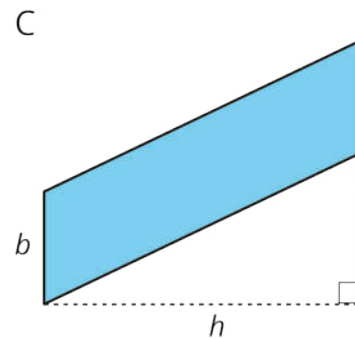
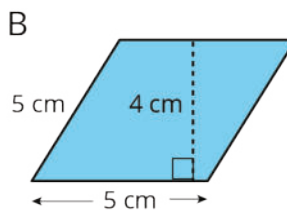
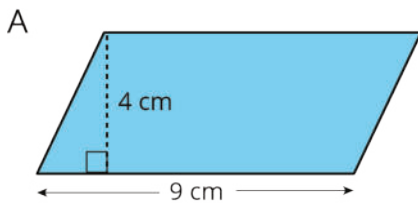


4. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



- A. 6 units
- B. 4.8 units
- C. 4 units
- D. 5 units

5. Find the area of each parallelogram.



6. Do you agree with each of these statements? Explain your reasoning.

- a. A parallelogram has six sides.
- b. Opposite sides of a parallelogram are parallel.
- c. A parallelogram can have one pair or two pairs of parallel sides.
- d. All sides of a parallelogram have the same length.
- e. All angles of a parallelogram have the same measure.

(From Unit 1, Lesson 4.)

7. A square with an area of 1 square meter is decomposed into 9 identical small squares. Each small square is decomposed into two identical triangles.

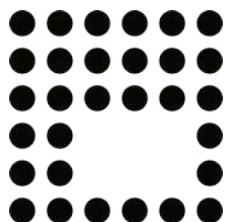
a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.

b. How many triangles are needed to compose a region that is  $1\frac{1}{2}$  square meters?

(From Unit 1, Lesson 2.)

# Lesson 6: Area of Parallelograms

## 6.1: Missing Dots

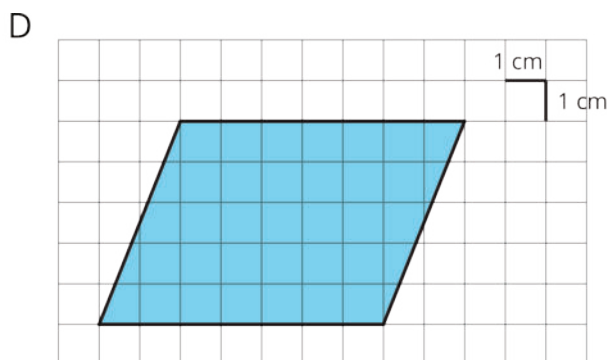
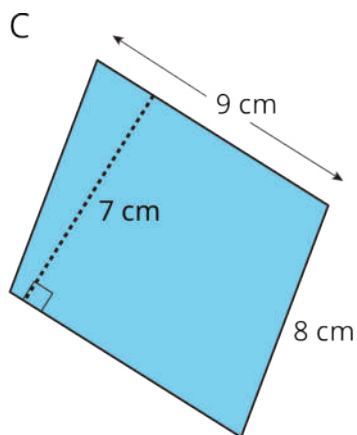
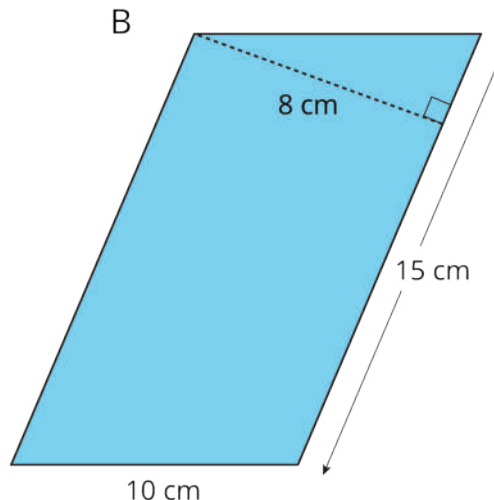
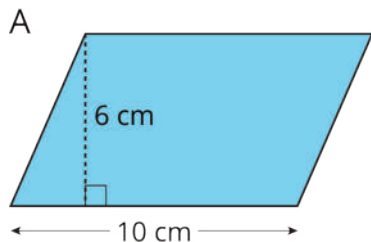


How many dots are in the image?

How do you see them?

## 6.2: More Areas of Parallelograms

1. Find the area of each parallelogram. Show your reasoning.



2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

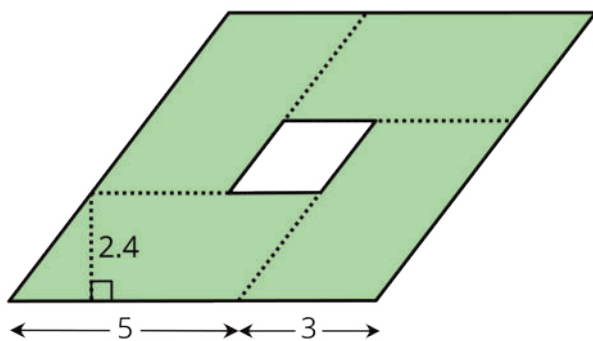
3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

On the grid, draw two parallelograms that could be P and Q.



**Are you ready for more?**

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

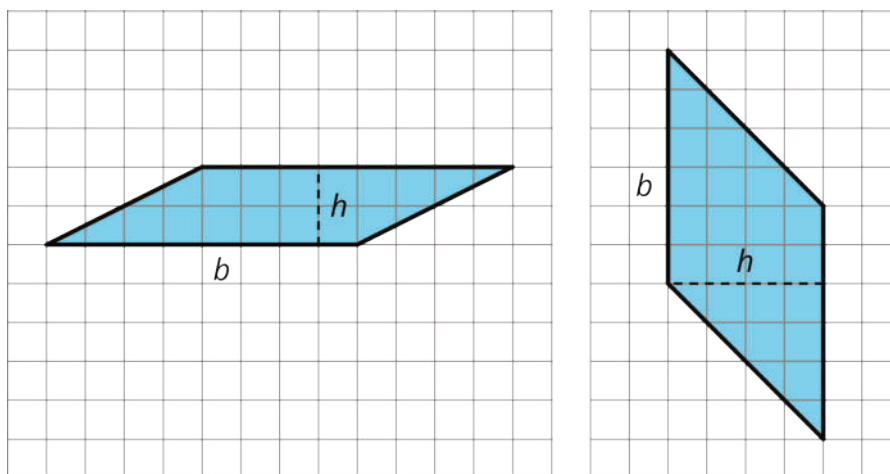


What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

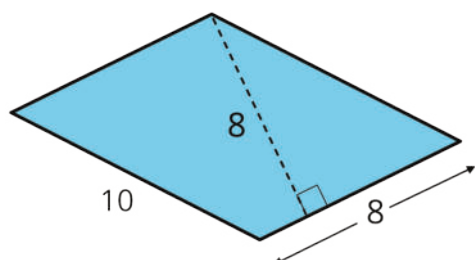
## Lesson 6 Summary

Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

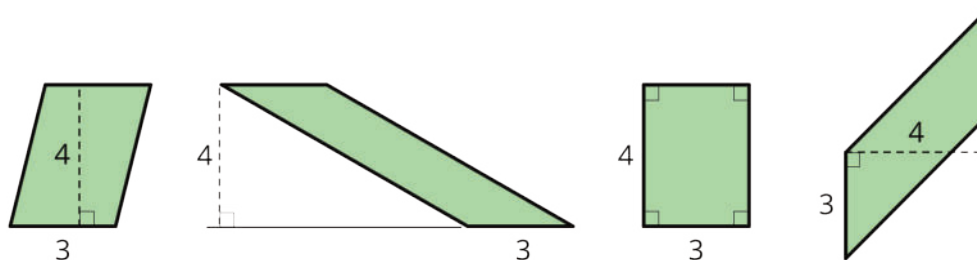


When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

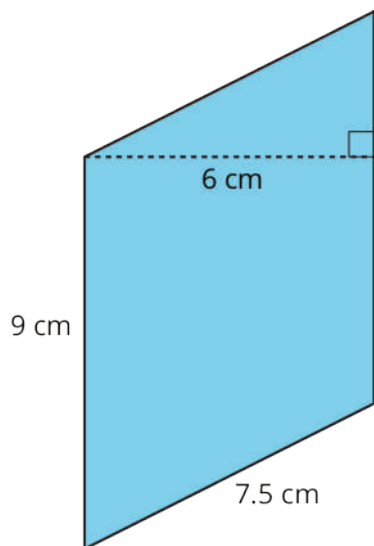
Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.





# Lesson 6: Area of Parallelograms

## Cool Down: One More Parallelogram

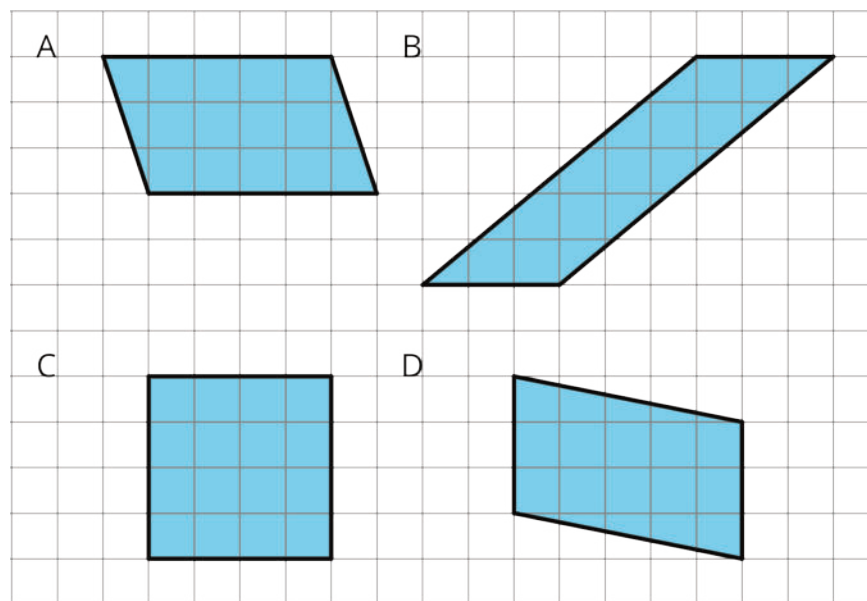


1. Find the area of the parallelogram. Explain or show your reasoning.

2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

## Unit 1 Lesson 6 Cumulative Practice Problems

1. Which three of these parallelograms have the same area as each other?



- A. A
- B. B
- C. C
- D. D

2. Which pair of base and height produces the greatest area? All measurements are in centimeters.

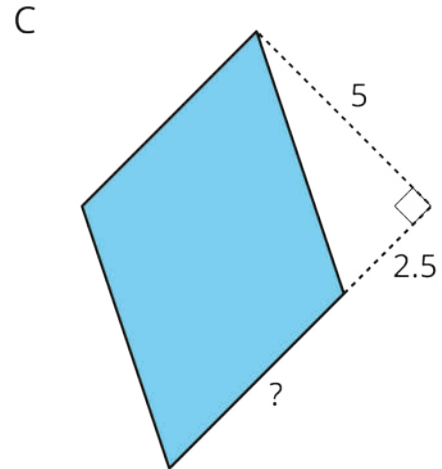
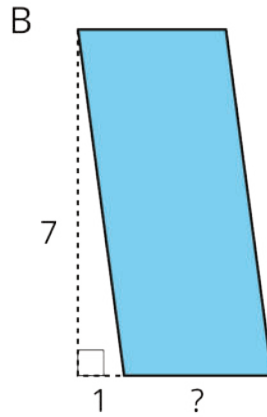
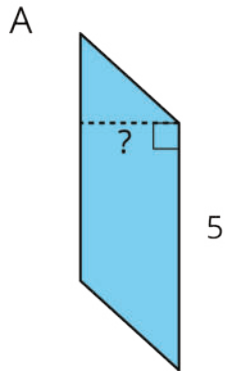
- A.  $b = 4, h = 3.5$
- B.  $b = 0.8, h = 20$
- C.  $b = 6, h = 2.25$
- D.  $b = 10, h = 1.4$

3. Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

A: 10 square units

B: 21 square units

C: 25 square units

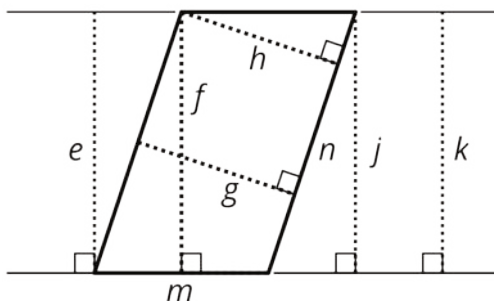


4. The Dockland Building in Hamburg, Germany is shaped like a parallelogram.



If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

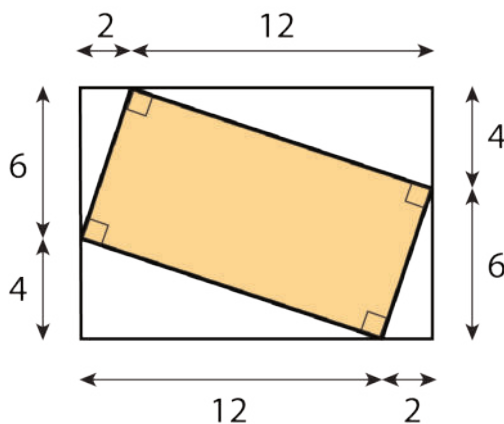
5. Select **all** segments that could represent a corresponding height if the side  $m$  is the base.



- A.  $e$
- B.  $f$
- C.  $g$
- D.  $h$
- E.  $j$
- F.  $k$
- G.  $n$

(From Unit 1, Lesson 5.)

6. Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



(From Unit 1, Lesson 3.)