# Fundamentals Ch 1

## Real numbers 1.1

# Real numbers, equations and the coordinate plane

- Suppose you get paid $9 an hour at your part-time job
- How many hours do you need to work to get paid $200?
- Since what you get paid depends on how many hours you work we can set x (independent variable) to represent an hour worked and y = your pay
- After working one hour you make $9
- After working 2 hours you make $18
- Therefore, we can model your pay with a linear graph on the coordinate plane.
- Draw it on the board

# Natural Numbers

- First you get the *natural* numbers which start at 1
  - {1,2,3,4,5, …… , (n-3), (n-2), n } =

$$\mathbb{N}$$

# Integers

- The **integers** consist of the natural numbers together with their negative additive inverses and 0.
- { -n, -n+1, n+2, ……0,1,2, …… ,(n-2), (n-1), n} =

$$\mathbb{Z}$$

# Rational numbers

- Meaningful quotients of integers
- Don't worry about dividing by zero for now

- $\left\{\frac{a}{b} \middle| \, a, b \in \; integers, \;\; b \neq 0\right\} = \mathbb{Q}$
- Examples
  - $\frac{1}{2}, -\frac{3}{7}, .017 = \frac{17}{1{,}000}, \; .17 = \frac{17}{100}$
- The decimal numbers are equivalent to the set of rational numbers.
- Properties:
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$

- $\frac{\sqrt{2}}{\pi} \notin \mathbb{Q}$
  - Not a quotient of integers

- a well-defined decimal number can be represented by a finite set of #’s and a decimal value or at least a repeating pattern of #’s
- #’s that **do not have** a repeating decimal pattern are considered **irrational #’s**
- Irrational # are still real though
    - Using the division algorithm fails to yield a repeating pattern for a well defined algebraic real #
- That makes up the real numbers
    - Irrational, rational, integers and natural numbers. We speak of natural numbers because some question are only meaningful when we consider positive integers for the possible inputs of a question

$$\mathbb{R} \setminus \mathbb{Q}$$

# Properties of

- Ordering Property

1. Ordering Property- The real numbers are well ordered.

$$a, b \in \mathbb{R} \Longrightarrow a < b, b < a \; or \; a = b$$

$\mathbb{R}$

# Properties of $\mathbb{R}$

2. Commutative properties
    a) For addition
        - a + b = b + a
    b) And multiplication
        - a * b = b * a
3. Associative properties
    - For addition
        a. (a + b) + c = a + (b + c)
    - And multiplication
        b. (a * b) * c = a * (b * c)
4. Distributive properties
    a. a(b + c) = a*b  +  a*c
    b. (b + c)a = a*b  +  a*c

# Negative properties

5. $-(a) = -a$
6. $-(-a) = a$
7. $-a(b) = a(-b) = -(ab) = -ab$
8. $(-a)(-b) = (ab)$
9. $-(a + b) = -a - b$
10. $-(a - b) = b - a$

# Fractional properties

| Property | Example | Description |
|---|---|---|
| 11. $\frac{a}{b} * \frac{c}{d} = \frac{ac}{bd}$ | | A product of fractions is a fraction of products |
| 12. $\frac{a}{b} \div \frac{c}{d} = \frac{a*d}{b*c}$ | | A quotient of fractions is a product of the numerator and the multiplicative inverse of the denominator |
| 13. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ | | A sum of fractions with the same denominator is a fraction with the numerator equal to the sum of the numerators and the denominator is the same |
| *14.* $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$<br>*This is useful when*<br>*b and d*<br>*are relatively prime* | Ex A $\frac{1}{2} + \frac{1}{3} = \frac{1}{2} * 1 + \frac{1}{3} * 1 = \frac{1*3}{2*3} + \frac{1*2}{3*2} = \frac{3+2}{6} = \frac{5}{6}$<br>Ex B $\frac{3}{4} + \frac{2}{3} = \frac{3}{4} * 1 + \frac{2}{3} * 1 = \frac{3}{4} * \frac{3}{3} + \frac{2}{3} * \frac{4}{4} = \frac{9}{12} + \frac{8}{12} =$<br>$\frac{9+8}{12} = \frac{17}{12}$ | Get both fractions to have the same denominator. Apply previous property |
| 15. $\frac{ac}{bc} = \frac{a}{b}$ | $\frac{12}{16} = \frac{3*4}{4*4} = \frac{3*\cancel{4}}{4*\cancel{4}} = \frac{3}{4}$ | Common factors cancel |
| 16. $-(a-b) = b-a$ | $-(x-3) = 3-x$<br>$-(-x^2+2xy-y^2) = y^2+x^2$ | A factor of -1 negates everything in polynomial when distributed |

A. $A$ ***set*** *is a collection of objects called* ***elements***
B. *An* ***element*** *is an object in a set*
   1. $a \in S$ *means a is an* ***element*** *of the* ***set*** $S$

**C. Simple Braces (good for a small finite sets of #'s)**
   A. $S = \{1, 2, 3, 4, 5, 6\}$

**D. Set builder notation (good for intervals)**
   A. $A = \{x |\ (description\ of\ x)\}$
   B. $S = \{x |\ 0 < x < 7\}$

*Let A be a set such that*

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 4, 6\}$

$C = \{7, 9, 11\}$

$A \cup B =$

$\{2,4\}$

$A \cup C =$

$\{7\}$

$A \cap B =$

$B$

$A \cap C =$

$\{7\}$

$B \cap C =$

$\emptyset$

# How do you convert a repeating decimal to a rational number? Is it possible?

Ex 1 Find a, b such $that\ 0.\overline{17} = \frac{a}{b} \in \mathbb{Q}$

1. Let $0.\overline{17} = fraction$

   $so\,\frac{a}{b} = 0.\overline{17}, \qquad b \neq 0$

   Call this equation 1
2. What do you have to multiply to your repeating decimal to put it on the left side of the decimal?
   - In this case it's 100
   - Multiplying both sides by 100 we get…

$100\frac{a}{b} = 17.\overline{17}$

*Call this equation 2*

3. We also know

$$\frac{a}{b} = 0.\overline{17}.$$

$Let's\ consider\ the\ equivalent\ equation$

$$1 * \frac{a}{b} = 0.\overline{17}.$$

Subtracting the bottom equation from the top we get

$100\frac{a}{b} = 17.\overline{17} \qquad EQUATION1$

$-1\frac{a}{b} = -0.\overline{17} \qquad -\ EQUATION2$

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$99\frac{a}{b} = 17$

$\frac{a}{b} = \frac{17}{99}$

Test on calculator or division algorithm

Ex 2

$$1.\overline{28574} = \frac{a}{b} \in \mathbb{Q}$$

1. Let
    - $\frac{a}{b} = 1.\overline{285714}$
2. What do you have to multiply to your repeating decimal to put in on the left of the decimal?
    - In this case it's 1,000,000
    - Multiplying both sides by 1,000,000 we get…
    - $1{,}000{,}000x = 1{,}285{,}714.\overline{285714}$
3. subtract the equation we're assuming from this equation

    - $$\begin{aligned} 1{,}000{,}000x &= 1{,}285{,}714.\overline{28574} \\ -1x &= -.\overline{28574} \end{aligned}$$
    - We get

      $$999{,}999x = 1{,}128{,}573$$
    - $\frac{a}{b} = \frac{1{,}285{,}713}{999{,}999} = \frac{999{,}999+285{,}714}{999{,}999} = \frac{999{,}999}{999{,}999} + \frac{128{,}574}{999{,}999} = 1\frac{285{,}714}{999{,}999} = 1\frac{2 * 3^3 * 11 * 13 * 37}{3^3 * 7 * 11 * 13 * 37} = 1\frac{2 * \cancel{3^3} * \cancel{11} * \cancel{13} * \cancel{37}}{\cancel{3^3} * 7 * \cancel{11} * \cancel{13} * \cancel{37}} = 1\frac{2}{7}$

- The purpose of the preceding examples is to show that every well-defined decimal number has a **rational representation** (and vice versa for that matter).
    - a well-defined decimal number can be represented by a finite set of #'s and a decimal value or at least a repeating pattern of natural #'s with a decimal
- #'s that **do not have** a repeating decimal pattern are considered **irrational #'s**
- Irrational # are still real though
    - These are the irrational numbers = $\mathbb{R} \setminus \mathbb{Q}$
    - Unable to use the division algorithm fails to yield a repeating pattern for a well defined algebraic real #
    
      Type equation here.
    - Irrational numbers still follow the real properties
- That makes up the real numbers
    - Irrational, rational, integers and natural numbers. We speak of natural numbers because some question are only meaningful when we consider positive integers for the possible inputs of a question