Plan for Grade 6 Unit 4: Dividing Fractions

Relevant Unit(s) to review: Grade 5 Unit 3: Fraction Multiplication and Division, Grade 3 Unit 3: Wrapping Up 1,000

Essential prior concepts to engage with this unit	• Understand division as an unknown-factor problem. • Understand and represent "How many groups?" and "How many in each group?" problems and understand division as an unknown factor problem. • Interpret division as a whole number
Brief narrative of approach	The unit's broad goal is for students to examine how the relative sizes of a numerator and denominator affect a quotient. Students use tape diagrams, equations, and expressions to represent situations involving partitive or quotitive interpretations of division with fractions before moving towards an algorithm for fraction division. The final lesson of the unit is recommended to be skipped given time constraints for other units which may need more intentional support and to accommodate additional tasks from Grade 5. Specifically, students may need more work interpreting whole number quotients and two types of division: "how many groups" (measurement division) and "how many in each group" (partitive division) . Tasks in Lessons 13–16 use division in the context of volume and area. Use these lessons if additional support for these geometric topics or if the additional more concrete examples would benefit students.

Lessons to Add	Lessons to Remove or Modify
1. Add Grade 5, Unit 3, Lesson 11: This lesson focuses on students determining the size of the piece when a unit fraction is divided into equally sized pieces. This supports students to be able to interpret a fraction divided into equal pieces and, in this context, and to connect multiplication with division. If the Check Your Readiness assessment shows that students need additional familiarity with interpreting division, consider also referring to Grade 3 Unit 3 where students interpret whole number division using grouping.	1. Remove 6.4.17: This lesson is an application of the concepts from the unit. It can be moved to outside of class. 2. Combine Lessons 6.4.12 and 6.4.13: Focus on the application of division to lengths of objects 3. Combine Lessons 6.4.14 and 6.4.15: Focus on the application of division to the volume of objects
Lessons added: 1	Lessons removed: 3

Modified Plan for Grade 6 Unit 4

Day	IM lesson	Notes
1	assessment	Check Your Readiness: Administer all items at least a couple of days before beginning instruction to gather data. Use the guidance provided with each problem to adjust instruction so that students can access the math in the unit. In particular, use questions 4, 5, and 6 to determine whether students need additional familiarity with understanding division. Use this information to decide whether to include the suggested lessons from Grade 5.
2	6.4.1	
3	6.4.2	If the Check Your Readiness assessment shows that students are familiar with interpreting division, consider moving more quickly through this lesson.

4	5.3.11	Dividing a unit fraction by a whole number.
5	6.4.3	
6	6.4.4	
7	6.4.5	
8	6.4.6	
9	Mid Unit Assessment	
10	6.4.7	
11	6.4.8	
12	6.4.9	
13	6.4.10	
14	6.4.11	
15	6.4.12 6.4.13	Consider combining these lessons: emphasize connecting division to finding length.
16	6.4.14 6.4.15	Consider combining these lessons: emphasize connecting division to finding volume.
17	6.4.16	
18	6.4 End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

Lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
6.4.1	_	E	This lesson focuses on exploring quotients of different sizes. The cool-down focuses on comparing values of 1.
6.4.2	0	E	This lesson focuses on exploring ways to think about division. The cool-down focuses on interpreting how to use multiplication algorithms in division.
6.4.3	0	D	This lesson focuses on division in context. The cool-down focuses on writing a division problem from a situation, then solving.
6.4.4	+	D	This lesson focuses on connecting concrete and pictorial representations and connecting them to division. The cool-down focuses on drawing patterns that represent division.
6.4.5	+	D	This lesson builds on Lesson 4 and connects concrete and visual representations of division with an equation. The cool-down focuses on matching an equation with a given situation.
6.4.6	+	D	This lesson focuses on the concept of how many groups. (measurement division) The cool-down focuses on how many groups by connecting to a multiplication algorithm.
6.4.7	+	D	This lesson focuses on the idea of “how many groups” when the number of groups is less than one. The cool-down focuses on this and uses a tape diagram.
6.4.8	+	D	This lesson focuses on using division problems to find the size of one group. (partitive) The cool-down focuses on showing the size of the group with an equation and a visual model.
6.4.9	+	D	This lesson focuses on finding one group in different situations. The cool-down provides students a diagram to help support demonstrating partitive division.

6.4.10	+	A	This lesson focuses on looking for patterns when dividing by a fraction. The cool-down focuses on finding a whole number divided by a fraction using both a diagram and a multiplication algorithm.
6.4.11	0	A	This lesson focuses on using an algorithm to divide fractions. The cool-down focuses on using an algorithm to solve a fraction divided by fraction problems.
6.4.12	0	D	This lesson focuses on solving fraction division problems involving length. The cool-down focuses on solving a fraction problem involving length in the context of a story problem.
6.4.13	0	D	This lesson focuses on exploring rectangles that have fractional measurements. The cool-down focuses on applying division by fractions to find the side length given area in a story problem.
6.4.14	-	A	This lesson focuses on exploring the area and volume where fractions are involved. The cool-down focuses on finding volume in cubic inches given fractional values.
6.4.15	-	D	This lesson focuses on solving volume problems that involve fractions. The cool-down focuses on finding a height given base and volume as well as problems with edge length to find the volume.
6.4.16	0	A	This lesson focuses on using fractional operations to solve word problems. The cool-down focuses on taking a problem in context, solving, and justifying one's reasoning.
6.4.17	-	A	This culminating activity is optional. In this task, students are given a scenario involving the Postal Service and flat-rate boxes. Students take what they have learned about fractions to find shipping costs.

Lesson 11: Patterns with Division of a Unit Fraction by a Whole Number

Standards Alignments

Addressing 5.NF.B.7.b

Teacher-facing Learning Goals

- Make sense of diagrams that represent division of a unit fraction by a whole number.

Student-facing Learning Goals

- Let's make sense of the division of a unit fraction by a whole number.

Lesson Purpose

The purpose of this lesson is to use diagrams and equations to represent division of a unit fraction by a whole number.

In the previous lesson, students solved problems about dividing a unit fraction by a whole number in a way that made sense to them. In this lesson, they use tape diagrams to evaluate division expressions. The tape diagram is also used in later lessons to represent dividing a whole number by a unit fraction. In the first activity, students may notice a relationship between dividing a unit fraction by a whole number and multiplying unit fractions. For example, when describing $\frac{1}{3} \div 4$, they may say, "That is the same as $\frac{1}{4} \times \frac{1}{3}$". Ask these students to explain where they see this relationship in the diagrams. In the second activity, students recognize the relationship between multiplication and division when they explain why $\frac{1}{3} \div 2$ is not equal to $\frac{1}{2}$. In the third activity, students recognize and explain the relationship between multiplication and division.

Instructional Routines

Estimation Exploration (Warm-up), MLR3 Clarify, Critique, Correct (Activity 2)

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	10 min
Activity 3	10 min

Teacher Reflection Question

What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson) 5 min

Evaluate Division Expressions

Standards Alignments

Addressing 5.NF.B.7.b

Student-facing Task Statement

Find the value of each expression. Use a diagram, if it is helpful.

1. $\frac{1}{5} \div 2$
2. $\frac{1}{5} \div 3$

Student Responses

1. $\frac{1}{10}$
2. $\frac{1}{15}$

Begin Lesson

Warm-up 10 min

Estimation Exploration: How Much is Shaded?

Standards Alignments

Addressing 5.NF.B.7.b

The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to share a

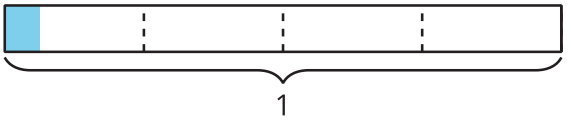
mathematical claim and the thinking behind it (MP3). Asking yourself, “Does this make sense?” is a component of making sense of problems (MP1), and making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

Instructional Routines

Estimation Exploration

Student-facing Task Statement

How much is shaded?



Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses

- Too low: $\frac{1}{100}$ - $\frac{1}{40}$
- About right: $\frac{1}{20}$ - $\frac{1}{16}$
- Too high: $\frac{1}{8}$ - $\frac{1}{12}$

Launch

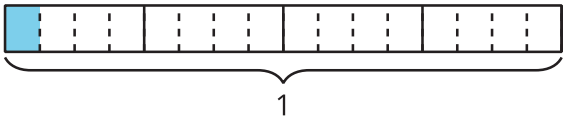
- Groups of 2
- Display the image/expression.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

Optional: Reveal the actual value and add it to the display.



Consider asking:

- “Is anyone’s estimate less than _____? Is anyone’s estimate greater than _____?”
- “Based on this discussion, does anyone want to revise their estimate?”

Activity 1

10 min

Diagrams, Equations, Situations

Standards Alignments

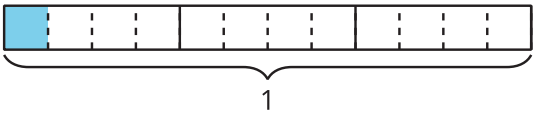
Addressing 5.NF.B.7.b

In this activity, students interpret division of a unit fraction by a whole number using tape diagrams. In future lessons, students will use tape diagrams to understand division of a whole number by a unit fraction. The first two activities are structured so students attend to the structure of the tape diagram and recognize how the tape is used to show both a fractional part of a whole being divided into a whole number of pieces and also the size of each resulting piece in relation to the whole. The third activity is meant for students to begin to notice structure in equations when dividing a fraction by a whole number. Students may still need to draw a diagram to solve the equations.

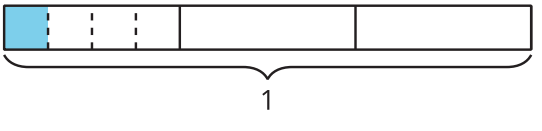
Student-facing Task Statement

Priya and Mai used the diagrams below to find the value of $\frac{1}{3} \div 4 = \_\_\_\_\_$.

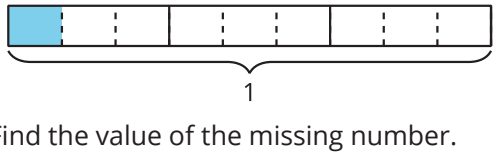
Priya's diagram:



Mai's diagram:



1. What is the same about the diagrams?
2. What is different?
3. Find the value of the missing number. $\frac{1}{3} \div 4 = \_\_\_\_\_$
4. Han drew the diagram below to represent $\frac{1}{3} \div 3$. Where do you see $\frac{1}{3} \div 3$?



5. Find the value of the missing number. Explain or show your reasoning.

Launch

- Groups of 2

Activity

- Monitor for students who:
  - can explain how Mai's diagram shows $\frac{1}{3}$ divided into 4 equal pieces
  - can explain how Priya's diagram shows that the size of each piece, after dividing $\frac{1}{3}$ into 4 equal pieces will be $\frac{1}{12}$

Synthesis

- Ask previously selected students to share how Priya and Mai's diagrams are similar and different.
- Display the diagrams that Priya and Mai drew and this equation: $\frac{1}{3} \div 4 = \frac{1}{12}$
- "Where do we see $\frac{1}{12}$ in Priya's diagram?" (It is the shaded piece. We know it is $\frac{1}{12}$ of the whole because Priya divided all the thirds into 4 pieces.)

$\frac{1}{3} \div 3 =$ _____

Student Responses

1. Sample responses: They both show 1 divided into 3 pieces. They both show a shaded blue piece. It looks like the shaded blue piece is the same size. They both show one third divided into 4 pieces.
2. Sample responses: Priya divided the other thirds into 4 pieces and Mai didn't.
3. $\frac{1}{3} \div 4 = \frac{1}{12}$
4. Sample response: $\frac{1}{3}$ is cut into 3 equal pieces. One of the pieces is shaded blue.
5. $\frac{1}{3} \div 3 = \frac{1}{9}$

- "How can we change Mai's diagram to show that $\frac{1}{3}$ divided into 4 equal pieces means that each piece will be $\frac{1}{12}$ of the whole?" (We can divide the other thirds into 4 pieces, too.)
- Mark Mai's diagram to divide the other thirds in Mai's drawing into 4 pieces.

Advancing Student Thinking

Students may not be able to see $\frac{1}{3} \div 4$ in Priya's or Mai's tape diagrams. Encourage students to draw $\frac{1}{3} \div 4$ in a way that makes sense to them. Then ask, "How does your diagram relate to Priya's or Mai's diagrams?"

Activity 2

10 min

Reason About Revisions

PLC Activity

Standards Alignments

Addressing 5.NF.B.7.b

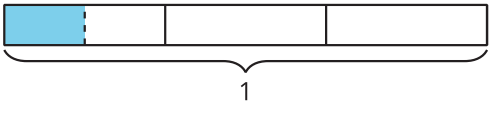
In the previous activity, students explain how tape diagrams represent equations and they use diagrams to solve division equations. In this activity, students examine a mistake in order to recognize the relationship between the number of pieces the fraction is being divided into and the size of the resulting piece. This activity uses *MLR3 Collect and Display. Advances: reading, writing, representing*

Instructional Routines

MLR3 Clarify, Critique, Correct

Student-facing Task Statement

1. Find the value of the expression. Show or explain your thinking. $\frac{1}{3} \div 2$_____
2. This is Priya's work for finding the value of $\frac{1}{3} \div 2$:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because I divided $\frac{1}{3}$ into 2 equal pieces and $\frac{1}{2}$ of $\frac{1}{3}$ is shaded in.

   a. What questions do you have for Priya?
   b. Priya's equation is incorrect. How can Priya revise her explanation?
   c. Revise your work if necessary.

Student Responses

1. $\frac{1}{6}$: Students may draw a diagram that shows $\frac{1}{3}$ divided into 2 equal pieces, each of which is the size of $\frac{1}{6}$ of the whole.
2. Sample responses:
   a. Why didn't you cut the other thirds? Why do you think $\frac{1}{3} \div 2$ $\frac{1}{2}$?
   b. Sample response: She should change her answer to $\frac{1}{6}$ and cut the other thirds into 2 equal pieces so you can see the sixths.
   c. Answers vary.

Launch

- Groups of 2

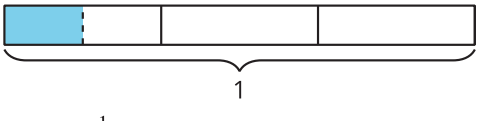
Activity

- 5 minutes

Synthesis

MLR3 Clarify, Critique, Correct

- Display the following partially correct answer and explanation:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because that is how much is shaded in.

- Read the explanation aloud.
- "What do you think Priya means?" (She shaded in $\frac{1}{2}$ of $\frac{1}{3}$.)
- "Is anything unclear?" (If you divide $\frac{1}{3}$ into 2 pieces, the answer will be smaller than $\frac{1}{3}$ and $\frac{1}{2}$ is larger than $\frac{1}{3}$.)
- "Are there any mistakes?" (The equation should be $\frac{1}{3} \div 2 = \frac{1}{6}$.)
- 1 minute: quiet think time
- 2 minutes: partner discussion
- "With your partner, work together to write a revised explanation."
- Display and review the following criteria:
  - explanation for each step
  - correct solution
  - labeled diagram

- 3–5 minutes: partner work time
- Select 1–2 groups to share their revised explanation with the class. Record responses as students share.
- "What is the same and different about the explanations?"
- Display a revised diagram for Priya's work or use the one from student responses.
- "Where do we see $\frac{1}{3} \div 2$?" (The shaded section shows one of the pieces if you divide $\frac{1}{3}$ into 2 equal pieces.)
- "Where do we see $\frac{1}{2} \times \frac{1}{3}$?" (The shaded section also shows $\frac{1}{2}$ of $\frac{1}{3}$.)
- "What fraction of the whole diagram is shaded in?" $\frac{1}{6}$
- Display: $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2}$
- "How do we know this is true?" (We can see both expressions in the diagram and they are both equal to $\frac{1}{6}$.)

Advancing Student Thinking

Students may not have a correct response to the first problem. Encourage students to draw a diagram to represent $\frac{1}{3} \div 2$, and shade the quotient. Then ask, "What is the size of the shaded amount?" Consider asking, "Is your response reasonable?" and "Is this quotient greater than or less than $\frac{1}{3}$?"

Activity 3

10 min

Look for Patterns

Standards Alignments

Addressing 5.NF.B.7.b

In the previous activity students recognize the relationship between multiplication and division when they explain why $\frac{1}{3} \div 2$ is not equal to $\frac{1}{2}$. In this activity, students notice as the divisor increases for a given dividend, the quotient gets smaller. Students also recognize and explain the relationship between multiplication and division.

Student-facing Task Statement

1. Solve the equations. Use a diagram if it is helpful.
   a. $\frac{1}{4} \div 2 =$ _____
   b. $\frac{1}{4} \div 3 =$ _____
   c. $\frac{1}{4} \div 4 =$ _____
2. What patterns do you notice?
3. Fill in the blanks to make the equation true. Show or explain your reasoning.

   $\frac{1}{3} \div$ _____ $= \frac{1}{\_\_} \times \frac{1}{3}$

Student Responses

1. $\frac{1}{4} \div 2 = \frac{1}{8}$ $\frac{1}{4} \div 3 = \frac{1}{12}$ $\frac{1}{4} \div 4 = \frac{1}{16}$
2. The quotient is getting smaller, the denominator in the quotient is the same as 4 times the number of pieces.
3. Answers vary. Sample responses: $\frac{1}{3} \div 2 = \frac{1}{2} \times \frac{1}{3}$, $\frac{1}{3} \div 3 = \frac{1}{3} \times \frac{1}{3}$, $\frac{1}{3} \div 4 = \frac{1}{4} \times \frac{1}{3}$, if I divide $\frac{1}{3}$ into 2 equal pieces, that is the same as finding $\frac{1}{2}$ of $\frac{1}{3}$, or I wrote a 4 in the first blank and figured out that $\frac{1}{3} \div 4 = \frac{1}{12}$ and then I figured out what I had to multiply $\frac{1}{3}$ by to get $\frac{1}{12}$.

Launch

- Groups of 2

Activity

- 1–2 minutes: independent think time
- 3–5 minutes: partner work time

Synthesis

- Display:
  $\frac{1}{4} \div 2 = \frac{1}{8}$
  $\frac{1}{4} \div 3 = \frac{1}{12}$
  $\frac{1}{4} \div 4 = \frac{1}{16}$
- "What patterns do you notice?" (The quotient is getting smaller. The denominator is getting bigger. The denominator in the quotient increases by 4. The denominator in the quotient is equal to 4 times the number you are dividing by.)
- "Why is the quotient getting smaller?" (Because we are dividing $\frac{1}{4}$ into more pieces each time, so the size of each piece will be smaller.)
- Display:
  $\frac{1}{4} \div 3$
  $\frac{1}{3} \times \frac{1}{4}$

- "What is the same about these expressions? What is different?" (They both have $\frac{1}{4}$ in them. They both represent $\frac{1}{4}$ being split into 3 pieces. They are both equal to $\frac{1}{12}$.)

Advancing Student Thinking

Students may not initially know how to complete the equation for the last problem. Have students review their responses for the first problem. Ask, "How does this relate to what you know about fraction multiplication?" Consider referring to one of their completed responses to the first problem: $\frac{1}{4} \div 4 = \frac{1}{16}$, and asking, "How would you complete this equation: $\_\_\_\_\_ \times \frac{1}{4} = \frac{1}{16}$?" Then ask, "How might this reasoning help you complete the equation in the last problem?"

Lesson Synthesis

10 min

Display:

$$\frac{1}{3} \div \_\_\_\_\_ = \frac{1}{\_\_\_\_\_} \times \frac{1}{3}$$

"What are some numbers we can write in the blanks to make the equation true?"

Record answers as new equations for all to see.

"Explain to your partner why the number in the blanks on each side of the equation is the same." (Dividing a unit fraction sized piece is the same as finding a fraction of a fraction.)

Record answers on a poster and keep the display to refer back to in future lessons.

Suggested Centers

- Rolling For Fractions, Stage 8: Division Involving a Whole Number and Unit Fraction

Complete Cool-Down

Response to Student Thinking

If a student responds with the following:

Students do not write $\frac{1}{5} \div 2 = \frac{1}{10}$ and $\frac{1}{5} \div 3 = \frac{1}{15}$.

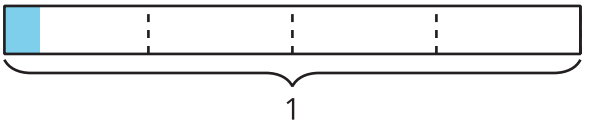
Next Day Support

- Create a poster with a diagram that represents this cool-down.

Lesson 11: Patterns with Division of a Unit Fraction by a Whole Number

Warm-up: Estimation Exploration: How Much is Shaded?

How much is shaded?



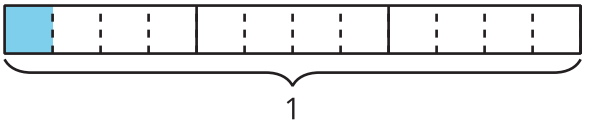
Record an estimate that is:

too low	about right	too high

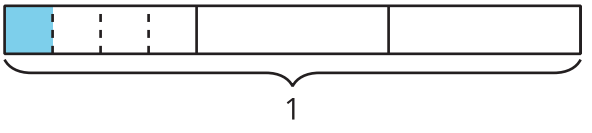
11.1: Diagrams, Equations, Situations

Priya and Mai used the diagrams below to find the value of $\frac{1}{3} \div 4 =$ _____.

Priya's diagram:



Mai's diagram:

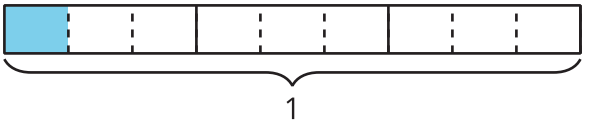


1. What is the same about the diagrams?

2. What is different?

3. Find the value of the missing number. $\frac{1}{3} \div 4 =$ _____

4. Han drew the diagram below to represent $\frac{1}{3} \div 3$. Where do you see $\frac{1}{3} \div 3$?



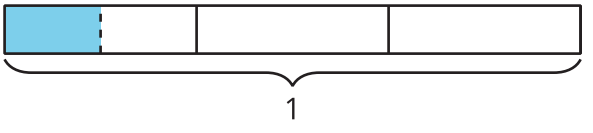
5. Find the value of the missing number. Explain or show your reasoning.

$\frac{1}{3} \div 3 =$ _____

11.2: Reason About Revisions

1. Find the value of the expression. Show or explain your thinking. $\frac{1}{3} \div 2$_____

2. This is Priya's work for finding the value of $\frac{1}{3} \div 2$:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because I divided $\frac{1}{3}$ into 2 equal pieces and $\frac{1}{2}$ of $\frac{1}{3}$ is shaded in.

   a. What questions do you have for Priya?

   b. Priya's equation is incorrect. How can Priya revise her explanation?

   c. Revise your work if necessary.

11.3: Look for Patterns

1. Solve the equations. Use a diagram if it is helpful.

   a. $\frac{1}{4} \div 2 =$ _____

   b. $\frac{1}{4} \div 3 =$ _____

   c. $\frac{1}{4} \div 4 =$ _____

2. What patterns do you notice?

3. Fill in the blanks to make the equation true. Show or explain your reasoning.

   $\frac{1}{3} \div$ _____ $= \frac{1}{\_\_} \times \frac{1}{3}$

Lesson 11: Patterns with Division of a Unit Fraction by a Whole Number

Cool Down: Evaluate Division Expressions

Find the value of each expression. Use a diagram, if it is helpful.

1. $\frac{1}{5} \div 2$

2. $\frac{1}{5} \div 3$

Unit 3 Lesson 11 Aligned Practice Problems

1. Mai has a strip of paper that is 3 feet long. She cuts it into $\frac{1}{4}$ foot strips. How many $\frac{1}{4}$ foot strips does Mai make? Write a division equation to represent your answer.