Plan for Grade 6 Unit 2: Introducing Ratios

Relevant Unit(s) to review: Grade 5 Units 3 and 6

Essential prior concepts to engage with this unit	• additive reasoning • use of a number line • dividing one whole number by another • multiplication as scaling
Brief narrative of approach	This unit will be taught with fidelity with some minor adjustments. The unit's broad goal, to introduce students to the concept of ratio, starts with an accessible entry point where students are introduced to the concept of ratios using direct modeling and counting strategies. From there, students learn multiple approaches to make sense of ratio thinking such as double number lines, tables, and tape diagrams.

Lessons to Add	**Lessons to Remove or Modify**
An introductory understanding of the use of a number line and tape diagram is helpful when engaging with this unit. These lessons might make a nice reference to activate prior knowledge: 1. IM Grade 5, Unit 3, Lessons 9 and 11 for division using algorithms. 2. IM Grade 5, Unit 6, Lesson 14 for the use of a number line and multiplication by scaling.	1. 6.2.1 and 6.2.2 combine to make sense of ratios. 2. 6.2.3 and 6.2.4 have similar goals. 6.2.3 also requires physical manipulatives and could be combined with a lesson from the Accelerated course Acc6.2.2. 3. 6.14 is optional and centers around using an additional strategy. 4. 6.1.17 is optional. It is a culminating task that could be done outside of class.
Lessons added: 3	Lessons removed: 4

Modified Plan for Grade 6 Unit 2

Day	IM lesson	Notes
	Check Your Readiness	
1	6.2.1 6.2.2	Combine these lessons with an emphasis on making sense of ratios.
2	5.3.9	Understand how the value of a quotient changes when the numerator or denominator changes.
3	5.3.11	Recall division of whole numbers including the use of tape diagrams.
4	Acc6.2.2	Use the Accelerated lesson to work with ratios.
5	6.2.5	
6	6.2.6	
7	5.6.14	Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
8	6.2.7	Introduction to the double number line.
9	6.2.8	Emphasize “per” and “at this rate”.
10	6.2.9	
11	6.2.10	
12	6.2.11	
13	6.2.12	Introduction to ratio tables.
14	6.2.13	
15	6.2.15	Introduction to tape diagrams

16	6.2.16	
17	End Assessment	

Priority and Category List for Lessons

High priority (+), Medium priority (0), Low priority (-)

E: Explore, Play, and Discuss, D: Deep Dive, A: Synthesize and Apply

IM lesson	Priority (+, 0, -)	Category (E, D, A)	Notes
6.2.1	+	E	In this lesson, students use collections of objects to make sense of the idea of a ratio and compare two quantities at the same time. Activity 2 is a high priority towards this outcome. The cool down asks students both to draw a diagram to express a ratio and to complete statements to describe a ratio correctly.
6.2.2	0	D	In this lesson, students use diagrams to represent situations involving ratios to continue to develop ratio language. The cool down asks students both to draw a diagram to express a ratio and to complete statements to describe a ratio correctly.
6.2.3	0	E	First of two lessons to develop the idea of equivalent ratios through physical experiences. Students see that scaling a ratio up or down requires multiplying the amount of each ingredient. The cool-down asks students to scale down a ratio.
6.2.4	0	D	This is the second of two lessons to develop the idea of equivalent ratios. The cool-down asks students to write equivalent ratios given a doubling or tripling of scale.
6.2.5	+	A	In this lesson, students work through different types of ratio problems that involve scaling. The cool-down asks students given a ratio to create an equivalent one.
6.2.6	+	E	In this lesson, students get introduced to the double number line. The cool down

			asks students to create a double number line and use it to answer further questions.
6.2.7	+	E	In this lesson, students create double number line diagrams from scratch. The cool-down gives students a ratio (using cats ears, paws, and tails) to represent using a double number line.
6.2.8	+	E	In this lesson, students are introduced to the idea of a unit price. In this context, they are introduced to the idea of "per" and "at this rate". The cool down gives students a double number line, and a ratio as a tool to find both a unit rate and an unknown quantity that requires scaling.
6.2.9	0	A	In this lesson, students are looking at the idea of ratios through the context of constant speed. The cool-down has students compare two different sets of double number lines that represent the speed of a train to figure out which one is moving faster.
6.2.10	+	A	In this lesson, students compare ratios to see if two situations in familiar contexts involve the same rate. The cool-down rationalizes whether two given rates with different quantities (2 people running) are the same rate.
6.2.11	+	E	In this lesson, students are introduced to the idea of using tables to solve ratio problems. The cool-down asks students to interpret a table for equivalent ratios and, given the table, to find unknown quantities.
6.2.12	+	D	In this lesson, students build on what they learned about tables and ratios in the previous lesson. Here, they use multipliers to solve ratio problems. The cool down asks students, given a rate with different quantities, to find different values of each quantity.
6.2.13	+	D	In this lesson, students explicitly connect and contrast double number lines and tables. The cool-down asks students given a rate, to find a value (in this case how far a cyclist travels in 3 seconds).
6.2.14	-	A	This lesson is optional. The purpose of this lesson is to give students further practice in solving equivalent ratio problems and introduce them to the info gap activity structure. Cool down asks given missing information, what a student would need to solve a ratio problem.
6.2.15	+	A	In this lesson, students are introduced to tape diagrams as a way to represent

			ratios. The cool-down asks students to find the total of each part. given a total amount (189 square feet) and a 3 part ratio (example 4:3:2).
6.2.16	+	A	In this lesson, students use all representations they have learned in this unit—double number lines, tables, and tape diagrams—to solve ratio problems that involve the sum of the quantities in the ratio. The cool-down gives a scenario where students are given a ratio (6 ounces dough, 4 ounces sauce) and total (130 ounces total) and are to find the values that equal the sum of the quantities.
6.2.17	-	A	The lesson is optional.

Lesson 9: Whole Number Division

Standards Alignments

Addressing	5.NBT.B.7, 5.NF.B.7
Building Towards	5.NBT.B.6, 5.NF.B.7

Teacher-facing Learning Goals

- Reason about the size of quotients in whole-number division problems.

Student-facing Learning Goals

- Let's think about quotients.

Lesson Purpose

The purpose of this lesson is for students to reason about the size of a quotient and consider the relationships between the dividend, divisor, and quotient.

In grade 3, students' work with whole number division focuses on two types of problems: "How many groups?" and "How many in each group?" In the previous unit, students connected division to fractions.

Students start this unit by revisiting the meaning of division through whole numbers. They examine the relationship between the numbers in division problems and compare the size of the quotient by reasoning about the relative sizes of the divisor and dividend. This prepares them to make sense of division involving whole numbers and unit fractions in subsequent lessons.

Students with Disabilities (SwD)

- Representation (Activity 1)

English Learners (EL)

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up), MLR2 Collect and Display (Activity 1)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What did you see or hear in your students' responses today that showed evidence of their understandings of division? How will you leverage this understanding in the rest of this section?

Cool-down (to be completed at the end of the lesson)

5 min

Reason About Division

Standards Alignments

Addressing 5.NBT.B.7, 5.NF.B.7

Student-facing Task Statement

The value of $392 \div 7$ is less than the value of $392 \div 28$.

1. Do you agree with this statement? Show or explain your reasoning.

Student Responses

1. No, because if you divide the same number into fewer groups, you will have more in each group. $392 \div 7$ is more than $392 \div 28$.

Begin Lesson

Warm-up

10 min

Number Talk: Same Dividend, Different Divisor

Standards Alignments

Building Towards 5.NBT.B.6

This Number Talk encourages students to think about the relationship between the size of the divisor and the size of the quotient and to rely on the structure of division expressions to mentally solve problems.

To find the value of division expressions with the same dividend, students need to look for and make use of structure (MP7). In explaining their answers, students need to be precise in their word choice and use of language (MP6).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $120 \div 12$
- $120 \div 6$
- $120 \div 3$
- $120 \div 2$

Student Responses

- 10: I know it.
- 20: There are twice as many groups as in the last problem, so there will be half as many in each group.
- 40: 12 divided by 3 is 4. 12 tens divided by 3 is 4 tens. 4 tens equals
- 60: I know half of 120 is 60.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

Focus question: “Why did the quotient get bigger with each problem?” (There are fewer groups so there will be more in each group.)

Consider asking:

- “Who can restate _____'s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone approach the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”

Activity 1

20 min

Share Pretzels

Standards Alignments

Building Towards 5.NBT.B.6, 5.NF.B.7

The purpose of this activity is to compare quotients of quantities based on the relative size of the dividend and the divisor. Students should be encouraged to use whatever strategy makes sense to them to order situations about sharing pretzels. The numbers were intentionally chosen so that students don't have to do any calculations to solve the problem. In upcoming lessons, students will divide a unit fraction by a whole number and a whole number by a unit fraction. This activity uses MLR2 Collect and Display. Advances: conversing, reading, writing. As students work, listen for the language they use to describe the relationship between the number of people sharing pretzels and the number of pretzels being shared. Record the authentic language students are using to describe the relationship. During the synthesis of the activity, connect the students' authentic language to more formal math vocabulary. Keep the display of language up throughout the section so students can refer to it in later lessons.

English Learners (EL)

MLR8 Discussion Supports. Activity: During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . ." Original speakers can agree or clarify for their partner. *Advances: Listening, Speaking*

Students with Disabilities (SwD)

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most useful to solve the problem. Display the sentence frame: "When comparing quotients of quantities, I will pay attention to . . ." *Supports accessibility for: Conceptual Processing; Language; Memory.*

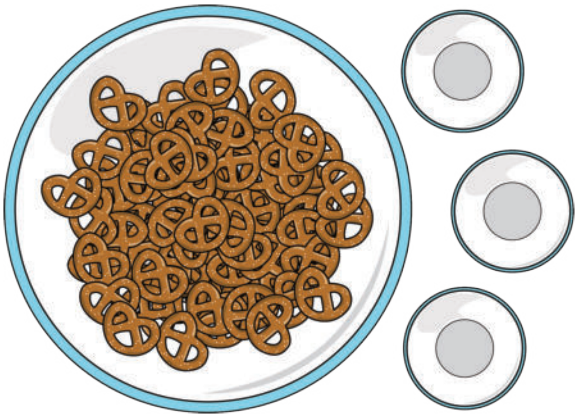
Instructional Routines

MLR2 Collect and Display

Student-facing Task Statement

Launch

- Groups of 2
- Display:

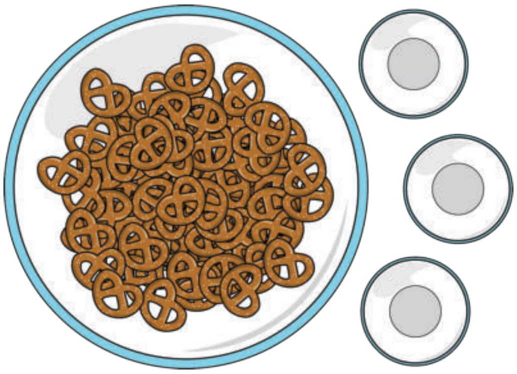


Order the situations from greatest to least based on the number of pretzels each student will get. Be prepared to explain your reasoning.

3 students equally share 42 pretzels
14 students equally share 42 pretzels
3 students equally share 24 pretzels
3 students equally share 45 pretzels
7 students equally share 42 pretzels
3 students equally share 6 pretzels
6 students equally share 42 pretzels

Student Responses

3 students equally share 45 pretzels
3 students equally share 42 pretzels
3 students equally share 24 pretzels
6 students equally share 42 pretzels
7 students equally share 42 pretzels
14 students equally share 42 pretzels
3 students equally share 6 pretzels



- "What do you notice? What do you wonder?"

Activity

- 1–2 minutes: quiet think time
- 10 minutes: partner work time

MLR2 Collect and Display

- Circulate, listen for, and collect the language students use to describe the relationship between the number of students sharing and the number of pretzels being shared. Listen for: number in each group, size of the group, number of pretzels each person gets, dividend, divisor, quotient.
- Record students' words and phrases on a visual display and update it throughout the lesson.

Synthesis

- "Are there any other words or phrases that are important to include on our display?"
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- Display:

3 students equally share 45 pretzels
3 students equally share 42 pretzels
3 students equally share 24 pretzels
3 students equally share 6 pretzels

- "What is the same? What is different?" (The same number of students are sharing different numbers of pretzels.)
- "How does the number of pretzels each person gets change in each situation?" (It gets smaller because there are fewer pretzels to share.)
- Display:
  14 students equally share 42 pretzels
  7 students equally share 42 pretzels
  6 students equally share 42 pretzels
- "What is the same? What is different?" (The number of pretzels being shared is the same. The number of students sharing is different.)
- "How does the number of pretzels each person gets change in each situation?" (When fewer people share the same number of pretzels, each person gets more pretzels.)

Advancing Student Thinking

Students may not immediately visualize the different situations. Encourage them to draw a tape diagram to represent each situation. Consider asking, "How might looking at the diagrams help you put the situations in order?"

Activity 2

15 min

Division Patterns

Standards Alignments

Building Towards 5.NBT.B.6, 5.NF.B.7

The purpose of this activity is to review division of whole numbers with whole number quotients. The numbers in these problems were intentionally chosen so students consider the size of the quotient in relation to the size of the dividend. Display the poster of the language students used to describe the relationship between quotient, dividend, and divisor during the previous activity.

Student-facing Task Statement

1. Solve.

   $36 \div 3$

   $12 \div 3$

   $9 \div 3$

   $6 \div 3$

   $3 \div 3$

   $1 \div 3$
2. What patterns do you notice?
3. Why is the quotient getting smaller?
4. What do you know about this expression: $\frac{1}{3} \div 3$?
5. Draw a diagram to show what $\frac{1}{3} \div 3$ might look like.

Student Responses

1. $36 \div 3 = 12$
   $12 \div 3 = 4$
   $9 \div 3 = 3$
   $6 \div 3 = 2$
   $3 \div 3 = 1$
   $1 \div 3 = \frac{1}{3}$
2. Sample responses: All the problems are about dividing by 3; the number that is being divided gets smaller, so does the quotient.
3. Sample response: It is getting smaller because the number of things being

Launch

- Groups of 2
- 5 minutes: quiet think time

Activity

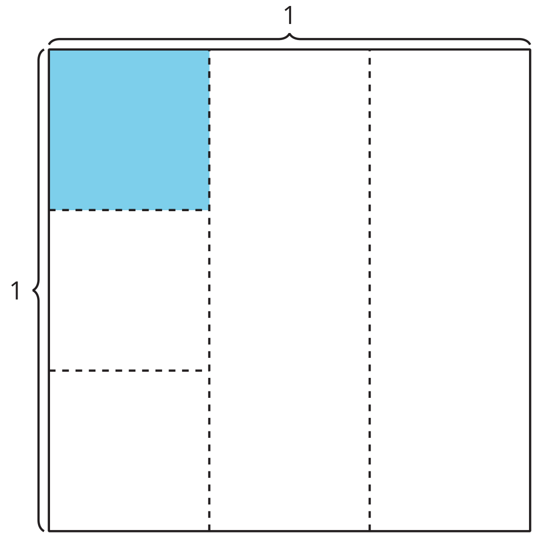
- Monitor for students who:
  - can explain why the quotient gets smaller when the dividend gets smaller
  - describe the quotient of $\frac{1}{3} \div 3$ as being smaller than $\frac{1}{3}$
  - draw a diagram to show $\frac{1}{3}$ divided into 3 equal sections

Synthesis

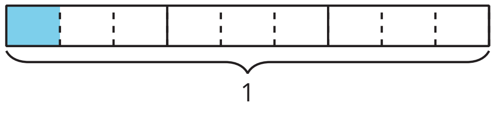
- Ask previously identified students to share their solutions.
- "Why is the quotient getting smaller as the dividend gets smaller?" (There are a smaller number of things being split into the same number of groups, so there will be fewer in each group.)
- "Why is $\frac{1}{3} \div 3$ going to be smaller than $\frac{1}{3}$?" ($\frac{1}{3}$ is being divided into 3 equal pieces.)
- Display student diagrams like the ones in student responses.
- "Where do we see $\frac{1}{3}$ being divided into 3 equal pieces in the diagrams?"

shared is smaller, so there will be fewer in each group.

4. Sample responses: It will be smaller than $\frac{1}{3}$, it will be $\frac{1}{3}$ of $\frac{1}{3}$



5.



Advancing Student Thinking

Students may not immediately visualize the patterns in the division expressions. Encourage them to draw a tape diagram for each expression, and ask them what they notice. Consider asking, “What is happening to the size of each group as the amount being divided gets smaller?”

Lesson Synthesis

10 min

“Today we noticed patterns while dividing whole numbers. What do we know about division?” (When you divide the same number into smaller groups, the number in each group gets larger. When you divide a larger number of things into the same size groups, there will be more in each group. Division is about sharing. When you divide a smaller number into the same size groups, the number in each group gets smaller.)

Record student responses for all to see.

“What do you still wonder about division?” (Can you divide fractions? When would you ever need to divide a fraction? Does the answer get smaller or bigger when you divide fractions?)

Record student responses for all to see. Keep the display visible. Refer back to it in future lessons.

Suggested Centers

- Rolling For Fractions, Stage 8: Division Involving a Whole Number and Unit Fraction

Complete Cool-Down

Response to Student Thinking

If a student responds with the following:

Students either agree with the statement or do not explain their response with a clear understanding of the relationship between the dividend, divisor and quotient.

Next Day Support

- Launch Warm-up or Activity 1 by highlighting key vocabulary from previous lessons.

If a student responds with the following:

Students do not choose a reasonable estimate.

Prior Unit Support

Grade 4, Unit 5, Section C: Section C

Lesson 9: Whole Number Division

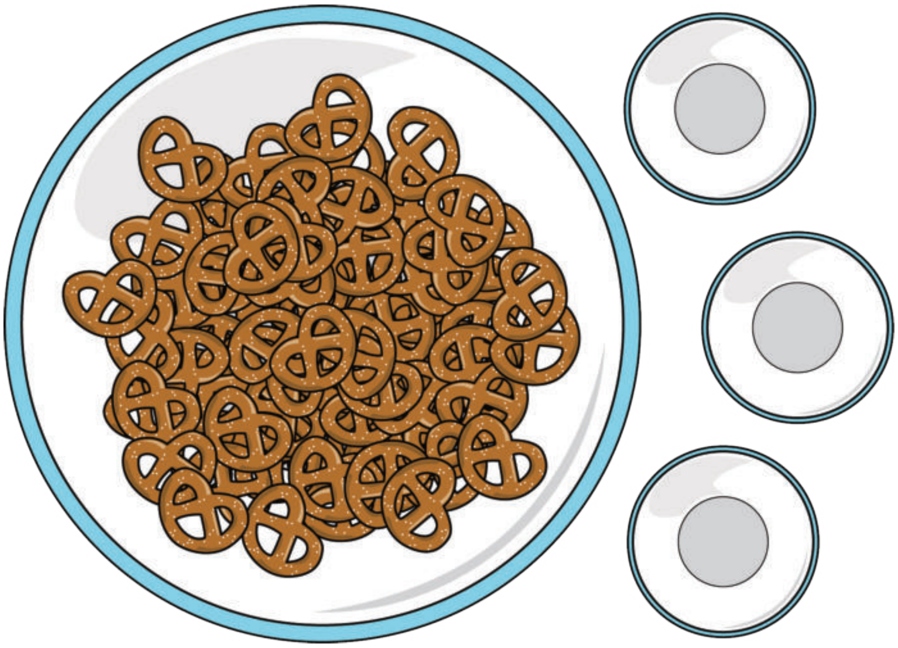
- Let’s think about quotients.

Warm-up: Number Talk: Same Dividend, Different Divisor

Find the value of each expression mentally.

- $120 \div 12$
- $120 \div 6$
- $120 \div 3$
- $120 \div 2$

9.1: Share Pretzels



Order the situations from greatest to least based on the number of pretzels each student will get. Be prepared to explain your reasoning.

3 students equally share 42 pretzels
14 students equally share 42 pretzels
3 students equally share 24 pretzels
3 students equally share 45 pretzels
7 students equally share 42 pretzels
3 students equally share 6 pretzels
6 students equally share 42 pretzels

9.2: Division Patterns

1. Solve.

   $36 \div 3$

   $12 \div 3$

   $9 \div 3$

   $6 \div 3$

   $3 \div 3$

   $1 \div 3$

2. What patterns do you notice?

   ___

3. Why is the quotient getting smaller?

   ___

4. What do you know about this expression: $\frac{1}{3} \div 3$?

   ___

5. Draw a diagram to show what $\frac{1}{3} \div 3$ might look like.

Lesson 11: Patterns with Division of a Unit Fraction by a Whole Number

Standards Alignments

Addressing 5.NF.B.7.b

Teacher-facing Learning Goals

- Make sense of diagrams that represent division of a unit fraction by a whole number.

Student-facing Learning Goals

- Let's make sense of the division of a unit fraction by a whole number.

Lesson Purpose

The purpose of this lesson is to use diagrams and equations to represent division of a unit fraction by a whole number.

In the previous lesson, students solved problems about dividing a unit fraction by a whole number in a way that made sense to them. In this lesson, they use tape diagrams to evaluate division expressions. The tape diagram is also used in later lessons to represent dividing a whole number by a unit fraction. In the first activity, students may notice a relationship between dividing a unit fraction by a whole number and multiplying unit fractions. For example, when describing $\frac{1}{3} \div 4$, they may say, "That is the same as $\frac{1}{4} \times \frac{1}{3}$". Ask these students to explain where they see this relationship in the diagrams. In the second activity, students recognize the relationship between multiplication and division when they explain why $\frac{1}{3} \div 2$ is not equal to $\frac{1}{2}$. In the third activity, students recognize and explain the relationship between multiplication and division.

Instructional Routines

Estimation Exploration (Warm-up), MLR3 Clarify, Critique, Correct (Activity 2)

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	10 min
Activity 3	10 min

Teacher Reflection Question

What did you say, do, or ask during the lesson synthesis that helped students be clear on the learning of the day? How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Evaluate Division Expressions

Standards Alignments

Addressing 5.NF.B.7.b

Student-facing Task Statement

Find the value of each expression. Use a diagram, if it is helpful.

1. $\frac{1}{5} \div 2$
2. $\frac{1}{5} \div 3$

Student Responses

1. $\frac{1}{10}$
2. $\frac{1}{15}$

Begin Lesson

Warm-up

10 min

Estimation Exploration: How Much is Shaded?

Standards Alignments

Addressing 5.NF.B.7.b

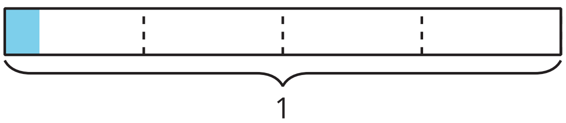
The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it (MP3). Asking yourself, “Does this make sense?” is a component of making sense of problems (MP1), and making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

Instructional Routines

Estimation Exploration

Student-facing Task Statement

How much is shaded?



Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses

- Too low: $\frac{1}{100}$ - $\frac{1}{40}$
- About right: $\frac{1}{20}$ - $\frac{1}{16}$
- Too high: $\frac{1}{8}$ - $\frac{1}{12}$

Launch

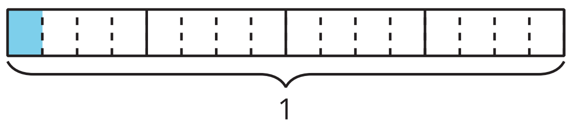
- Groups of 2
- Display the image/expression.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

Optional: Reveal the actual value and add it to the display.



Consider asking:

- “Is anyone’s estimate less than _____? Is anyone’s estimate greater than _____?”
- “Based on this discussion, does anyone want to revise their estimate?”

Activity 1

10 min

Diagrams, Equations, Situations

Standards Alignments

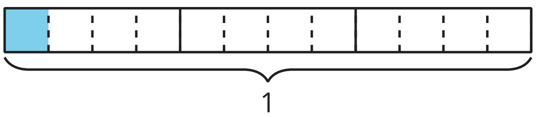
Addressing 5.NF.B.7.b

In this activity, students interpret division of a unit fraction by a whole number using tape diagrams. In future lessons, students will use tape diagrams to understand division of a whole number by a unit fraction. The first two activities are structured so students attend to the structure of the tape diagram and recognize how the tape is used to show both a fractional part of a whole being divided into a whole number of pieces and also the size of each resulting piece in relation to the whole. The third activity is meant for students to begin to notice structure in equations when dividing a fraction by a whole number. Students may still need to draw a diagram to solve the equations.

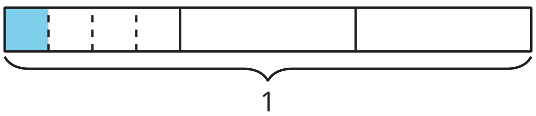
Student-facing Task Statement

Priya and Mai used the diagrams below to find the value of $\frac{1}{3} \div 4 =$ ______.

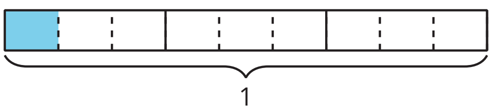
Priya's diagram:



Mai's diagram:



1. What is the same about the diagrams?
2. What is different?
3. Find the value of the missing number.
   $\frac{1}{3} \div 4 =$ ______
4. Han drew the diagram below to represent $\frac{1}{3} \div 3$. Where do you see $\frac{1}{3} \div 3$?



5. Find the value of the missing number. Explain or show your reasoning.

Launch

- Groups of 2

Activity

- Monitor for students who:
  - can explain how Mai's diagram shows $\frac{1}{3}$ divided into 4 equal pieces
  - can explain how Priya's diagram shows that the size of each piece, after dividing $\frac{1}{3}$ into 4 equal pieces will be $\frac{1}{12}$

Synthesis

- Ask previously selected students to share how Priya and Mai's diagrams are similar and different.
- Display the diagrams that Priya and Mai drew and this equation: $\frac{1}{3} \div 4 = \frac{1}{12}$
- "Where do we see $\frac{1}{12}$ in Priya's diagram?" (It is the shaded piece. We know it is $\frac{1}{12}$ of the whole because Priya divided all the thirds into 4 pieces.)

$\frac{1}{3} \div 3 = $ ______

Student Responses

1. Sample responses: They both show 1 divided into 3 pieces. They both show a shaded blue piece. It looks like the shaded blue piece is the same size. They both show one third divided into 4 pieces.
2. Sample responses: Priya divided the other thirds into 4 pieces and Mai didn't.
3. $\frac{1}{3} \div 4 = \frac{1}{12}$
4. Sample response: $\frac{1}{3}$ is cut into 3 equal pieces. One of the pieces is shaded blue.
5. $\frac{1}{3} \div 3 = \frac{1}{9}$

- "How can we change Mai's diagram to show that $\frac{1}{3}$ divided into 4 equal pieces means that each piece will be $\frac{1}{12}$ of the whole?" (We can divide the other thirds into 4 pieces, too.)
- Mark Mai's diagram to divide the other thirds in Mai's drawing into 4 pieces.

Advancing Student Thinking

Students may not be able to see $\frac{1}{3} \div 4$ in Priya's or Mai's tape diagrams. Encourage students to draw $\frac{1}{3} \div 4$ in a way that makes sense to them. Then ask, "How does your diagram relate to Priya's or Mai's diagrams?"

Activity 2

10 min

Reason About Revisions

PLC Activity

Standards Alignments

Addressing 5.NF.B.7.b

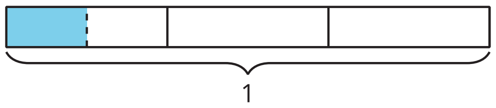
In the previous activity, students explain how tape diagrams represent equations and they use diagrams to solve division equations. In this activity, students examine a mistake in order to recognize the relationship between the number of pieces the fraction is being divided into and the size of the resulting piece. This activity uses *MLR3 Collect and Display. Advances: reading, writing, representing*

Instructional Routines

MLR3 Clarify, Critique, Correct

Student-facing Task Statement

1. Find the value of the expression. Show or explain your thinking. $\frac{1}{3} \div 2$______
2. This is Priya's work for finding the value of $\frac{1}{3} \div 2$:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because I divided $\frac{1}{3}$ into 2 equal pieces and $\frac{1}{2}$ of $\frac{1}{3}$ is shaded in.

   a. What questions do you have for Priya?
   b. Priya's equation is incorrect. How can Priya revise her explanation?
   c. Revise your work if necessary.

Student Responses

1. $\frac{1}{6}$: Students may draw a diagram that shows $\frac{1}{3}$ divided into 2 equal pieces, each of which is the size of $\frac{1}{6}$ of the whole.
2. Sample responses:
   a. Why didn't you cut the other thirds? Why do you think $\frac{1}{3} \div 2$ $\frac{1}{2}$?
   b. Sample response: She should change her answer to $\frac{1}{6}$ and cut the other thirds into 2 equal pieces so you can see the sixths.
   c. Answers vary.

Launch

- Groups of 2

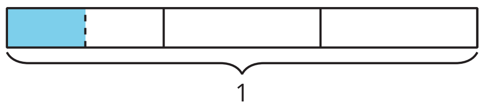
Activity

- 5 minutes

Synthesis

MLR3 Clarify, Critique, Correct

- Display the following partially correct answer and explanation:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because that is how much is shaded in.

- Read the explanation aloud.
- "What do you think Priya means?" (She shaded in $\frac{1}{2}$ of $\frac{1}{3}$.)
- "Is anything unclear?" (If you divide $\frac{1}{3}$ into 2 pieces, the answer will be smaller than $\frac{1}{3}$ and $\frac{1}{2}$ is larger than $\frac{1}{3}$.)
- "Are there any mistakes?" (The equation should be $\frac{1}{3} \div 2 = \frac{1}{6}$.)
- 1 minute: quiet think time
- 2 minutes: partner discussion
- "With your partner, work together to write a revised explanation."
- Display and review the following criteria:
  - explanation for each step
  - correct solution
  - labeled diagram

- 3–5 minutes: partner work time
- Select 1–2 groups to share their revised explanation with the class. Record responses as students share.
- "What is the same and different about the explanations?"
- Display a revised diagram for Priya's work or use the one from student responses.
- "Where do we see $\frac{1}{3} \div 2$?" (The shaded section shows one of the pieces if you divide $\frac{1}{3}$ into 2 equal pieces.)
- "Where do we see $\frac{1}{2} \times \frac{1}{3}$?" (The shaded section also shows $\frac{1}{2}$ of $\frac{1}{3}$.)
- "What fraction of the whole diagram is shaded in?" $\frac{1}{6}$
- Display: $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2}$
- "How do we know this is true?" (We can see both expressions in the diagram and they are both equal to $\frac{1}{6}$.)

Advancing Student Thinking

Students may not have a correct response to the first problem. Encourage students to draw a diagram to represent $\frac{1}{3} \div 2$, and shade the quotient. Then ask, "What is the size of the shaded amount?" Consider asking, "Is your response reasonable?" and "Is this quotient greater than or less than $\frac{1}{3}$?"

Activity 3

10 min

Look for Patterns

Standards Alignments

Addressing 5.NF.B.7.b

In the previous activity students recognize the relationship between multiplication and division when they explain why $\frac{1}{3} \div 2$ is not equal to $\frac{1}{2}$. In this activity, students notice as the divisor increases for a given dividend, the quotient gets smaller. Students also recognize and explain the relationship between multiplication and division.

Student-facing Task Statement

1. Solve the equations. Use a diagram if it is helpful.
   a. $\frac{1}{4} \div 2 =$ ______
   b. $\frac{1}{4} \div 3 =$ ______
   c. $\frac{1}{4} \div 4 =$ ______
2. What patterns do you notice?
3. Fill in the blanks to make the equation true. Show or explain your reasoning.

   $\frac{1}{3} \div$ ______ $= \frac{1}{\_\_\_} \times \frac{1}{3}$

Student Responses

1. $\frac{1}{4} \div 2 = \frac{1}{8}$ $\frac{1}{4} \div 3 = \frac{1}{12}$ $\frac{1}{4} \div 4 = \frac{1}{16}$
2. The quotient is getting smaller, the denominator in the quotient is the same as 4 times the number of pieces.
3. Answers vary. Sample responses: $\frac{1}{3} \div 2 = \frac{1}{2} \times \frac{1}{3}$, $\frac{1}{3} \div 3 = \frac{1}{3} \times \frac{1}{3}$, $\frac{1}{3} \div 4 = \frac{1}{4} \times \frac{1}{3}$, if I divide $\frac{1}{3}$ into 2 equal pieces, that is the same as finding $\frac{1}{2}$ of $\frac{1}{3}$, or I wrote a 4 in the first blank and figured out that $\frac{1}{3} \div 4 = \frac{1}{12}$ and then I figured out what I had to multiply $\frac{1}{3}$ by to get $\frac{1}{12}$.

Launch

- Groups of 2

Activity

- 1–2 minutes: independent think time
- 3–5 minutes: partner work time

Synthesis

- Display:
  $\frac{1}{4} \div 2 = \frac{1}{8}$
  $\frac{1}{4} \div 3 = \frac{1}{12}$
  $\frac{1}{4} \div 4 = \frac{1}{16}$
- "What patterns do you notice?" (The quotient is getting smaller. The denominator is getting bigger. The denominator in the quotient increases by 4. The denominator in the quotient is equal to 4 times the number you are dividing by.)
- "Why is the quotient getting smaller?" (Because we are dividing $\frac{1}{4}$ into more pieces each time, so the size of each piece will be smaller.)
- Display:
  $\frac{1}{4} \div 3$
  $\frac{1}{3} \times \frac{1}{4}$

- "What is the same about these expressions? What is different?" (They both have $\frac{1}{4}$ in them. They both represent $\frac{1}{4}$ being split into 3 pieces. They are both equal to $\frac{1}{12}$.)

Advancing Student Thinking

Students may not initially know how to complete the equation for the last problem. Have students review their responses for the first problem. Ask, "How does this relate to what you know about fraction multiplication?" Consider referring to one of their completed responses to the first problem: $\frac{1}{4} \div 4 = \frac{1}{16}$, and asking, "How would you complete this equation: $\_\_\_\_ \times \frac{1}{4} = \frac{1}{16}$?" Then ask, "How might this reasoning help you complete the equation in the last problem?"

Lesson Synthesis

10 min

Display:

$$\frac{1}{3} \div \_\_\_\_ = \frac{1}{\_\_\_\_} \times \frac{1}{3}$$

"What are some numbers we can write in the blanks to make the equation true?"

Record answers as new equations for all to see.

"Explain to your partner why the number in the blanks on each side of the equation is the same." (Dividing a unit fraction sized piece is the same as finding a fraction of a fraction.)

Record answers on a poster and keep the display to refer back to in future lessons.

Suggested Centers

- Rolling For Fractions, Stage 8: Division Involving a Whole Number and Unit Fraction

Complete Cool-Down

Response to Student Thinking

If a student responds with the following:

Students do not write $\frac{1}{5} \div 2 = \frac{1}{10}$ and $\frac{1}{5} \div 3 = \frac{1}{15}$.

Next Day Support

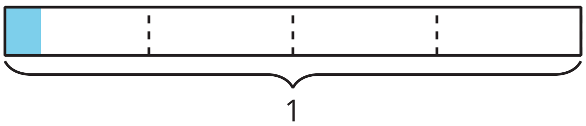
- Create a poster with a diagram that represents this cool-down.

Lesson 11: Patterns with Division of a Unit Fraction by a Whole Number

- Let's make sense of the division of a unit fraction by a whole number.

Warm-up: Estimation Exploration: How Much is Shaded?

How much is shaded?



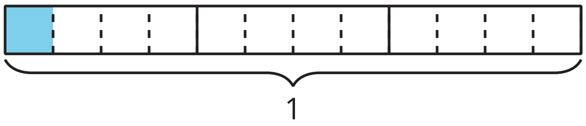
Record an estimate that is:

too low	about right	too high

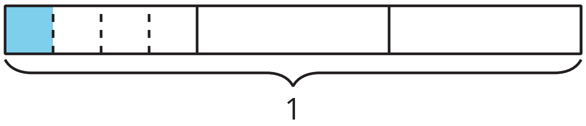
11.1: Diagrams, Equations, Situations

Priya and Mai used the diagrams below to find the value of $\frac{1}{3} \div 4 =$ ______.

Priya's diagram:



Mai's diagram:



1. What is the same about the diagrams?

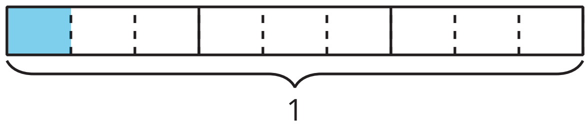
____

2. What is different?

____

3. Find the value of the missing number. $\frac{1}{3} \div 4 =$ ______

4. Han drew the diagram below to represent $\frac{1}{3} \div 3$. Where do you see $\frac{1}{3} \div 3$?

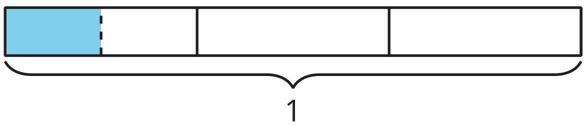


5. Find the value of the missing number. Explain or show your reasoning.
$\frac{1}{3} \div 3 =$ ______

11.2: Reason About Revisions

1. Find the value of the expression. Show or explain your thinking. $\frac{1}{3} \div 2$_______

2. This is Priya's work for finding the value of $\frac{1}{3} \div 2$:



$\frac{1}{3} \div 2 = \frac{1}{2}$ because I divided $\frac{1}{3}$ into 2 equal pieces and $\frac{1}{2}$ of $\frac{1}{3}$ is shaded in.

a. What questions do you have for Priya?

b. Priya's equation is incorrect. How can Priya revise her explanation?

c. Revise your work if necessary.

11.3: Look for Patterns

1. Solve the equations. Use a diagram if it is helpful.

   a. $\frac{1}{4} \div 2 =$ ______

   b. $\frac{1}{4} \div 3 =$ ______

   c. $\frac{1}{4} \div 4 =$ ______

2. What patterns do you notice?

3. Fill in the blanks to make the equation true. Show or explain your reasoning.

$\frac{1}{3} \div \_\_\_\_\_ = \frac{1}{\_\_\_} \times \frac{1}{3}$

Unit 6: Bringing Fractions Back	**Lesson 14:** Compare Without Multiplying

Teacher-facing Learning Goals

- Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Building on CCSS:
Addressing CCSS: 5.NF.B.5

Lesson Purpose

The purpose of this lesson is for students to compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Materials Needed

Gather	**Copy**
•	•

Cool-down: Raising Money

Diego, Kiran, Elena, and Mai were raising money for a school trip.

- Diego raised $40.00.
- Elena sold $\frac{7}{8}$ as much lemonade as Diego.
- Kiran sold $2\frac{1}{2}$ times as much lemonade as Diego.
- Mai sold $\frac{2}{3}$ as much lemonade as Diego.

Write the names of the students in order of who raised the least amount of money to who raised the most amount of money:

_______________ ________________ __________________ __________________

Student Responses

Mai Elena Diego Kiran

Teacher Reflection Question

What evidence did you see today that each of your students extended their previous understanding of multiplication?

Lesson Narrative
In previous grades students have interpreted whole number multiplication equations as "groups of", and as area. In previous units, they apply and extend understandings of whole number multiplication to multiply a fraction by a whole number, a whole number by a fraction, and a fraction by a fraction. In previous grades, students have also interpreted whole number multiplication equations as comparison statements. In this lesson, students interpret multiplication as scaling (resizing). The work in this lesson is connected to the comparison work students did with whole numbers in earlier grades, but it is different because students interpret multiplication expressions as a quantity that is resized or scaled by a factor. The quantity and factor may be a whole number or a fraction. Students do not need to use the term "scale". The purpose of this lesson is for students to compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Student-facing Learning Goal: Let's compare factors without multiplying.

Warm-up Narrative: Estimation Exploration: Fraction of a Whole Number	**Addressing CCSS:** 5.NF.B.4.A **Building Toward CCSS:**

10 min	The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to share a mathematical claim and the thinking behind it (MP3). Asking yourself "Does this make sense?" is a component of making sense of problems (MP1), and making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

Task Statement

$\frac{5}{3} \times 9{,}625$

Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses

- Too low: 3,000 - 12,000
- About right: 12,000 - 16,000
- Too high: 18,000

Launch/Activity

- Groups of 2
- Display image/expression.
- "What is an estimate that's too high?" "Too low?" "About right?"
- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.

Synthesis

"How do we know the product is going to be greater than 9,625?"

Optional: Reveal the actual value and add it to the display.

Consider asking:

- "Is anyone's estimate less than ___?

	Is anyone's estimate greater than ___?" ● "Based on this discussion does anyone want to revise their estimate?"
Activity 1 Narrative: Going the Distance	**Addressing CCSS:** 5.NF.B.4.A

15 min	The purpose of this activity is for students to compare the size of different products where one factor stays the same on the basis of the size of the other factor. Students should be encouraged to use whatever strategies and representations make sense to them. Monitor for students who use number lines, use multiplication to compute the total distance, and reason about the relationship between the fraction of the trail and the total distance without doing any computations.

Task Statement

Kiran, Noah, and Elena were trying to run as far as they could in one hour.

- Kiran ran $1\frac{1}{4}$ of an 8 mile trail.
- Noah ran $\frac{1}{2}$ of an 8 mile trail.
- Elena ran $\frac{3}{4}$ of an 8 mile trail.

1. Sequence the students in order from who ran the least amount of miles to who ran the farthest.
2. Fill in the blanks to make the statement true:
   a. Diego ran farther than Noah, but not as far as Kiran. Diego ran _________ of an 8 mile trail.
   b. Lin ran farther than Kiran, but not twice as far as Kiran. Lin ran __________ of an 8 mile trail.
   c. Tyler ran farther than Noah, but not as far as Elena. Tyler ran __________ of an 8 mile trail.

Student Responses

1. Noah ran the least because he ran $\frac{1}{2}$ of an 8 mile trail and Elena ran $\frac{3}{4}$ of an 8 mile trail - so Elena ran farther. Kiran ran the farthest because he ran the 8 mile trail more than once.
1. Answers vary. Possible solutions:

Launch/Activity

- Display:

- "Have you ever gone walking or running on a trail? Describe what it was like."

Synthesis

- "How did you decide how to sequence the lengths that each student ran?" (The length of the trail is always 8 so we can just compare the fractions.)
- "How did you decide which numbers to use in problem 2?" (Answers vary. Sample responses include references to fractions greater than or less than the fraction representing how far the student ran.)

- Diego ran $1\frac{1}{8}$ of an 8 mile trail.
- Lin ran $1\frac{1}{2}$ of an 8 mile trail.
- Tyler ran $\frac{4}{4}$ of a 4 mile trail.

- Display problem 2c.
- Poll the class.
- “What numbers work?”
- “What numbers do not work?”
- Record answers for all to see.
- “What can we say about the numbers that work?” (They are all between $\frac{1}{2}$ and $\frac{3}{4}$.)
- Display: $1\frac{1}{4}$ x 8
- “We can write this multiplication expression to represent how far Kiran ran.”
- “What multiplication expression represents how far Noah ran?” ($\frac{1}{2}$ x 8)

Activity 2 Narrative: Who ran the farthest?

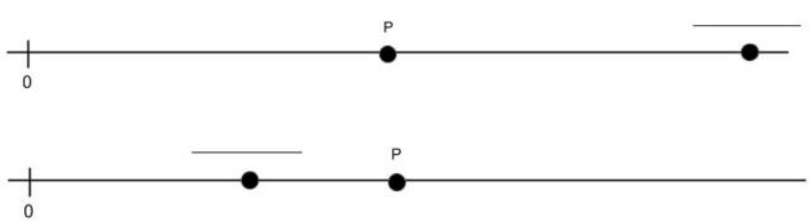
Addressing CCSS: 5.NF.B.4.A

20 min — The purpose of this activity is for students to consider how a factor that is greater than 1, equal to one, and less than one impacts a product.

Task Statement

- Priya ran to her grandmother’s house.
- Jada ran twice as far as Priya.
- Han ran $\frac{5}{7}$ as far as Priya.
- Clare ran $\frac{14}{8}$ as far as Priya.
- Mai ran $\frac{3}{5}$ times as far as Priya.

1. Which students ran further than Priya?
2. Which students did not run as far as Priya?
3. The point labeled P represents how far Priya ran. Write the name of the student above the point that represents how far they ran, compared to Priya.



Launch

Activity

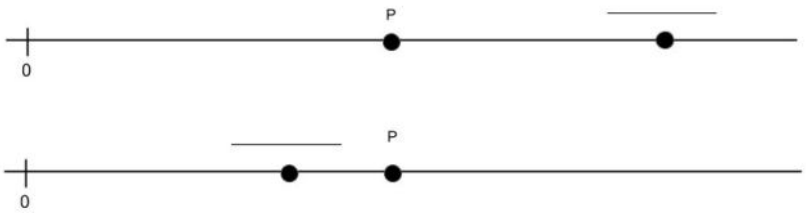
- Groups of 2
- 1-2 minutes: quiet think time
- 6-8 minutes: partner work time

Synthesis

- Display:

Mai ran:	Han ran:	Priya ran:	Clare ran:	Jada ran:
$\frac{3}{5}$ x 2	$\frac{5}{7}$ x 2	2	$\frac{14}{8}$ x 2	2 x 2
		4		

- “What do you notice about these expressions?” (They represent the amount that each person ran, compared to Priya.)
- “What if Priya ran 4 miles? What multiplication expressions can we

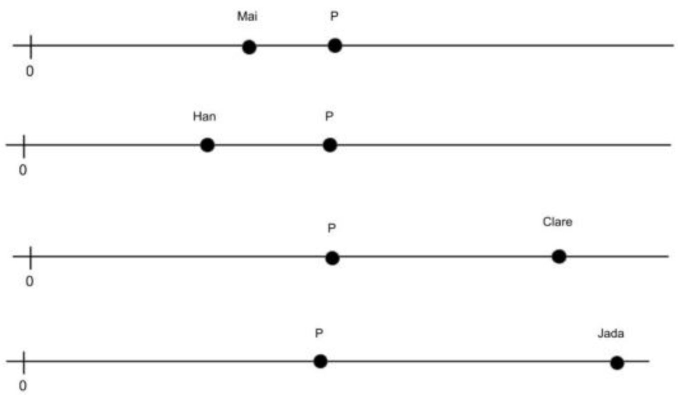


4. Sequence the runners from who ran the least to who ran the most.

_________ _________ _________ _________ _________

Student Responses

1. Jada and Clare ran farther than Priya.
2. Han and Mai did not run as far as Priya.



3. Mai, Han, Priya, Clare, Jada

write to represent how far each of the other students ran?"

Mai ran:	Han ran:	Priya ran:	Clare ran:	Jada ran:
$\frac{3}{5}$ x 2	$\frac{5}{7}$ x 2	2	$\frac{14}{8}$ x 2	2 x 2
$\frac{3}{5}$ x 4	$\frac{5}{7}$ x 4	4	$\frac{14}{8}$ x 4	4 x 2

- "Does the sequence change when Priya's distance changes? Why or why not?" (We can always look at the first factor to compare the distance of another student to the distance that Priya ran.)

Lesson Synthesis

"Today we compared factors without multiplying."

$\frac{5}{8}$ x 4	$\frac{7}{6}$ x 4	$\frac{1}{2}$ x 4

"Which of these expressions represents the largest product?" ($\frac{7}{6}$ x 4)
Give 30 seconds of think time.
"Share your reasoning with a partner." (It is the only fraction that is greater than 1.)
Give 1 minute of partner discussion.
"Which of these expressions represents the smallest product?" ($\frac{1}{2}$ x 4)
Give 30 seconds of think time.
"Share your reasoning with a partner." ($\frac{5}{8}$ is greater than $\frac{1}{2}$.)
Give 1 minute of partner discussion.
"What if we changed the second factor in all of the expressions to 6? 10? 100?"

“Why doesn’t the sequence of expressions change when we change the second factor to a different whole number?” (As long as the second factor is the same whole number, we can compare the size of the product by comparing the size of the first factor in each expression.)

Warm-up: Fraction of a Whole Number

$\frac{5}{3}$ x 9,625

Record an estimate that is:

too low	about right	too high

Activity 1: Going the Distance

Kiran, Noah, and Elena were trying to run as far as they could in one hour.

- Kiran ran 1 ¼ of an 8 mile trail.
- Noah ran ¾ of a 4 mile trail.
- Elena ran ¾ of an 8 mile trail.

1. Sequence the students in order from who ran the least amount of miles to who ran the farthest.

2. Fill in the blanks to make the statement true:

   Diego ran farther than Noah, but not as far as Kiran.
   Diego ran __________ of an 8 mile trail.

   Lin ran farther than Kiran, but not twice as far as Kiran.
   Lin ran ___________ of an 8 mile trail.

   Tyler ran farther than Noah, but not as far as Elena.
   Tyler ran ___________ of a ________ mile trail.

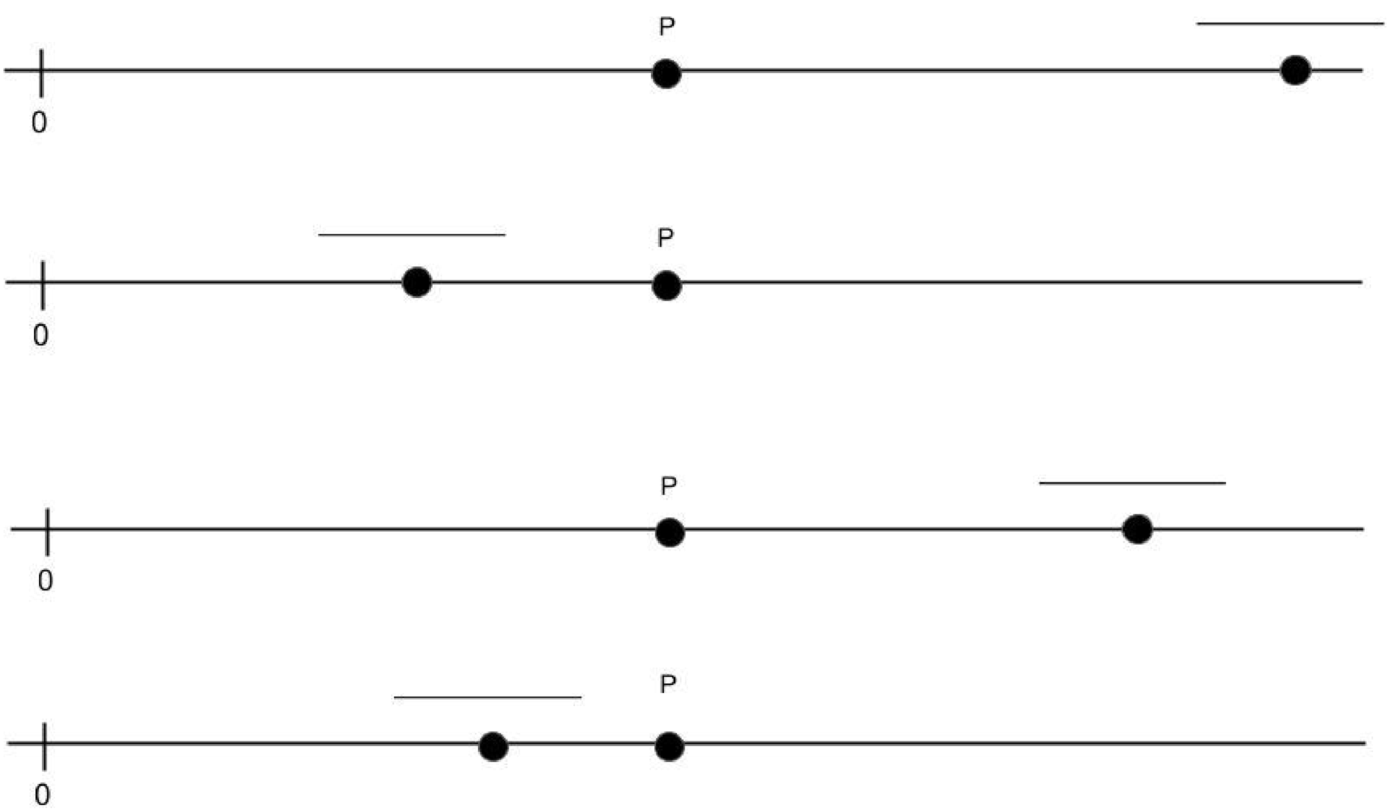
Activity 2: Who ran the farthest?

- Priya ran to her grandmother's house.
- Jada ran twice as far as Priya.
- Han ran 5/7 as far as Priya.
- Clare ran 14/8 as far as Priya.
- Mai ran 3/5 times as far as Priya.

1. Which students ran further than Priya?

2. Which students did not run as far as Priya?

3. The point labeled P represents how far Priya ran. Write the name of the student above the point that represents how far they ran, compared to Priya.



5. Sequence the runners from who ran the least to who ran the most.

Cool-down: Raising Money

Diego, Kiran, Elena, and Mai were raising money for a school trip.

- Diego raised $40.00.
- Kiran sold 2
- $\frac{1}{2}$ times as much lemonade as Diego.
- Elena sold $\frac{7}{8}$ as much lemonade as Diego.
- Mai sold $\frac{2}{3}$ as much lemonade as Diego.

Write the names of the students in order of who raised the least amount of money to who raised the most amount of money:

____________ ____________ ____________ ____________