UNIT ONE

for

Content Area of

MATHEMATICS

MS Band
Pre-Algebra

Mathematics Curriculum

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Similarity

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[1] Each lesson is ONE day and ONE day is considered a 45 minute period.

Grade 8 • Module 3

Similarity

OVERVIEW

In Module 3, students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The module begins with the definition of dilation, properties of dilations, and compositions of dilations. The instruction regarding dilation in Module 3 is structured similarly to the instruction regarding concepts of basic rigid motions in Module 2. One overarching goal of this module is to replace the common idea of "same shape, different sizes" with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles.

In this module, students describe the effect of dilations on two-dimensional figures in general and using coordinates. Building on prior knowledge of scale drawings (**7.G.A.1**), Module 3 demonstrates the effect dilation has on a figure when the scale factor is greater than zero but less than one (shrinking of figure), equal to one (congruence), and greater than one (magnification of figure). Once students understand how dilation transforms figures in the plane, they examine the effect that dilation has on points and figures in the coordinate plane. Beginning with points, students learn the multiplicative effect that dilation has on the coordinates of the ordered pair. Then students apply the knowledge about points to describe the effect dilation has on figures in the coordinate plane, in terms of their coordinates.

Additionally, Module 3 demonstrates that a two-dimensional figure is similar to another if the second can be obtained from a dilation followed by congruence. Knowledge of basic rigid motions is reinforced throughout the module, specifically when students describe the sequence that exhibits a similarity between two given figures. In Module 2, students used vectors to describe the translation of the plane. Module 3 begins in the same way, but once figures are bound to the coordinate plane, students will describe translations in terms of units left or right and/or units up or down. When figures on the coordinate plane are rotated, the center of rotation is the origin of the graph. In most cases, students will describe the rotation as having center O and degree d, unless the rotation can be easily identified, i.e., a rotation of 90° or 180°. Reflections remain reflections across a line, but when possible, students should identify the line of reflection as the x-axis or y-axis.

It should be noted that congruence, together with similarity, is *the* fundamental concept in planar geometry. It is a concept defined without coordinates. In fact, it is most transparently understood when introduced without the extra conceptual baggage of a coordinate system. This is partly because a coordinate system picks out a preferred point (the origin), which then centers most discussions of rotations, reflections, and translations at or in reference to that point. These discussions are further restricted to only the "nice" rotations, reflections, or translations that are easy to do in a coordinate plane. Restricting to "nice" transformations is a huge mistake mathematically because it is antithetical to the main point that must be made about congruence: that rotations, translations, and reflections are abundant in the plane—that for every point in the plane, there are an *infinite number* of rotations up to 360°, that for every line in the plane there is a reflection, and for every directed line segment there is a translation. It is this abundance that helps students realize that every congruence transformation (i.e., the act of "picking up a figure" and moving it to another location) can be accomplished through a sequence of translations, rotations, and reflections and further, that similarity is a congruence transformation in addition to dilation.

In Grades 6 and 7, students learned about unit rate, rates in general (**6.RP.A.2**), and how to represent and use proportional relationships between quantities (**7.RP.A.2**, **7.RP.A.3**). In Module 3, students apply this knowledge of proportional relationships and rates to determine if two figures are similar, and if so, by what scale factor one can be obtained from the other. By looking at the effect of a scale factor on the length of a segment of a given figure, students will write proportions to find missing lengths of similar figures.

Module 3 provides another opportunity for students to learn about the Pythagorean Theorem and its applications. With the concept of similarity firmly in place, students are shown a proof of the Pythagorean Theorem that uses similar triangles.

Focus Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
8.G.A.5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b≠0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

7.G.A.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Focus Standards for Mathematical Practice

MP.6 **Attend to precision.** To communicate precisely, students will use clear definitions in discussions with others and in their own reasoning with respect to similar figures. Students will use the basic properties of dilations to prove or disprove claims about a pair of figures. Students will incorporate their knowledge about basic rigid motions as it relates to similarity, specifically in the description of the sequence that is required to prove two figures are similar.

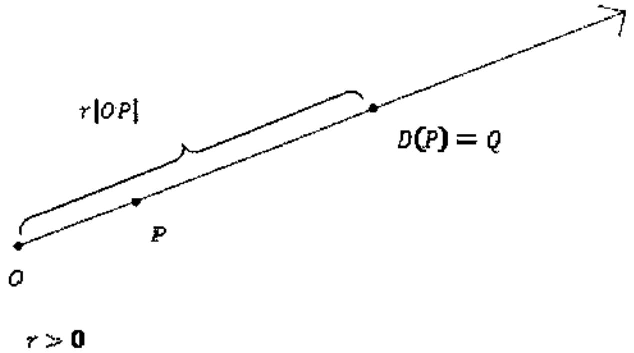
MP.4 **Model with mathematics.** This module provides an opportunity for students to apply their knowledge of dilation and similarity in real-world applications. Students will use shadow lengths and a known height to find the height of trees, the distance across a lake, and the height of a flagpole.

MP.3 **Construct viable arguments and critiques the reasoning of others.** Many times in this module, students are exposed to the reasoned logic of proofs. Students are called on to make conjectures about the effect of dilations on angles, rays, lines, and segments, and then must evaluate the validity of their claims based on evidence. Students also make conjectures about the effect of dilation on circles, ellipses, and other figures. Students are encouraged to participate in discussions and evaluate the claims of others.

Terminology

New or Recently Introduced Terms

- **Dilation** (Dilation, D, is a transformation of the plane with center O and scale factor r ($r > 0$) if $D(O) = O$ and if $P \neq O$, then the point $D(P)$, to be denoted by Q, is the point on the ray $\overrightarrow{OP}$ so that $|OQ| = r|OP|$. A dilation in the coordinate plane is a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by the scale factor.)



- **Congruence** (A finite composition of basic rigid motions—reflections, rotations, translations—of the plane. Two figures in a plane are *congruent* if there is a congruence that maps one figure onto the other figure.)
- **Similar** (Two figures in the plane are similar if there exists a similarity transformation taking one figure to the other.)
- **Similarity Transformation** (A *similarity transformation*, or *similarity*, is a composition of a finite number of basic rigid motions or dilations. The *scale factor* of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1.)
- **Similarity** (A similarity is an example of a transformation.)

Familiar Terms and Symbols[2]

- Scale Drawing
- Degree-Preserving

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[2] These are terms and symbols students have seen previously.

Suggested Tools and Representations

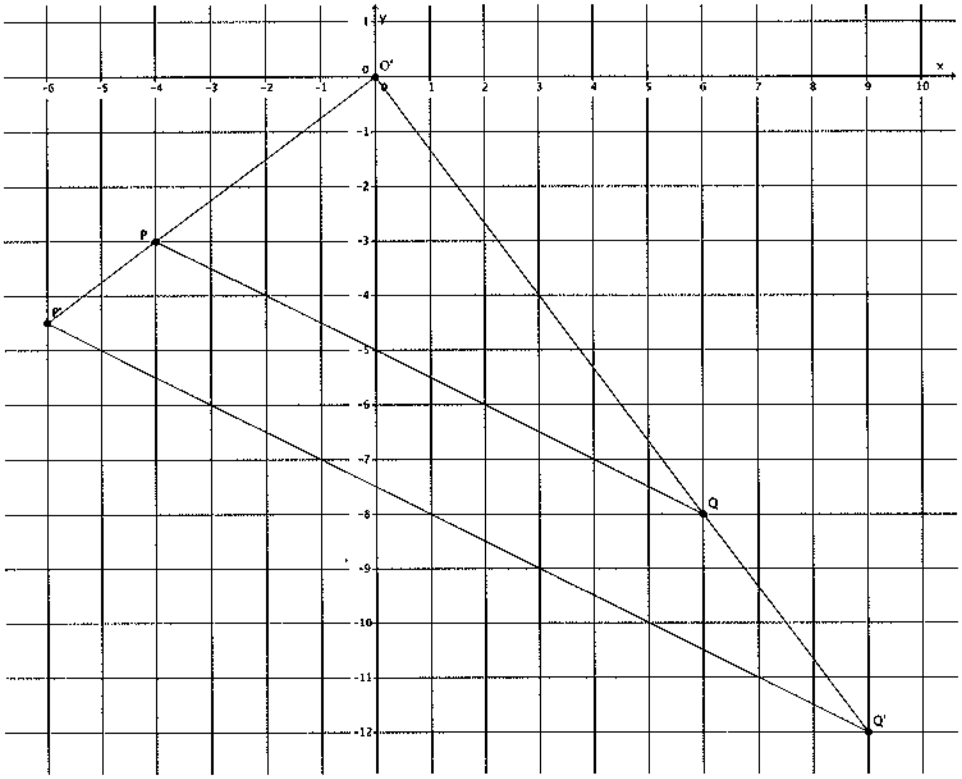
- Compass (Required)
- Transparency or patty paper
- Wet or dry erase markers for use with transparency
- Optional: geometry software
- Ruler
- Video that demonstrates Pythagorean Theorem proof using similar triangles: http://www.youtube.com/watch?v=QCyvxYLFSfU

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	8.G.A.3
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	8.G.A.3, 8.G.A.4, 8.G.A.5

2. Use the diagram below to answer the questions that follow.

Let D be the dilation with center O and scale factor $r > 0$ so that $D(P) = P'$ and $D(Q) = Q'$.



a. Use lengths $|OQ| = 10$ units and $|OQ'| = 15$ units, to determine the scale factor r, of dilation D. Describe how to determine the coordinates of P' using the coordinates of P.

b. If $|OQ| = 10$ units, $|OQ'| = 15$ units, and $|P'Q'| = 11.2$ units, determine the length of $|PQ|$. Round your answer to the tenths place, if necessary.

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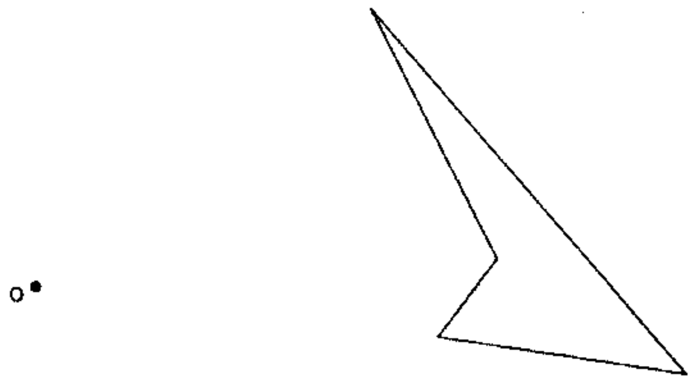
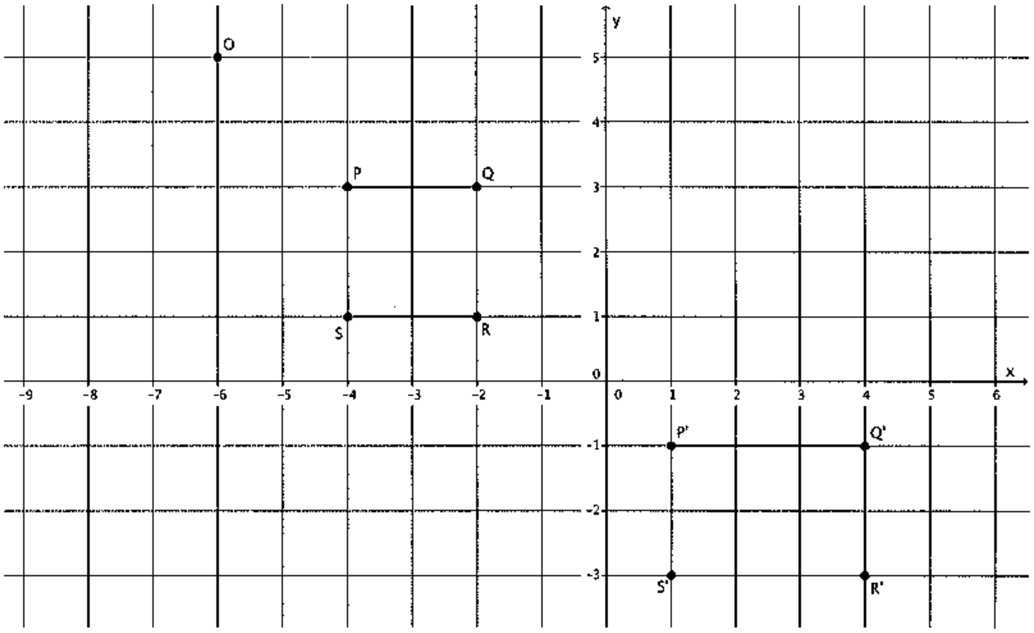
1. Use the figure below to complete parts (a) and (b).



a. Use a compass and ruler to produce an image of the figure with center O and scale factor $r = 2$.

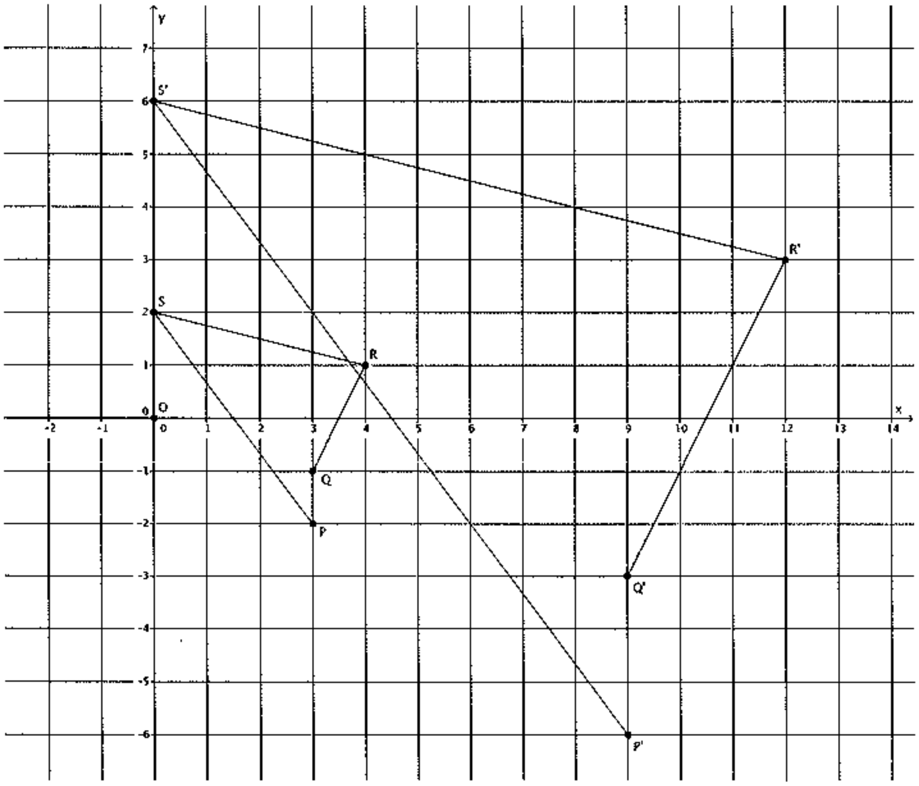
b. Use a ruler to produce an image of the figure with center O and scale factor $r = \frac{1}{2}$.

b. Is there a dilation D with center O that would map figure $PQRS$ to figure $P'Q'R'S'$? If yes, describe the dilation in terms of coordinates of corresponding points.



3. Use a ruler and compass, as needed, to answer parts (a) and (b).

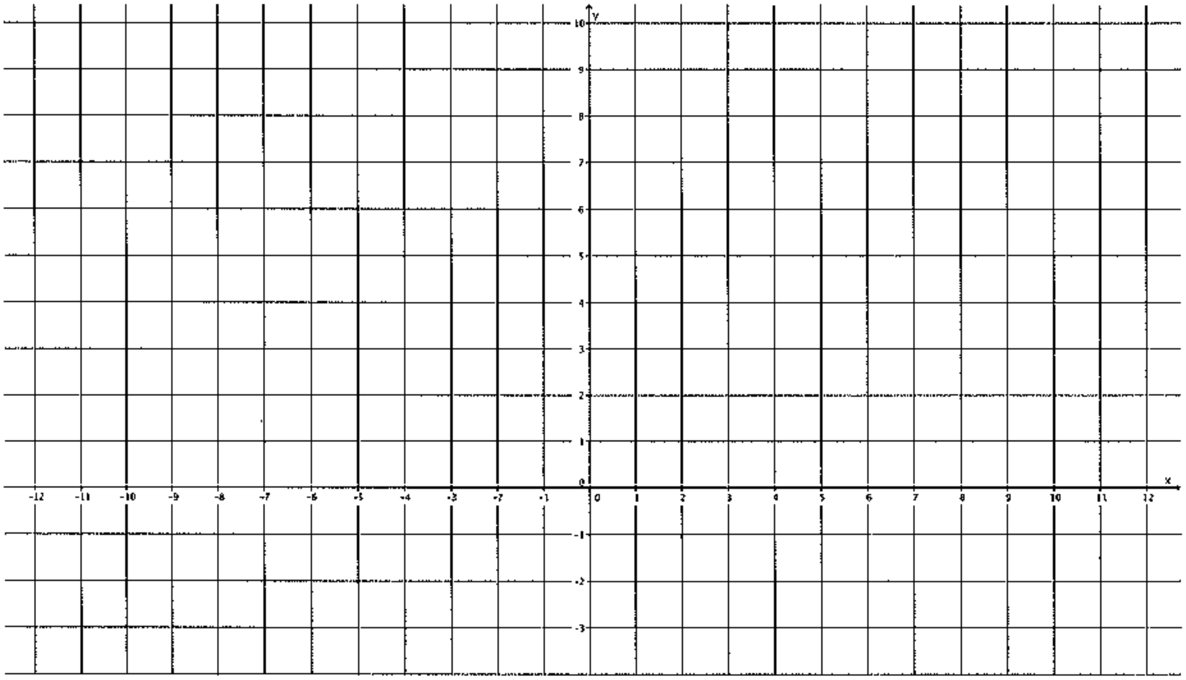
   a. Is there a dilation D with center O that would map figure $PQRS$ to figure $P'Q'R'S'$? If yes, describe the dilation in terms of coordinates of corresponding points.



A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a** **8.G.A.3**	Student did not use compass and ruler to dilate the figure, i.e., the dilated figure is drawn free hand or not drawn. Student used an incorrect or no scale factor. The dilated figure is smaller than the original figure. The corresponding segments are not parallel.	Student may or may not have used a compass and/or ruler to dilate the figure, i.e., some of the work was done by free hand. The dilated figure is larger than the original figure. Student may or may not have solid or dotted rays drawn from the center O through most of the vertices. Student may have used an incorrect scale factor for parts of the dilated figure, i.e., the length from the center O to all dilated vertices is not two times the length from the center to the corresponding vertices. Some of the corresponding segments are parallel.	Student used a compass to dilate the figure, evidenced by arcs on the figure to measure the appropriate lengths. The dilated figure is larger than the original figure. Student has solid or dotted rays drawn from the center O through most of the vertices. Student may have used an incorrect scale factor for parts of the dilated figure, i.e., the length from the center O to all dilated vertices is not two times the length from the center to the corresponding vertices. Most of the corresponding segments are parallel.	Student used a compass to dilate the figure, evidenced by arcs on the figure to measure the appropriate lengths. The dilated figure is larger than the original figure. Student has solid or dotted rays drawn from the center O through all of the vertices. The length from the center O to all dilated vertices is two times the length from the center to the corresponding vertices. All of the corresponding segments are parallel.

c. Triangle ABC is located at points $A = (-4, 3)$, $B = (3, 3)$, and $C = (2, -1)$ and has been dilated from the origin by a scale factor of 3. Draw and label the vertices of triangle ABC. Determine the coordinates of the dilated triangle $A'B'C'$ and draw and label it on the coordinate plane.

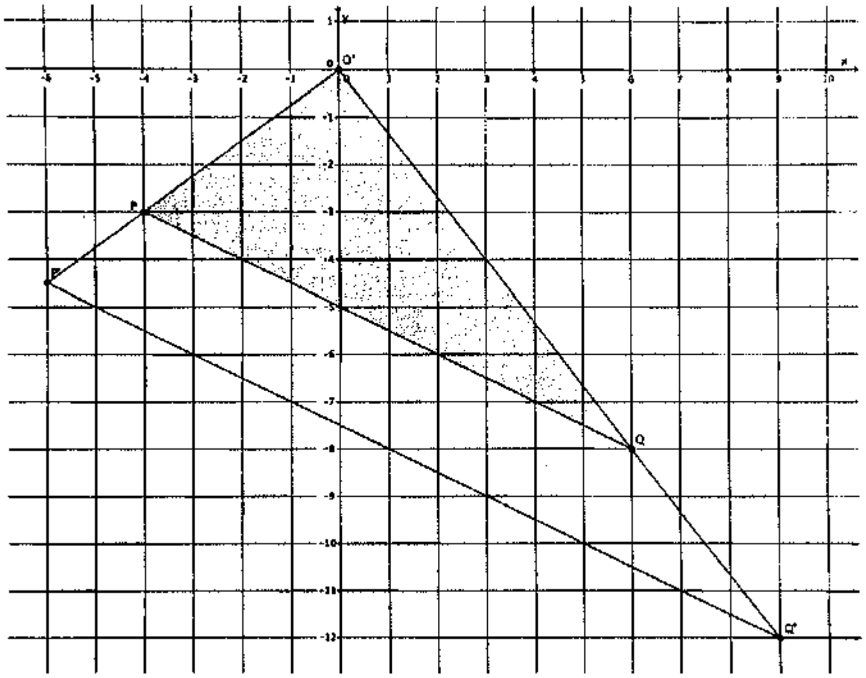


3	a 8.G.A.3	Student did not attempt the problem. Student wrote a number for scale factor without showing any work or providing an explanation for how the scale factor was determined. Student did not describe the dilation.	Student made an error in calculation leading to an incorrect scale factor. Student did not describe the dilation in terms of coordinates of corresponding points.	Student may have identified the scale factor of dilation as $r = \frac{1}{3}$ instead of $r = 3$. Student attempted to describe the dilation with some evidence of mathematical vocabulary and/or reasoning.	Student correctly identified the scale factor as $r = 3$. Student clearly described the dilation in terms of the coordinates of at least one pair of corresponding points. There is strong evidence of mathematical reasoning and use of related vocabulary.
	b 8.G.A.3	Student did not attempt the problem. Student answered with yes or no only. Student did not give any explanation or reasoning.	Student answered yes or no. Student explanation and/or reasoning is not based on mathematics, e.g., "doesn't look like there is." Student attempted to solve problem by showing measurements. Student may or may not have attempted to solve problem by drawing in a solid or dotted ray from center O through one vertex, e.g., from center O through P and P'.	Student answered no correctly. Student used mathematical vocabulary in the explanation. Basis for explanation relies heavily on the diagram, e.g., "look at drawing." Student attempted to solve problem by drawing a solid or dotted ray from center O through one or more vertex, e.g., from center O through P and P'.	Student answered no correctly. Student used mathematical vocabulary in the explanation. Explanation included the fact that the corresponding vertices and center O must be on the same line, e.g., "center O, P, and P' would be on the same ray if a dilation was possible." Student drew solid or dotted rays from center O through multiple vertices. Diagram enhanced explanation.
	c 8.G.A.3	Student did not attempt the problem. Student may have drawn $\triangle ABC$ using incorrect coordinates, or student did not label coordinates correctly.	Student correctly drew and labeled $\triangle ABC$. Student may or may not have identified the correct coordinates of the dilated points. For example, student may have only multiplied one coordinate of each ordered pair to determine location of image point. Student may have placed the image of $\triangle ABC$ at the wrong coordinates.	Student correctly drew and labeled $\triangle ABC$. Student may have minor calculation errors when identifying the coordinates of A', B', C'. For example, student multiplied -4 and 3 and wrote 12. Student drew and labeled $\triangle A'B'C'$ using the incorrect coordinates that were calculated.	Student correctly drew and labeled $\triangle ABC$. Student correctly identified the image of the points as $A' = (-12, 9)$, $B' = (9,9$, and $C' = (6, -3)$. Student correctly drew and labeled $\triangle A'B'C'$.

	b **8.G.A.3**	Student did not use a ruler to dilate the figure, i.e., the dilated figure is drawn free hand or not drawn. Student used an incorrect or no scale factor. The dilated figure is larger than the original figure. The corresponding segments are not parallel.	Student may or may not have used a ruler to dilate the figure, i.e., parts of the work were done by free hand. The dilated figure is smaller than the original figure. Student may or may not have solid or dotted rays drawn from the center O through most of the vertices. Student may have used an incorrect scale factor for parts of the dilated figure, e.g., the length from the center O to all dilated vertices is not one half of the length from the center to the corresponding vertices. Some of the corresponding segments are parallel.	Student used a ruler to dilate the figure. The dilated figure is smaller than the original figure. Student has solid or dotted rays drawn from the center O through most of the vertices. Student may have used an incorrect scale factor for parts of the dilated figure, e.g., the length from the center O to all dilated vertices is not one half of the length from the center to the corresponding vertices. Most of the corresponding segments are parallel.	Student used a ruler to dilate the figure. The dilated figure is smaller than the original figure. Student has solid or dotted rays drawn from the center O through all of the vertices. The length from the center O to all dilated vertices is one half of the length from the center to the corresponding vertices. All of the corresponding segments are parallel.
2	**a** **8.G.A.3**	Student did not attempt the problem. Student may or may not have calculated the scale factor correctly.	Student used the definition of dilation with the given side lengths to calculate the scale factor. Student may have made calculation errors when calculating the scale factor. Student may or may not have attempted to find the coordinates of P'.	Student used the definition of dilation with the given side lengths to calculate the scale factor $r = 1.5$ or equivalent. Student determined the coordinates of point P', but did not explain or relate it to scale factor and point P.	Student used the definition of dilation with the given side lengths to calculate the scale factor $r = 1.5$ or equivalent. Student explained that the coordinates of P' are found by multiplying the coordinates of P by the scale factor. Student determined the coordinates of $Q' = (-6, -4.5)$.
	b **8.G.A.3**	Student did not attempt the problem. Student wrote a number for the length of $\lvert PQ \rvert$ without showing any work as to how he/she arrived at the answer.	Student may have inverted one of the fractions of the equal ratios leading to an incorrect answer. Student may have made a calculation error in finding the length of $\lvert PQ \rvert$. Student did not answer the question in a complete sentence.	Student correctly set up ratios to find the length of $\lvert PQ \rvert$. Student may have made a rounding error in stating the length. Student did not answer the question in a complete sentence or did not include units in the answer.	Student correctly set up ratios to find the length of $\lvert PQ \rvert$. Student correctly identified the length of $\lvert PQ \rvert \approx 7.5$ units. Student answered the question in a complete sentence and identified the units.

2. Use the diagram below to answer the questions that follow.

Let D be the dilation with center O and scale factor $r > 0$ so that $D(P) = P'$ and $D(Q) = Q'$.



a. Use lengths $|OQ| = 10$ units and $|OQ'| = 15$ units, to determine the scale factor r, of dilation D. Describe how to determine the coordinates of P' using the coordinates of P.

$|OQ|r = |OQ'|$

$r = \frac{|OQ'|}{|OQ|}$

$r = \frac{15}{10}$

$r = \frac{3}{2}$

THE SCALE FACTOR IS $r = \frac{3}{2}$.

SINCE THE COORDINATES OF $P=(-4,-3)$ THE COORDINATES OF THE DILATED POINT P' WILL BE THE SCALE FACTOR TIMES THE COORDINATES OF P. THEREFORE $P' = \left(\frac{3}{2}\times(-4), \frac{3}{2}\times(-3)\right) = (-6, -4.5)$.

b. If $|OQ| = 10$ units, $|OQ'| = 15$ units, and $|P'Q'| = 11.2$ units, determine the length of $|PQ|$. Round your answer to the tenths place, if necessary.

$\frac{15}{10} = \frac{11.2}{|PQ|}$

$15(|PQ|) = 112$

$|PQ| = \frac{112}{15} \approx 7.5$

THE LENGTH OF PQ IS ABOUT 7.5 UNITS

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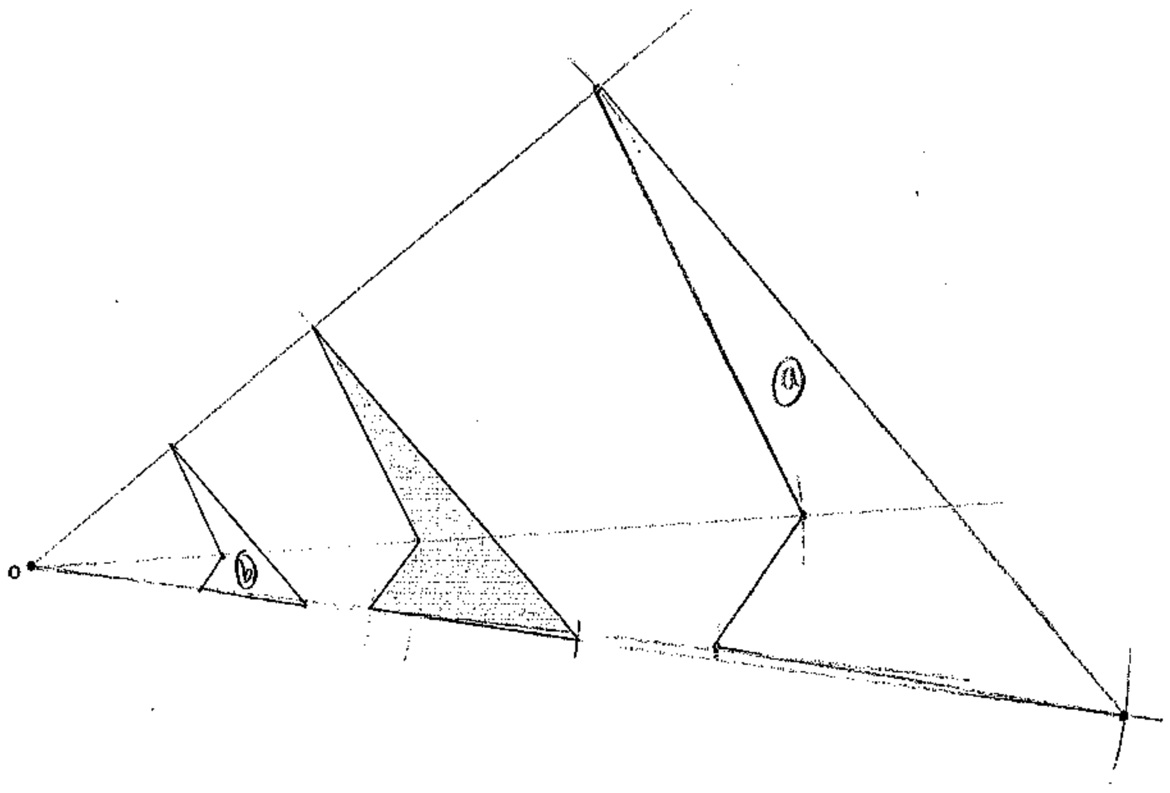
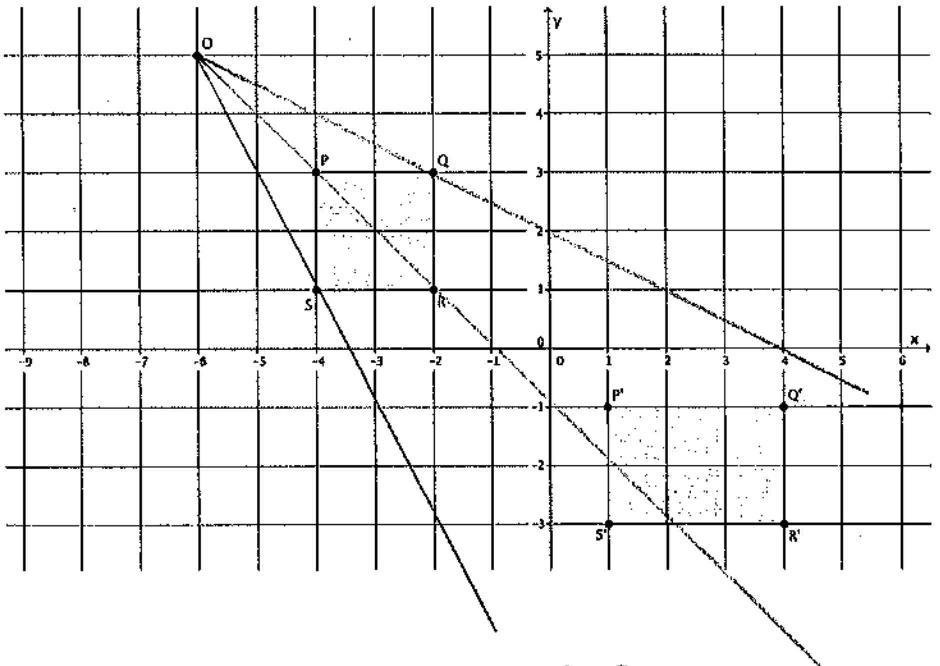
1. Use the figure below to complete parts (a) and (b).



a. Use a compass and ruler to produce an image of the figure with center O and scale factor, $r = 2$.

b. Use a ruler to produce an image of the figure with center O and scale factor, $r = \frac{1}{2}$.

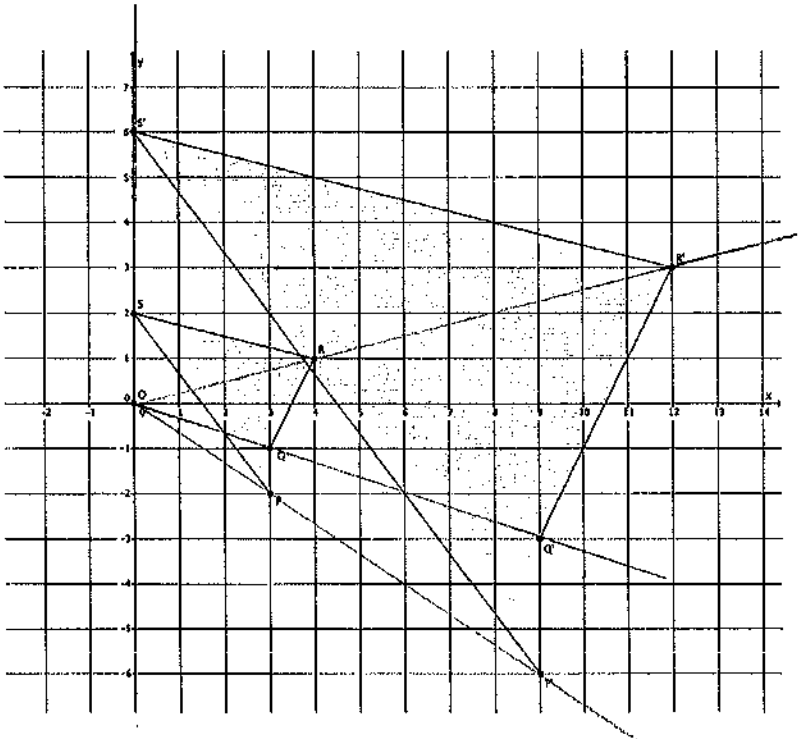
b. Is there a dilation D with center O that would map figure $PQRS$ to figure $P'Q'R'S'$? If yes, describe the dilation in terms of coordinates of corresponding points.



NO, THERE IS NOT A DILATION D THAT WILL MAP PQRS TO P'Q'R'S'. A DILATION WILL MOVE A POINT, S, TO ITS IMAGE S' ON THE RAY $\overrightarrow{OS}$. IN THE PICTURE ABOVE O, S, S' ARE NOT ON THE SAME RAY. A SIMILAR STATEMENT CAN BE MADE FOR POINTS P, Q, AND R. THEREFORE, THERE IS NO DILATION THAT MAPS PQRS TO P'Q'R'S'.

3. Use a ruler and compass, as needed, to answer parts (a) and (b).

   a. Is there a dilation D with center O that would map figure $PQRS$ to figure $P'Q'R'S'$? If yes, describe the dilation in terms of coordinates of corresponding points.

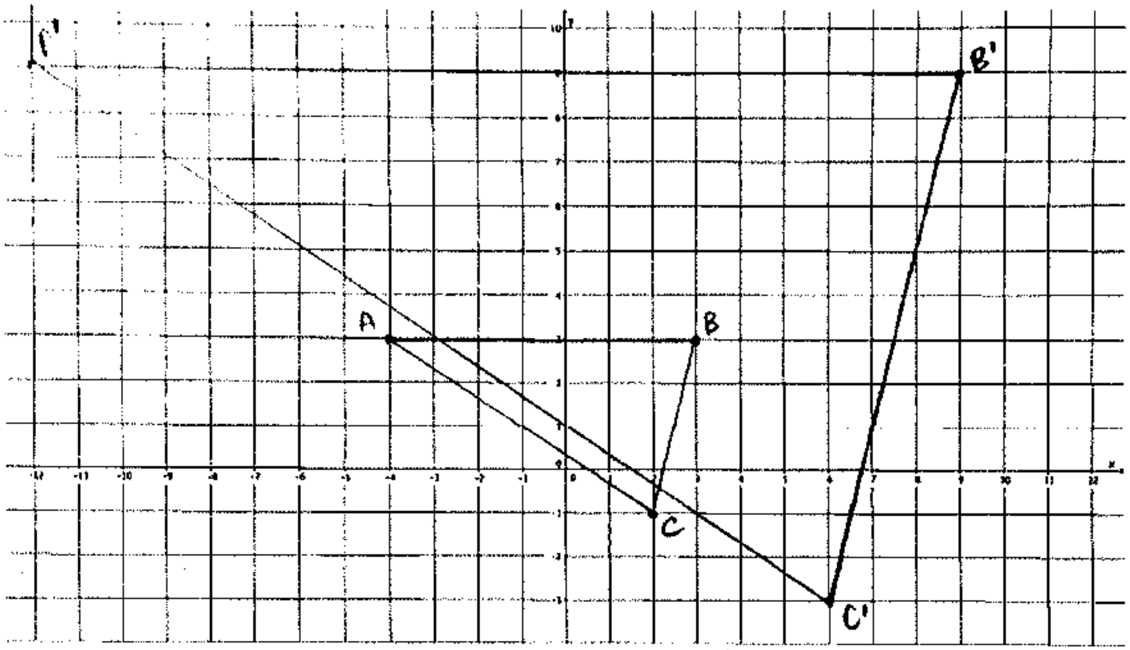


P=(3,-2)
Q = (3,-1)
R = (4,1)
S = (0,2)

P' = (9,-6)
Q' = (9,-3)
R' = (12,3)
S' = (0,6)

YES, THERE IS A DILATION D WITH GENTER O THAT MAPS PQRS TO PQ'R'S'. THE SCALE FACTOR IS 3. THE IMAGE OF EACH POINT IS 3 TIMES THE COORDINATES OF THE ORIGINAL IMAGE. FOR EXAMPLE, P=(3,-2) AND P' = (3×3, 3×(-2))=(9,-6).

c. Triangle $\triangle ABC$ is located at points $A = (-4, 3)$, $B = (3, 3)$ and $C = (2, -1)$ has been dilated from the origin by a scale factor of 3. Draw and label the vertices of $\triangle ABC$. Determine the coordinates of the dilated triangle, $\triangle A'B'C'$ and draw and label it on the coordinate plane.



A = (-4,3)	A' = (-4×3, 3×3) = (-12,9)
B = (3,3)	B' = (3×3, 3×3) = (9,9)
C = (2,-1)	C' = (2×3, -1×3) = (6,-3)

c. Are $\angle OQP$ and $\angle OQ'P'$ equal in measure? Explain.

d. What is the relationship between the lines PQ and $P'Q'$? Explain in terms of similar triangles.

e. If the length of segment $|OQ| = 9.8$ units, what is the length of segment $|OQ'|$? Explain in terms of similar triangles.

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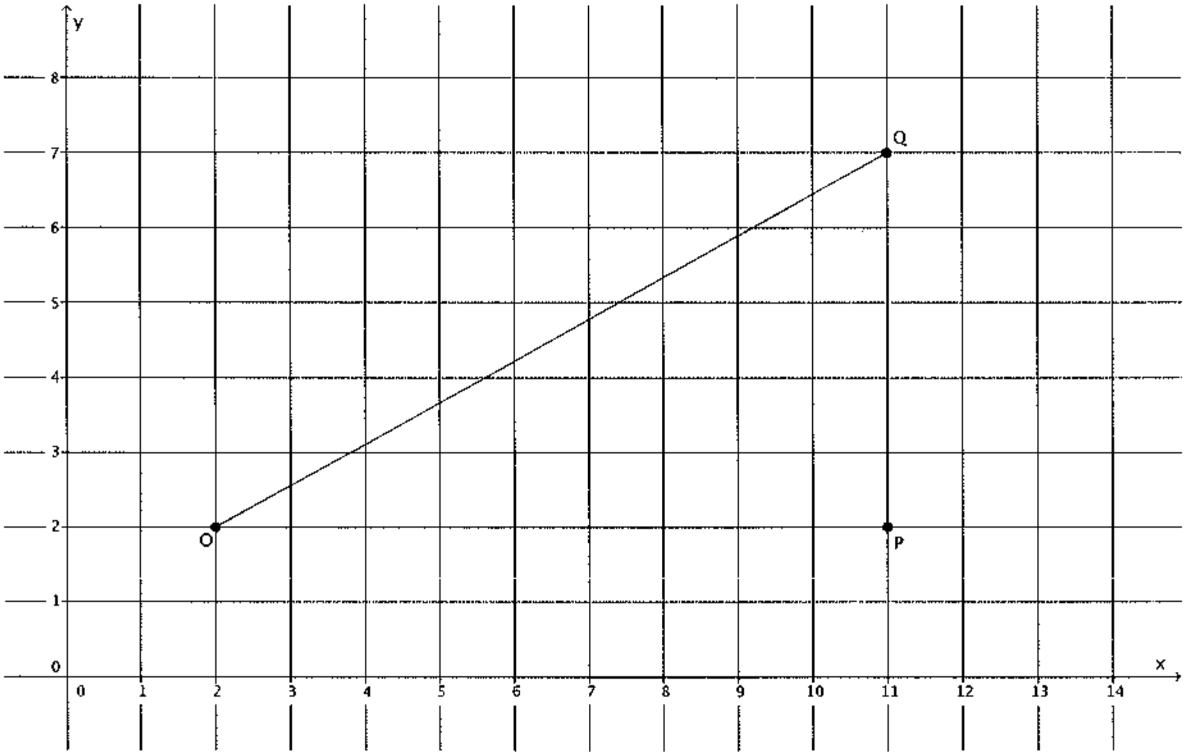
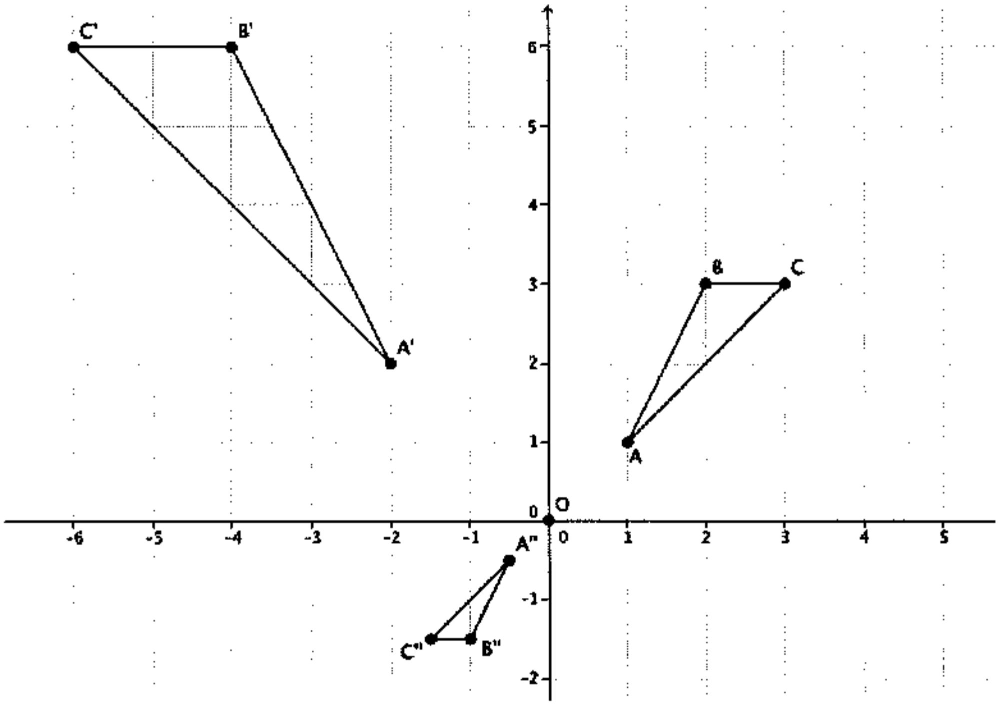
1. Use the diagram below to answer the questions that follow.



a. Dilate triangle ΔOPQ from center O and scale factor $r = \frac{4}{9}$. Label the image $\Delta OP'Q'$.

b. Find the coordinates of P' and Q'.

3. Given $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$ in the diagram below, answer parts (a)–(c).

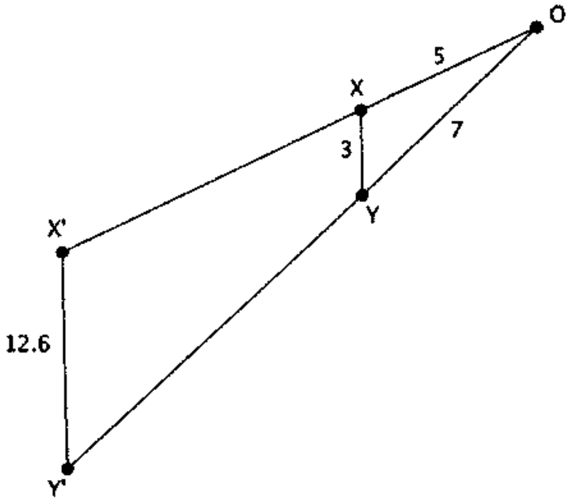


a. Describe the sequence that shows the similarity for ΔABC and $\Delta A'B'C'$.

b. Describe the sequence that shows the similarity for ΔABC and $\Delta A''B''C''$.

c. Is $\Delta A'B'C'$ similar to $\Delta A''B''C''$? How do you know?

2. Use the diagram below to answer the questions that follow. The length of each segment is as shown: segment OX is 5 units, segment OY is 7 units, segment XY is 3 units, and segment $X'Y'$ is 12.6 units.



a. Suppose XY is parallel to $X'Y'$. Is triangle ΔOXY similar to triangle $\Delta OX'Y'$? Explain.

b. What is the length of segment OX'? Show your work.

c. What is the length of segment OY'? Show your work.

	d **8.G.A.5**	Student does not attempt the problem or leaves the problem blank. Student may state that $PQ \parallel P'Q'$. Student does not attempt any explanation or reasoning.	Student may state that $PQ \parallel P'Q'$. Student may not use mathematical language in explanation or reasoning. For example, student may write: "they look like they won't touch," or "the angles are the same." Reasoning may include some facts. Reasoning may not be complete. There are significant gaps in explanation.	Student states that $PQ \parallel P'Q'$. Student uses some mathematical language in explanation or reasoning. Reasoning includes some of the following facts: $\angle O = \angle O$, $\angle OQP = \angle OQ'P'$ and $\angle OPQ = \angle OP'Q'$, then by AA criterion for similarity, $\Delta OPQ \sim \Delta OP'Q'$. Then, by FTS $PQ \parallel P'Q'$. Reasoning may not be complete.	Student states that $PQ \parallel P'Q'$. Student uses mathematical language in explanation or reasoning. Reasoning includes the following facts: At least two pairs of corresponding angles are equal, e.g., $\angle O = \angle O$ and/or $\angle OQP = \angle OQ'P'$ and/or $\angle OPQ = \angle OP'Q'$, then by AA criterion for similarity, $\Delta OPQ \sim \Delta OP'Q'$. Then, by FTS $PQ \parallel P'Q'$. Reasoning is thorough and complete.
	e **8.G.A.5**	Student does not attempt the problem or leaves the problem blank.	Student answers incorrectly. Student may not use mathematical language in explanation or reasoning. Student reasoning does not include a reference to similar triangles. Student reasoning may or may not include that the ratio of lengths are equal to scale factor. There are significant gaps in explanation.	Student answers correctly that $OQ' \approx \mathbf{4.4}$ units. Student uses some mathematical language in explanation or reasoning. Student may or may not have referenced similar triangles in reasoning. Student reasoning includes that the ratio of lengths are equal to scale factor. Explanation or reasoning may not be complete.	Student answers correctly that $OQ' \approx \mathbf{4.4}$ units. Student uses mathematical language in explanation or reasoning. Student referenced similar triangles in reasoning. Student reasoning includes that the ratio of lengths are equal to scale factor. Reasoning is thorough and complete.
2	**a** **8.G.A.5**	Student does not attempt the problem or leaves the problem blank. Student answers yes or no only. Student does not attempt to explain reasoning.	Student may or may not answer correctly. Student may use some mathematical language in explanation or reasoning. Explanation or reasoning is not mathematically based, e.g., "they look like they are." There are significant gaps in explanation.	Student answers yes correctly. Student uses some mathematical language in explanation or reasoning. Explanation includes some of the following facts: Since $XY \parallel X'Y'$, then corresponding angles of parallel lines are	Student answers yes correctly. Student uses mathematical language in explanation or reasoning. Explanation includes the following facts: Since $XY \parallel X'Y'$, then corresponding angles of parallel lines are congruent by AA

A Progression Toward Mastery

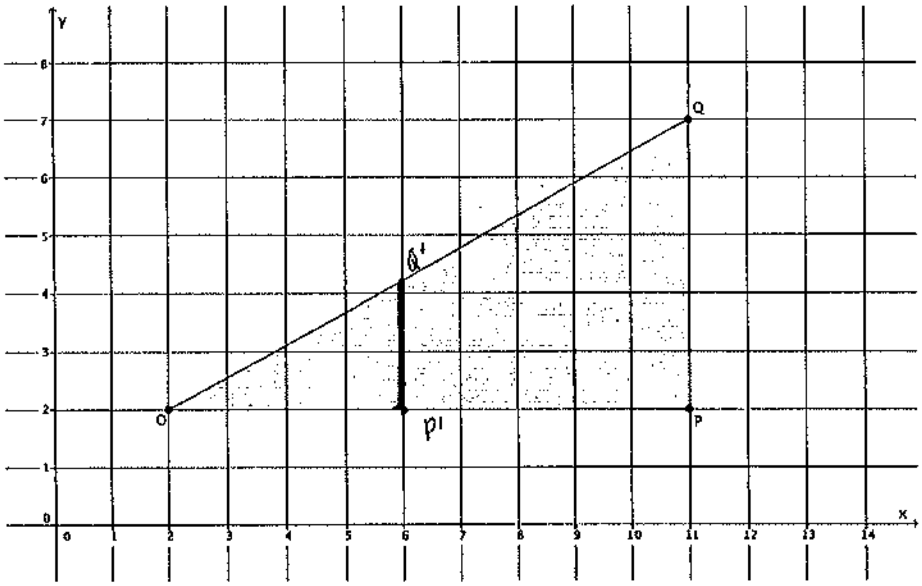
Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 8.G.A.4	Student does not mark any points on the drawing.	Student drew an arbitrary triangle that is not a dilation according to the scale factor and is not labeled.	Student drew a triangle $\Delta OQ'P'$ and labeled the points, but it was not a dilation according to the scale factor.	Student drew a triangle $\Delta OQ'P'$ according to the scale factor and labeled the points.
	b 8.G.A.4	Student does not attempt the problem or leaves the problem blank.	Student identifies both of the coordinates of P' or Q' incorrectly <u>OR</u> student may have transposed the coordinates of P' as $(2,6)$.	Student identifies one of the coordinates of P' correctly and the x -coordinate of Q' correctly. A calculation error may have led to to an incorrect y -coordinate of Q'.	Student correctly identifies the coordinates of Q' as $\left(\frac{6,38}{9}\right)$. Student correctly identifies the coordinates of P' as $(6,2)$.
	c 8.G.A.4	Student does not attempt the problem or leaves the problem blank. Student states that $\angle OQP \neq \angle OQ'P'$.	Student states that $\angle OQP = \angle OQ'P'$. Student does not attempt any explanation or reasoning. Explanation or reasoning is not mathematically based. For example, student may write: "it looks like they are the same."	Student states that $\angle OQP = \angle OQ'P'$. Student explanation includes mathematical language. Student explanation may not be complete, e.g., stating dilations are degree persevering without explaining $D(\angle OQP) = \angle OQ'P'$.	Student states that $\angle OQP = \angle OQ'P'$. Student explanation includes mathematical language. Reasoning includes that $D(\angle OQP) = \angle OQ'P'$, and dilations are degree preserving.

			or scale factor. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Explanation or reasoning is not mathematically based, e.g., "one looks about half the size of the other."	$0 < r < 1$, but does not give center. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Student uses some mathematical language in explanation or reasoning.	there is a congruence of rotation of $180°$ centered at the origin. Student uses mathematical language in explanation or reasoning, such as: $R(D(\Delta ABC)) = \Delta A''B''C$ and $\Delta ABC \sim \Delta A''B''C''$. Reasoning is thorough and complete.
	c **8.G.5**	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student does not attempt any explanation or reasoning. Student does not reference the AA Criterion for similarity. Explanation or reasoning is not mathematically based, e.g., "They don't look like they are the same."	Student may or may not have answered correctly. Student states that only one set of angles is congruent. Student uses some mathematical language in explanation or reasoning. Student may or may not reference the AA Criterion for similarity.	Student answers correctly that yes, $\Delta A'B'C' \sim \Delta A''B''C''$. Student states that dilations are angle preserving. Student shows that since $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$, at least two corresponding angles are congruent (i.e., $\angle A \cong \angle A' \cong \angle A''$). Student references the AA Criterion for similarity. Student uses mathematical language in explanation or reasoning. Reasoning is thorough and complete.

				congruent by AA criterion for similar triangles; therefore, $\Delta OXY \sim \Delta OX'Y'$. Reasoning may not be complete.	criterion for similar triangles; therefore, $\Delta OXY \sim \Delta OX'Y'$. Reasoning is thorough and complete.
	b 8.G.A.5	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student uses some method other than proportion to solve problems, e.g., guessing. Student may have made calculation errors.	Student may or may not have answered correctly. Student uses a proportion to solve problem. Student may have set up proportion incorrectly. Student may have made calculation errors.	Student answers correctly with length of $OX' = 21$ units. Student uses a proportion to solve problem.
	c 8.G.A.5	Student does not attempt the problem or leaves the problem blank.	Student may or may not have answered correctly. Student uses some method other than proportion to solve problems, e.g., guessing. Student may have made calculation errors.	Student may or may not have answered correctly. Student uses a proportion to solve problem. Student may have set up proportion incorrectly. Student may have made calculation errors.	Student answers correctly with length of $OY' = 29.4$ units. Student uses a proportion to solve problem.
3	a 8.G.A.5	Student does not attempt the problem or leaves the problem blank.	Student does not attempt any explanation or reasoning. Student may or may not have stated dilation and does not give any center or scale factor. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Explanation or reasoning is not mathematically based, e.g., "one looks about three times bigger than the other."	Student states dilation. Student states dilation is centered at origin, but does not give scale factor, $r > 1$, or states scale factor of $r > 1$, but does not give center. Student may or may not have stated the congruence. Student may have stated the incorrect congruence. Student uses some mathematical language in explanation or reasoning.	Student states correctly there is a dilation with center at the origin and has a scale factor, $r = 2$. Student states correctly there is a congruence of reflection across the Y-axis. Student uses mathematical language in explanation or reasoning such as $\Lambda(D(\Delta ABC)) = \Delta A'B'C'$ and $\Delta ABC \sim \Delta A'B'C'$. Reasoning is thorough and complete.
	b 8.G.A.5	Student does not attempt the problem or leaves the problem blank.	Student does not attempt any explanation or reasoning. Student may or may not have stated dilation and does not give any center	Student states dilation. Student states dilation is centered at origin, but does not give scale factor, $0 < r < 1$ or states scale factor of	Student states correctly there is a dilation with center at the origin and has a scale factor, $0 < r < 1$. Student states correctly

Name ______________________ Date ____________

1. Use the diagram below to answer the questions that follow.



a. Dilate triangle ΔOPQ from center O and scale factor $r = \frac{4}{9}$. Label the image $\Delta OP'Q'$.

b. Find the coordinates of P' and Q'.

$P' = (6,2)$

$Q' = \left(6, \frac{38}{9}\right)$

$$\frac{|P'Q'|}{|PQ|} = \frac{4}{9}$$

$$\frac{|P'Q'|}{5} = \frac{4}{9}$$

$$|P'Q'| = \frac{20}{9}$$

$$\frac{20}{9} + 2 = \frac{20}{9} + \frac{18}{9}$$

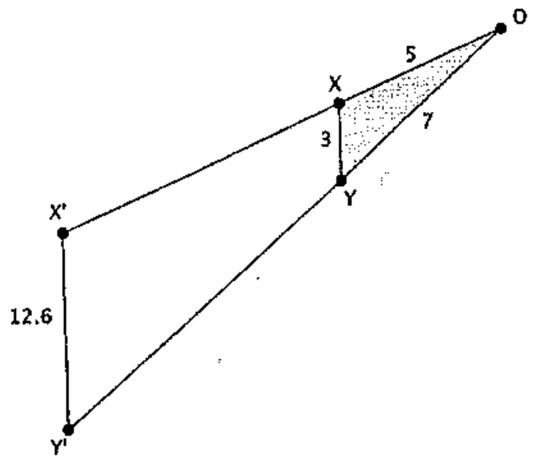
$$= \frac{38}{9}$$

c. Are $\angle OQP$ and $\angle OQ'P'$ equal in measure? Explain.

YES $\angle OQP = \angle OQ'P'$ SINCE $D(\Delta OQP) = \Delta OQ'P'$ AND DILATIONS ARE DEGREE PRESERVING, THEN $\angle OQP = \angle OQ'P'$.

$\angle OQP$ & $\angle OQ'P'$ ARE CORRESPONDING ANGLES OF PARALLEL LINES PQ & P'Q', THEREFORE $\angle OQP = \angle OQ'P'$.

2. Use the diagram below to answer the questions that follow. The length of each segment is as shown: segment OX is **5** units, segment OY is **7** units, segment XY is **3** units, and segment $X'Y'$ is 12.6 units.



a. Suppose XY is parallel to $X'Y'$. Is triangle ΔOXY similar to triangle $\Delta OX'Y'$? Explain.

YES, $\Delta OXY \sim OX'Y'$. SINCE $XY \parallel X'Y'$ THEN $\angle OXY = \angle OX'Y'$ AND $\angle OYX = \angle OY'X'$. BECAUSE CORRESPONDING ANGLES OF PARALLEL LINES ARE EQUAL, BY AA $\Delta OXY \sim \Delta OX'Y'$.

b. What is the length of segment OX'? Show your work.

$$\frac{12.6}{3} = \frac{|OX'|}{5}$$

$$5(12.6) = 3(|OX'|)$$

$$63 = 3(|OX'|)$$

$$21 = |OX'|$$

THE LENGTH OF $|OX'|$ IS 21 UNITS.

c. What is the length of segment OY'? Show your work.

$$\frac{12.6}{3} = \frac{|OY'|}{7}$$

$$12.6(7) = 3(|OY'|)$$

$$88.2 = 3(|OY'|)$$

$$29.4 = |OY'|$$

THE LENGTH OF $|OY'|$ IS 29.4 UNITS.

d. What is the relationship between the lines PQ and $P'Q'$? Explain in terms of similar triangles.

THE LINES PQ AND P'Q' ARE PARALLEL. $\triangle OPQ \sim \triangle OP'Q'$ BY THE AA CRITERION ($\angle O = \angle O$, $\angle OPQ = \angle OP'Q'$), THEREFORE BY THE FUNDAMENTAL THEOREM OF SIMILARITY $PQ \parallel P'Q'$.

e. If the length of segment $|OQ| = 9.8$ units, what is the length of segment $|OQ'|$? Explain in terms of similar triangles.

SINCE $\triangle OPQ \sim \triangle OP'Q'$, THEN THE RATIOS OF LENGTHS OF CORRESPONDING SIDES WILL BE EQUAL TO THE SCALE FACTOR. THEN

$$\frac{|OP'|}{|OP|} = \frac{|OQ'|}{|OQ|} = \frac{4}{9}$$

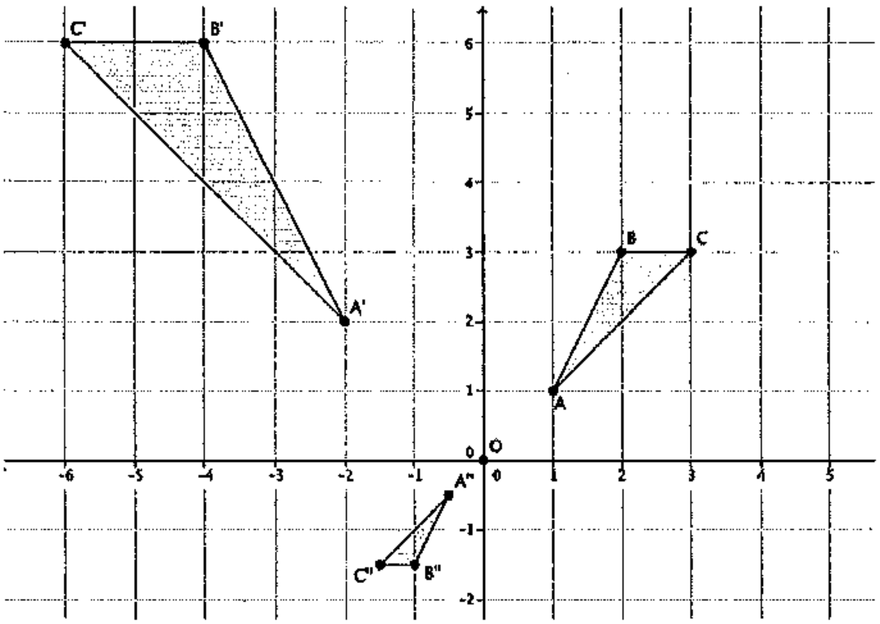
$$\frac{4}{9} = \frac{|OQ'|}{9.8}$$

$$39.2 = 9(|OQ'|)$$

$$4.36 = |OQ'|$$

THE LENGTH OF $|OQ'|$ IS APPROXIMATELY 4.4 UNITS.

3. Given $\Delta ABC \sim \Delta A'B'C'$ and $\Delta ABC \sim \Delta A''B''C''$ in the diagram below, answer parts (a)-(c).



a. Describe the sequence that shows the similarity for ΔABC and $\Delta A'B'C'$. $\frac{B'C'}{BC} = \frac{2}{1} = 2 = r$

LET D BE THE DILATION FROM CENTER O AND SCALE FACTOR $r = 2$. LET THERE BE A REFLECTION ACROSS THE Y-AXIS. THEN THE DILATION FOLLOWED BY THE REFLECTION MAPS ΔABC ONTO $\Delta A'B'C'$.

b. Describe the sequence that shows the similarity for ΔABC and $\Delta A''B''C''$.

LET D BE THE DILATION FROM CENTER O AND SCALE FACTOR $0 < r < 1$. LET THERE BE A ROTATION OF 180° AROUND CENTER O. THEN THE DILATION FOLLOWED BY THE ROTATION MAPS ΔABC ONTO $\Delta A''B''C''$.

c. Is $\Delta A'B'C'$ similar to $\Delta A''B''C''$? How do you know?

YES. $\Delta A'B'C' \sim \Delta A''B''C''$. DILATIONS PRESERVE ANGLE MEASURES AND SINCE $\Delta ABC \sim \Delta A'B'C$ AND $\Delta ABC \sim \Delta A''B''C''$, WE KNOW $\angle A = \angle A' = \angle A''$, $\angle B = \angle B' = \angle B''$. BY AA CRITERION FOR SIMILARITY $\Delta A'B'C' \sim \Delta A''B''C''$. ALSO SIMILARITY IS TRANSITIVE.

UNIT TWO

for

Content Area of

MATHEMATICS

MS Band
Pre-Algebra

Table of Contents[1]

The Concept of Congruence

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[1] Each lesson is ONE day, and ONE day is considered a 45 minute period.

Grade 8 • Module 2

The Concept of Congruence

OVERVIEW

In this module, students learn about translations, reflections, and rotations in the plane and, more importantly, how to use them to precisely define the concept of *congruence*. Up to this point, "congruence" has been taken to mean, intuitively, "same size and same shape." Because this module begins a serious study of geometry, this intuitive definition must be replaced by a precise definition. This module is a first step; its goal is to provide the needed intuitive background for the precise definitions that are introduced in this module for the first time.

Translations, reflections, and rotations are examples of *rigid motions*, which are, intuitively, rules of moving points in the plane in such a way that preserves distance. For the sake of brevity, these three rigid motions will be referred to exclusively as the *basic rigid motions*. Initially, the exploration of these basic rigid motions is done via hands-on activities using an overhead projector transparency, but with the availability of geometry software, the use of technology in this learning environment is inevitable, and some general guidelines for this usage will be laid out at the end of Lesson 2. What needs to be emphasized is that the importance of these basic rigid motions lies not in the fun activities they bring but in the *mathematical* purpose they serve in clarifying the meaning of congruence.

Throughout Topic A, on the definitions and properties of the basic rigid motions, students verify experimentally their basic properties and, when feasible, deepen their understanding of these properties using reasoning. In particular, what students learned in Grade 4 about angles and angle measurement (**4.MD.5**) will be put to good use here: they learn that the basic rigid motions preserve angle measurements, as well as segment lengths.

Topic B is a critical foundation to the understanding of congruence. All the lessons of Topic B demonstrate to students the ability to sequence various combinations of rigid motions while maintaining the basic properties of individual rigid motions. Lesson 7 begins this work with a sequence of translations. Students verify experimentally that a sequence of translations have the same properties as a single translation. Lessons 8 and 9 demonstrate sequences of reflections and translations and sequences of rotations. The concept of sequencing a combination of all three rigid motions is introduced in Lesson 10; this paves the way for the study of congruence in the next topic.

In Topic C, on the definition and properties of congruence, students learn that congruence is just a sequence of basic rigid motions. The fundamental properties shared by all the basic rigid motions are then inherited by congruence: congruence moves lines to lines and angles to angles, and it is both distance- and degree-preserving (Lesson 11). In Grade 7, students used facts about supplementary, complementary, vertical, and adjacent angles to find the measures of unknown angles (**7.G.5**). This module extends that knowledge to angle relationships that are formed when two parallel lines are cut by a transversal. In Topic C, on angle relationships related to parallel lines, students learn that pairs of angles are congruent because they are angles that have been translated along a transversal, rotated around a point, or reflected across a line.

Students use this knowledge of angle relationships in Lessons 13 and 14 to show why a triangle has a sum of interior angles equal to 180° and why the exterior angles of a triangle is the sum of the two remote interior angles of the triangle.

Optional Topic D begins the learning of Pythagorean Theorem. Students are shown the "square within a square" proof of the Pythagorean Theorem. The proof uses concepts learned in previous topics of the module, i.e., the concept of congruence and concepts related to degrees of angles. Students begin the work of finding the length of a leg or hypotenuse of a right triangle using $a^2 + b^2 = c^2$. Note that this topic will not be assessed until Module 7.

Focus Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.

b. Angles are taken to angles of the same measure.

c. Parallel lines are taken to parallel lines.

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.

8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Foundational Standards

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.1 Draw points, lines, line segments, rays, angles, and perpendicular and parallel lines. Identify these in two-dimensional figures.

4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** This module is rich with notation that requires students to decontextualize and contextualize throughout. Students work with figures and their transformed images using symbolic representations and need to attend to the meaning of the symbolic notation to contextualize problems. Students use facts learned about rigid motions in order to make sense of problems involving congruence.

MP.3 **Construct viable arguments and critique the reasoning of others.** Throughout this module, students construct arguments around the properties of rigid motions. Students make assumptions about parallel and perpendicular lines and use properties of rigid motions to directly or indirectly prove their assumptions. Students use definitions to describe a sequence of rigid motions to prove or disprove congruence. Students build a logical progression of statements to show relationships between angles of parallel lines cut by a transversal, the angle sum of triangles, and properties of polygons like rectangles and parallelograms.

MP.5 **Use appropriate tools strategically**. This module relies on students' fundamental understanding of rigid motions. As a means to this end, students use a variety of tools but none as important as an overhead transparency. Students verify experimentally the properties of rigid motions using physical models and transparencies. Students use transparencies when learning about translation, rotation, reflection, and congruence in general. Students determine when they need to use the transparency as a tool to justify conjectures or when critiquing the reasoning of others.

MP.6 **Attend to precision**. This module begins with precise definitions related to transformations and statements about transformations being distance and angle preserving. Students are expected to attend to the precision of these definitions and statements consistently and appropriately as they communicate with others. Students describe sequences of motions precisely and carefully label diagrams so that there is clarity about figures and their transformed images. Students attend to precision in their verbal and written descriptions of rays, segments, points, angles, and transformations in general.

Terminology

New or Recently Introduced Terms

- **Transformation** (A rule, to be denoted by F, that assigns each point P of the plane a unique point which is denoted by $F(P)$.)
- **Basic Rigid Motion** (A basic rigid motion is a rotation, reflection, or translation of the plane.
    - Basic rigid motions are examples of transformations. Given a transformation, the image of a point A is the point the transformation maps the point A to in the plane.)
- **Translation** (A basic rigid motion that moves a figure along a given vector.)
- **Rotation** (A basic rigid motion that moves a figure around a point, d degrees.)
- **Reflection** (A basic rigid motion that moves a figure across a line.)
- **Image of a point, image of a figure** (Image refers to the location of a point, or figure, after it has been transformed.)
- **Sequence (Composition) of Transformations** (More than one transformation. Given transformations G and F, $G \circ F$ is called the composition of F and G.)
- **Vector** (A Euclidean vector (or directed segment) $\overrightarrow{AB}$ is the line segment AB together with a direction given by connecting an initial point A to a terminal point B.)
- **Congruence** (A congruence is a sequence of basic rigid motions (rotations, reflections, translations) of the plane.)
- **Transversal** (Given a pair of lines L and M in a plane, a third line T is a transversal if it intersects L at a single point and intersects M at a single but different point.)

Familiar Terms and Symbols[2]

- Ray, line, line segment, angle
- Parallel and perpendicular lines
- Supplementary, complementary, vertical, and adjacent angles
- Triangle, quadrilateral
- Area and perimeter

Suggested Tools and Representations

- Transparency or patty paper
- Wet or dry erase markers for use with transparency
- Optional: geometry software
- Composition of Rigid Motions: http://youtu.be/O2XPy3ZLU7Y
- ASA: http://www.youtube.com/watch?v=-ylZdenw5U4

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	8.G.A.1
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	8.G.A.2, 8.G.A.5

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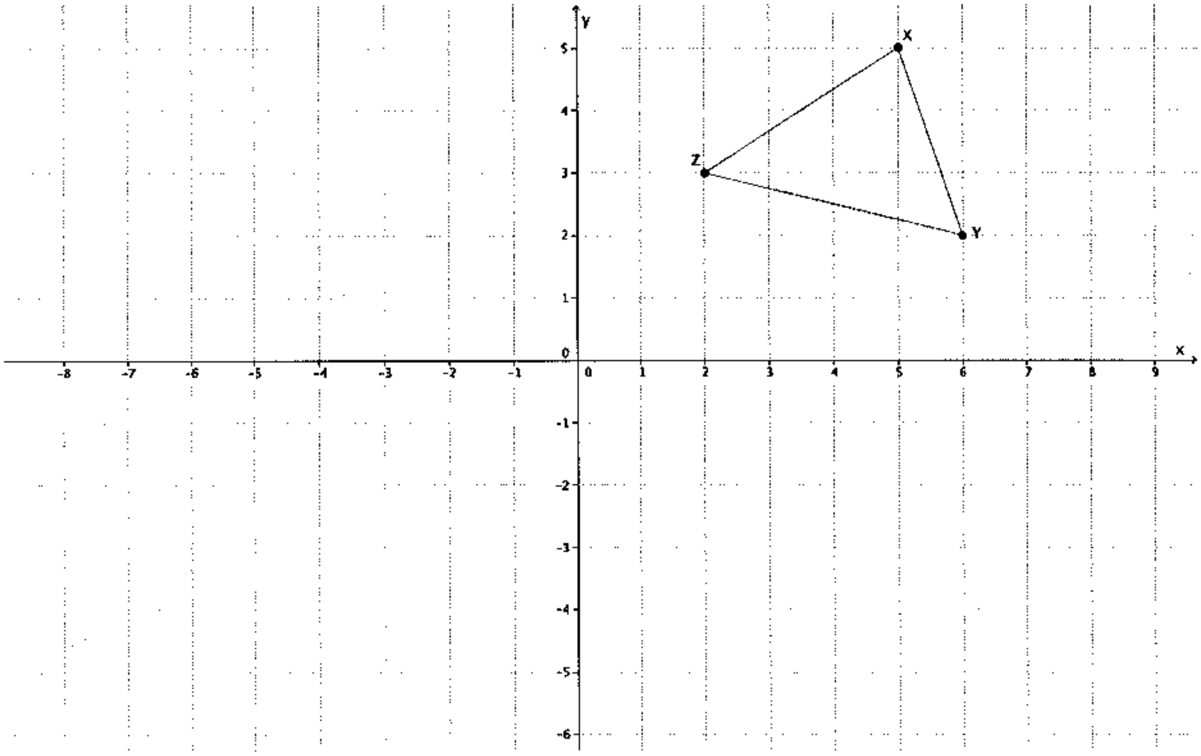
[2] These are terms and symbols students have seen previously.

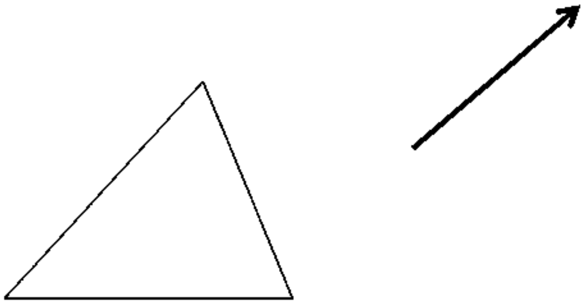
c. Rotate $\triangle XYZ$ around the point $(1,0)$, clockwise 90°. Label the image of the triangle with X', Y', and Z'.



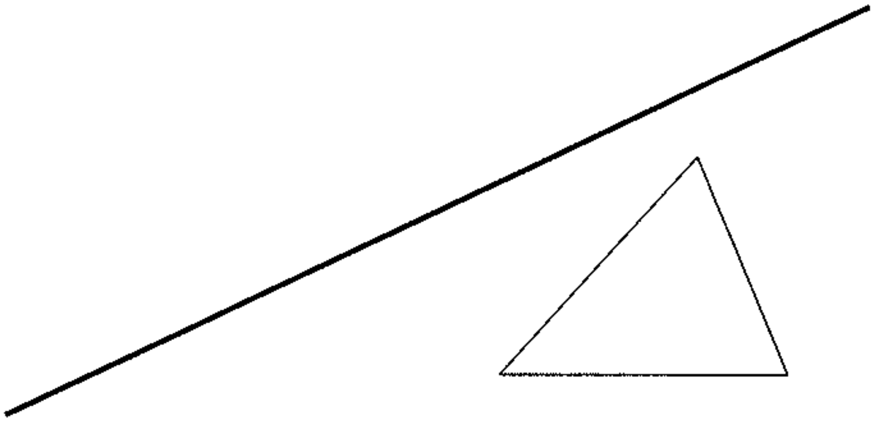
Name ______________________________ Date ____________________

1.

a. Translate ΔXYZ along $\overrightarrow{AB}$. Label the image of the triangle with X', Y', and Z'.

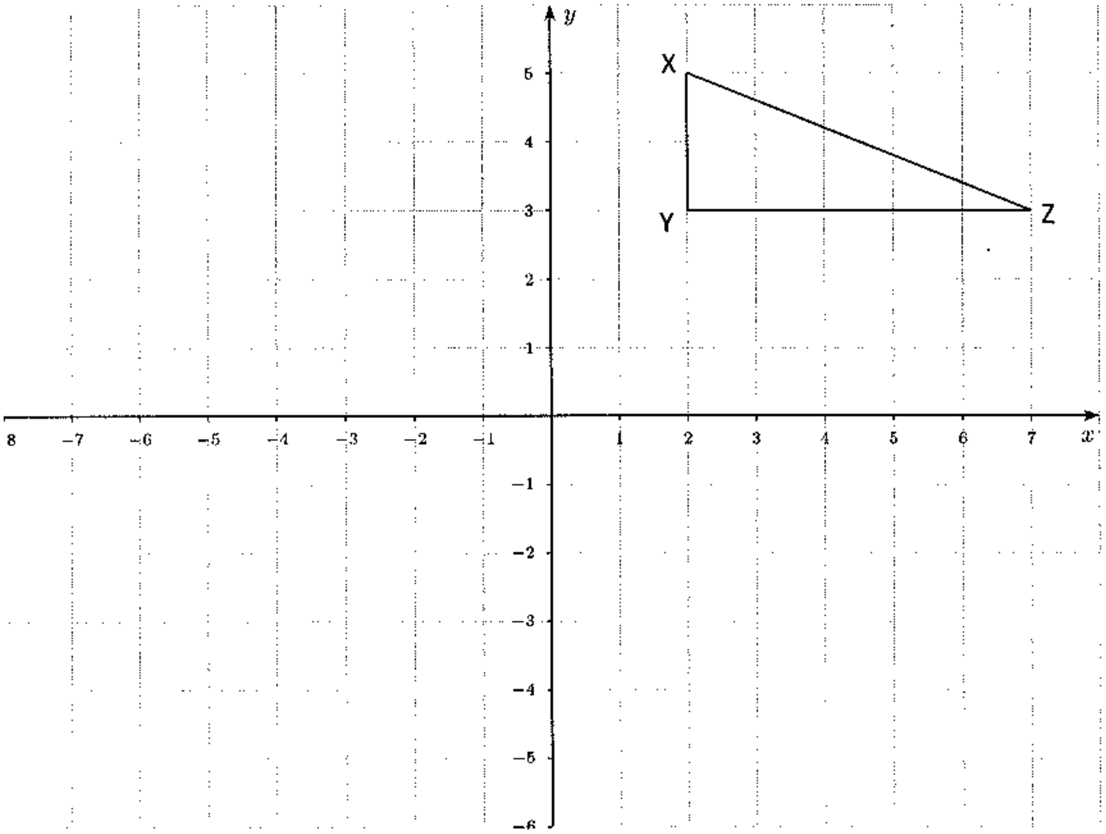


b. Reflect ΔXYZ across the line of reflection, l. Label the image of the triangle with X', Y', and Z'.



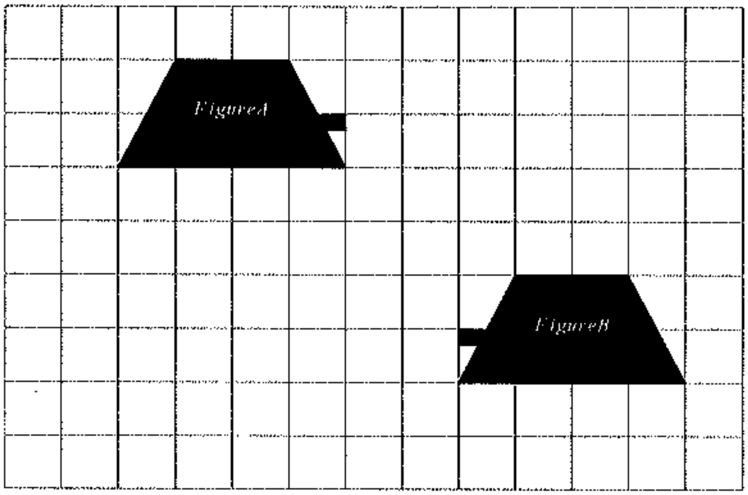
3. Use the graphs below to answer parts (a) and (b).

   a. Reflect $\triangle XYZ$ over the horizontal line (parallel to the x-axis) through point $(0,1)$. Label the reflected image with $X'Y'Z'$.



2. Use the picture below to answer the questions.

Figure A has been transformed to Figure B.

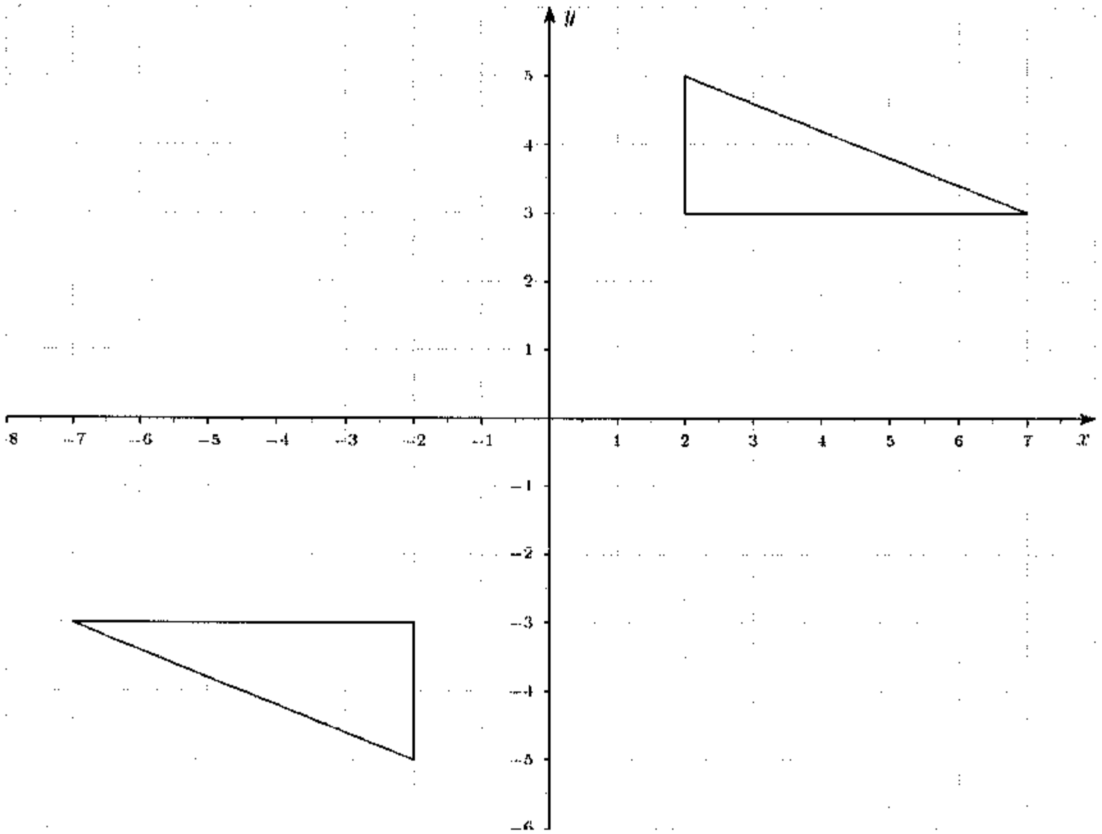


a. Can Figure A be mapped onto Figure B using only translation? Explain. Use drawings, as needed, in your explanation.

b. Can Figure A be mapped onto Figure B using only reflection? Explain. Use drawings, as needed, in your explanation.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 8.G.A.1	Student was unable to respond to the question or left item blank. Student may have enlarged or shrunk image. Student may have reflected or rotated image.	Student did translate along a vector. Student may have used a different vector than what was given. Student may have shortened or lengthened given vector. Student did not label image or may have labeled image incorrectly.	Student translated correctly along vector. Student did not label image or may have labeled image incorrectly.	Student translated correctly along vector <u>AND</u> student labeled image correctly.
	b 8.G.A.1	Student was unable to respond to the question or left item blank. Student may have enlarged or shrunk image. Student may have translated or rotated the image.	Student did reflect across line. Student may have reflected across a different line than what was given. Student did not label image or may have labeled image incorrectly. The orientation of the image may be incorrect.	Student reflected correctly across line. Student did not label image or may have labeled image incorrectly.	Student reflected correctly across line <u>AND</u> student labeled image correctly.
	c 8.G.A.1	Student was unable to respond to the question or left item blank. Student may have translated the triangle to the correct quadrant. Student may have reflected the triangle to the correct quadrant.	Student did rotate about the point (1,0). Student may have rotated the triangle counter-clockwise 90°. Student may have rotated more or less than 90°. Student did not label image or may have labeled image	Student did rotate about the point (1,0) clockwise 90°. Student did not label image or may have labeled image incorrectly.	Student did rotate about the point (1,0) clockwise 90° <u>AND</u> student labeled image correctly.

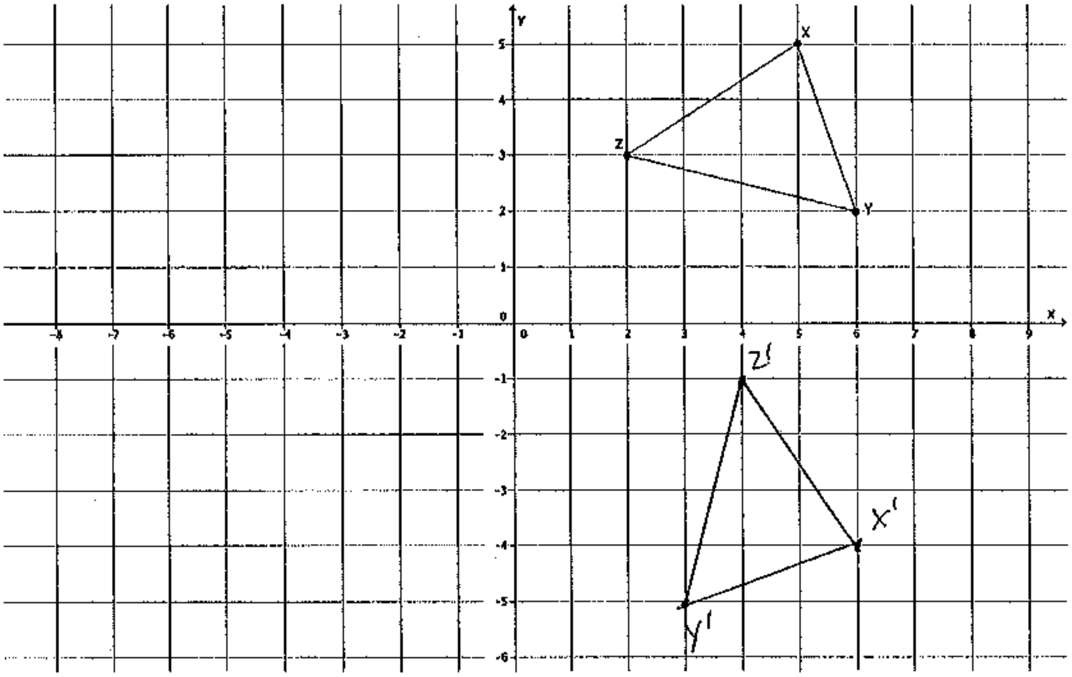
b. One triangle in the diagram below can be mapped onto the other using two reflections. Identify the lines of reflection that would map one onto the other. Can you map one triangle onto the other using just one basic rigid motion? If so, explain.



	b **8.G.A.1**	Student was unable to respond to the questions or left items blank. Student answered with yes or no only. Student may or may not have identified the lines of reflection. No evidence of mathematical reasoning used in written explanation.	Student answered with yes or no. Student may or may not have identified the lines of reflection. Student identified a rotation as the rigid motion. Student may or may not have identified the degree of rotation or the center of rotation. Some evidence mathematical reasoning used in written explanation.	Student answered correctly with yes. Student identified the lines of reflection. Student identified a rotation as the rigid motion. Student identified the degree of rotation. Student may or may not have identified the center of rotation. Some evidence mathematical reasoning used in written explanation.	Student answered correctly with yes. Student correctly identified the lines of reflection as $y = 0$, then $x = 0$ OR as $x = 0$, then $y = 0$ AND student identified a rotation as the rigid motion AND student identified the degree of rotation as 180 AND student identified the center of rotation as the origin AND substantial evidence mathematical reasoning used in written explanation.

			incorrectly.		
2	a 8.G.A.1	Student answered with yes or no only. Student was unable to give any explanation (pictorially or written).	Student answered with yes or no. Student showed some reasoning (pictorially or written) to solve the problem. Student showed no application of mathematics to solve the problem.	Student answered correctly with no. Student used a pictorial explanation only as evidence of reasoning. Some evidence of mathematical reasoning in explanation. Student did not use mathematical vocabulary in explanation.	Student answered correctly with no AND student used mathematical vocabulary in explanation. Student may have used pictorial explanation to enhance mathematical explanation.
	b 8.G.A.1	Student answered with yes or no only. Student was unable to give any explanation (pictorially or written).	Student answered with yes or no. Student showed some reasoning (pictorially or written) to solve the problem. Student showed no application of mathematics to solve the problem.	Student answered correctly with no. Student used a pictorial explanation only as evidence of reasoning. Some evidence of mathematical reasoning in explanation. Student did not use mathematical vocabulary in explanation.	Student answered correctly with no AND student used mathematical vocabulary in explanation. Student may have used pictorial explanation to enhance mathematical explanation.
3	a 8.G.A.1	Student was unable to respond to the question or left item blank. Student showed no reasoning or application of mathematics to solve the problem.	Student reflected triangle across any line other than the line $y = 1$. The orientation of the triangle may or may not be correct. Student may or may not have labeled the triangle correctly.	Student reflected triangle across the line $y = 1$. The orientation of the triangle is correct. Student may or may not have labeled the triangle correctly.	Student reflected triangle across the line $y = 1$ AND the orientation of the triangle is correct. AND Student labeled the triangle correctly.

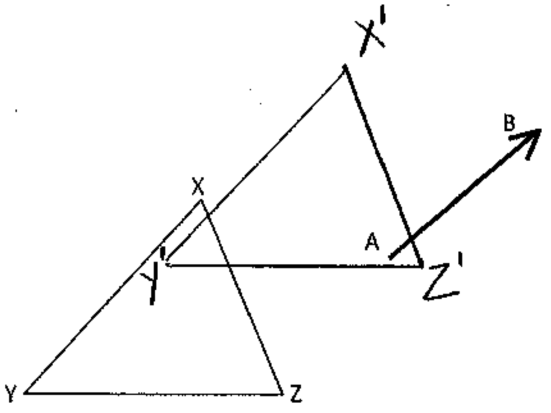
c. Rotate $\triangle XYZ$ around the point (1,0), clockwise, 90°. Label the image of the triangle with X′, Y′, and Z′.



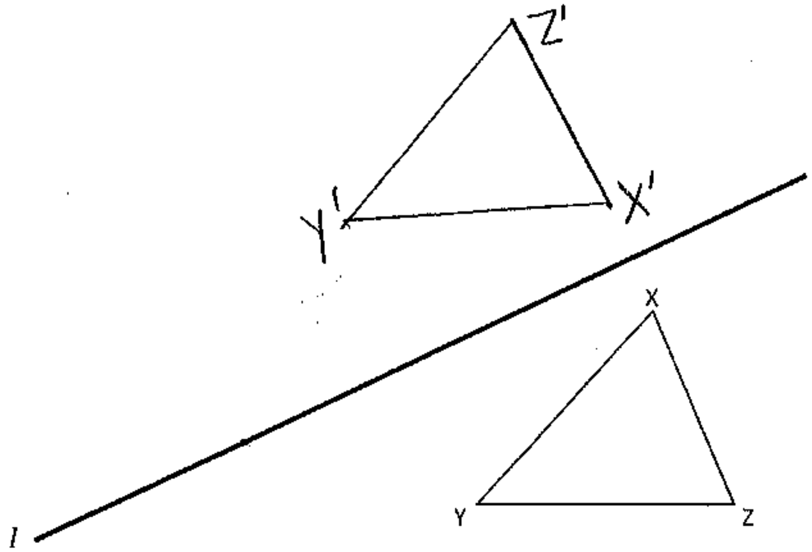
Name ______________________________ Date ______________

1.

a. Translate ΔXYZ along $\overrightarrow{AB}$. Label the image of the triangle with X′, Y′, and Z′.

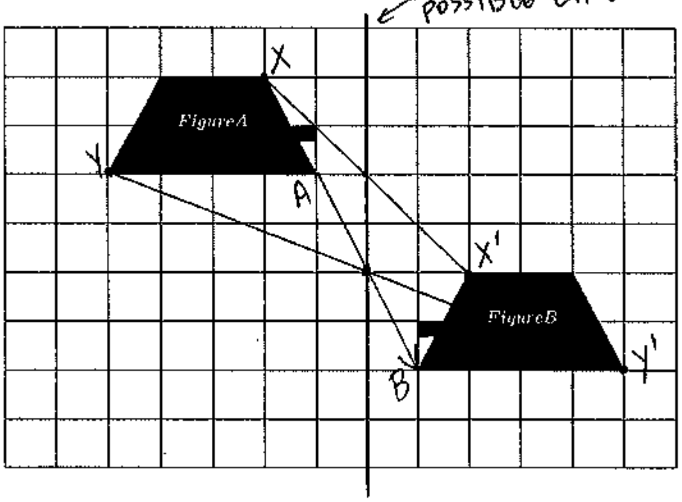


b. Reflect ΔXYZ across the line of reflection, l. Label the image of the triangle with X′, Y′, and Z′.



2. Use the picture below to answer the questions.

Figure A has been transformed to Figure B.



← POSSIBLE LINE OF REFLECTION

a. Can Figure A be mapped onto Figure B using only translation? Explain. Use drawings, as needed, in your explanation.

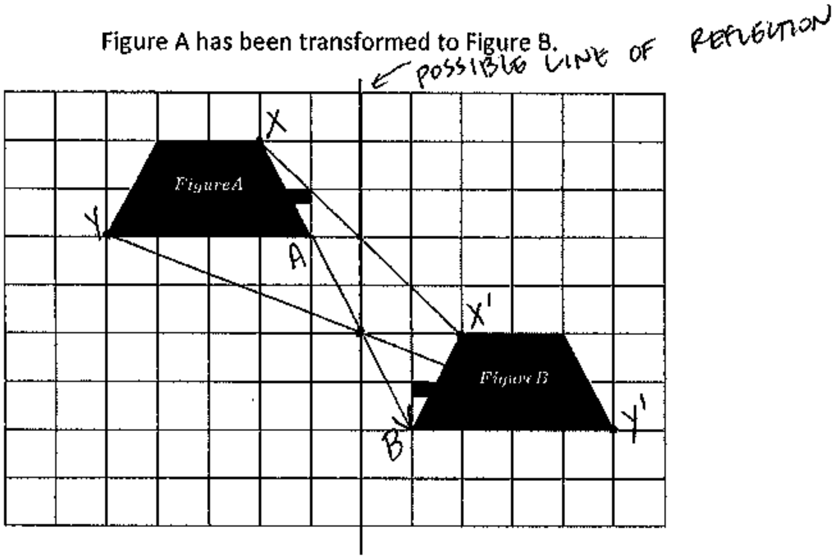
NO, IF I TRANSLATE ALONG VECTOR $\overrightarrow{AB}$ I CAN GET THE LOWER POINT OF FIGURE A TO MAP ONTO THE LOWER LEFT POINT OF FIGURE B (ONE PAIR OF CORRESPONDING POINTS) BUT NO OTHER POINTS OF THE FIGURES COINCIDE.

b. Can Figure A be mapped onto Figure B using only reflection? Explain. Use drawings, as needed, in your explanation.

NO, WHEN I CONNECT A POINT OF FIGURE A TO ITS IMAGE ON FIGURE B, THE LINE OF REFLECTION SHOULD BISECT THE SEGMENT. WHEN I CONNECT MIDPOINTS OF $\overline{XX'}$ & $\overline{YY'}$ I GET A POSSIBLE LINE OF REFLECTION, BUT WHEN I CHECK, FIGURE A DOES NOT MAP ONTO FIGURE B.

2. Use the picture below to answer the questions.

Figure A has been transformed to Figure B.



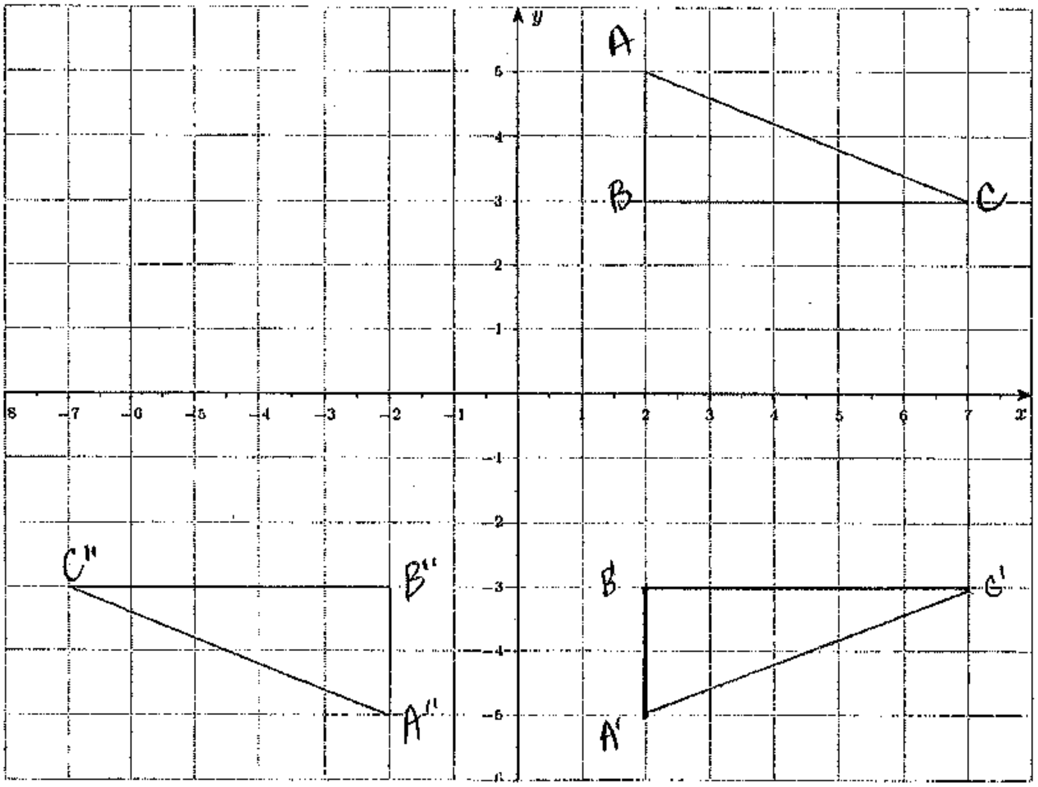
a. Can Figure A be mapped onto Figure B using only translation? Explain. Use drawings, as needed, in your explanation.

NO, IF I TRANSLATE ALONG VECTOR $\overrightarrow{AB}$ I CAN GET THE LOWER POINT OF FIGURE A TO MAP ONTO THE LOWER LEFT POINT OF FIGURE B (ONE PAIR OF CORRESPONDING POINTS) BUT NO OTHER POINTS OF THE FIGURES COINCIDE.

b. Can Figure A be mapped onto Figure B using only reflection? Explain. Use drawings, as needed, in your explanation.

NO, WHEN I CONNECT A POINT OF FIGURE A TO ITS IMAGE ON FIGURE B, THE LINE OF REFLECTION SHOULD BISECT THE SEGMENT. WHEN I CONNECT MIDPOINTS OF $\overline{XX'}$ & $\overline{YY'}$ I GET A POSSIBLE LINE OF REFLECTION, BUT WHEN I CHECK, FIGURE A DOES NOT MAP ONTO FIGURE B.

b. One triangle in the diagram below can be mapped onto the other using two reflections. Identify the lines of reflection that would map one onto the other. Can you map one triangle onto the other using just one basic rigid motion? If so, explain.

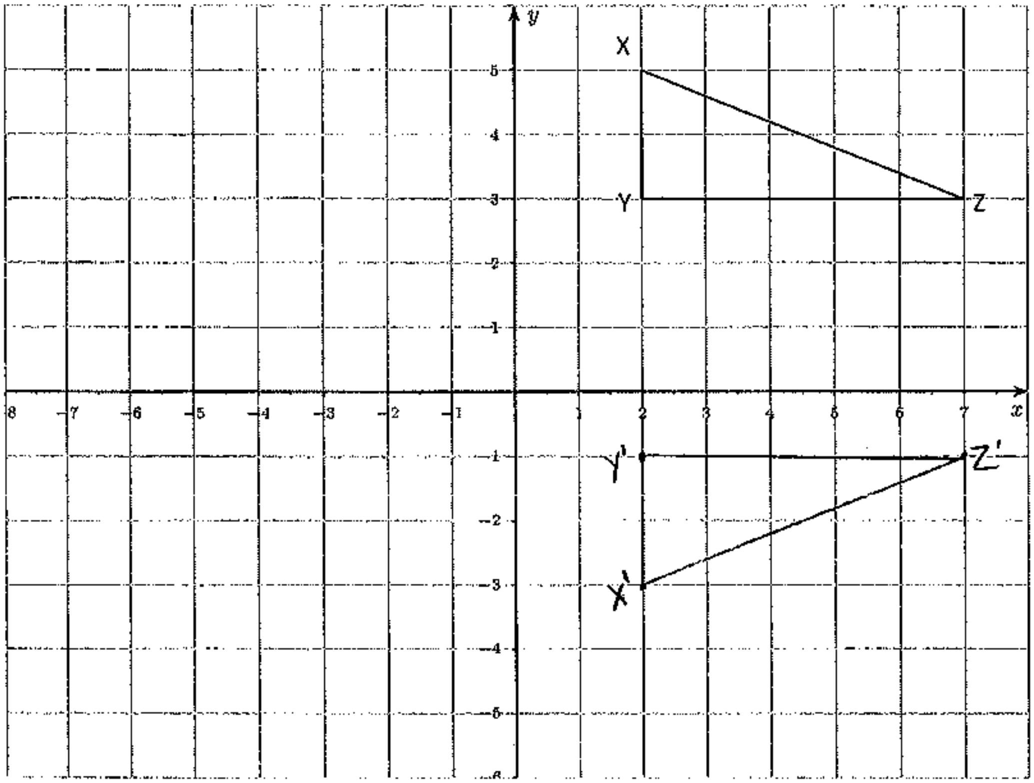


A REFLECTION ACROSS THE X-AXIS MAPS △ABC TO △A'B'C' AND A REFLECTION ACROSS THE Y-AXIS MAPS △A'B'C' TO △A"B"C".

SINCE AB∥A"B", BC∥B"C", AND AC∥A"C" AND THE LENGTHS AB = A"B", BC = B"C", AC = A"C", THEN A 180° ROTATION ABOUT THE ORIGIN WILL MAP △ABC TO △A"B"C".

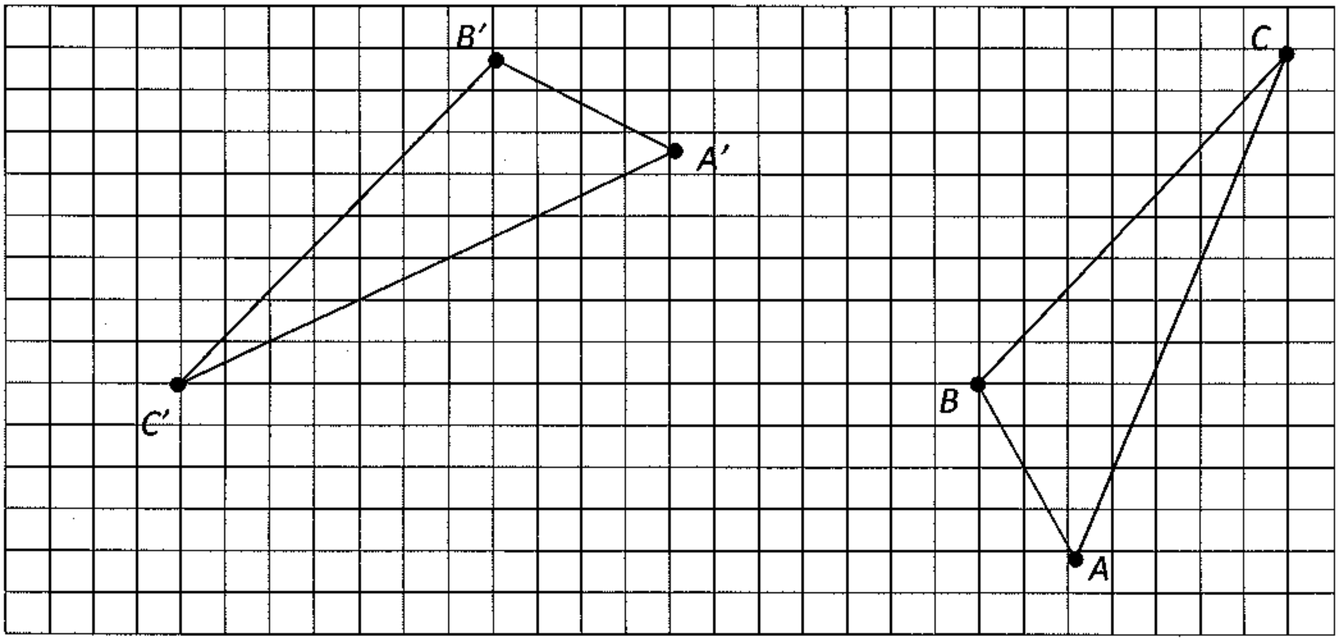
3. Use the graphs below to answer parts (a) and (b).

a. Reflect ΔXYZ over the line $y = 1$. Label the reflected image of X as X′, Y as Y′, and Z as Z′.



Name ______________________________________ Date ____________________

1. $\triangle ABC \cong \triangle A'B'C'$. Use the picture below to answer parts (a) and (b).

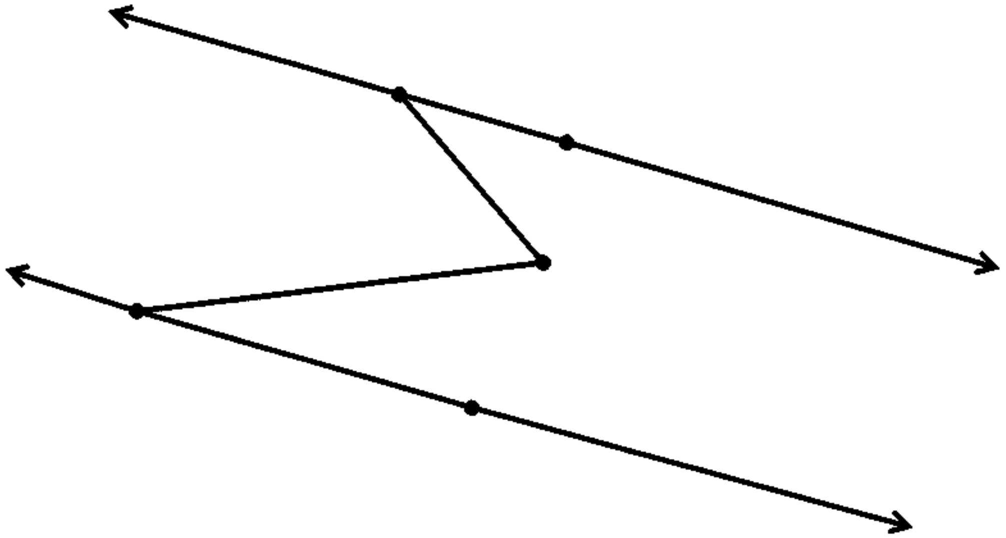


a. Is it possible to show a congruence between $\triangle ABC$ and $\triangle A'B'C'$ using only one translation and one reflection? Explain.

b. Describe a sequence of rigid motions that would prove a congruence between $\triangle ABC$ and $\triangle A'B'C'$.

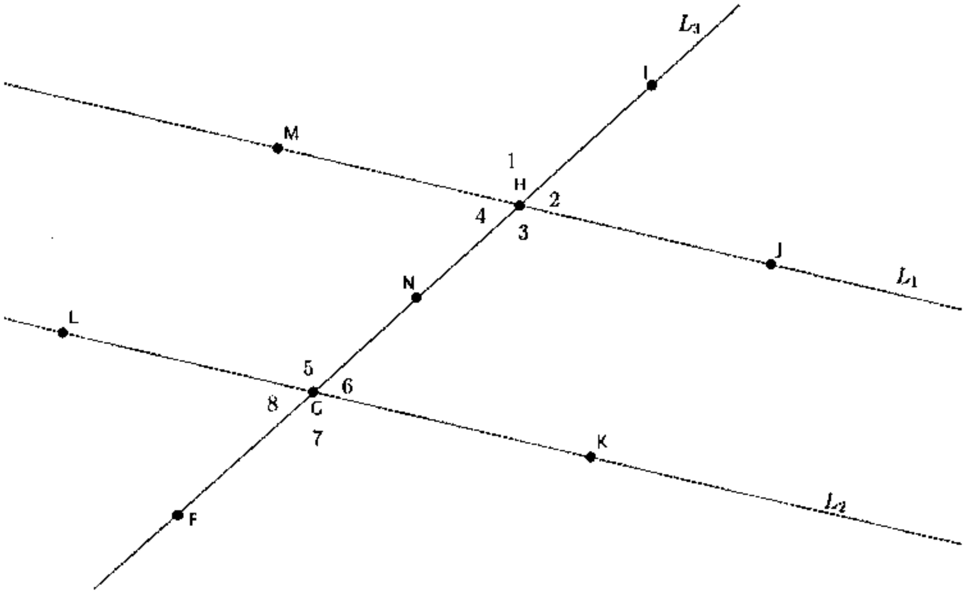
2. Use the diagram to answer the question below.

$k \| l$



Line k is parallel to line l. $m\angle EDC = 41°$ and $m\angle ABC = 32°$. Find the $m\angle BCD$. Explain in detail how you know you are correct. Add additional lines and points as needed for your explanation.

3. Use the diagram below to answer the questions that follow. Lines L_1 and L_2 are parallel, $L_1 \parallel L_2$. Point N is the midpoint of segment GH.



a. If $\angle IHM = 125°$, what is the measure of $\angle IHJ$? $\angle JHN$? $\angle NHM$?

b. What can you say about the relationship between $\angle 4$ and $\angle 6$? Explain using a basic rigid motion. Name another pair of angles with this same relationship.

c. What can you say about the relationship between $\angle 1$ and $\angle 5$? Explain using a basic rigid motion. Name another pair of angles with this same relationship.

A Progression Toward Mastery

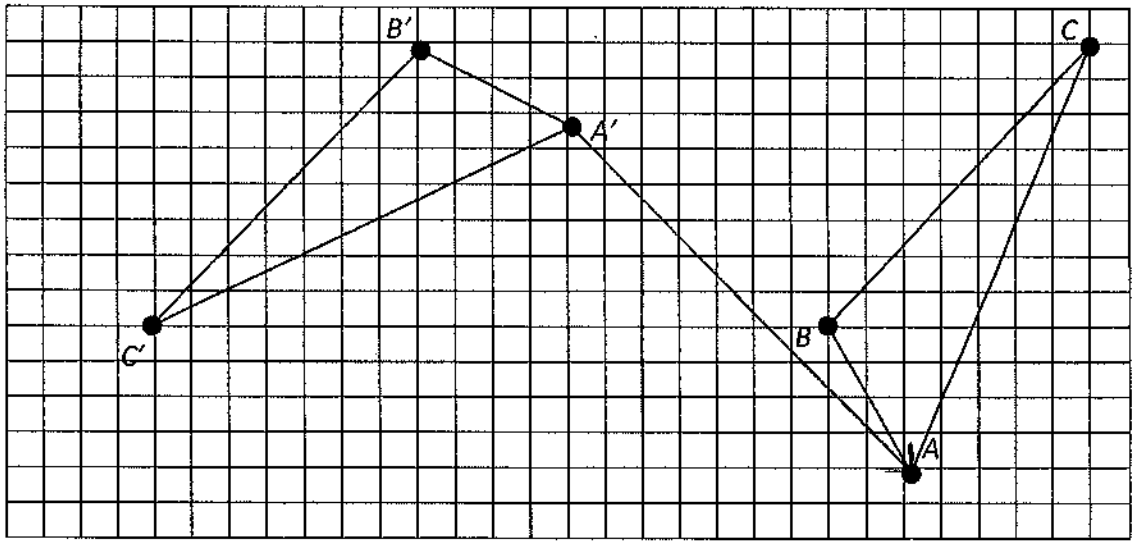
Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a 8.G.A.2	Student was unable to respond to the question or left item blank. Student answered with yes or no only. Student may or may not have answered correctly. Student shows no reasoning or application of mathematics to solve the problem.	Student answered with yes or no. Student may or may not have answered correctly. Student used little or no mathematical vocabulary or notation (e.g., map, image, prime notation, etc.) in written explanation. Some evidence mathematical reasoning used in written explanation.	Student answered correctly that one translation and one reflection will not work. Student may or may not have used mathematical vocabulary or notation (e.g., map, image, prime notation, etc.) in written explanation in attempt to translate or reflect. Some evidence mathematical reasoning used in written explanation.	Student answered correctly that one translation and one reflection will not work. Student used mathematical vocabulary and notation (e.g., map, image, prime notation, etc.) in written explanation. AND Substantial evidence mathematical reasoning used in written explanation.
	b 8.G.A.2	Student was unable to respond to the question or left item blank. Student did not describe a sequence. Student showed no reasoning or application of mathematics to solve the problem.	Student identified an incorrect sequence of rigid motions. Student used little or no mathematical vocabulary or notation in sequence. Some evidence mathematical reasoning used in sequence.	Student identified a correct sequence of rigid motions but lacked precision. Student may or may not have used mathematical vocabulary or notation sequence. Some evidence mathematical reasoning used in sequence.	Student identified a correct sequence of rigid motions with precision. Student used mathematical vocabulary and notation in sequence. Substantial evidence mathematical reasoning used in sequence.

2	**8.G.A.5**	Student was unable to respond to the questions or left items blank. Student showed no reasoning or application of mathematics to solve the problem.	Student calculated the measurement of the angle. Student may have made calculation errors. Student attempted to use auxiliary lines to solve the problem. Student showed little or no reasoning in written explanation. Student did not use any theorem in written explanation.	Student calculated the measurement of the angle. Student may have made calculation errors. Student used auxiliary lines to solve the problem. Student showed some reasoning in written explanation. Student may or may not have used the correct theorem in the written explanation.	Student calculated the measurement of the angle correctly as 73°. Student used auxiliary lines to solve the problem. AND Student shows substantial reasoning in written explanation including information about congruent angles being equal, straight angles having 180°, triangle sum being 180°, sum of remote interior angles equal to exterior angle of a triangle, etc.
3	**a** **8.G.A.5**	Student was unable to respond to the questions or left items blank. Student shows no reasoning or application of mathematics to solve the problem.	Student may have made calculation errors. Student may have answered part of the question correctly, i.e., $\angle IHM = \angle JHN = 125^\circ$ but omitted $\angle IHJ = \angle NHM = 55^\circ$, OR answered with all four angles are the same measure.	Student showed some application of mathematics to solve the problem. Student may have made calculation errors. Student may have reversed the answers, i.e., $\angle IHM = \angle JHN = 55^\circ$ or $\angle IHJ = \angle NHM = 125^\circ$.	Student answered correctly with $\angle IHM = \angle JHN = 125^\circ$ and $\angle IHJ = \angle NHM = 55^\circ$ for measures of ALL four angles.
	b **8.G.A.5**	Student was unable to respond to the questions or left items blank. Student showed no reasoning or application of mathematics to solve the problem. Student did not include a written explanation.	Student may have answered the name of the angles incorrectly. Student may have identified incorrectly the other angles with the same relationship. Student included a written explanation. Student referenced a rigid motion, translation, rotation, reflection. Written explanation is not mathematically based, i.e., "they look the same."	Student may have answered the name of the angles incorrectly but did identify correctly the other angles with the same relationship. Student used some mathematical vocabulary in written explanation. Student referenced rotation but may not have referenced all of the key points in written explanation.	Student answered correctly by calling the angles Alternate Interior Angles. AND Student named $\angle 3$ and $\angle 5$ as angles with the same relationship. AND Student used mathematical vocabulary in written explanation. AND Student referenced ALL of the following key points: N is the midpoint of HG, rotation of 180° around N, and rotation is degree preserving ine written explanation. Written explanation is

					thorough and complete.
	c **8.G.A.5**	Student was unable to respond to the questions or left items blank. Student showed no reasoning or application of mathematics to solve the problem. Student did not include a written explanation.	Student may have answered the name of the angles incorrectly. Student may have identified incorrectly the other angles with the same relationship. Student included a written explanation. Student referenced a rigid motion, translation, rotation, reflection. Written explanation is not mathematically based, i.e. "they look the same."	Student may have answered the name of the angles incorrectly but did identify correctly the other angles with the same relationship. Student used some mathematical vocabulary in written explanation. Student referenced translation but may not have referenced all of the key points in written explanation.	Student answered correctly by calling the angles Corresponding Angles. Student named $\angle 2$ and $\angle 6$ (or $\angle 3$ and $\angle 7$ or $\angle 4$ and $\angle 8$) as angles with the same relationship. <u>AND</u> Student used mathematical vocabulary in written explanation. <u>AND</u> Student referenced <u>ALL</u> of the following key points: translation along vector HG, translation maps parallel lines to parallel lines, and translation is degree preserving in written explanation. Written explanation is thorough and complete.

Name ______________________________ Date ____________________

1. $\Delta ABC \cong \Delta A'B'C'$. Use the picture below to answer parts (a) and (b).



a. Is it possible to show a congruence between ΔABC and $\Delta A'B'C'$ using only one translation and one reflection? If so, explain how.

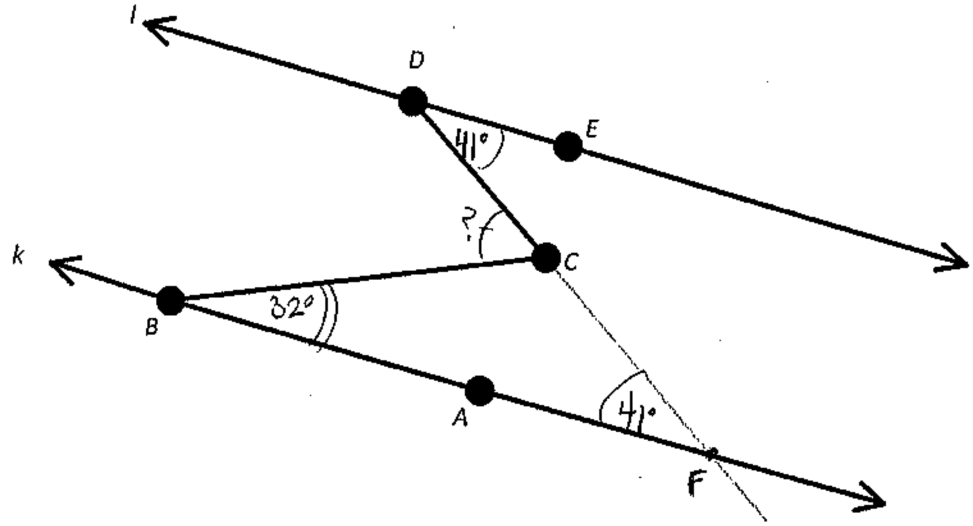
NO. A TRANSLATION CAN MAP A' TO A. BUT A REFLECTION WILL NOT MAP $\Delta A'B'C'$ TO ΔABC.

b. Describe a sequence of rigid motions that would prove a congruence between ΔABC and $\Delta A'B'C'$.

LET T BE THE TRANSLATION ALONG $\overrightarrow{A'A}$ SO THAT $T(A')=A$.
LET R BE THE ROTATION AROUND A, d DEGREES SO THAT $R(A'B')=AB$. BY HYPOTHESIS $|AB|=|A'B'|$.
LET Λ BE THE REFLECTION ACROSS L_{AB}. AGAIN BY HYPOTHESIS $|\angle A|=|\angle A|$, $|\angle B|=|\angle B|$, SO THE COMPOSITION $\Lambda \circ R \circ T$ WILL MAP $\Delta A'B'C'$ TO ΔABC, ie., $\Lambda(R(T(\Delta A'B'C')))=\Delta ABC$.

2. Use the diagram to answer the question below.

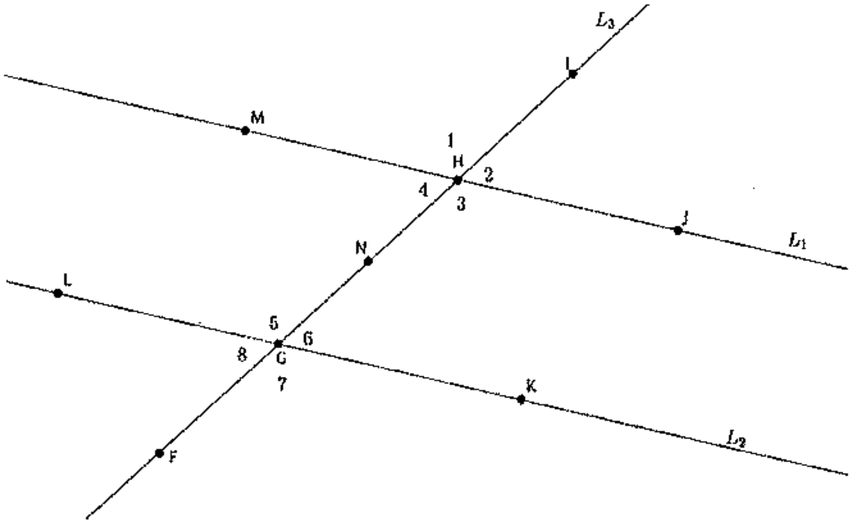
$k \| l$



Line k is parallel to line l. $m\angle EDC = 41^\circ$ and $m\angle ABC = 32^\circ$. Find the $m\angle BCD$. Explain in detail how you know you are correct. Add auxiliary lines and points as needed for your explanation.

LET F BE A POINT ON LINE k SO THAT $\angle DCF$ IS A STRAIGHT ANGLE. THEN BECAUSE k//l, $\angle EDC \cong \angle CFA$ AND HAVE EQUAL MEASURE. $\angle ABC$ AND $\angle CFA$ ARE THE REMOTE INTERIOR ANGLES OF $\triangle BCF$ WHICH MEANS $\angle BCD = \angle ABC + CFA$. THEREFORE $\angle BCD = 32 + 41 = 73^\circ$.

3. Use the diagram below to answer the questions that follow. Lines L_1 and L_2 are parallel, $L_1 \| L_2$. Point N is the midpoint of segment GH.



a) If $\angle IHM = 125°$, what is the measure of $\angle IHJ$? $\angle JHN$? $\angle NHM$?

∠IHJ = 55° ∠JHN = 125° ∠NHM = (55°

b) What can you say about the relationship between ∠4 and ∠6? Explain using a basic rigid motion. Name another pair of angles with this same relationship.

∠4 & ∠6 ARE ALTERNATE INTERIOR ANGLES THAT ARE EQUAL BECAUSE $L_1 // L_2$. LET R BE A ROTATION OF 180° AROUND POINT N. THEN $R(N) = N$; $R(L_3) = L_3$; AND $R(L_1) = L_2$. ROTATIONS ARE DEGREE PRESERVING SO $R(\angle 4) = \angle 6$.

∠3 & ∠5 ARE ALSO ALTERNATE INTERIOR ANGLES THAT ARE EQUAL.

c) What can you say about the relationship between ∠1 and ∠5? Explain using a basic rigid motion. Name another pair of angles with this same relationship.

∠1 & ∠5 ARE CORRESPONDING ANGLES THAT ARE EQUAL BECAUSE $L_1 // L_2$. LET T BE THE TRANSLATION ALONG VECTOR $\overrightarrow{GH}$. THEN $T(L_2) = L_1$ AND $T(\angle 5) = \angle 1$.

∠3 & ∠7 ARE ALSO CORRESPONDING ANGLES THAT ARE EQUAL.

UNIT THREE

for

Content Area of

MATHEMATICS

MS Band
Pre-Algebra

Lesson 1: Writing Equations Using Symbols

Classwork

Exercises

Write each of the following statements using symbolic language.

1. The sum of four consecutive even integers is -28.

2. A number is four times larger than the square of half the number.

3. Steven has some money. If he spends nine dollars, then he will have $\frac{3}{5}$ of the amount he started with.

4. The sum of a number squared and three less than twice the number is 129.

5. Miriam read a book with an unknown number of pages. The first week she read five less than $\frac{1}{3}$ of the pages. The second week she read 171 pages and finished the book. Write an equation that represents the total number of pages in the book.

Lesson Summary

Begin all word problems by defining your variables. State clearly what you want each symbol to represent.

Written mathematical statements can be represented as more than one correct symbolic statement.

Break complicated problems into smaller parts or try working them with simpler numbers.

Problem Set

Write each of the following statements using symbolic language.

1. Bruce bought two books. One book costs four dollars more than three times the other. Together, the two books cost him $\$72$.
2. Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is 55.
3. The sum of three consecutive integers is $1,623$.
4. One number is six more than another number. The sum of their squares is 90.
5. When you add 18 to $\frac{1}{4}$ of a number, you get the number itself.
6. When a fraction of 12 is taken away from 17, what remains exceeds one-third of seventeen by six.
7. The sum of two consecutive even integers divided by four is 189.5.
8. Subtract seven more than twice a number from the square of one-third of the number to get zero.
9. The sum of three consecutive integers is 42. Let x be the middle of the three integers. Transcribe the statement accordingly.

Lesson 2: Linear and Non-Linear Expressions in x

Classwork

Exercises

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or non-linear. If it is non-linear, then explain why.

1. The sum of a number and four times the number.

2. The product of five and a number.

3. Multiply six and the reciprocal of the quotient of a number and seven.

4. Twice a number subtracted from four times a number, added to 15.

5. The square of the sum of six and a number.

6. The cube of a positive number divided by the square of the same positive number.

7. The sum of four consecutive numbers.

8. Four subtracted from the reciprocal of a number.

9. Half of the product of a number multiplied by itself, three times.

10. The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and $\frac{2}{3}$ of the remaining pages today.

11. An admission fee of $\$10$ plus an additional $\$2$ per game.

12. Five more than four times a number, then twice that sum.

Lesson Summary

Linear expressions are sums of constants and products of constants and x raised to a power of 0 or 1. For example, $4 + 3x$, $7x + x - 15$, and $\frac{1}{2}x + 7 - 2$ are all linear expressions in x.

Non-linear expressions are also sums of constants and products of constants and a x raised to a power that is not 1

Problem Set

Write each of the following statements as a mathematic expression. State whether the expression is linear or non-linear. If it is non-linear, then explain why.

1. A number decreased by three squared.
2. The quotient of two and a number, subtracted from seventeen.
3. The sum of thirteen and twice a number.
4. 5.2 more than the product of seven and a number.
5. The sum that represents tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday.
6. The product of 19 and a number, subtracted from the reciprocal of the number cubed.
7. The product of 15 and number, multiplied by itself four times.
8. A number increased by five, divided by two.
9. Eight times the result of subtracting three from a number.
10. The sum of twice a number and four times a number subtracted from the number squared.
11. One-third of the result of three times a number that is increased by 12.
12. Five times the sum of one-half and a number.
13. Three-fourths of a number multiplied by seven.
14. The sum of a number and negative three, multiplied by the number.

15. The square of the difference between a number and 10.

Lesson 3: Linear Equations in x

Classwork

Exercises

Is the equation true when $x = -3$; in other words, is -3 a solution to the equation: $6x + 5 = 5x + 8 + 2x$? Explain.

Does $x = 12$ satisfy the equation: $16 - \frac{1}{2}x = \frac{3}{4}x + 1$? Explain.

Chad solved the equation $24x + 4 + 2x = 3(10x - 1)$ and is claiming that $x = 2$ makes the equation true. Is Chad correct? Explain.

Lisa solved the equation $x + 6 = 8 + 7x$ and claimed that the solution is: $x = -\frac{1}{3}$. Is she correct? Explain.

Angel transformed the following equation from $6x + 4 - x = 2(x + 1)$ to $10 = 2(x + 1)$. He then stated that the solution to the equation is $x = 4$. Is he correct? Explain.

Claire was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of $x = 3$.

Does an equation always have a solution? Could you come up with an equation that does not have a solution?

Lesson Summary

Equations are statements about equality. If the expression on the left side of the equal sign has the same value as the expression on the right side of the equal sign, then you have a true equation.

A solution to a linear equation in x is a number, such that when all instances of x are replaced with the number, the left side will equal the right side. For example, $x = 2$ is a solution to $3x + 4 = x + 8$ because the left side of the equation is

$3x + 4 = 3(2) + 4$

and the right side of the equation is

$x + 8 = 2 + 8$

Since $10 = 10$, then $x = 2$ is a solution to the linear equation $3x + 4 = x + 8$.

Problem Set

1. Given that $2x + 7 = 27$ and $3x + 1 = 28$, does $2x + 7 = 3x + 1$? Explain.

Is -5 a solution to the equation: $6x + 5 = 5x + 8 + 2x$? Explain.

Does $x = 1.6$ satisfy the equation: $6 - 4x = -\frac{x}{4}$? Explain.

Use the linear equation $3(x + 1) = 3x + 3$ to answer parts (a)–(d).

a. Does $x = 5$ satisfy the equation above? Explain.

b. Is $x = -8$ a solution the equation above? Explain.

c. Is $x = \frac{1}{2}$ a solution the equation above? Explain.

d. What interesting fact about the equation $3(x + 1) = 3x + 3$ is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?

Lesson 4: Solving a Linear Equation

Classwork

Exercises

For each problem, show your work, and check that your solution is correct.

1. Solve the linear equation: $x + x + 2 + x + 4 + x + 6 = -28$. State the property that justifies your first step and why you chose it.

2. Solve the linear equation: $2(3x + 2) = 2x - 1 + x$. State the property that justifies your first step and why you chose it.

3. Solve the linear equation: $x - 9 = \frac{3}{5}x$. State the property that justifies your first step and why you chose it.

4. Solve the linear equation: $29 - 3x = 5x + 5$. State the property that justifies your first step and why you chose it.

5. Solve the linear equation: $\frac{1}{3}x - 5 + 171 = x$. State the property that justifies your first step and why you chose it.

Lesson Summary

The properties of equality, shown below, are used to transform equations into simpler forms. If A, B, C are rational numbers, then

- If $A = B$, then $A + C = B + C$ Addition Property of Equality
- If $A = B$, then $A - C = B - C$ Subtraction Property of Equality
- If $A = B$, then $A \cdot C = B \cdot C$ Multiplication Property of Equality

A B

Problem Set

For each problem, show your work and check that your solution is correct.

1. Solve the linear equation: $x + 4 + 3x = 72$. State the property that justifies your first step and why you chose it.
2. Solve the linear equation: $x + 3 + x - 8 + x = 55$. State the property that justifies your first step and why you chose it.
3. Solve the linear equation: $\frac{1}{2}x + 10 = -\frac{1}{4}x + 54$. State the property that justifies your first step and why you chose it.
4. Solve the linear equation: $\frac{1}{4}x + 18 = x$. State the property that justifies your first step and why you chose it.
5. Solve the linear equation: $17 - x = \frac{1}{3} \cdot 15 + 6$. State the property that justifies your first step and why you chose it.
6. Solve the linear equation: $\frac{x + x + 2}{4} = 189.5$. State the property that justifies your first step and why you chose it.
7. Alysha solved the linear equation: $2x - 3 - 9x = 14 + x - 1$. Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.

$$2x - 3 - 9x = 14 + x - 1$$

$-6x-3+3=13+3+2x$

$-6x=16+2x$

$-6x+2x=16$

$-4x=16$

$\frac{-4}{-4}x=\frac{16}{-4}$
$x=-4$

Lesson 5: Writing and Solving Linear Equations

Classwork

Example 1

One angle is five less than three times the size of another angle. Together they have a sum of 143°. What are the sizes of each angle?

Example 2

Given a right triangle, find the size of the angles if one angle is ten more than four times the other angle and the third angle is the right angle.

Exercises

For each of the following problems, write an equation and solve.

1. A pair of congruent angles are described as follows: the measure of one angle is three more than twice a number and the other angle's measure is 54.5 less than three times the number. Determine the size of the angles.

2. The measure of one angle is described as twelve more than four times a number. Its supplement is twice as large. Find the measure of each angle.

3. A triangle has angles described as follows: the first angle is four more than seven times a number, another angle is four less than the first and the third angle is twice as large as the first. What are the sizes of each of the angles?

4. One angle measures nine more than six times a number. A sequence of rigid motions maps the angle onto another angle that is described as being thirty less than nine times the number. What is the measure of the angles?

5. A right triangle is described as having an angle of size "six less than negative two times a number," another angle that is "three less than negative one-fourth the number", and a right angle. What are the measures of the angles?

6. One angle is one less than six times the size of another. The two angles are complementary angles. Find the size of each angle.

Problem Set

For each of the following problems, write an equation and solve.

1. The measure of one angle is thirteen less than five times the measure of another angle. The sum of the measures of the two angles is $140°$. Determine the measures of each of the angles.

2. An angle measures seventeen more than three times a number. Its supplement is three more than seven times the number. What is the measure of each angle?

3. The angles of a triangle are described as follows: $\angle A$ is the largest angle, its measure is twice the measure of $\angle B$. The measure of $\angle C$ is 2 less than half the measure of $\angle B$. Find the measures of the three angles.

4. A pair of corresponding angles are described as follows: the measure of one angle is five less than seven times a number and the measure of the other angle is eight more than seven times the number. Are the angles congruent? Why or why not?

5. The measure of one angle is eleven more than four times a number. Another angle is twice the first angle's measure. The sum of the measures of the angles is $195°$. What is the measure of each angle?

6. Three angles are described as follows: $\angle B$ is half the size of $\angle A$. The measure of $\angle C$ is equal to one less than 2 times the measure of $\angle B$. The sum of $\angle A$ and $\angle B$ is 114. Can the three angles form a triangle? Why or why not?

Lesson 1: Writing Equations Using Symbols

Student Outcomes

- Students write mathematical statements using symbols to represent numbers.
- Students know that written statements can be written as more than one correct mathematical sentence.

Lesson Notes

The content of this lesson will continue to develop the skills and concepts presented in Grades 6 and 7. Specifically, this lesson builds on both 6.EE.B.7 (Solve real world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$), and 7.EE.B.4 (Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.)

Classwork

Discussion (4 minutes)

MP.

Show students the text of a mathematical statement compared to the equation.

The number $1{,}157$ is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers.	$1{,}157 = \dfrac{x^2 + (x+2)^2}{(x+2) - x}$

Ask students to write or share aloud (a) how these two are related, (b) which representation they prefer, and (c) why. Then, continue with the discussion that follows.

- Using letters to represent numbers in mathematical statements was introduced by René Descartes in the 1600s. In that era, people used only words to describe mathematical statements. Use of letters, or symbols, to represent numbers not only brought clarity to mathematical statements, it also expanded the horizons of mathematics.
- The reason we want to learn how to write a mathematical statement using symbols is to save time and labor.

  Imagine having to write the sentence: "The number $1{,}157$ is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers." Then, imagine having to write the subsequent sentences necessary to solve it; compare that to:

  Let x represent the first odd integer. Then,

  $$1{,}157 = \frac{x^2 + (x+2)^2}{(x+2) - x}$$
- Notice that x is just a number. That means the square of x is also a number, along with the square of the next odd integer and the difference between the numbers. This is a symbolic statement about numbers.
- Writing in symbols is simpler than writing in words, as long as everyone involved is clear about what the

symbols mean. This lesson focuses on accurately transcribing written statements into mathematical symbols. When we write mathematical statements using letters, we say we are using symbolic language.

- All of the "mathematical statements" in this lesson are equations. Recall that an equation is a statement of equality between two expressions. Developing equations from written statements forms an important basis for problem solving and is one of the most vital parts of algebra. Throughout this module, there will be work with written statements and symbolic language. Students work first with simple expressions, then with equations that gradually increase in complexity, and finally with systems of equations (more than one equation at a time).

Example 1 (3 minutes)

Throughout Example 1, have students write their thoughts on personal white boards or a similar tool and show you their transcription(s).

- We want to express the following statement using symbolic language: A whole number has the property that when the square of half the number is subtracted from five times the number, we get back the number itself.
- Do the first step, and hold up your white board.
  - *First, we define the variable. Let x be the whole number.*
- Using x to represent the whole number, write "the square of half the number."
  - $\left(\frac{x}{2}\right)^2$ *or* $\left(\frac{1}{2}x\right)^2$ *or* $\left(\frac{x^2}{4}\right)$ *or* $\left(\frac{1}{4}x^2\right)$

Scaffolding:

- Alternative written statement:
- A whole number has the property that when half the number is added to 15, we get the number itself.

$\frac{1}{2}x + 15 = x$

Ask students to write their expression in more than one way. Then, have students share their expressions of "the square of half the number." Elicit the above responses from students (or provide them). Ask students why they are all correct.

- Write the entire statement: A whole number has the property that when the square of half the number is subtracted from five times the number, we get back the number itself.
  - $5x - \left(\frac{x}{2}\right)^2 = x$

Challenge students to write this equation in another form. Engage in a conversation about why they are both correct. For example, when a number is subtracted from itself, the difference is zero. For that reason, the equation above can be written as $5x - \left(\frac{x}{2}\right)^2 - x = 0$.

Example 2 (4 minutes)

Throughout Example 2, have students write their thoughts on personal white boards or a similar tool and show you their transcription(s).

- We want to express the following statement using symbolic language: Paulo has a certain amount of money. If he spends six dollars, then he has $\frac{1}{4}$ of the original amount left.
- What is the first thing that must be done before we express this situation using symbols?
  - *We need to define our variables; that is, we must decide what symbol to*

Scaffolding:

You may need to remind students that we do not have to put the multiplication symbol between a number and a symbol. It is not wrong if we include it, but by convention (a common agreement), it is not necessary.

use and state what it is going to represent.

- Suppose we decide to use the symbol x. We will let x be the amount of money Paulo had originally. How do we show Paulo's spending 6 dollars using symbols?
  - *To show that Paulo spent* 6 *dollars, we write:* $x - 6$.
- How do we express: "he has $\frac{1}{4}$ of the original amount"?
  - *We can express it as:* $\frac{1}{4}x$.
- Put the parts together to express: "Paulo has a certain amount of money. If he spends 6 dollars, then he has $\frac{1}{4}$ of the original amount left." Use x to represent the amount of money Paulo had originally.
  - $x - 6 = \frac{1}{4}x.$

Challenge students to write this equation in another form. Engage in a conversation about why they are both correct. For example, students may decide to show that the six dollars plus what he has left is equal to the amount of money he now has. In symbols: $x = \frac{1}{4}x + 6.$

Example 3 (8 minutes)

Throughout Example 3, have students write their thoughts on personal white boards or a similar tool and show you their transcription(s).

- We want to write the following statement using symbolic language: When a fraction of 57 is taken away from 57, what remains exceeds $\frac{2}{3}$ of 57 by 4.
- The first step is to clearly state what we want our symbol to represent. If we choose the letter x, then we would say: "Let x be the fraction of 57" because that is the number that is unknown to us in the written statement. It is acceptable to use any letter to represent the unknown number, but regardless of which letter we use to symbolize the unknown number, we must clearly state what it means. This is called defining our variables.

Scaffolding:

- Alternative written statement:
- When a number is taken away from 57, what remains is four more than 5 times the number.
- $57 - x = 5x + 4$

- The hardest part of transcription is figuring out exactly how to write the statement so that it is accurately represented in symbols. Begin with the first part of the statement: "When a fraction of 57 is taken away from 57"—how can we capture that information in symbols?
  - *Students should write:* $57 - x$.
- How do we write $\frac{2}{3}$ of 57?
  - *If we are trying to find* $\frac{2}{3}$ *of* 57, *then we multiply* $\frac{2}{3} \cdot 57$.

Scaffolding:

If students have a hard time thinking about these transcriptions, give them something easier to think about. One number, say 10, exceeds another number, say 6, by 4. Is it accurate to represent this by:

$10 - 4 = 6$?

$10 = 6 + 4$?

$10 - 6 = 4$?

- Would it be accurate to write: $57 - x = \frac{2}{3} \cdot 57$?
  - *No, because we are told that "what remains exceeds* $\frac{2}{3}$ *of* 57 *by* 4*."*

- Where does the 4 belong? "What remains exceeds $\frac{2}{3}$ of 57 by 4." Think about what the word *exceeds* means in the context of the problem. Specifically, which is bigger: $57 - x$ or $\frac{2}{3}$ of 57? How do you know?
  - $57 - x$ *is bigger because* $57 - x$ *exceeds* $\frac{2}{3}$ *of* 57 *by* 4*. That means that* $57 - x$ *is* 4 *more than* $\frac{2}{3}$ *of* 57*.*
- We know that $57 - x$ is bigger than $\frac{2}{3} \cdot 57$ by 4. What would make the two numbers equal?
  - *We either have to subtract* 4 *from* $57 - x$, *or add* 4 *to* $\frac{2}{3} \cdot 57$ *to make them equal.*
- Now, if x is the fraction of 57, then we could write $(57 - x) - 4 = \frac{2}{3} \cdot 57$, or $57 - x = \frac{2}{3} \cdot 57 + 4$. Which is correct?
  - *Both transcriptions are correct because both express the written statement accurately.*
- Consider this transcription: $(57 - x) - \frac{2}{3} \cdot 57 = 4$. Is it an accurate transcription of the information in the written statement?
  - *Yes, because* $57 - x$ *is bigger than* $\frac{2}{3} \cdot 57$ *by* 4*. That means that the difference between the two numbers is* 4*. If we subtract the smaller number from the bigger number, we have a difference of* 4 *and that is what this version of the transcription shows.*

Example 4 (4 minutes)

Throughout Example 4, have students write their thoughts on personal white boards or a similar tool and show you their transcription(s).

- We want to express the following statement using symbolic language: The sum of three consecutive integers is 372.
- Do the first step, and hold up your white board.
  - *Let* x *be the first integer.*
- If we let x represent the first integer, what do we need to do to get the next consecutive integer?
  - *If* x *is the first integer, we add* 1 *to* x *to get the next integer.*

Scaffolding:

Explain that *consecutive* means one after the next. For example, 18, 19, and 20 are consecutive integers. Consider giving students a number and asking them what the next consecutive integer

- In symbols, the next integer would be $x + 1$. What do we need to do now to get the next consecutive integer?
    - *We need to add* 1 *to that integer, or* $x + 1 + 1$ *; this is the same as* $x + 2$.
- Now express the statement: The sum of three consecutive integers is 372 .
    - $x + x + 1 + x + 2 = 372$.

Students may also choose to rewrite the above equation as $3x + 3 = 372$. Transforming equations, such as this, will be a focus of the next few lessons when students begin to solve linear equations. Ask students why both equations are correct.

Example 5 (3 minutes)

Throughout Example 5, have students write their thoughts on personal white boards or a similar tool and show you their transcription(s).

Scaffolding:

Consider giving kids an odd number, then asking them what the next consecutive odd number is. Then ask student what they need to add to go from the first number to the second number.

- We want to express the following statement using symbolic language: The sum of three consecutive odd integers is 93 .
- Do the first step, and hold up your white board.
    - *Let* x *be the first integer.*
- If we let x represent the first integer, what do we need to do to get the next consecutive odd integer?
    - *If* x *is the first integer, we add* 2 *to* x *to get the next odd integer.*
- In symbols, the next odd integer would be $x + 2$. What do we need to do now to get the next odd consecutive integer?
    - *We need to add* 2 *to that integer, or* $x + 2 + 2$ *; this is the same as* $x + 4$.
- Now express the statement: The sum of three consecutive odd integers is 372 .
    - $x + x + 2 + x + 4 = 93$.
- Represent the statement "The sum of three consecutive odd integers is 93 " in another way. Be prepared to explain why both are correct.
    - *Answers will vary. Accept any correct answers and justifications. For example, students may write* $(x + x + x) + 6 = 93$ *and state that it is equivalent to the equation* $x + x + 2 + x + 4 = 93$. *Because the Associative and Commutative properties of addition were applied, those properties do not change the value of an expression.*

Exercises 1–5 (10 minutes)

Students complete Exercises 1–5 independently or in pairs.

Exercises 1–5

Write each of the following statements using symbolic language.

1. The sum of four consecutive even integers is -28.

   Let x *be the first integer. Then,* $x + x + 2 + x + 4 + x + 6 = -28$.

2. A number is four times larger than the square of half the number.

   Let x *be the number. Then,* $x = 4\left(\frac{x}{2}\right)^2$.

3. Steven has some money. If he spends $nine$ dollars, then he will have $\frac{3}{5}$ of the amount he started with.

   Let x *be the amount of money Steven started with. Then,* $x - 9 = \frac{3}{5}x$.

4. The sum of a number squared and three less than twice the number is 129.

   Let x *be the number. Then,* $x^2 + 2x - 3 = 129$.

Lesson Summary

Begin all word problems by defining your variables. State clearly what you want each symbol to represent.

Written mathematical statements can be represented as more than one correct symbolic statement.

Break complicated problems into smaller parts or try working them with simpler numbers.

5. Miriam read a book with an unknown number of pages. The first week she read five less than $\frac{1}{3}$ of the pages. The second week she read 171 pages and finished the book. Write an equation that represents the total number of pages in the book.

   Let x *be the total number of pages in the book. Then,* $\frac{1}{3}x - 5 + 171 = x$.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to write mathematical statements using symbolic language.
- Written mathematical statements can be represented as more than one correct symbolic statement.
- We must always begin writing a symbolic statement by defining our symbols (variables).
- Complicated statements should be broken into parts or attempted with simple numbers to make the representation in symbolic notation easier.

Exit Ticket (5 minutes)

Name ______________________ Date______________

Lesson 1: Writing Equations Using Symbols

Exit Ticket

Write each of the following statements using symbolic language.

1. When you square five times a number you get three more than the number.

2. Monica had some cookies. She gave seven to her sister. Then, she divided the remainder in half, and she still had five cookies.

Exit Ticket Sample Solutions

Write each of the following statements using symbolic language.

1. **When you square five times a number you get three more than the number.**

   Let x *be the number. Then,* $(5x)^2 = x + 3$.

2. **Monica had some cookies. She gave seven to her sister. Then, she divided the remainder in half, and she still had five cookies.**

   Let x *be the original amount of cookies. Then,* $\frac{1}{2}(x - 7) = 5$.

Problem Set Sample Solutions

Students practice transcribing written statements into symbolic language.

Write each of the following statements using symbolic language.

1. **Bruce bought two books. One book costs four dollars more than three times the other. Together, the two books cost him** $\$72$.

   Let x *be the cost of the less expensive book. Then,* $x + 4 + 3x = 72$.

2. **Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is** 55.

   Let x *be Julie's age. Then,* $x + 3 + x - 8 + x = 55$.

3. **The sum of three consecutive integers is** $1,623$.

   Let x *be the first integer. Then,* $x + x + 1 + x + 2 = 1,623$.

4. **One number is six more than another number. The sum of their squares is** 90.

   Let x *be the smaller number. Then* $x^2 + (x + 6)^2 = 90$.

5. **When you add** 18 **to** $\frac{1}{4}$ **of a number, you get the number itself.**

   Let x *be the number. Then,* $\frac{1}{4}x + 18 = x$.

6. **When a fraction of** 12 **is taken away from** 17**, what remains exceeds one-third of seventeen by six.**

   Let x *be the fraction of* 12*. Then,* $17 - x = \frac{1}{3} \cdot 17 + 6$.

7. **The sum of two consecutive even integers divided by four is** 189.5.

Let x *be the first even integer. Then,* $\frac{x+x+2}{4} = 189.5$.

8. Subtract seven more than twice a number from the square of one-third of the number to get zero.

   Let x *be the number. Then,* $\left(\frac{1}{3}x\right)^2 - (2x+7) = 0.$

9. The sum of three consecutive integers is 42 . Let x be the middle of the three integers. Transcribe the statement accordingly.

   $x - 1 + x + x + 1 = 42.$

Lesson 2: Linear and Non-Linear Expressions in x

Student Outcomes

- Students know the properties of linear and non-linear expressions in x.
- Students transcribe and identify expressions as linear or non-linear.

Classwork

Discussion (4 minutes)

- A symbolic statement in x with an equal sign is called an equation in x. The equal sign divides the equation into two parts, the left side and the right side. The two sides are called *expressions*.
- For sake of simplicity, we will only discuss expressions in x, but know that we can write expressions in any symbol.
- The following chart contains both linear and non-linear expressions in x. Sort them into two groups and be prepared to explain what is different about the two groups.

$5x+3$	$-8x+\frac{7}{9}-3$	$9-x^2$
$4x^2-9$	$0.31x+7-4.2x$	$\left(\frac{x}{2}\right)^3+1$
$11(x+2)$	$-(6-x)+15-9x$	$7+x^{-4}+3x$

MP.

Linear expressions are noted in red in the table below.

$5x+3$	$-8x+\frac{7}{9}-3$	$9-x^2$
$4x^2-9$	$0.31x+7-4.2x$	$\left(\frac{x}{2}\right)^3+1$
$11(x+2)$	$-(6-x)+15-9x$	$7+x^{-4}+3x$

- Identify which equations you placed in each group. Explain your reasoning for grouping the equations.
  - *Equations that contained an exponent of x other than 1 were put into one group. The other equations were put into another group. That seemed to be the only difference between the types of equations given.*
- Linear expressions in x are a special type of expression. Linear expressions are expressions that are sums of constants and products of a constant and x raised to a power of 0, which simplifies to a value of 1. Non-linear expressions are also sums of constants and products of a constant and a power of x. However, non-linear expressions will have a power of x that is not equal to 1 or 0.

- The reason we want to be able to distinguish linear expressions from non-linear expressions is because we will soon be solving linear equations. Non-linear equations will be a set of equations you learn to solve in Algebra I, though we will begin to solve simple non-linear equations later this year (Module 7). We also want to be able to recognize linear equations in order to predict the shape of their graph, which is a concept we will learn more about later in this module.

Example 1 (3 minutes)

- A linear expression in x is an expression where each term is either a constant, an x, or a product of a constant and x. For example, the expression $(57 - x)$ is a linear expression. However, the expression $2x^2 + 9x + 5$ is not a linear expression. Why is $2x^2 + 9x + 5$ not a linear expression in x?
    - *Students should say that $2x^2 + 9x + 5$ is not a linear expression because the terms of linear expressions must either be a constant, an x, or the product of a constant and x. The term $2x^2$ does not fit the definition of a linear expression in x.*

Scaffolding:

- Terms are any product of an integer power of x and a constant or just a constant.
- Constants are fixed numbers.
- When a term is the product of a constant(s) and a power of x, the constant is called a

Example 2 (4 minutes)

- Let's examine the expression $4 + 3x^5$ more deeply. To begin, we want to identify the terms of the expression. How many terms are there, and what are they?
    - *There are two terms, 4 and $3x^5$.*
- How many terms are comprised of just constants, and what are they?
    - *There is one constant term, 4.*
- How many terms have coefficients, and what are they?
    - *There is one term with a coefficient, 3.*
- Is $4 + 3x^5$ a linear or non-linear expression in x? Why or why not?
    - *The expression $4 + 3x^5$ is a non-linear expression in x because it is the sum of a constant and the product of a constant and positive integer power of $x > 1$.*

Example 3 (4 minutes)

- How many terms does the expression $7x + 9 + 6 + 3x$ have? What are they?
    - *As is, this expression has 4 terms: $7x, 9, 6,$ and $3x$.*
- This expression can be transformed using some of our basic properties of numbers. For example, if we apply the Commutative Property of Addition, we can rearrange the terms from $7x + 9 + 6 + 3x$ to $7x + 3x + 9 + 6$. First, we can apply the Associative Property of Addition:

  $(7x + 3x) + (9 + 6)$

  Next, we apply the Distributive Property:

  $(7 + 3)x + (9 + 6)$

Finally,

$10x + 15$

- How many terms with coefficients does the expression $10x + 15$ have? What are they?
    - *The expression has one term with a coefficient,* $10x$ *. For this term, the coefficient is* 10 *.*
- Is $10x + 15$ a linear or non-linear expression in x ? Why or why not?
    - *The expression* $10x + 15$ *is a linear expression in* x *because it is the sum of constants and products that contain* x *to the 1st power.*

Example 4 (2 minutes)

- How many terms does the expression $5 + 9x \cdot 7 + 2x^9$ have? What are they?
    - *The expression has three terms:* 5 *,* $9x \cdot 7$ *, and* $2x^9$*.*
- How many terms with coefficients does the expression $5 + 9x \cdot 7 + 2x^5$ have? What are they?
    - *The expression has two terms with coefficients:* $63x$ *and* $2x^5$*. The coefficients are* 63 *and* 2 *.*
- Is $5 + 9x \cdot 7 + 2x^9$ a linear or non-linear expression in x ? Why or why not?
    - *The expression* $5 + 9x \cdot 7 + 2x^9$ *is a non-linear expression in* x *because it is the sum of constants and products that contain* x *raised to a power that is greater than* 1 *.*

Example 5 (2 minutes)

- Is $94 + x + 4x^{-6} - 2$ a linear or non-linear expression in x ? Why or why not?
    - *Students may first say that it is not a linear nor non-linear expression in* x *because of the* -2 *. Remind them that subtraction can be rewritten as a sum, i.e.,* $+(-2)$*; therefore, this expression does fit the definition of non-linear.*

Example 6 (2 minutes)

- Is the expression $x^1 + 9x - 4$ a linear expression in x?
    - *Yes,* $x^1 + 9x - 4$ *is a linear expression in* x *because* x^1 *is the same as* x.
- What powers of x are acceptable in the definition of a linear expression in x?
    - *Only the power of* 1 *is acceptable because* x^1 *is, by definition, just* x.

Exercises 1–12 (14 minutes)

Students complete Exercises 1–12 independently.

Exercises 1–12

Write each of the following statements in Exercises 1–12 as a mathematical expression. State whether or not the expression is linear or non-linear. If it is non-linear, then explain why.

1. **The sum of a number and four times the number.**

   Let x *be a number; then,* $x + 4x$ *is a linear expression.*

2. **The product of five and a number.**

   Let x *be a number, then* $5x$ *is a linear expression.*

3. **Multiply six and the reciprocal of the quotient of a number and seven.**

   Let x *be a number, then* $6 \cdot \frac{7}{x}$ *is a non-linear expression. The number* $\frac{7}{x} = 7 \cdot \frac{1}{x} = 7 \cdot x^{-1}$ *is the reason it is not a linear expression. The exponent of the* x *is the reason it is not linear.*

4. **Twice a number subtracted from four times a number, added to** 15**.**

   Let x *be a number, then* $15 + (4x - 2x)$ *is a linear expression.*

5. **The square of the sum of six and a number.**

   Let x *be a number, then* $(x + 6)^2$ *is a non-linear expression. When you multiply* $(x + 6)^2$*, you get* $x^2 + 12x + 36$*. The* x^2 *is the reason it is not a linear expression.*

6. **The cube of a positive number divided by the square of the same positive number.**

   Let x *be a number, then* $\frac{x^3}{x^2}$ *is a non-linear expression. However, if you simplify the expression to just* x*, then it is linear.*

7. **The sum of four consecutive numbers.**

   Let x *be a number, then* $x + x + 1 + x + 2 + x + 3$ *is a linear expression.*

8. **Four subtracted from the reciprocal of a number.**

   Let x *be a number, then* $\frac{1}{x} - 4$ *is a non-linear expression. The term* $\frac{1}{x}$ *is the same as* x^{-1}*, which is why this expression is not linear. It is possible that a student may let* x *be the reciprocal of a number,* $\frac{1}{x}$*, which would make the expression linear.*

9. **Half of the product of a number multiplied by itself, three times.**

Lesson Summary

Linear expressions are sums of constants and products of constants and x raised to a power of 0 or 1. For example, $4+3x$, $7x+x-15$, and $\frac{1}{2}x+7-2$ are all linear expressions in x.

Non-linear expressions are also sums of constants and products of constants and a x raised to a power that is not

Let x be a number; then, $\frac{1}{2}\cdot x\cdot x\cdot x$ is not a linear expression. The term $\frac{1}{2}\cdot x\cdot x\cdot x$ is the same as $\frac{1}{2}x^3$, which is why this expression is not linear.

10. **The sum that shows how many pages Maria read if she read 45 pages of a book yesterday and $\frac{2}{3}$ of the remaining pages today.**

    Let x be the number of remaining pages of the book, then $45+\frac{2}{3}x$ is a linear expression.

11. **An admission fee of $\$10$ plus an additional $\$2$ per game.**

    Let x be the number of games, then $10+2x$ is a linear expression.

12. **Five more than four times a number, then twice that sum.**

    Let x be the number, then $2(4x+5)$ is a linear expression.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We have definitions for linear and non-linear expressions.
- We know how to use the definitions to identify expressions as linear or non-linear.
- We can write expressions that are linear and non-linear.

Exit Ticket (5 minutes)

Name ______________________________ Date______________

Lesson 2: Linear and Non-Linear Expressions in x

Exit Ticket

Write each of the following statements as a mathematic expression. State whether the expression is a linear or non-linear expression in x.

1. Seven subtracted from five times a number, then the difference added to nine times a number.

2. Three times a number subtracted from the product of fifteen and the reciprocal of a number.

3. Half of the sum of two and a number multiplied by itself three times.

Exit Ticket Sample Solutions

Write each of the following statements as a mathematic expression. State whether the expression is a linear or non-linear expression in x.

1. **Seven subtracted from five times a number, then the difference added to nine times a number.**

   Let x *be a number.* $(5x - 7) + 9x$.

   The expression is a linear expression.

2. **Three times a number subtracted from the product of fifteen and the reciprocal of a number.**

   Let x *be a number.* $15 \cdot \frac{1}{x} - 3x$.

   The expression is a non-linear expression.

3. **Half of the sum of two and a number multiplied by itself three times.**

   Let x *be a number.* $\frac{1}{2}(2 + x^3)$.

   The expression is a non-linear expression.

Problem Set Sample Solutions

Students practice writing expressions and identifying them as linear or non-linear.

Write each of the following statements as a mathematic expression. State whether the expression is linear or non-linear. If it is non-linear, then explain why.

1. **A number decreased by three squared.**

   Let x *be a number, then* $x - 3^2$ *is a linear expression.*

2. **The quotient of two and a number, subtracted from seventeen.**

   Let x *be a number, then* $17 - \frac{2}{x}$ *is a non-linear expression. The term* $\frac{2}{x}$ *is the same as* $2 \cdot \frac{1}{x}$ *and* $\frac{1}{x} = x^{-1}$. *That is why it is not a linear expression.*

3. **The sum of thirteen and twice a number.**

   Let x *be a number, then* $13 + 2x$ *is a linear expression.*

4. 5.2 **more than the product of seven and a number.**

Let x *be a number, then* $5.2 + 7x$ *is a linear expression.*

5. The sum that represents tickets sold if 35 tickets were sold Monday, half of the remaining tickets were sold on Tuesday, and 14 tickets were sold on Wednesday.

   Let x *be the remaining number of tickets, then* $35 + \frac{1}{2}x + 14$ *is a linear expression.*

6. The product of 19 and a number, subtracted from the reciprocal of the number cubed.

   Let x *be a number, then* $\frac{1}{x^3} - 19x$ *is a non-linear expression. The term* $\frac{1}{x^3}$ *is the same as* x^{-3}*. That is why it is not a linear expression.*

7. The product of 15 and number, multiplied by itself four times.

   Let x *be a number, then* $(15x)^4$ *is a non-linear expression. The expression can be written as* $15^4 \cdot x^4$*. The exponent of* 4 *with a base of* x *is the reason it is not linear.*

8. A number increased by five, divided by two.

   Let x *be a number, then* $\frac{x + 5}{2}$ *is a linear expression.*

9. Eight times the result of subtracting three from a number.

   Let x *be a number, then* $8(x - 3)$ *is a linear expression.*

10. The sum of twice a number and four times a number subtracted from the number squared.

    Let x *be a number, then* $x^2 - (2x + 4x)$ *is a non-linear expression. The term* x^2 *is the reason it is not linear.*

11. One-third of the result of three times a number that is increased by 12 .

    Let x *be a number, then* $\frac{1}{3}(3x + 12)$ *is a linear expression.*

12. Five times the sum of one-half and a number.

    Let x *be a number, then* $5\left(\frac{1}{2} + x\right)$ *is a linear expression.*

13. Three-fourths of a number multiplied by seven.

    Let x *be a number, then* $\frac{3}{4}x \cdot 7$ *is a linear expression.*

14. **The sum of a number and negative three, multiplied by the number.**

    Let x *be a number, then* $(x+(-3))x$ *is a non-linear expression because* $(x+(-3))x = x^2 - 3x$ *after using the distributive property. It is non-linear because the power of* x *in the term* x^2 *is greater than* 1 *.*

15. **The square of the difference between a number and** 10 **.**

    Let x *be a number, then* $(x-10)^2$ *is a non-linear expression because* $(x-10)^2 = x^2 - 20x + 100.$ *The term* x^2 *is a positive power of* $x > 1$ *; therefore, this is a not a linear expression.*

Lesson 3: Linear Equations in x

Student Outcomes

- Students know that a linear equation is a statement of equality between two expressions.
- Students know that a linear equation in x is actually a question: Can you find all numbers x, if they exist, that satisfy a given equation? Students know that those numbers x that satisfy a given equation are called *solutions*.

Classwork

Concept Development (7 minutes)

- We want to define "linear equation in x." Here are some examples of linear equations in x. Using what you know about the words *linear* (from Lesson 2) and *equation* (from Lesson 1), develop a mathematical definition of "linear equation in x."

Scaffolding:

Consider developing a word bank or word wall to be used throughout the module.

Show students the examples below, and provide them time to work individually or in small groups to develop an appropriate definition. Once students share their definitions, continue with the definition and discussion that follows.

$x+11=15$	$5+3=8$	$-\frac{1}{2}x=22$
$15-4x=x+\frac{4}{5}$	$3-(x+2)=-12x$	$\frac{3}{4}x+6(x-1)=9(2-x)$

- When two linear expressions are equal, they can be written as a linear equation in x.
- Consider the following equations. Which are true, and how do you know?

  $4+1=5$

  $6+5=16$

  $21-6=15$

  $6-2=1$

  - *The first and third equations are true because the value on the left side is equal to the number on the right side.*

- Is $4+15x=49$ true? How do you know?

Have a discussion that leads to students developing a list of values for x that make it false, along with one value of x that makes it true. Then, conclude the discussion by making the two points below.

- A linear equation in x is a statement about equality, but it is also an invitation to find all of the numbers x, if they exist, that make the equation true. Sometimes the question is asked in this way: What number(s)

x satisfy the equation? The question is often stated more as a directive: Solve. When phrased as a directive, it is still considered a question. Is there a number(s) x that make the statement true? If so, what is the number(s) x ?

- Equations that contain a variable do not have a definitive truth value; in other words, there are values of the variable that make the equation true and values that make it false. When we say that we have "solved an equation," what we are really saying is that we have found a number (or numbers) x that makes the linear equation true. That number x is called the solution to the linear equation.

Example 1 (4 minutes)

- Here is a linear equation in x : $4 + 15x = 49$. The question is, is there a number x that makes the linear expression $4 + 15x$ equal to the linear expression 49 ? Suppose you are told this number x has a value of 2 , i.e., $x = 2.$ We replace any instance of x in the linear equation with the value of 2 , as shown:

  $4 + 15 \cdot 2 = 49$

  Next, we evaluate each side of the equation. The left side is

$$4 + 15 \cdot 2 = 4 + 30$$
$$= 34$$

  The right side of the equation is 49 . Clearly, $34 \neq 49$. Therefore, the number 2 is not a solution to this equation.

Scaffolding:

Remind students that when a number and a symbol are next to one another, such as $15x$, we do not need to use a symbol to represent the multiplication (it is a convention). For clarity, when two numbers are being multiplied, we do use a multiplication symbol. It is necessary to tell the difference between the number, 152

- Is the number 3 a solution to the equation, i.e., is this equation true when $x = 3$?
    - *Yes, because the left side of the equation equals the right side of the equation:*

      The left side is

$$4 + 15 \cdot 3 = 4 + 45$$
$$= 49$$

  The right side is 49 . *Since* $49 = 49$, *then we can say that* $x = 3$ *is a solution to the equation* $4 + 15x = 49.$

- 3 is a solution to the equation because it is a value of x that makes the equation true.

Example 2 (4 minutes)

- Here is a linear equation in x : $8x - 19 = -4 - 7x$.
- Is 5 a solution to the equation? That is, is $x = 5$ a solution to the equation?
    - *No, because the left side of the equation does not equal the right side of the equation:*
      The left side is

$$8 \cdot 5 - 19 = 40 - 19$$
$$= 21$$

The right side is

$$-4 - 7 \cdot 5 = -4 - 35$$
$$= -39$$

Since $21 \neq -39$ *, then* $x \neq 5$.

- Is 1 a solution to the equation? That is, is this equation true when $x = 1$?
    - *Yes, the left side and right side of the equation are equal to the same number.*
      The left side is

$$8 \cdot 1 - 19 = 8 - 19$$
$$= -11$$

The right side is

$$-4 - 7 \cdot 1 = -4 - 7$$
$$= -11$$

Since $-11 = -11$ *, then* $x = 1$.

Example 3 (4 minutes)

- Here is a linear equation in x : $3(x + 9) = 4x - 7 + 7x$.

- We can make our work simpler if we use some properties to transform the expression on the right side of the equation into an expression with fewer terms.

Provide students time to transform the equation into fewer terms, then proceed with the points below.

MP.

- For example, notice that on the right side there are two terms that contain x. First, we will use the Commutative Property to rearrange the terms to better see what we are doing:

  $4x + 7x - 7$

  Next, we will use the distributive property to collect the terms that contain x:

  $$4x + 7x - 7 = (4 + 7)x - 7$$
  $$= 11x - 7$$

  Finally, the transformed (but still the same) equation can be written as: $3(x + 9) = 11x - 7$.

- Is $\frac{5}{4}$ a solution to the equation? That is, is this equation true when $x = \frac{5}{4}$?
  - *No, because the left side of the equation does not equal the right side of the equation:*

    The left side is

$$3\left(\frac{5}{4} + 9\right) = 3\left(\frac{41}{4}\right)$$
$$= \frac{123}{4}$$

The right side is

$$11 \cdot \frac{5}{4} - 7 = \frac{55}{4} - 7$$
$$= \frac{27}{4}$$

Since $\frac{123}{4} \neq \frac{27}{4}$, *then* $x \neq \frac{5}{4}$.

Example 4 (4 minutes)

- Here is a linear equation in x: $-2x + 11 - 5x = 5 - 6x$.
- We want to check to see if 6 is a solution to the equation, i.e., is $x = 6$ a solution to the equation? Before we do that, what would make our work easier?
  - *We could use the Commutative and Distributive Properties to transform the left side of the equation into an expression with fewer terms.*

$$-2x + 11 - 5x = -2x - 5x + 11$$

$$= (-2 - 5)x + 11$$

$$= -7x + 11$$

- The transformed equation can be written as: $-7x + 11 = 5 - 6x$. Is 6 a solution to the equation, i.e., is this equation true when $x = 6$?
    - *Yes, because the left side of the equation is equal to the right side of the equation:*

      The left side is

      $$-7x + 11 = -7 \cdot 6 + 11$$

      $$= -42 + 11$$
      $$= -31$$

      The right side is

      $$5 - 6x = 5 - 6 \cdot 6$$

      $$= 5 - 36$$
      $$= -31$$

      Since $-31 = -31$ *, then* $x = 6$.

Exercises 1–6 (12 minutes)

Students complete Exercises 1–6 independently.

Exercises 1–6

1. Is the equation true when $x = -3$; in other words, is -3 a solution to the equation: $6x + 5 = 5x + 8 + 2x$? Explain.

If we replace x *with the number* -3 *, then the left side of the equation is*

$$6 \cdot (-3) + 5 = -18 + 5$$
$$= -13$$

and the right side of the equation is

$$5 \cdot (-3) + 8 + 2 \cdot (-3) = -15 + 8 - 6$$

$$= -7 - 6$$
$$= -13$$

Since $-13 = -13$ *, then* $x = -3$ *is a solution to the equation* $6x + 5 = 5x + 8 + 2x$.

Note: Some students may have transformed the equation.

2. Does $x = 12$ satisfy the equation: $16 - \frac{1}{2}x = \frac{3}{4}x + 1?$ Explain.

If we replace x *with the number* 12 *, then the left side of the equation is*

$$16 - \frac{1}{2}x = 16 - \frac{1}{2} \cdot (12)$$

$$= 16 - 6$$
$$= 10$$

and the right side of the equation is

$$\frac{3}{4}x + 1 = \frac{3}{4} \cdot (12) + 1$$

$= 9 + 1$

$= 10$

Since $10 = 10$ *, then* $x = 12$ *is a solution to the equation* $16 - \frac{1}{2}x = \frac{3}{4}x + 1$.

3. Chad solved the equation $24x + 4 + 2x = 3(10x - 1)$ and is claiming that $x = 2$ makes the equation true. Is Chad correct? Explain.

If we replace x *with the number* 2 *, then the left side of the equation is*

$24x + 4 + 2x = 24 \cdot 2 + 4 + 2 \cdot 2$

$= 48 + 4 + 4$
$= 56$

and the right side of the equation is

$3(10x - 1) = 3(10 \cdot 2 - 1)$

$= 3(20 - 1)$

$= 3(19)$

$= 57$

Since $56 \neq 57$ *, then* $x = 2$ *is not a solution to the equation* $24x + 4 + 2x = 3(10x - 1)$*, and Chad is not correct.*

4. Lisa solved the equation $x + 6 = 8 + 7x$ and claimed that the solution is $x = -\frac{1}{3}$. Is she correct? Explain.

If we replace x *with the number* $-\frac{1}{3}$*, then the left side of the equation is*

$$x+6=-\frac{1}{3}+6$$
$$=5\frac{2}{3}$$

and the right side of the equation is

$$8+7x=8+7\cdot\left(-\frac{1}{3}\right)$$

$$=8-\frac{7}{3}$$

$$=\frac{24}{3}-\frac{7}{3}$$

$$=\frac{17}{3}$$

Since $5\frac{2}{3}=\frac{17}{3}$*, then* $x=-\frac{1}{3}$ *is a solution to the equation* $x+6=8+7x$ *, and Lisa is correct.*

5. **Angel transformed the following equation from $6x + 4 - x = 2(x + 1)$ to $10 = 2(x + 1)$. He then stated that the solution to the equation is $x = 4$. Is he correct? Explain.**

No, Angel is not correct. He did not transform the equation correctly. The expression on the left side of the equation $6x + 4 - x = 2(x + 1)$ would transform to

$$6x + 4 - x = 6x - x + 4$$

$$= (6 - 1)x + 4$$
$$= 5x + 4$$

If we replace x with the number 4, then the left side of the equation is

$$5x + 4 = 5 \cdot 4 + 4$$

$$= 20 + 4$$
$$= 24$$

and the right side of the equation is

$$2(x + 1) = 2(4 + 1)$$

$$= 2(5)$$

$$= 10$$

Since $24 \neq 10$, then $x = 4$ is not a solution to the equation $6x + 4 - x = 2(x + 1)$, and Angel is not correct.

6. **Claire was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of $x = 3$.**

Answers will vary. Ask students to share their equations and justifications as to how they knew $x = 3$ would

Lesson Summary

Equations are statements about equality. If the expression on the left side of the equal sign has the same value as the expression on the right side of the equal sign, then you have a true equation.

A solution to a linear equation in x is a number, such that when all instances of x are replaced with the number, the left side will equal the right side. For example, $x = 2$ is a solution to $3x + 4 = x + 8$ because the left side of the equation is

$$\begin{aligned} 3x + 4 &= 3(2) + 4 \\ &= 6 + 4 \\ &= 10 \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} x + 8 &= 2 + 8 \\ &= 10 \end{aligned}$$

make a true number sentence.

7. **Does an equation always have a solution? Could you come up with an equation that does not have a solution?**

   Answers will vary Expect students to write equations that are false. Ask students to share their equations and justifications as to how they knew the equation they wrote did not have a solution. The concept of "no solution" is introduced in Lesson 6 and solidified in Lesson 7.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that equations are statements about equality. That is, the expression on the left side of the equal sign is equal to the expression on the right side of the equal sign.
- We know that a solution to a linear equation in x will be a number, and that when all instances of x are replaced with the number, the left side will equal the right side.

Exit Ticket (5 minutes)

Name ____ Date__________________

Lesson 3: Linear Equations in x

Exit Ticket

1. Is 8 a solution to $\frac{1}{2}x + 9 = 13$? Explain.

2. Write three different equations that have $x = 5$ as a solution.

3. Is -3 a solution to the equation $3x - 5 = 4 + 2x$? Explain.

Exit Ticket Sample Solutions

1. Is 8 a solution to $\frac{1}{2}x + 9 = 13$? Explain.

   If we replace x *with the number* 8 *, then the left side is* $\frac{1}{2}(8) + 9 = 4 + 9 = 13$ *and the right side is* 13 *. Since* $13 = 13$ *, then* $x = 8$ *is a solution.*

2. Write three different equations that have $x = 5$ as a solution.

   Answers will vary. Accept equations where $x = 5$ *makes a true number sentence.*

3. Is -3 a solution to the equation $3x - 5 = 4 + 2x$? Explain.

   If we replace x *with the number* -3 *, then the left side is* $3(-3) - 5 = -9 - 5 = -14$. *The right side is*

   $4 + 2(-3) = 4 - 6 = -2$ *. Since* $-14 \neq -2$ *, then* $x = -3$ *is not a solution to the equation.*

Problem Set Sample Solutions

Students practice determining whether or not a given number is a solution to the linear equation.

1. Given that $2x + 7 = 27$ and $3x + 1 = 28$, does $2x + 7 = 3x + 1$? Explain.

   No, because a linear equation is a statement about equality. We are given that $2x + 7 = 27$ *, but* $3x + 1 = 28$ *. Since each linear expression is equal to a different number,* $2x + 7 \neq 3x + 1$.

2. Is -5 a solution to the equation: $6x + 5 = 5x + 8 + 2x$? Explain.

   If we replace x *with the number* -5 *, then the left side of the equation is*

   $$\begin{aligned} &6 \cdot (-5) + 5 = -30 + 5 \\ &= -25 \end{aligned}$$

   and the right side of the equation is

   $$5 \cdot (-5) + 8 + 2 \cdot (-5) = -25 + 8 - 10$$

$= -17 - 10$
$= -27$

Since $-25 \neq -27$ *, then* $x \neq -5$ *is not a solution to the equation* $6x + 5 = 5x + 8 + 2x$.

Note: ***Some students may have transformed the equation.***

3. Does $x = 1.6$ satisfy the equation: $6 - 4x = -\frac{x}{4}$? Explain.

If we replace x *with the number* 1.6 *, then the left side of the equation is*

$6 - 4 \cdot 1.6 = 6 - 6.4$
$= -0.4$

and the right side of the equation is

$-\frac{1.6}{4} = -0.4$

Since $-0.4 = -0.4$ *, then* $x = 1.6$ *is a solution to the equation* $6 - 4x = -\frac{x}{4}$.

4. Use the linear equation $3(x + 1) = 3x + 3$ to answer parts (a)–(d).

a. Does $x = 5$ satisfy the equation above? Explain.

If we replace x *with the number* 5 *, then the left side of the equation is*

$3(5 + 1) = 3(6)$
$= 18$

and the right side of the equation is

$3x + 3 = 3 \cdot 5 + 3$

$= 15 + 3$

$= 18$

Since $18 = 18$ *, then* $x = 5$ *is a solution to the equation* $3(x + 1) = 3x + 3.$

b. Is $x = -8$ a solution the equation above? Explain.

If we replace x *with the number* -8 *, then the left side of the equation is*

$3(-8 + 1) = 3(-7)$

$= -21$

and the right side of the equation is

$3x + 3 = 3 \cdot (-8) + 3$

$= -24 + 3$

$= -21$

Since $-21 = -21$ *, then* $x = -8$ *is a solution to the equation* $3(x + 1) = 3x + 3.$

c. Is $x = \frac{1}{2}$ a solution the equation above? Explain.

If we replace x *with the number* $\frac{1}{2}$*, then the left side of the equation is*

$3\left(\frac{1}{2} + 1\right) = 3\left(\frac{1}{2} + \frac{2}{2}\right)$

$= 3\left(\frac{3}{2}\right)$

$= \frac{9}{2}$

and the right side of the equation is

$$3x + 3 = 3 \cdot \left(\frac{1}{2}\right) + 3$$

$$= \frac{3}{2} + 3$$

$$= \frac{3}{2} + \frac{6}{2}$$

$$= \frac{9}{2}$$

Since $\frac{9}{2} = \frac{9}{2}$*, then* $x = \frac{1}{2}$ *is a solution to the equation* $3(x + 1) = 3x + 3$.

d. **What interesting fact about the equation $3(x + 1) = 3x + 3$ is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?**

Note to teacher: Ideally students will notice that the equation $3(x + 1) = 3x + 3$ *is an identity under the distributive law. The purpose of this problem is to prepare students for the idea that linear equations can have more than one solution, which is a topic of Lesson 7.*

Lesson 4: Solving a Linear Equation

Student Outcomes

- Students extend the use of the properties of equality to solve linear equations having rational coefficients.

Classwork

Concept Development (13 minutes)

- To solve an equation means to find all of the numbers x, if they exist, so that the given equation is true.
- In some cases, some simple guess work can lead us to a solution. For example, consider the following equation:

$4x + 1 = 13$

What number x would make this equation true? That is, what value of x would make the left side equal to the right side? (Give students a moment to guess a solution.)

  - *When $x = 3$, we get a true statement. The left side of the equal sign is equal to 13 and so is the right side of the equal sign.*

In other cases, guessing the correct answer is not so easy. Consider the following equation:

$3(4x - 9) + 10 = 15x + 2 + 7x$

Can you guess a number for x that would make this equation true? (Give students a minute to guess.)

- Guessing is not always an efficient strategy for solving equations. In the last example, there are several terms in each of the linear expressions comprising the equation. This makes it more difficult to easily guess a solution. For this reason, we want to use what we know about the properties of equality to transform equations into equations with fewer terms.
- The ultimate goal of solving any equation is to get it into the form of x (or whatever symbol is being used in the equation) equal to a constant.

MP.

Complete the activity described to remind students of the properties of equality, then proceed with the discussion that follows.

Give students the equation: $4 + 1 = 7 - 2$ and ask them the following questions.

1. Is this equation true?
2. Perform each of the following operations, and state whether or not the equation is still true:
   a. Add three to both sides of the equal sign.
   b. Add three to the left side of the equal sign, and add two to the right side of the equal sign.
   c. Subtract six from both sides of the equal sign.
   d. Subtract three from one side of the equal sign and subtract three from the other side.
   e. Multiply both sides of the equal sign by ten.
   f. Multiply the left side of the equation by ten and the right side by four.
   g. Divide both sides of the equation by two.

h. Divide the left side of the equation by two and the right side of the equation by five.

3. What do you notice? Describe any patterns you see.

- There are four properties of equality that will allow us to transform an equation into the form we want. If A, B, and C are any rational numbers, then
  - If $A = B$, then $A + C = B + C$.
  - If $A = B$, then $A - C = B - C$.
  - If $A = B$, then $A \cdot C = B \cdot C$.
  - If $A = B$, then $\frac{A}{C} = \frac{B}{C}$, where C is not equal to zero.

  All four of the properties require us to start off with $A = B$. That is, we have to assume that a given equation has an expression on the left side that is equal to the expression on the right side. Working under that assumption, each time we use one of the properties of equality, we are transforming the equation into another equation that is also true, i.e., left side equals right side.

Example 1 (3 minutes)

- Solve the linear equation $2x - 3 = 4x$ for the number x.
- Examine the properties of equality. Choose "something" to add, subtract, multiply, or divide on both sides of the equation.

Validate the use of the properties of equality by having students share their thoughts. Then, discuss the "best" choice for the first step in solving the equation with the points below. Be sure to remind students throughout this and the other examples that our goal is to get x equal to a constant; therefore, the "best" choice is one that gets us to that goal most efficiently.

- First, we must assume that there is a number x that makes the equation true. Working under that assumption, when we use the property, If $A = B$, then $A - C = B - C$, we get an equation that is also true:

$2x - 3 = 4x$
$2x - 2x - 3 = 4x - 2x$

Now, using the distributive property, we get another set of equations that is also true:

$(2 - 2)x - 3 = (4 - 2)x$

$0x - 3 = 2x$
$-3 = 2x$

Using another property, If $A = B$, then $\frac{A}{C} = \frac{B}{C}$, we get another true equation:

$\frac{-3}{2} = \frac{2x}{2}$

After simplifying the fraction $\frac{2}{2}$, we have

$\frac{-3}{2} = x$

is also true.

- The last step is to check to see if $x = -\frac{3}{2}$ satisfies the equation $2x - 3 = 4x$.

  The left side of the equation is equal to $2 \cdot \left(-\frac{3}{2}\right) - 3 = -3 - 3 = -6$.

  The right side of the equation is equal to $4 \cdot \left(-\frac{3}{2}\right) = 2 \cdot (-3) = -6$.

  Since the left side equals the right side, we know we have found the correct number x that solves the equation $2x - 3 = 4x$.

Example 2 (4 minutes)

- Solve the linear equation $\frac{3}{5}x - 21 = 15$. Keep in mind that our goal is to transform the equation so that it is in the form of x equal to a constant. If we assume that the equation is true for some number x, which property should we use to help us reach our goal and how should we use it?

Again, provide students time to decide which property is "best" to use first.

  - *We should use: If $A = B$, then $A + C = B + C$, where the number C is 21.*

Note to teacher: If students suggest that we subtract 15 from both sides, i.e., make C be -15, then remind them that we want the form of x equal to a constant. Subtracting 15 from both sides of the equal sign puts the x and all of the constants on the same side of the equal sign. There is nothing mathematically incorrect about subtracting 15; it just doesn't get us any closer to reaching our goal.

- If we use $A + C = B + C$, then we have the true statement:

$\frac{3}{5}x - 21 + 21 = 15 + 21$

and

$$\frac{3}{5}x = 36.$$

Which property should we use to reach our goal, and how should we use it?

  - *We should use: If* $A = B$, *then* $A \cdot C = B \cdot C$, *where C is* $\frac{5}{3}$.
- If we use $A \cdot C = B \cdot C$, then we have the true statement:

$$\frac{3}{5}x \cdot \frac{5}{3} = 36 \cdot \frac{5}{3}$$

and by the commutative property and the cancellation law we have:

$$x = 12 \cdot 5 = 60.$$

- Does $x = 60$ satisfy the equation $\frac{3}{5}x - 21 = 15$?
  - *Yes, because the left side of the equation is equal to* $\frac{180}{5} - 21 = 36 - 21 = 15$. *Since the right side is also* 15, *then we know that* $x = 60$ *is a solution to* $\frac{3}{5}x - 21 = 15$.

Example 3 (5 minutes)

- The properties of equality are not the only properties we can use with equations. What other properties do we know that could make solving an equation more efficient?
  - *We know the distributive property, which allows us to expand and simplify expressions.*
  - *We know the commutative and associative properties, which allow us to rearrange and group terms within expressions.*

Now we will solve the linear equation $\frac{1}{5}x + 13 + x = 1 - 9x + 22$. Is there anything we can do to the linear expression on the left side to transform it into an expression with fewer terms?

- *Yes, we can use the commutative and distributive properties:*

$$\frac{1}{5}x + 13 + x = \frac{1}{5}x + x + 13$$
$$= \frac{6}{5}x + 13$$

Note to Teacher:

There are many ways to solve this equation. Any of the actions listed below are acceptable. In fact, a student could say: "add 100 to both sides of the equal sign," and that, too, would be an acceptable action. It may not lead us directly to our answer, but it is still an action that would make a mathematically correct statement. Make clear to students that it doesn't matter which option they choose or in which order; what matters is that they use the properties of equality to make true statements that lead to a solution in the form of x equal

- Is there anything we can do to the linear expression on the right side to transform it into an expression with fewer terms?
  - *Yes, we can use the commutative property:*

$$1 - 9x + 22 = 1 + 22 - 9x$$
$$= 23 - 9x$$

- Now we have the equation: $\frac{6}{5}x + 13 = 23 - 9x$. What should we do now to solve the equation?

Students should come up with the following four responses as to what should be done first. A case can be made for each of them being the "best" move. In this case, each first move gets us one step closer to our goal of having the solution in the form of x equal to a constant. Select one option and move forward with solving the equation (the notes that follow align to the first choice, subtracting 13 from both sides of the equal sign).

- *We should subtract 13 from both sides of the equal sign.*
- *We should subtract 23 from both sides of the equal sign.*
- *We should add $9x$ to both sides of the equal sign.*
- *We should subtract $\frac{6}{5}x$ from both sides of the equal sign.*

- Let's choose to subtract 13 from both sides of the equal sign. Though all options were generally equal with respect to being the "best" first step, I chose this one because when I subtract 13 on both sides, the value of the constant on the left side is positive. I prefer to work with positive numbers. Then we have

$$\frac{6}{5}x + 13 - 13 = 23 - 13 - 9x$$

$\frac{6}{5}x = 10 - 9x$

- What should we do next? Why?
  - *We should add* $9x$ *to both sides of the equal sign. We want our solution in the form of* x *equal to a constant, and this move puts all terms with an* x *on the same side of the equal sign.*
- Adding $9x$ to both sides of the equal sign, we have:

Note to Teacher:

We still have options. If students say we should subtract $\frac{6}{5}x$ from both sides of the equal sign, remind them of our goal of obtaining the x equal to a constant

$\frac{6}{5}x + 9x = 10 - 9x + 9x$

$\frac{51}{5}x = 10$

- What do we need to do now?
  - *We should multiply* $\frac{5}{51}$ *on both sides of the equal sign.*
- Then we have:

$\frac{51}{5}x \cdot \frac{5}{51} = 10 \cdot \frac{5}{51}$

By the commutative property and the fact that $\frac{5}{51} \times \frac{51}{5} = 1$, we have

$x = \frac{50}{51}$.

- Since all transformed versions of the original equation are true, we can select any of them to check our answer. However, it is best to check the solution in the original equation because we may have made a mistake transforming the equation.

$\frac{1}{5}x + 13 + x = 1 - 9x + 22$

$\frac{1}{5}\left(\frac{50}{51}\right) + 13 + \frac{50}{51} = 1 - 9\left(\frac{50}{51}\right) + 22$

$\frac{6}{5}\left(\frac{50}{51}\right) + 13 = 23 - 9\left(\frac{50}{51}\right)$

$$\frac{300}{255}+13=23-\frac{450}{51}$$
$$\frac{3615}{255}=\frac{723}{51}$$
$$\frac{723}{51}=\frac{723}{51}$$

Since both sides of our equation equal $\frac{723}{51}$, then we know our answer is correct.

Exercises 1–5 (10 minutes)

Students work on Exercises 1–5 independently.

Exercises 1–5

For each problem, show your work and check that your solution is correct.

1. **Solve the linear equation: $x+x+2+x+4+x+6=-28$. State the property that justifies your first step and why you chose it.**

   The left side of the equation can be transformed from $x+x+2+x+4+x+6$ *to* $4x+12$ *using the commutative and distributive properties. Using these properties decreases the number of terms of the equation. Now we have the equation:*

$$4x+12=-28$$

$$4x+12-12=-28-12$$

$$4x=-40$$

$$\frac{1}{4}\cdot 4x=-40\cdot\frac{1}{4}$$
$$x=-10$$

The left side of the equation is equal to $(-10)+(-10)+2+(-10)+4+(-10)+6$ *which is* -28. *Since the left side is equal to the right side, then* $x=-10$ *is the solution to the equation.*

Note: Students could use the division property in the last step to get the answer.

2. Solve the linear equation: $2(3x+2)=2x-1+x$. State the property that justifies your first step and why you chose it.

Both sides of equation can be rewritten using the distributive property. I have to use it on the left side to expand the expression. I have to use it on the right side to collect like terms.

The left side is:

$$2(3x+2)=6x+4$$

The right side is:

$$2x-1+x=2x+x-1$$
$$=3x-1$$

The equation is:

$$6x+4=3x-1$$
$$6x+4-4=3x-1-4$$
$$6x=3x-5$$
$$6x-3x=3x-3x-5$$
$$3x=-5$$
$$\frac{1}{3}\cdot 3x=\frac{1}{3}\cdot(-5)$$
$$x=-\frac{5}{3}$$

The left side of the equation is equal to $2(-5+2)=2(-3)=-6.$ *The right side of the equation is equal to* $-5-1=-6.$ *Since both sides are equal to* -6 *, then* $x=-\frac{5}{3}$ *is a solution to* $2(3x+2)=2x-1+x.$

Note: Students could use the division property in the last step to get the answer.

3. Solve the linear equation: $x-9=\frac{3}{5}x.$ State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all terms with an x *on one side of the equal sign.*

$$x-9=\frac{3}{5}x$$

$$x-x-9=\frac{3}{5}x-x$$

$$-9=-\frac{2}{5}x$$

$-\frac{5}{2}\cdot(-9)=-\frac{5}{2}\cdot-\frac{2}{5}x$

$\frac{45}{2}=x$

The left side of the equation is $\frac{45}{2}-\frac{18}{2}=\frac{27}{2}$. *The right side is* $\frac{3}{5}\cdot\frac{45}{2}=\frac{3}{1}\cdot\frac{9}{2}=\frac{27}{2}$. *Since both sides are equal to the same number, then* $x=\frac{45}{2}$ *is a solution to* $x-9=\frac{3}{5}x$.

4. Solve the linear equation: $29-3x=5x+5$. State the property that justifies your first step and why you chose it.

I chose to use the addition property of equality to get all terms with an x *on one side of the equal sign.*

$29-3x=5x+5$

$29-3x+3x=5x+3x+5$

$29=8x+5$

$29-5=8x+5-5$

$24=8x$

$\frac{1}{8}\cdot 24=\frac{1}{8}\cdot 8x$

$3=x$

The left side of the equal sign is $29-3(3)=29-9=20$. *The right side is equal to* $5(3)+5=15+5=20$. *Since both sides are equal,* $x=3$ *is a solution to* $29-3x=5x+5$.

***Note:** Students could use the division property in the last step to get the answer.*

5. Solve the linear equation: $\frac{1}{3}x-5+171=x$. State the property that justifies your first step and why you chose it.

I chose to combine the constants -5 *and* 171. *Then, I used the subtraction property of equality to get all terms with an* x *on one side of the equal sign.*

$$\frac{1}{3}x-5+171=x$$

$$\frac{1}{3}x+166=x$$

$$\frac{1}{3}x-\frac{1}{3}x+166=x-\frac{1}{3}x$$

$$166=\frac{2}{3}x$$

$$166\cdot\frac{3}{2}=\frac{3}{2}\cdot\frac{2}{3}x$$

$$83\times3=x$$

$$249=x$$

The left side of the equation is $\frac{1}{3}\cdot249-5+171=83-5+171=78+171=249$, *which is exactly equal to the right side. Therefore,* $x=249$ *is a solution to* $\frac{1}{3}x-5+171=x$.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that properties of equality, when used to transform equations, make equations with fewer terms that are simpler to solve.
- When solving an equation, we want the answer to be in the form of the symbol x equal to a constant.

Exit Ticket (5 minutes)

Lesson Summary

The properties of equality, shown below, are used to transform equations into simpler forms. If A, B, C are rational numbers, then

- If $A = B$, then $A + C = B + C$ **Addition Property of Equality**
- If $A = B$, then $A - C = B - C$ **Subtraction Property of Equality**
- If $A = B$, then $A \cdot C = B \cdot C$ **Multiplication Property of Equality**

Name ______________________________ Date ______________

Lesson 4: Solving a Linear Equation

Exit Ticket

1. Guess a number for x that would make the equation true. Check your solution.

   $5x - 2 = 8$

2. Use the properties of equality to solve the equation: $7x - 4 + x = 12$. State which property justifies your first step and why you chose it. Check your solution.

3. Use the properties of equality to solve the equation: $3x + 2 - x = 11x + 9$. State which property justifies your first step and why you chose it. Check your solution.

Exit Ticket Sample Solutions

1. **Guess a number for x that would make the equation true. Check your solution.**

   $5x - 2 = 8$

   When $x = 2$, the left side of the equation is 8, which is the same as the right side. Therefore, $x = 2$ is the solution to the equation.

2. **Use the properties of equality to solve the equation: $7x - 4 + x = 12$. State which property justifies your first step and why you chose it. Check your solution.**

   I used the commutative and distributive properties on the left side of the equal sign to simply the expression to fewer terms.

$7x - 4 + x = 12$

$8x - 4 = 12$

$8x - 4 + 4 = 12 + 4$

$8x = 16$

$\frac{8}{8}x = \frac{16}{8}$

$x = 2$

The left side of the equation is $7(2) - 4 + 2 = 14 - 4 + 2 = 12$. The right side is also 12. Since the left side equals the right side, $x = 2$ is the solution to the equation.

3. **Use the properties of equality to solve the equation: $3x + 2 - x = 11x + 9$. Check your solution.**

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$3x + 2 - x = 11x + 9$

$$2x + 2 = 11x + 9$$

$$2x - 2x + 2 = 11x - 2x + 9$$

$$2 = 9x + 9$$

$$2 - 9 = 9x + 9 - 9$$

$$-7 = 9x$$

$$\frac{-7}{9} = \frac{9}{9}x$$
$$-\frac{7}{9} = x$$

The left side of the equation is $3\left(\frac{-7}{9}\right) + 2 - \frac{-7}{9} = -\frac{21}{9} + \frac{18}{9} + \frac{7}{9} = \frac{4}{9}$. *The right side is* $11\left(-\frac{7}{9}\right) + 9 = \frac{-77}{9} + \frac{81}{9} = \frac{4}{9}$.

Since the left side equals the right side, $x = -\frac{7}{9}$ *is the solution to the equation.*

Problem Set Sample Solutions

Students solve equations using properties of equality.

For each problem, show your work and check that your solution is correct.

1. **Solve the linear equation:** $x + 4 + 3x = 72$**. State the property that justifies your first step and why you chose it.**

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

   $x + 4 + 3x = 72$

   $4x + 4 = 72$

   $4x + 4 - 4 = 72 - 4$

   $4x = 68$

   $\frac{4}{4}x = \frac{68}{4}$

   $x = 17$

   The left side is equal to $17 + 4 + 3(17) = 21 + 51 = 72$, *which is what the right side is. Therefore,* $x = 17$ *is a solution to the equation* $x + 4 + 3x = 72$.

2. **Solve the linear equation:** $x + 3 + x - 8 + x = 55$**. State the property that justifies your first step and why you chose it.**

   I used the commutative and distributive properties on the left side of the equal sign to simplify the expression to fewer terms.

$x + 3 + x - 8 + x = 55$

$3x - 5 = 55$

$$3x - 5 + 5 = 55 + 5$$

$$3x = 60$$

$$\frac{3}{3}x = \frac{60}{3}$$
$$x = 20$$

The left side is equal to $20 + 3 + 20 - 8 + 20 = 43 - 8 + 20 = 35 + 20 = 55$, *which is equal to the right side. Therefore,* $x = 20$ *is a solution to* $x + 3 + x - 8 + x = 55$.

3. Solve the linear equation: $\frac{1}{2}x + 10 = -\frac{1}{4}x + 54$. State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all of the constants on one side of the equal sign.

$$\frac{1}{2}x + 10 = \frac{1}{4}x + 54$$

$$\frac{1}{2}x + 10 - 10 = \frac{1}{4}x + 54 - 10$$

$$\frac{1}{2}x = -\frac{1}{4}x + 44$$

$$\frac{1}{2}x - \frac{1}{4}x = \frac{1}{4}x - \frac{1}{4}x + 44$$

$$\frac{1}{4}x = 44$$

$$4 \cdot \frac{1}{4}x = 4 \cdot 44$$
$$x = 176$$

The left side of the equation is $\frac{1}{2}(176) + 10 = 88 + 10 = 98$. *The right side of the equation is*

$\frac{1}{4}(176)+54=44+54=98.$ Since both sides equal 98, $x=176$ is a solution to the equation $\frac{1}{2}x+10=\frac{1}{4}x+54.$

4. Solve the linear equation: $\frac{1}{4}x+18=x.$ State the property that justifies your first step and why you chose it.

I chose to use the subtraction property of equality to get all terms with an x on one side of the equal sign.

$$\frac{1}{4}x+18=x$$

$$\frac{1}{4}x-\frac{1}{4}x+18=x-\frac{1}{4}x$$

$$18=\frac{3}{4}x$$

$$\frac{4}{3}\cdot 18=\frac{4}{3}\cdot\frac{3}{4}x$$

$$24=x$$

The left side of the equation is $\frac{1}{4}(24)+18=6+18=24,$ which is what the right side is equal to. Therefore, $x=24$ is a solution to $\frac{1}{4}x+18=x.$

5. Solve the linear equation: $17-x=\frac{1}{3}\cdot 15+6.$ State the property that justifies your first step and why you chose it.

The right side of the equation can be simplified to 11. Then the equation is

$$17-x=11$$

and $x=6$. Both sides of the equation equal 11; therefore, $x=6$ is a solution to the equation $17-x=\frac{1}{3}\cdot 15+6$. I was able to solve the equation mentally without using the properties of equality.

6. Solve the linear equation: $\frac{x+x+2}{4}=189.5$. State the property that justifies your first step and why you chose it.

$$\frac{x+x+2}{4}=189.5$$

$$x+x+2=4(189.5)$$

$$2x+2=758$$

$$2x+2-2=758-2$$

$$2x=756$$

$$\frac{2}{2}x=\frac{756}{2}$$
$$x=378$$

The left side of the equation is $\frac{378+378+2}{4}=\frac{758}{4}=189.5$ *, which is equal to the right side of the equation. Therefore,* $x=378$ *is a solution to* $\frac{x+x+2}{4}=189.5$.

7. **Alysha solved the linear equation: $2x - 3 - 9x = 14 + x - 1$. Her work is shown below. When she checked her answer, the left side of the equation did not equal the right side. Find and explain Alysha's error, and then solve the equation correctly.**

$2x - 3 - 9x = 14 + x - 1.$

$-6x - 3 = 13 + 2x$

$-6x - 3 + 3 = 13 + 3 + 2x$

$-6x = 16 + 2x$

$-6x + 2x = 16$

$-4x = 16$

$$\frac{-4}{-4}x = \frac{16}{-4}$$
$x = -4$

Alysha made a mistake on the 5^{th} line. She added $2x$ to the left side of the equal sign and subtracted $2x$ on the right side of the equal sign. To use the property correctly, she should have subtracted $2x$ on both sides of the equal sign, making the equation at that point:

$-6x - 2x = 16 + 2x - 2x$

$-8x = 16$

$$\frac{-8}{-8}x = \frac{16}{-8}$$
$x = -2$

Lesson 5: Writing and Solving Linear Equations

Student Outcomes

- Students apply knowledge of geometry to writing and solving linear equations.

Lesson Notes

All of the problems in this lesson relate to what students have learned about geometry in recent modules and previous years. The purpose is two-fold, first, we want to reinforce what students have learned about geometry, and second, we want to demonstrate a need for writing and solving an equation in a context that is familiar. Throughout the lesson students solve mathematical problems that relate directly to what students have learned about angle relationships, congruence, and the triangle sum theorem. Encourage students to draw a diagram to represent the situation presented in the word problems.

Classwork

Example 1 (5 minutes)

MP.

- Solve the following:

Example 1

One angle is five less than three times the size of another angle. Together they have a sum of 143°. What are the sizes of each angle?

Provide students with time to make sense of the problem and persevere in solving it. They could begin their work by guessing and checking, drawing a diagram, or other methods as appropriate. Then move to the algebraic method shown below.

- What do we need to do first to solve this problem?
  - *First we need to define our variable (symbol). Let x be the size of the first angle.*
- If x is the size of the first angle, how do you represent the size of the second angle?
  - *The second angle is $3x - 5$.*
- What is the equation that represents this situation?
  - *The equation is: $x + 3x - 5 = 143$.*
- The equation that represents this situation is: $x + 3x - 5 = 143$. Solve for x, then determine the

Scaffolding:

Model for students how to use diagrams to help make sense of the problems throughout this lesson. Encourage students to use diagrams to help them understand the situation.

size of each angle.

As students share their solutions for this and subsequent problems, ask them a variety of questions to reinforce the learning of the last few lessons. For example, you can ask students whether or not this is a linear equation and how they know, to justify their steps and explain why they chose their particular first step, what the solution means, or how they know their answer is correct.

Student work:

$$x + 3x - 5 = 143$$

$$(1 + 3)x - 5 = 143$$

$$4x - 5 = 143$$

$$4x - 5 + 5 = 143 + 5$$

$$4x = 148$$

$$x = 37$$

The size of the first angle is 37° ***. The second angle is*** $3(37) - 5 = 111 - 5 = 106^\circ$.

- Compare the method you tried at the beginning of the problem with the algebraic method. What advantage does writing and solving an equation have?
  - *Writing and solving an equation is a more direct method than the one I tried before. It allows me to find the answer more quickly.*
- Could we have defined x to be the size of the second angle? If so, what, if anything, would change?
  - *If we let* x *be the size of the second angle, then the equation would change, but the answers for the sizes of the angles should remain the same.*
- If x is the size of the second angle, how would we write the size of the first angle?

- *The first angle would be* $\frac{x+5}{3}$.

- The equation that represents the situation is $x+\frac{x+5}{3}=143.$ How should we solve this equation?
  - *We could add the fractions together, then solve for* x.
  - *We could multiply every term by* 3 *to change the fraction to a whole number.*
- Using either method, solve the equation. Verify that the sizes of the angles are the same as before.
  -

Scaffolding:

You may need to remind students how to add fractions by rewriting term(s) as equivalent fractions then adding the numerators. Provide support as needed.

$$x+\frac{x+5}{3}=143$$

$$\frac{3x}{3}+\frac{x+5}{3}=143$$

$$\frac{3x+x+5}{3}=143$$

$$3x+x+5=143(3)$$

$$(3+1)x+5=429$$

$$4x+5=429$$

$$4x+5-5=429-5$$

$4x = 424$

$x = 106$

- *The size of the second angle is* $106°$. *The measure of the first angle is* $\frac{x+5}{3} = \frac{106+5}{3} = \frac{111}{3} = 37°$.

  OR

  $x + \frac{x+5}{3} = 143$

  $3\left(x + \frac{x+5}{3} = 143\right)$

  $3x + x + 5 = 429$

  $4x + 5 = 429$

  $4x + 5 - 5 = 429 - 5$

  $4x = 424$

  $x = 106$

  The size of the second angle is $106°$. *The measure of the first angle is* $\frac{x+5}{3} = \frac{106+5}{3} = \frac{111}{3} = 37°$.

- Whether we let x represent the first angle or the second angle does not change our answers.

Whether we solve the equation using the first or second method does not change our answers. What matters is that we accurately write the information in the problem and correctly use the properties of equality. You may solve a problem differently than your classmates or teachers. Again, what matters most is that what you do is accurate and correct.

Example 2 (12 minutes)

- Solve the following:

Example 2

Given a right triangle, find the size of the angles if one angle is ten more than four times the other angle and the third angle is the right angle.

Give students time to work. As they work, walk around and identify students who are writing and solving the problem in different ways. The instructional goal of this example is to make clear that there are different ways to solve a linear equation as opposed to one "right way". Select students to share their work with the class. If students don't come up with different ways of solving the equation, talk them through the following student work samples.

Again, as students share their solutions, ask them a variety of questions to reinforce the learning of the last few lessons. For example, you can ask students whether or not this is a linear equation and how they know, or to justify their steps and explain why they chose their particular first step, what the solution means, or how they know their answer is correct.

<u>Solution One</u>

Let x *be the size of the first angle. Then the second angle is* $4x + 10$. *The sum of the measures for the angles for this right triangle is as follows:* $x + 4x + 10 + 90 = 180$.

$x + 4x + 10 + 90 = 180$

$(1 + 4)x + 100 = 180$

$5x + 100 = 180$

$5x + 100 - 100 = 180 - 100$

$5x = 80$
$x = 16$

The measure of the first angle is 16° *, the measure of the second angle is* $4(16) + 10 = 64 + 10 = 74^\circ$, *and the third angle is* 90° .

<u>Solution Two</u>

Let x *be the size of the first angle. Then the second angle is* $4x + 10$. *Since we have a right triangle we already know that one angle is* 90° *which means that the sum of the other two angles is* 90 *:* $x + 4x + 10 = 90$.

$x + 4x + 10 = 90$

$(1 + 4)x + 10 = 90$

$5x + 10 = 90$

$5x + 10 - 10 = 90 - 10$

$5x = 80$
$x = 16$

The measure of the first angle is 16° *, the measure of the second angle is* $4(16) + 10 = 64 + 10 = 74^\circ$*, and the third angle is* 90° *.*

__Solution Three__

Let x *be the size of the second angle. Then the first angle is* $\frac{x-10}{4}$*. Since we have a right triangle we already know that one angle is* 90° *which means that the sum of the other two angles is* 90 *:* $x + \frac{x-10}{4} = 90.$

$$x + \frac{x - 10}{4} = 90$$

$$4\left(x + \frac{x - 10}{4} = 90\right)$$

$\mathbf{4x + x - 10 = 360}$

$\mathbf{(4 + 1)x - 10 = 360}$

$\mathbf{5x - 10 = 360}$

$\mathbf{5x - 10 + 10 = 360 + 10}$

$\mathbf{5x = 370}$

$\mathbf{x = 74}$

The measure of the second angle is 74° ***, the measure of the first angle is*** $\frac{74-10}{4} = \frac{64}{4} = 16^\circ$***, and the third angle is*** 90° ***.***

Solution Four

Let x *be the size of the second angle. Then the first angle is* $\frac{x-10}{4}$. *The sum of the three angles is as follows:* $x+\frac{x-10}{4}+90=180.$

$$x+\frac{x-10}{4}+90=180$$

$$x+\frac{x-10}{4}+90-90=180-90$$

$$x+\frac{x-10}{4}=90$$

$$\frac{4x}{4}+\frac{x-10}{4}=90$$

$$\frac{4x+x-10}{4}=90$$

$$4x+x-10=360$$

$$5x-10+10=360+10$$

$5x = 370$
$x = 74$

The second angle is $74°$ *, the first angle is* $\frac{74-10}{4} = \frac{64}{4} = 16°$*, and the third angle is* $90°$.

Make sure students see at least four different methods of solving the problem. Conclude this example with the statements below.

- Each method was slightly different either in terms of how the variable was defined or how the properties of equality were used to solve the equation. The way you find the answer may be different than your classmates or your teacher.
- As long as you write accurately and do what is mathematically correct, you will find the correct answer.

Example 3 (4 minutes)

- A pair of alternate interior angles are described as follows. One angle is fourteen more than half a number. The other angle is six less than half a number. Are the angles congruent?
- We will begin by assuming that the angles are congruent. If the angles are congruent, what does that mean about their measures?
    - *It means that they are equal in measure.*
- Write an expression that describes each angle.
    - *One angle is* $\frac{x}{2} + 14$ *and the other angle is* $\frac{x}{2} - 6$.
- If the angles are congruent, we can write the equation is $\frac{x}{2} + 14 = \frac{x}{2} - 6$. We know that our properties of equality allow us to transform equations while making sure that they remain true.

$$\frac{x}{2} + 14 = \frac{x}{2} - 6$$

$$\frac{x}{2} - \frac{x}{2} + 14 = \frac{x}{2} - \frac{x}{2} - 6,$$

$14 \neq -6.$

Therefore, our assumption was not correct and the angles are not congruent.

Exercises 1–6 (16 minutes)

Students complete Exercises 1–6 independently or in pairs.

Exercises 1–6

For each of the following problems, write an equation and solve.

1. **A pair of congruent angles are described as follows: the measure of one angle is three more than twice a number and the other angle's measure is 54.5 less than three times the number. Determine the size of the angles.**

   Let x be the number. Then the measure of one angle is $3 + 2x$ and the measure of the other angle is $3x - 54.5$. Because the angles are congruent, their measures are equal. Therefore,

   $$3 + 2x = 3x - 54.5$$

   $$3 + 2x - 2x = 3x - 2x - 54.5$$

   $$3 = x - 54.5$$

   $$3 + 54.5 = x - 54.5 + 54.5$$
   $$57.5 = x$$

   Then each angle is $3 + 2(57.5) = 3 + 115 = 118^{\circ}$.

2. **The measure of one angle is described as twelve more than four times a number. Its supplement is twice as large. Find the measure of each angle.**

   Let x be the number. Then the measure of one angle is $4x + 12$. The other angle is $2(4x + 12) = 8x + 24.$ *Since the angles are supplementary, their sum must be 180:*

   $$4x + 12 + 8x + 24 = 180$$

$12x + 36 = 180$

$12x + 36 - 36 = 180 - 36$

$12x = 144$
$x = 12$

One angle is $4(12) + 12 = 48 + 12 = 60°$ *and the other angle is* $180 - 60 = 120°$.

3. A triangle has angles described as follows: the first angle is four more than seven times a number, another angle is four less than the first and the third angle is twice as large as the first. What are the sizes of each of the angles?

Let x *be the number. The measure of the first angle is* $7x + 4$*. The measure of the second angle is* $7x + 4 - 4 = 7x$*. The measure of the third angle is* $2(7x + 4) = 14x + 8$*. The sum of the angles of a triangle must be* $180°$.

$7x + 4 + 7x + 14x + 8 = 180$

$28x + 12 = 180$

$28x + 12 - 12 = 180 - 12$

$28x = 168$
$x = 6$

The measure of the first angle is $7(6) + 4 = 42 + 4 = 46°$ *. The measure of the second angle is* $7(6) = 42°$ *. The measure of the third angle is* $2(46) = 92°$.

4. One angle measures nine more than six times a number. A sequence of rigid motions maps the angle onto another angle that is described as being thirty less than nine times the number. What is the measure of the angles?

Let x *be the number. Then the measure of one angle is* $6x + 9$ *. The measure of the other angle is* $9x - 30$ *. Since rigid motions preserve the measures of angles, then the measure of these angles is equal.*

$$6x + 9 = 9x - 30$$

$$6x + 9 - 9 = 9x - 30 - 9$$

$$6x = 9x - 39$$

$$6x - 9x = 9x - 9x - 39$$

$$-3x = -39$$
$$x = 13$$

The angles measure $6(13) + 9 = 78 + 9 = 87^\circ$.

5. A right triangle is described as having an angle of size "six less than negative two times a number," another angle that is "three less than negative one-fourth the number", and a right angle. What are the measures of the angles?

Let x *be a number. Then one angle is* $-2x - 6$ *. The other angle is* $-\frac{x}{4} - 3$. *The sum of the two angles must be* 90° .

$$-2x - 6 + \left(-\frac{x}{4}\right) - 3 = 90$$

$$\left(-\frac{8x}{4}\right) + \left(-\frac{x}{4}\right) - 9 = 90$$

$$\left(-\frac{9x}{4}\right) - 9 + 9 = 90 + 9$$

$$-\frac{9x}{4} = 99$$

$$-9x = 396$$
$$x = -44$$

One of the angles is $-2(-44)-6=88-6=82^{\circ}$*. The other angle is* $90-82=8^{\circ}$.

6. **One angle is one less than six times the size of another. The two angles are complementary angles. Find the size of each angle.**

   Let x *be the measure of the first angle. Then the measure of the second angle is* $6x-1$*. The sum of the measures will be* 90 *because the angles are complementary.*

   $$x+6x-1=90$$

   $$7x-1=90$$

   $$7x-1+1=90+1$$

   $$7x=91$$
   $$x=13$$

   One angle is 13° *and the other angle is* $6(13)-1=78-1=77^{\circ}$.

Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that an algebraic method for solving equations is more efficient than guessing and checking.
- We know how to write and solve equations that relate to angles, triangles, and geometry in general.
- We know that drawing a diagram can sometimes make it easier to understand a problem and that there is more than one way to solve an equation.

Exit Ticket (4 minutes)

Name ______________________________ Date ______________

Lesson 5: Writing and Solving Linear Equations

Exit Ticket

For each of the following problems, write an equation and solve.

1. Given a right triangle, find the measures of all the angles if one angle is a right angle and the measure of the second angle is six less than seven times the measure of the third angle.

2. In a triangle, the measure of the first angle is six times a number. The measure of the second angle is nine less than the first angle. The measure of the third angle is three times the number more than the measure of the first angle. Determine the measures of each of the angles.

Exit Ticket Sample Solutions

1. **Given a right triangle, find the measures of all of the angles if one angle is a right angle and the measure of a second angle is six less than seven times the measure of the third angle.**

   Let x *represent the measure of the third angle. Then* $7x - 6$ *can represent the measure of the second angle. The sum of the two angles in the right triangles will be* $90°$ *.*

   $$7x - 6 + x = 90$$

   $$8x - 6 = 90$$

   $$8x - 6 + 6 = 90 + 6$$

   $$8x = 96$$

   $$\frac{8}{8}x = \frac{96}{8}$$
   $$x = 12$$

   The measure of the third angle is $12°$ *and the measure of the second angle is* $7(12) - 6 = 84 - 6 = 78°$. . *The measure of the third angle is* $90°$ *.*

2. **In a triangle, the measure of the first angle is six times a number. The measure of the second angle is nine less than the first angle. The measure of the third angle is three times the number more than the measure of the first angle. Determine the measures of each of the angles.**

   Let x *be the number. Then the measure of the first angles is* $6x$ *, the measure of the second angle is* $6x - 9$ *, and the measure of the third angle is* $3x + 6x$ *. The sum of the measures of the angles in a triangle is* $180°$ *.*

   $$6x + 6x - 9 + 3x + 6x = 180$$

   $$21x - 9 = 180$$

$21x - 9 + 9 = 180 + 9$

$21x = 189$

$\frac{21}{21}x = \frac{189}{21}$
$x = 9$

The measure of the first angle is $6(9) = 54^\circ$. *The measure of the second angle is* $54 - 9 = 45^\circ$. *The measure of the third angle is* $54 + 3(9) = 54 + 27 = 81^\circ$.

Note to teacher: There are several ways to solve problems like these. For example, a student may let x be the size of the first angle, and write the size of the other angles accordingly. Either way, make sure that students are defining their symbols and correctly using the properties of equality to solve.

Problem Set Sample Solutions

Students practice writing and solving linear equations.

For each of the following problems, write an equation and solve.

1. **The measure of one angle is thirteen less than five times the measure of another angle. The sum of the measures of the two angles is 140°. Determine the measures of each of the angles.**

Let x *be the size of the one angle. Then the measure of the other angle is* $5x - 13$.

$x + 5x - 13 = 140$

$6x - 13 = 140$

$6x - 13 + 13 = 140 + 13$

$6x = 153$
$x = 25.5$

The measure of one angle is 25.5° *and the measure of the other angle is* $140 - 25.5 = 114.5^\circ$.

2. **An angle measures seventeen more than three times a number. Its supplement is three more than seven times the number. What is the measure of each angle?**

Let x *be the number. Then one angle is* $3x + 17$. *The other angle is* $7x + 3$. *Since the angles are supplementary, the sum of their measures will be* 180.

$$3x + 17 + 7x + 3 = 180$$

$$10x + 20 = 180$$

$$10x + 20 - 20 = 180 - 20$$

$$10x = 160$$
$$x = 16$$

One angle is $3(16) + 17 = 65^\circ$. *The other angle is* $180 - 65 = 115^\circ$.

3. **The angles of a triangle are described as follows:** $\angle A$ **is the largest angle, its measure is twice the measure of** $\angle B$. **The measure of** $\angle C$ **is** 2 **less than half the measure of** $\angle B$. **Find the measures of the three angles.**

Let x *be the measure of* $\angle B$. *Then the measure of* $\angle A = 2x$ *and* $\angle C = \frac{x}{2} - 2$. *The sum of the measures of the angles must be* 180.

$$x + 2x + \frac{x}{2} - 2 = 180$$

$$3x + \frac{x}{2} - 2 + 2 = 180 + 2$$

$$3x + \frac{x}{2} = 182$$

$$\frac{6x}{2} + \frac{x}{2} = 182$$

$$\frac{7x}{2} = 182$$

$$7x = 364$$
$$x = 52$$

The measures of the angles are as follows:

$$\angle A = 104^\circ, \angle B = 52^\circ, and\ \angle C = \frac{52}{2} - 2 = 26 - 2 = 24^\circ.$$

4. A pair of corresponding angles are described as follows: the measure of one angle is five less than seven times a number and the measure of the other angle is eight more than seven times the number. Are the angles congruent? Why or why not?

Let x *be the number. Then measure of one angle is* $7x - 5$ *and the measure of the other angle is* $7x + 8$ *. Assume they are congruent, which means their measures are equal.*

$$7x - 5 = 7x + 8$$

$$7x - 7x - 5 = 7x - 7x + 8$$
$$-5 \neq 8$$

Since $|5 \neq 8,$ *the angles are not congruent.*

5. The measure of one angle is eleven more than four times a number. Another angle is twice the first angle's measure. The sum of the measures of the angles is 195° . What is the measure of each angle?

Let x *be the number. The measure of one angle can be represented with* $4x + 11$ *and the other angle's measure can be represented as* $2(4x + 11) = 8x + 22.$

$$4x + 11 + 8x + 22 = 195$$

$$12x + 33 = 195$$

$$12x + 33 - 33 = 195 - 33$$

$12x = 162$
$x = 13.5$

The measure of one angle is $4(13.5) + 11 = 54 + 11 = 65^\circ$ *and the measure of the other angle is* 130°.

6. Three angles are described as follows: $\angle B$ is half the size of $\angle A$. The measure of $\angle C$ is equal to one less than 2 times the measure of $\angle B$. The sum of $\angle A$ and $\angle B$ is 114. Can the three angles form a triangle? Why or why not?

Let x *represent the measure of* $\angle A$. *Then the measure of* $\angle B = \frac{x}{2}$ *and the measure of* $\angle C = 2\left(\frac{x}{2}\right) - 1 = x - 1$.

The sum of $\angle A + \angle B = 114$.

$x + \frac{x}{2} = 114$

$\frac{3x}{2} = 114$

$3x = 228$
$x = 76$

The measure of $\angle A = 76^\circ$, $\angle B = \frac{76}{2} = 38^\circ$, *and* $\angle C = 75^\circ$. *The sum of the three angles is* $76 + 38 + 75 = 189$. *Since the sum of the measures of the interior angles of a triangle must sum to* 180°, *these angles do not form a triangle. Their sum is too large.*

UNIT ONE

for

Content Area of

MATHEMATICS

HS Band
Algebra I

New York State Common Core

Mathematics Curriculum

Table of Contents[1]

Relationships Between Quantities and Reasoning with Equations and Their Graphs

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[1] Each lesson is ONE day, and ONE day is considered a 45 minute period.

Algebra I • Module 1

Relationships Between Quantities and Reasoning with Equations and Their Graphs

OVERVIEW

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to non-linear equations and their graphs. They formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Grade 9: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise (**A-CED.2**). As they graph, they reason abstractly and quantitatively as they choose and interpret units to solve problems related to the graphs they create (**N-Q.1, N-Q.2, N-Q.3**).

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions (**6.EE.3, 6.EE.4, 7.EE.1, 8.EE.1**). Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves (**A-SSE.2**). They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions (**A-APR.1**). The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations (**6.EE.5, 6.EE.7, 7.EE.4, 8.EE.7**) and systems of linear equations (**8.EE.8**). Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations (**A-REI.1, A-REI.3, A-CED.4**). Students take their experience solving systems of linear equations further as they prove the validity of the addition method, learn a formal definition for the graph of an equation and use it to explain the reasoning of solving systems graphically, and graphically represent the solution to systems of linear inequalities (**A-CED.3, A-REI.5, A-REI.6, A-REI.10, A-REI.12**).

In Topic D, students are formally introduced to the modeling cycle (see page 61 of the CCLS) through problems that can be solved by creating equations and inequalities in one variable, systems of equations, and graphing (**N-Q.1, A-SSE.1, A-CED.1, A-CED.2, A-REI.3**). The End-of-Module Assessment follows Topic D.

Focus Standards

Reason quantitatively and use units to solve problems.

N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.★

N-Q.2[2] Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Interpret the structure of expressions

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.★

- a. Interpret parts of an expression, such as terms, factors, and coefficients.
- b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

Perform arithmetic operations on polynomials

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Create equations that describe numbers or relationships

A-CED.1[3] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

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[2] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.

[3] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.★

Understand solving equations as a process of reasoning and explain the reasoning

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6[4] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Foundational Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.*

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[4] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.*

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or depending on the purpose at hand, any number in a specified set.

6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

6.EE.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Work with radicals and integer exponents.

8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.

MP.2 **Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students reason about solving equations using "if-then" moves based on equivalent expressions and properties of equality and inequality. They analyze when an "if-then" move is not reversible.

MP.4 **Model with mathematics.** Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system.

MP.6 **Attend to precision.** Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation.

MP.7 **Look for and make use of structure.** Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$, etc.

MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: $ax + b = cx + d$. They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.

Terminology

New or Recently Introduced Terms

- **Piecewise-Linear Function** (Given a finite number of non-overlapping intervals on the real number line, a *(real) piecewise-linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)
- **Numerical Symbol** (A *numerical symbol* is a symbol that represents a specific number.)
- **Variable Symbol** (A variable symbol is a symbol that is a placeholder for a number. It is possible that a question may restrict the type of number that a placeholder might permit, maybe integers only or a positive real number, for instance.)
- **Numerical Expression** (A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols) and that evaluates to a single number.)
- **Algebraic Expression** (An *algebraic expression* is either (1) a numerical symbol or a variable symbol or (2) the result of placing previously generated algebraic expressions into the two blanks of one of the four operators ((__)+(__), (__)–(__), (__)×(__), (__)÷(__)) or into the base blank of an exponentiation with an exponent that is a rational number.)
- **Equivalent Numerical Expressions** (Two numerical expressions are *equivalent* if they evaluate to the same number.)
- **Equivalent Algebraic Expressions** (Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression.)
- **Polynomial Expression** (A *polynomial expression* is either (1) a numerical expression or a variable symbol or (2) the result of placing two previously generated polynomial expressions into the blanks of the addition operator (__+__) or the multiplication operator (__×__).)
- **Monomial** (A *monomial* is a polynomial expression generated using only the multiplication operator (__×__). Monomials are products whose factors are numerical expressions or variable symbols.)
- **Degree of a Monomial** (The *degree* of a non-zero monomial is the sum of the exponents of the variable symbols that appear in the monomial.)
- **Standard Form of a Polynomial Expression in One Variable** (A polynomial expression with one variable symbol x is in *standard form* if it is expressed as $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, where n is a non-negative integer, and $a_0, a_1, a_2, \ldots, a_n$ are constant coefficients with $a_n \neq 0$. A polynomial expression in x that is in standard form is often called a *polynomial in* x.)
- **Degree of a Polynomial in Standard Form** (The *degree of a polynomial in standard form* is the highest degree of the terms in the polynomial, namely n.)
- **Leading Term and Leading Coefficient of a Polynomial in Standard Form** (The term $a_n x^n$ is called the *leading term*, and a_n is called the *leading coefficient.*)
- **Constant Term of a Polynomial in Standard Form** (The *constant term* is the value of the numerical expression found by substituting 0 into all the variable symbols of the polynomial, namely a_0.)

- **Solution** (A *solution* to an equation with one variable is a number in the domain of the variable that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence.)
- **Solution Set** (The set of solutions of an equation is called its *solution set.*)
- **Graph of an Equation in Two Variables** (The set of all points in the coordinate plane that are solutions to an equation in two variables is called the *graph of the equation.*)
- **Zero Product Property** (The Zero Product Property states that given real numbers, a and b, if $a \cdot b = 0$ then either $a = 0$ or $b = 0$, or both a and $b = 0$.)

Familiar Terms and Symbols[5]

- Equation
- Identity
- Inequality
- System of Equations
- Properties of Equality
- Properties of Inequality
- Solve
- Linear Function
- Formula
- Term

Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.1, N-Q.2, N-Q.3, A-APR.1, A-SSE.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	N-Q.1, A-SSE.1, A-SSE.2, A-APR.1, A-CED.1, A-CED.2, A-CED.3, A-CED.4, A-REI.1, A-REI.3,

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[5] These are terms and symbols students have seen previously.

			A-REI.5, A-REI.6, A-REI.10, A-REI.12

Name ______________________________ Date ________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$

   $mB + nK$

   $$\frac{mB + nK}{m + n}$$

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$(x + 3)(3x + 8) - 3x(x + 3)$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1+((2+3) \cdot 4)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

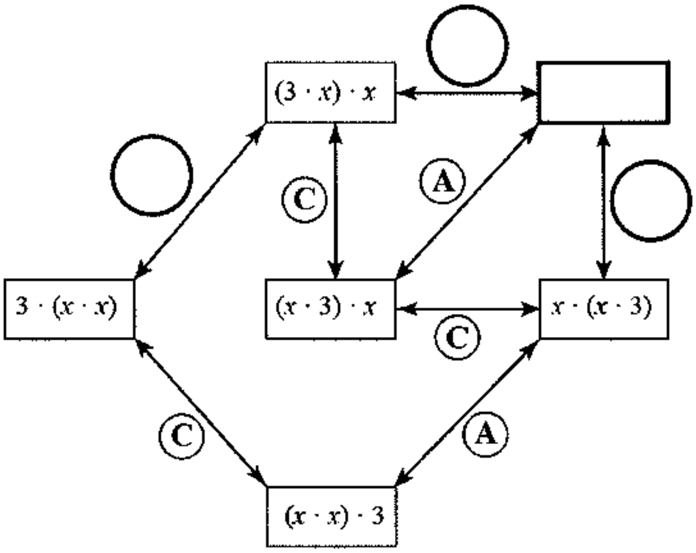
Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

  (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for a, b, c, and d into each expression, the expressions evaluate to **different numbers,** and

  (2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for Associative Property and **C** for Commutative Property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.

b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813×10 , he says, is 28,130 . He doesn't understand why this "rule" is true.

a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x ?

b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

8.

a. Find the following products:

i. $(x-1)(x+1)$

ii. $(x-1)\left(x^2 + x + 1\right)$

iii. $(x-1)(x^3 + x^2 + x + 1)$

iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

b. Substitute $x = 10$ into each of the products and your answers to show how each of the products appears as a statement in arithmetic.

c. If we substituted $x = 10$ into the product $(x-2)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and computed the product, what number would result?

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$, and express your answer in standard form.

Substitute $x = 10$ into your answer, and see if you obtain the same result that you obtained in part (c).

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x = 10$, multiplying by "$x-9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$

ii. Put $x = 10$ into your answer.

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x = 10$?

iii. Was Francois right?

A Progression Toward Mastery

Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**a** **N-Q.1** **N-Q.2**	Student was unable to respond to question, <u>OR</u> student provided a minimal attempt to create an incorrect graph.	Graph reflects something related to the problem, but the axes do not depict the correct units of distance from the house on the y-axis and a measurement of time on the *x*-axis, <u>OR</u> the graph indicates significant errors in calculations or reasoning.	Student created axes that depict distance from the house on the y -axis and some measurement of time on the x -axis, and the graph represents a reflection of what occurred but with errors in calculations, missing or erroneous axis labels, <u>or</u> choice of units that makes the graph difficult to obtain information from.	Student created and labeled the y -axis to represent distance from the house in miles and an x -axis to represent time (in minutes past 1:00 p.m.) <u>AND</u> created a graph based on solid reasoning and correct calculations.
	b **N-Q.1**	Student answered incorrectly with no evidence of reasoning to support the answer, <u>OR</u> student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 1:21 p.m. but did not either refer to a correct graph or provide sound reasoning to support the answer. <u>OR</u> Student answered incorrectly because	Student answered 1:21 p.m. <u>AND</u> either referred to a correct graph from part (a) or provided reasoning and calculations to explain the answer.

				either the graph in part (a) was incorrect and the graph was referenced or because a minor calculation error was made but sound reasoning was used.	
	c N-Q.1	Student answered incorrectly with no evidence of reasoning to support the answer, OR student left item blank.	Student answered incorrectly but demonstrated some reasoning in explaining the answer.	Student answered 6 miles but did not either refer to the work in part (a) or provide sound reasoning in support of the answer. OR Student answered incorrectly because either the work in part (a) was referenced, but the work was incorrect or because a minor calculation error was made but sound reasoning was used.	Student answered 6 miles and either referenced correct work from part (a) or provided reasoning and calculations to support the answer.
2	a N-Q.3	Student left the question blank, OR student provided an answer that reflected no or very little reasoning.	Student either began with an assumption that was not based on the evidence of water being used at a rate of approximately 10 liters/second at noon, OR student used poor reasoning in extending that reading to consider total use across 24 hours.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon but made an error in the calculations to extend and combine that rate to consider usage across 24 hours. OR Student did not defend the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.	Student answered beginning with the idea that water was being used at a rate of approximately 10 liters/second at noon AND made correct calculations to extend and combine that rate to consider usage across 24 hours. AND Student defended the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.

	b **N-Q.3**	Student left the question blank, OR student provided an answer that reflected no or very little reasoning.	Student answer is outside of the range from "to the nearest ten" to "to the nearest hundred", OR student answer is within that range but is not supported by an explanation.	Student answer ranges from "to the nearest ten" to "to the nearest hundred" but is not well supported by sound reasoning, OR student answer contains an error in the way the explanation was written, even if it was clear what the student meant to say.	Student answer ranges from "to the nearest ten" to "to the nearest hundred" AND is supported by correct reasoning that is expressed accurately.
	c **N-Q.3**	Student left the question blank, OR student provided an answer that reflected no or very little reasoning.	Student answer is not in the range of 6 to 48 checks but provides some reasoning to justify the choice, OR student answer is in that range, perhaps written in the form of 'every *x* minutes' or 'every *x* hours' but is not supported by an explanation with solid reasoning.	Student answer is in the range of 6 to 48 checks but is only given in the form of *x* checks per minute or *x* checks per hour; the answer is well supported by a written explanation, OR student answer is given in terms of number of checks but is not well supported by a written explanation.	Student answer is in the range of 6 to 48 checks, AND student provided solid reasoning to support the answer.
3	**a** **A-SSE.1a** **A-SSE.1b**	Student either did not answer, OR student answered incorrectly for all three expressions.	Student answered one or two of the three correctly but left the other one blank or made a gross error in describing what it represented.	Student answered two of the three correctly AND made a reasonable attempt at describing what the other one represented.	Student answered all three correctly.
	b **A-SSE.1a** **A-SSE.1b**	Student either did not answer, OR student answered incorrectly for all three parts of the question.	Student understood that the expressions represented a portion of the orders for each color but mis-assigned the colors and/or incorrectly determined which one would be larger.	Student understood that the expressions represented a portion of the orders for each color AND correctly determined which one would be larger but had errors in the way the answer was worded OR did not provide support for	Student understood that the expressions represented a portion of the orders for each color, correctly determined which one would be larger, AND provided a well written explanation for why.

				why $\frac{n}{m+n}$ would be larger.	
4	A-SSE.1b A-SSE.2	Student left the question blank, <u>OR</u> student was not able to re-write the expression successfully, even by multiplying out the factors first.	Student got to the correct re-written expression of 8(x + 3) but did so by multiplying out the factors first <u>OR</u> did not show the work needed to demonstrate how 8(x + 3) was determined.	Student attempted to use structure to re-write the expression as described, showing the process, but student made errors in the process.	Student correctly used the process described to arrive at 8(x + 3) without multiplying out linear factors <u>AND</u> demonstrated the steps for doing so.
5	a – b A-SSE.2	Student was unable to respond to many of the questions, <u>OR</u> student left several items blank.	Student was only able to come up with one option for part (a) and, therefore, had only partial work for part (b), <u>OR</u> student answered "Yes" for the question about equivalent expressions.	Student successfully answered part (a) <u>AND</u> identified that the expressions created in part (b) were not equivalent, but there were minor errors in the answering of the remaining questions.	Student answered all four parts correctly <u>AND</u> completely.
6	a A-SSE.2	Student left at least three items blank, <u>OR</u> student answered at least three items incorrectly.	Student answered one or two items incorrectly or left one or more items blank.	Student completed circling task correctly, <u>AND</u> provided a correct ordering of symbols in the box, but the answer did not use parentheses or multiplication dots.	Student completed all four item correctly, including exact placement of parentheses <u>AND</u> symbols for the box: $x \cdot (3 \cdot x)$.
	b A-SSE.2	Student did not complete either proof successfully.	Student attempted both proofs but made minor errors in both, <u>OR</u> student only completed one proof, with or without errors.	Student attempted both proofs but made an error in one of them.	Student completed both proofs correctly, <u>AND</u> the two proofs were different from one another.
7	a A-APR.1	Student's work is blank or demonstrates no understanding of multiplication of polynomials.	Student made more than one error in his multiplication but demonstrates some understanding of multiplication of	Student made a minor error in the multiplication.	Student multiplied correctly and expressed the resulting polynomial as a sum of monomials.

			polynomials.		
	b **A-APR.1**	Student's explanation is missing or did not demonstrate a level of thinking that was higher than what was given in the problem's description of Ahmed's thinking.	Student used language that did not indicate an understanding of base x and/or the place value system. Student may have used language such as shifting or moving.	Student made only minor errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.	Student made no errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.
8	**a–c** **A-APR.1**	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student completed all products correctly, expressing each as a sum of monomials with like terms collected, and evaluated correctly when x is 10.
	d **A-APR.1**	Student showed limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student made multiple errors but showed some understanding of polynomial multiplication. Student may not have combined like terms to present the product as the sum of monomials. Student may have gotten an incorrect result when evaluating with $x = 10$.	Student made one or two minor errors but demonstrated knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also showed understanding of evaluating a polynomial for the given value of x.	Student correctly multiplied the polynomials and expressed the product as a polynomial in standard form. Student correctly evaluated with a value of 10 and answered "Yes".

	e **A-APR.1**	Student was not able to demonstrate an understanding that part iii is "No" and/or demonstrated limited or no understanding of polynomial multiplication.	Student may have some errors as he multiplied the polynomials and expressed the product as a sum of monomials. Student may have some errors in the calculation of the value of the polynomial when x is 10. Student incorrectly answered part iii or applied incorrect reasoning.	Student may have made minor errors in multiplying the polynomials and expressing the product as a sum of monomials. Student may have made minor errors in calculating the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.	Student correctly multiplied the polynomials and expressed the product as a sum of monomials with like terms collected. Student correctly calculated the value of the polynomial when x is 10. Student explained that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explained that the two expressions are not algebraically equivalent.

Name ______________________________________ Date ____________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late, and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

Walk Post Office Run Grocery Store End

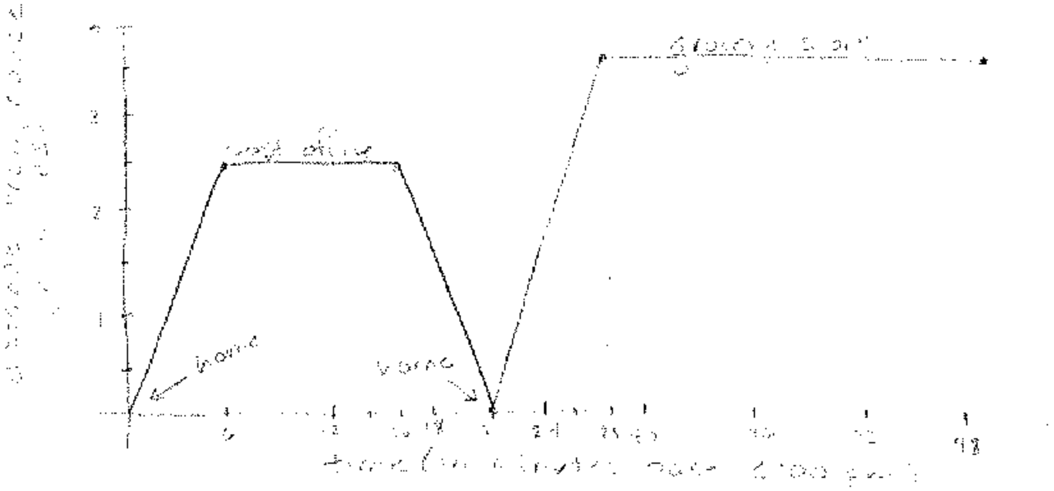
$\frac{25 \text{ miles}}{\text{hour}} \times 6 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 2.5$ miles from house to post office.

$\frac{30 \text{ miles}}{\text{hour}} \times 12 \text{ min} \times \frac{1 \text{ hr}}{60 \text{ min}} = 6$ miles from post office to store

6 − 2.5 = 3.5 miles from home to store

6 miles in 12 min is 1 mile in 2 min

So 2.5 miles takes 5 min and 3.5 mi takes 7 min

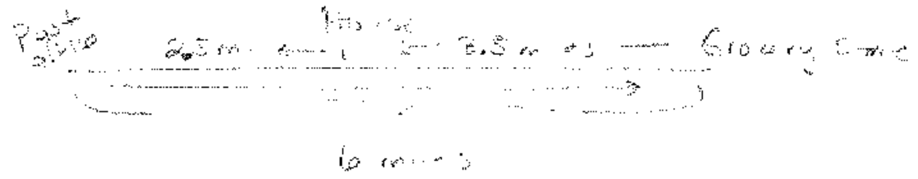


b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

It is 1:21. The graph shows the time as 21 minutes past 1:00pm. He spent 6 minutes getting to the post office 10 minutes at the post office and 5 min getting from the post office to the point of passing by his house. You know it took 5 minutes for the last part because he traveled 30 miles per hour & went 2.5 miles. $2.5 \text{ mi} \cdot \frac{60 \text{ min}}{30 \text{ miles}} = 5 \text{ min}$

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

12 miles.



2.5 mi + 6 mi + [illegible]

You know that it is 2.5 miles from the house to the post office because $25 \frac{\text{mi}}{\text{hr}} \times 6 \text{min} \times \frac{1 \text{hr}}{60 \text{min}} = 2.5 \text{ miles}$.

You know it is 6 miles from the post office to the store because $20 \frac{\text{mi}}{\text{hr}} \times 12 \text{min} \times \frac{1 \text{hr}}{60 \text{min}} = 6 \text{ miles}$.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and

that the digit in the **tens** place changes every second or so.

a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate.

1
36
× 18
288
360
648

$10 \frac{\text{liters}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times 60 \frac{\text{min}}{\text{hr}} \times 18 \text{ hr} = 648{,}000$ liters.

Since water is probably only used from about 5:00 am to 11:00 pm, I did not multiply by 24 hours, but by 18 hours instead.

b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

It can be reported within ± 10 liters, since he can read the 10's place, but it is changing by a 10 during the second he reads it

c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

24 checks.

Every hour would be good to show the peaks in usage during morning and evening hours from those in the apartments. And it might also show that businesses stop using it after business hours. It would depend on what portion of the building is business vs. apartments.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders

of blue ink and n orders of black ink.

a. What quantities could the following expressions represent in terms of the problem context?

$m + n$ total number of ink orders over a one month period.

$mB + nK$ total gallons of ink ordered over a one month period.

$\frac{mB + nK}{m + n}$ average number of gallons of ink per order.

b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

$$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

and explain which expression must be greater using those interpretations.

$\frac{m}{m+n}$ is the fraction of orders that are for blue ink.

$\frac{n}{m+n}$ is the fraction of orders that are for black ink.

$\frac{n}{m+n}$ would be bigger, 2 times as big as $\frac{m}{m+n}$

because they ordered twice as many orders for black ink than blue ink.

4. Sam says that he knows a clever set of steps to rewrite the expression

$(x + 3)(3x + 8) - 3x(x + 3)$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

$((3x+8) - 3x)(x+3)$

$8(x+3)$

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, 1+((2+3) · 4) is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

$(1+2)\cdot(3+4) = 21$

$((2+4)\cdot 1)+3 = 21$

b. In both of your expressions, replace 1 with *a*, 2 with *b*, 3 with *c*, and 4 with *d* to get two algebraic expressions. For example, $a + ((b + c) \cdot d)$ shows the replacements for the example given.

[illegible]

[illegible]

Are your algebraic expressions equivalent? Circle: Yes [✓ No]

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

(1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for *a, b, c,* and *d* into each expression, the expressions evaluate to **different numbers**, and

[illegible]

[illegible]

[illegible]

(2) Find four different, non-zero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

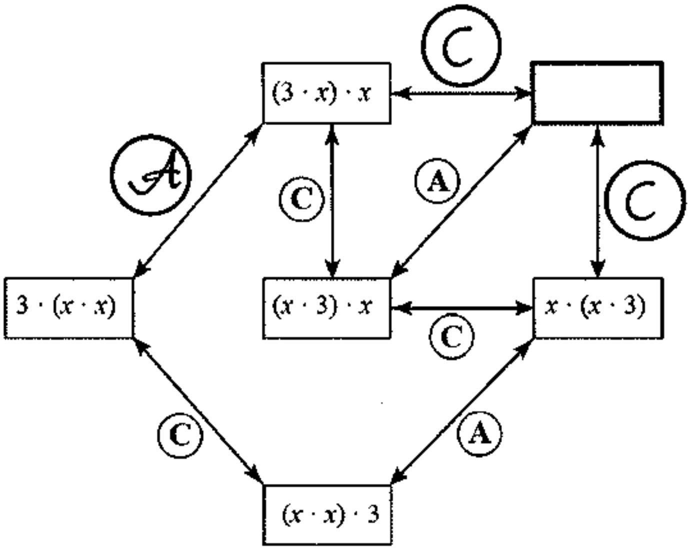
[illegible]

[illegible]

[illegible]

[illegible]

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for Associative Property and **C** for Commutative Property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



   a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.
   b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

① [illegible]

[illegible]

[illegible]

② [illegible]

[illegible]

7. Ahmed learned: "To multiply a whole number by ten, just place a zero at the end of the number." For example, 2813 × 10 , he says, is 28,130 . He doesn't understand why this "rule" is true.

   a. What is the product of the polynomial, $2x^3 + 8x^2 + x + 3$, times the polynomial, x ?

   $2x^4 + 8x^3 + x^2 + 3x$

   b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

   When you multiply by the same number as the base, it creates a new number where each digit in the original number is now one place-value higher so that there is nothing left (no numbers) to represent the ones' digit, which leads to a trailing "0" in the ones' digit.

8.

a. Find the following products:

i. $(x-1)(x+1)$

$x^2 + x - x - 1$

$x^2 - 1$

ii. $(x-1)(x^2 + x + 1)$

$x^3 + x^2 + x - x^2 - x - 1$

$x^3 - 1$

iii. $(x-1)(x^3 + x^2 + x + 1)$

$x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$

$x^4 - 1$

iv. $(x-1)(x^4 + x^3 + x^2 + x + 1)$

$x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$

$x^5 - 1$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

$x^{n+1} - 1$

b. Substitute $x = 10$ into each of the products and your answers to show how each of the products appears as a statement in arithmetic.

i. $(10 - 1)(10 + 1) = (100 - 1)$

$9(11) = 99$

ii. $(10 - 1)(100 + 10 + 1) = (1000 - 1)$

$9(111) = 999$

iii. $(10 - 1)(1000 + 100 + 10 + 1) = (10{,}000 - 1)$

$9(1111) = 9999$

iv. $(10 - 1)(10{,}000 + 1000 + 100 + 10 + 1) = (100{,}000 - 1)$

$9(11{,}111) = 99{,}999$

c. If we substituted $x = 10$ into the product $(x-2)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and computed the product, what number would result?

$8(11{,}111{,}111) = 88{,}888{,}888$

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and express your answer in standard form.

$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 2x^7 - 2x^6 - 2x^5 - 2x^4 - 2x^3 - 2x^2 - 2x - 2$

$x^8 - x^7 - x^6 - x^5 - x^4 - x^3 - x^2 - x - 2$

Substitute $x = 10$ into your answer and see if you obtain the same result as you obtained in part (c).

$10^8 - 10^7 - 10^6 - 10^5 - 10^4 - 10^3 - 10^2 - 10 - 2 = 88{,}888{,}888$. *Yes, I get the same answer.*

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x = 10$, multiplying by "$x-9$" is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$

$x^8 - 8x^7 - 8x^6 - 8x^5 - 8x^4 - 8x^3 - 8x^2 - 8x - 9$

ii. Put $x = 10$ into your answer.

$100{,}000{,}000 - 80{,}000{,}000 - 8{,}000{,}000 - 800{,}000 - 80{,}000 - 8{,}000 - 800 - 80 - 9$

$100{,}000{,}000 - 88{,}888{,}889 = 11{,}111{,}111$

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x = 10$?

Yes.

iii. Was Francois right?

No, just because it is true when x is 10, doesn't make it true for all real x. The two expressions are not algebraically equivalent.

Name ______________________________ Date ________________

1. Solve the following equations for x. Write your answer in set notation.

   a. $3x - 5 = 16$

   b. $3(x + 3) - 5 = 16$

   c. $3(2x - 3) - 5 = 16$

   d. $6(x + 3) - 10 = 32$

   e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

2. Let c and d be real numbers.

   a. If $c = 42 + d$ is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

   b. If $c = 42 - d$ is true, then which is greater: c or d, or are you not able to tell? Explain how you know your choice is correct.

3. If $a < 0$ and $c > b$, circle the expression that is greater:

$$a(b - c) \quad \text{or} \quad a(c - b)$$

Use the properties of inequalities to explain your choice.

4. Solve for x in each of the equations or inequalities below, and name the property and/or properties used:

   a. $\frac{3}{4}x = 9$

   b. $10 + 3x = 5x$

   c. $a + x = b$

   d. $cx = d$

   e. $\frac{1}{2}x - g < m$

   f. $q + 5x = 7x - r$

g. $\frac{3}{4}(x+2) = 6(x+12)$

h. $3(5 - 5x) > 5x$

5. The equation $3x + 4 = 5x - 4$ has the solution set {4}.

   a. Explain why the equation $(3x + 4) + 4 = (5x - 4) + 4$ also has the solution set {4}.

b. In Part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent expressions? Why does changing an expression on one side of an equation to an equivalent expression leave the solution set unchanged?

c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4\ .$$

Use the fact that the solution set to the original equation is {4} to find the solution set to this new equation.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m , the number of minutes the customer used that month.

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc. were not allowed.

   Student tickets cost \$5 each, and adult tickets cost \$10 each.
   A total of \$4500 was collected. 700 tickets were sold.

   a. Write a system of equations that can be used to find the number of student tickets, s, and the number of adult tickets, a, that were sold at the playoff game.

   b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?

   c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?

8. Alexus is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexus assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.

   a. Create a table that shows the total number of bacteria in the Petri dish at 1/3 hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

   b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.

   c. If Alexus starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.

d. For her experiment, Alexus plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexus expect to find in the Petri dish right after she adds the anti-bacterial chemical?

9. Jack is 27 years older than Susan. In 5 years time he will be 4 times as old as her.

a. Find the present ages of Jack and Susan.

b. What calculations would you do to check if your answer is correct?

10.

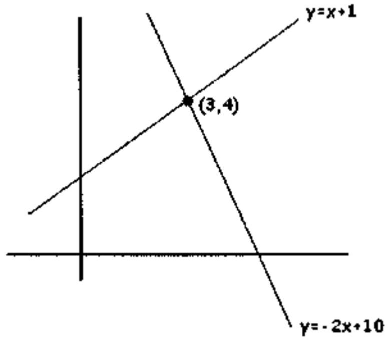
a. Find the product: $(x^2 - x + 1)(2x^2 + 3x + 2)$.

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

11. Consider the following system of equations with the solution $x = 3$, $y = 4$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: ______________

Equation B2: ______________

b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

c. What is the solution to the system given in part (b)?

d. For any real number m, the line $y = m(x - 3) + 4$ passes through the point $(3,4)$.

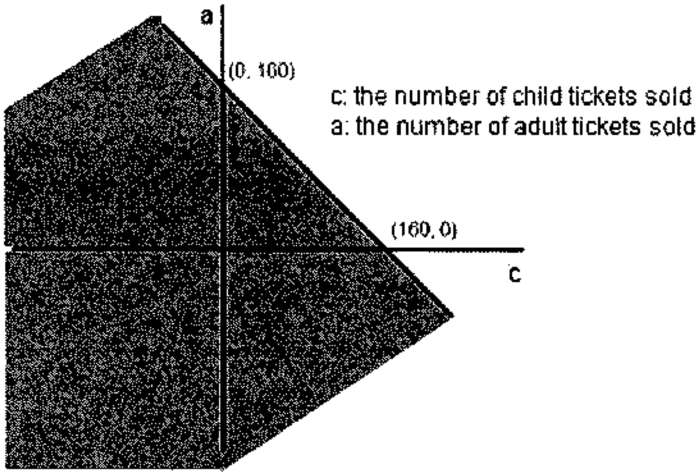
Is it certain, then, that the system of equations

Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3$, $y = 4$? Explain.

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.



   a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

b. Use equations, inequalities, graphs, and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

c. The theater charges $9 for each adult ticket and $6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was $1164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a – d** **A-REI.1**	Student gave a short incorrect answer OR left the question blank.	Student showed at least one correct step, but the solution was incorrect.	Student solved the equation correctly (every step that was shown was correct) but did not express the answer as a solution set.	Student solved the equation correctly (every step that was shown was correct) AND expressed the answer as a solution set.
	e **A-SSE.1b** **A-REI.3**	Student did not answer OR answered incorrectly with something other than b and d.	Student answered b and d but did not demonstrate solid reasoning in the explanation.	Student answered b and d but made minor misstatements in the explanation.	Student answered b and d AND articulated solid reasoning in the explanation.
2	**a – b** **A-CED.3**	Student responded incorrectly OR left the question blank.	Student responded correctly that c must be greater but did not use solid reasoning to explain the answer.	Student responded correctly that c must be greater but gave an incomplete or slightly incorrect explanation of why.	Student responded correctly that c must be greater AND supported the statement with solid, well-expressed reasoning.
3	**a** **A-SSE.1b**	Student responded incorrectly OR left the question blank.	Student responded correctly by circling the expression on the left but did not use solid reasoning to explain the answer.	Student responded correctly by circling the expression on the left but gave limited explanation OR did not use the properties	Student responded correctly by circling the expression on the left AND gave a complete explanation that used the

				of inequality in the explanation.	properties of inequality.
	b **A-SSE.1b**	Student was unable to respond to question OR left items blank.	Part (b) was answered incorrectly.	Student provided limited expression of the reasoning.	Student provided solid reasoning with examples.
4	**a – h** **A-REI.1** **A-REI.3**	Student answered incorrectly with no correct steps shown.	Student answered incorrectly but had one or more correct steps.	Student answered correctly but did not express the answer as a solution set.	Student answered correctly AND expressed the answer as a solution set.
5	**a** **A-REI.1**	Student did not answer OR demonstrated incorrect reasoning throughout.	Student demonstrated only limited reasoning.	Student demonstrated solid reasoning but fell short of a complete answer or made a minor misstatement in the answer.	Student answer was complete AND demonstrated solid reasoning throughout.
	b **A-REI.1**	Student did not answer OR did not demonstrate understanding of what the question was asking.	Student made more than one misstatement in the definition.	Student provided a mostly correct definition with a minor misstatement.	Student answered completely AND used a correct definition without error or mis-statement.
	c **A-REI.1**	Student made mistakes in both verifications and demonstrated incorrect reasoning OR left the question blank.	Student conducted both verifications but fell short of articulating reasoning to answer the question.	Student conducted both verifications AND articulated valid reasoning to answer the question but made a minor error in the verification or a minor misstatement in the explanation.	Student conducted both verifications without error AND articulated valid reasoning to answer the question.
	d **A-REI.1**	Student answered incorrectly OR does not answer.	Student identified one or both solutions but was unable to convey how the solutions could be found using the fact that 4 is a solution to the original equation.	Student identified only one solution correctly but articulated the reasoning of using the solution to the original equation to find the solution to the new equation.	Student identified both solutions correctly AND articulated the reasoning of using the solution to the original equation to find the solution to the new equation.

6	**A-CED.4**	Student did not answer OR showed no evidence of reasoning.	Student made more than one error in the solution process but showed some evidence of reasoning.	Student answer showed valid steps but with one minor error.	Student answered correctly.
7	**a – c** **A-CED.3**	Student was unable to answer any portion correctly.	Student answered one part correctly OR showed some evidence of reasoning in more than one part.	Student showed solid evidence of reasoning in every part but may have made minor errors.	Student answered every part correctly AND demonstrated and expressed valid reasoning throughout.
8	**a** **A-CED.2**	Student provided no table OR a table with multiple incorrect entries.	Data table is incomplete OR has more than one minor error.	Data table is complete but may have one error OR slightly inaccurate headings.	Data table is complete AND correct with correct headings.
	b **A-CED.2**	Student provided no equation OR an equation that did not represent exponential growth.	Student provided an incorrect equation but one that modeled exponential growth.	Student provided a correct answer in the form of $T = B(2)^{3h}$.	Student provided a correct answer in the form of $T = B8^h$ OR in more than one form such as $T = B(2)^{3h}$ and $T = B8^h$.
	c **A-CED.2**	Student provided no graph OR a grossly inaccurate graph.	Student provided a graph with an inaccurate shape but provided some evidence of reasoning in labeling the axes and/or data points.	Student created a graph with correct general shape but may have left off OR made an error on one or two axes or data points.	Student created complete graph with correctly labeled axes AND correctly labeled data points (or a data table) showing the values for h = 0, 1, 2, 3, 4.
	d **A-CED.2**	Student provided no answer OR an incorrect answer with no evidence of reasoning in arriving at the answer.	Student provided limited evidence of reasoning AND an incorrect answer.	Student answered that 409.6 bacteria would be alive.	Student answered that 410 or about 410 bacteria would be alive.
9	**a** **A-CED.1**	Student wrote incorrect equations OR did not provide equations.	Student answers were incorrect, but at least one of the equations was correct. Student may have made a gross error in the solution, made more than one minor error in the solution	Both equations were correct, but student made a minor mistake in finding the solution.	Both equations were correct AND student solved them correctly to arrive at the answer that Jack is 31 and Susan is 4.

			process, OR may have had one of the two equations incorrect.		
	b **A-REI.3**	Student did not answer OR gave a completely incorrect answer.	Student articulated only one of the calculations correctly.	Student articulated the two calculations but with a minor mis-statement in one of the descriptions.	Student articulated both calculations correctly.
10	**a-b** **A-APR.1**	Student work is blank or demonstrates no understanding of multiplication of polynomials, nor how to apply part (a) to arrive at an answer for part (b).	Student made more than one error in the multiplication but demonstrated some understanding of multiplication of polynomials. Student may not have been able to garner or apply information from part *a* to use in answering part *b* correctly.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part *a* to solve part *b*. There may be minor errors.	Student demonstrated the ability to multiply the polynomials (expressing the product as a sum of monomials with like terms combined) and to apply the structure from part *a* to solve part *b* as: $91(232)$.
11	**a** **A-REI.6**	Student was unable to demonstrate the understanding that two equations with (3, 4) as a solution are needed.	Student provided two equations that have (3, 4) as a solution (or attempted to provide such equations) but made one or more errors. Student may have provided an equation with a negative slope.	Student showed one minor error in the answer but attempted to provide two equations both containing (3, 4) as a solution and both with positive slope.	Student provided two equations both containing (3, 4) as a solution and both with positive slope.
	b **A-REI.6**	Student was unable to identify the multiple correctly.	Student identified the multiple as 3.	N/A	Student correctly identified the multiple as 2.
	c **A-REI.6**	Student was unable to demonstrate even a partial understanding of how to find the solution to the system.	Student showed some reasoning required to find the solution but made multiple errors.	Student made a minor error in finding the solution point.	Student successfully identified the solution point as $(3, 4)$.

	d **A-REI.5** **A-REI.6** **A-REI.10**	Student was unable to answer or to support the answer with any solid reasoning.	Student concluded yes or no but was only able to express limited reasoning in support of the answer.	Student correctly explained that all the systems would have the solution point $(3, 4)$ but incorrectly assumed this is true for all cases of m.	Student correctly explained that while in most cases this is true, that if $m = 1$, the two lines are coinciding lines, resulting in a solution set consisting of all the points on the line.
12	**a** **MP.2** **A-REI.12**	Student was unable to articulate any sound reasons.	Student was only able to articulate one sound reason.	Student provided two sound reasons but made minor errors in the expression of reasoning.	Student was able to articulate at least 2 valid reasons. Valid reasons include the following: the graph assumes x could be less than zero, the graph assumes y could be less than zero, the graph assumes a and b could be non-whole numbers, the graph assumes 160 children could attend with no adults.
	b **A-CED.2** **A-REI.10** **A-REI.12**	Student was unable to communicate a relevant requirement of the solution set.	Student provided a verbal description that lacked precision and accuracy but demonstrated some reasoning about the solution within the context of the problem.	Student made minor errors in communicating the idea that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.	Student communicated effectively that both (a) and (b) must be whole numbers whose sum is less than or equal to 160.
	c **A-CED.2** **A-REI.6**	Student was unable to demonstrate any substantive understanding in how to create the equations and solve the system of equations.	Student made multiple errors in the equations and/or solving process but demonstrated some understanding of how to create equations to represent a context and/or solve the system of equations.	Student made minor errors in the equations but solved the system accurately, or the student created the correct equations but made a minor error in solving the system of equations.	Student correctly wrote the equations to represent the system. Student solved the system accurately and summarized by defining or describing the values of the variable in the context of the problem: that there were 100 adult tickets and 44 child

					tickets sold.

Name ______________________ Date ____________

1. Solve the following equations for x. Write your answer in set notation.

   a. $3x - 5 = 16$

   $3x = 21$ Solution set: $\{7\}$

   $x = 7$

   b. $3(x+3) - 5 = 16$

   $3x + 9 - 5 = 16$ Solution set: $\{4\}$

   $3x = 12$

   $x = 4$

   c. $3(2x-3) - 5 = 16$

   $6x - 9 - 5 = 16$ Solution set: $\{5\}$

   $6x - 14 = 16$

   $6x = 30$

   $x = 5$

   d. $6(x+3) - 10 = 32$

   $6x + 18 - 10 = 32$ Solution set: $\{4\}$

   $6x = 24$

   $x = 4$

e. Which two equations above have the same solution set? Write a sentence explaining how the properties of equality can be used to determine the pair without having to find the solution set for each.

Problems (b) and (d) have the same solution set. The expressions on each side of the equal sign for d are twice those for b. So, if (left side) = (right side) is true for only some x-values, then 2(left side) = 2(right side) will be true for exactly the same x-values. Or simply, applying the multiplicative property of equality does not change the solution set.

2. Let c and d be real numbers.

a. If $c = 42 + d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

c must be greater because c is always 42 more than d.

b. If $c = 42 - d$ is true, then which is greater: c or d or are you not able to tell? Explain how you know your choice is correct.

There is no way to tell. We only know that the sum of c and d is 42. If d were 10, c would be 32 and, therefore, greater than d. But if d were 40, c would be 2 and, therefore, less than d.

3. If $a < 0$ and $c > b$, circle the expression that is greater:

[✓ $a(b-c)$] or $a(c-b)$

Use the properties of inequalities to explain your choice.

Since c > b,
it follows that 0 > b − c.
So (b−c) is negative.
And since a < 0, a is negative.
And the product of two negatives will be positive.

Since c > b,
it follows that c − b > 0.
So (c−b) is positive.
And since a is negative,
the product of a(c−b) is negative.
So, a(c−b) < a(b−c).

4. Solve for x in each of the equations or inequalities below and name the property and/or properties used:

a. $\frac{3}{4}x = 9$ $x = 9\ (3/4)$ Multiplication Property of Equality

$x = 12$

b. $10 + 3x = 5x$ $10 = 2x$ Addition Property of Equality

$5 = x$

c. $a + x = b$ $x = b - a$ Addition Property of Equality

d. $cx = d$ $x = d/c,\ c \neq 0$ Multiplication Property of Equality

e. $\frac{1}{2}x - g < m$ $\frac{1}{2}x < m + g$ Addition Property of Equality

$x < 2(m+g)$ Multiplication Property of Equality

$q + r = 2x$ Addition Property of Equality

$(q + r)/2 = x$ Multiplication property of Equality

f. $q + 5x = 7x - r$

g. $\frac{3}{4}(x + 2) = 6(x + 12)$

3(x+2) = 24(x + 12)	*Multiplication Property of Equality*
3x + 6 = 24x + 288	*Distributive Property*
-282/21 = x	*Addition Property of Equality*
-94/7 = x	

h. $3(5 - 5x) > 5x$

15 – 15x > 5x	*Distributive Property*
15 > 20 x	*Addition Property of Inequality*
¾ > x	

5. The equation, $3x + 4 = 5x - 4$, has the solution set {4}.

   a. Explain why the equation, $(3x + 4) + 4 = (5x - 4) + 4$, also has the solution set {4}.

Since the new equation can be created by applying the additive property of equality, the solution set does not change.

Or:

Each side of this equation is 4 more than the sides of the original equation. Whatever value(s) make 3x + 4 = 5x − 4 true would also make 4 more than 3x + 4 equal to 4 more than 5x − 4.

b. In Part (a), the expression $(3x + 4) + 4$ is equivalent to the expression $3x + 8$. What is the definition of equivalent algebraic expressions? Describe why changing an expression on one side of an equation to an equivalent expression leaves the solution set unchanged?

Algebraic expressions are equivalent if (possibly repeated) use of the Distributive, Associative, and Commutative Properties, and/or the properties of rational exponents can be applied to one expression to convert it to the other expression.

When two expressions are equivalent, assigning the same value to x in both expressions will give an equivalent numerical expression which then evaluates to the same number. Therefore, changing the expression to something equivalent will not change the truth value of the equation once values are assigned to x.

c. When we square both sides of the original equation, we get the following new equation:

$$(3x + 4)^2 = (5x - 4)^2.$$

Show that 4 is still a solution to the new equation. Show that 0 is also a solution to the new equation but is not a solution to the original equation. Write a sentence that describes how the solution set to an equation may change when both sides of the equation are squared.

$(3\cdot 4 + 4)^2 = (5\cdot 4-4)^2$ *gives* $16^2 = 16^2$ *which is true.*

$(3\cdot 0 + 4)^2 = (5\cdot 0-4)^2$ *gives* $4^2 = (-4)^2$ *which is true.*

But, $(3\cdot 0 + 4) = (5\cdot 0-4)$ *gives* $4 = -4$ *which is false.*

When both sides are squared, you might introduce new numbers to the solution set because statements like $4 = -4$ *are false, but statements like* $4^2 = (-4)^2$ *are true.*

d. When we replace x by x^2 in the original equation, we get the following new equation:

$$3x^2 + 4 = 5x^2 - 4 .$$

Use the fact that the solution set to the original equation is {4} to find the solution set to this new equation.

Since the original equation $3x + 4 = 5x - 4$ was true when $x = 4$, the new equation $3x^2 + 4 = 5x^2 - 4$ should be true when $x^2 = 4$. And, $x^2 = 4$ when $x = 2$ or when $x = -2$, so the solution set to the new equation is $\{-2, 2\}$.

6. The Zonda Information and Telephone Company (ZI&T) calculates a customer's total monthly cell phone charge using the formula,

$$C = (b + rm)(1 + t),$$

where C is the total cell phone charge, b is a basic monthly fee, r is the rate per minute, m is the number of minutes used that month, and t is the tax rate.

Solve for m, the number of minutes the customer used that month.

$$C = b + bt + rm + rmt$$

$$C - b - bt = m(r + rt)$$

$$\frac{C - b - bt}{r + rt} = m \qquad t \neq -1$$

$$r \neq 0$$

7. Students and adults purchased tickets for a recent basketball playoff game. All tickets were sold at the ticket booth—season passes, discounts, etc. were not allowed.

   Student tickets cost \$5 each, and adult tickets cost \$10 each.
   A total of \$4500 was collected. 700 tickets were sold.

   a. Write a system of equations that can be used to find the number of student tickets, s, and the number of adult tickets, a, that were sold at the playoff game.

$$\begin{cases} 5s + 10a = 4500 \\ s + a = 700 \end{cases}$$

   b. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the ticket booth charged students and adults the same price of \$10 per ticket?

700 × \$10 = \$7000

\$7000 - \$4500 = \$2500 more.

   c. Assuming that the number of students and adults attending would not change, how much more money could have been collected at the playoff game if the student price was kept at \$5 per ticket and adults were charged \$15 per ticket instead of \$10?

First solve for a & s

$5s + 10a = 4500$
$-5s - 5a = -3500$

$5a = 1000$
$a = 200$
$s = 500$

\$5(500) + \$15(200) = \$5500

\$1000 more

or

\$5 more per adult tkt

200 × \$5
= \$1000 more

8. Alexus is modeling the growth of bacteria for an experiment in science. She assumes that there are B bacteria in a Petri dish at 12:00 noon. In reality, each bacterium in the Petri dish subdivides into two new bacteria *approximately* every 20 minutes. However, for the purposes of the model, Alexus assumes that each bacterium subdivides into two new bacteria *exactly* every 20 minutes.

   a. Create a table that shows the total number of bacteria in the Petri dish at 1/3 hour intervals for 2 hours starting with time 0 to represent 12:00 noon.

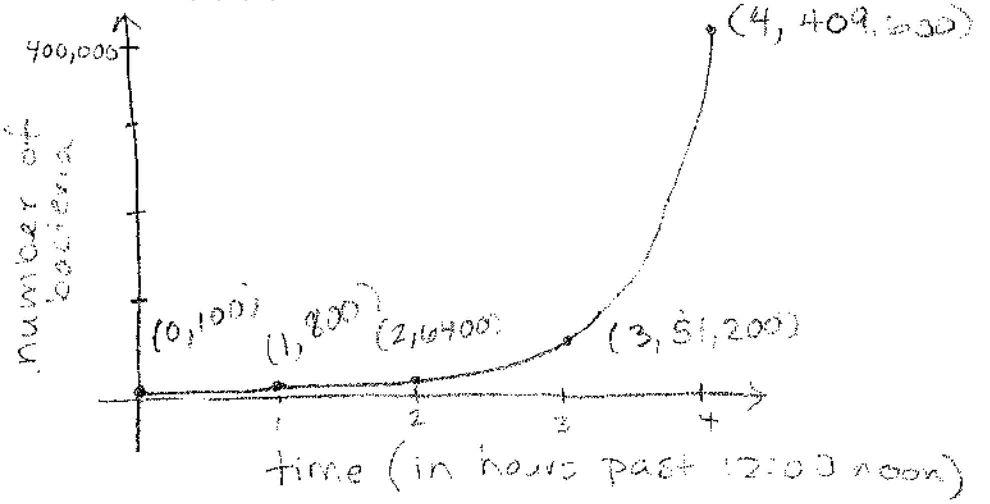
time	number of bacteria
0	B
1/3 hr	2 B
2/3 hr	4 B
1 hr	8 B
1 1/3 hr	16 B
1 2/3 hr	32 B
2 hr	64 B

   b. Write an equation that describes the relationship between total number of bacteria T and time h in hours, assuming there are B bacteria in the Petri dish at $h = 0$.

$$T = B(2)^{3h}$$

or $T = B \cdot 8^h$

   c. If Alexus starts with 100 bacteria in the Petri dish, draw a graph that displays the total number of bacteria with respect to time from 12:00 noon ($h = 0$) to 4:00 p.m. ($h = 4$). Label points on your graph at time $h = 0, 1, 2, 3, 4$.



d. For her experiment, Alexus plans to add an anti-bacterial chemical to the Petri dish at 4:00 p.m. that is supposed to kill 99.9% of the bacteria instantaneously. If she started with 100 bacteria at 12:00 noon, how many live bacteria might Alexus expect to find in the Petri dish right after she adds the anti-bacterial chemical?

$$(1-.999)\,409{,}600 = 409.6$$

about 410 live bacteria.

9. Jack is 27 years older than Susan. In 5 years time he will be 4 times as old as her.

a. Find Jack and Susan's present age.

$$\begin{cases} J = S+27 \\ J+5 = 4(S+5) \end{cases}$$

$J = 4+27$

$J = 31$

$S+27+5 = 4S+20$

$S+32 = 4S+20$

$12 = 3S$ $\quad S = 4$

Jack is 31 and Susan is 4.

b. What calculations would you do to check if your answer is correct?

Is Jack's age − Susan's age = 27?
add 5 years to Jack & Susan's ages and see if that makes Jack 4 times as old as Susan.

10.

a. Find the product:

$2x^4 + 3x^3 + 2x^2 - 2x^3 - 3x^2 - 2x + 2x^2 + 3x + 2$

$2x^4 + x^3 + x^2 + x + 2$

b. Use the results of part (a) to factor 21,112 as a product of a two-digit number and a three-digit number.

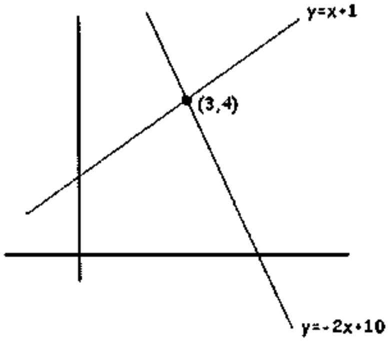
$(100 - 10 + 1)(200 + 30 + 2)$

$(91)(232)$

11. Consider the following system of equations with the solution $x = 3$, $y = 4$.

Equation A1: $y = x + 1$

Equation A2: $y = -2x + 10$



a. Write a unique system of two linear equations with the same solution set. This time make both linear equations have positive slope.

Equation B1: $y = 4/3\ x$

$y = x + 1$

Equation B2: ________________

b. The following system of equations was obtained from the original system by adding a multiple of equation A2 to equation A1.

Equation C1: $y = x + 1$

Equation C2: $3y = -3x + 21$

What multiple of A2 was added to A1?

2

c. What is the solution to the system given in part (b)?

(3, 4)

d. For any real number m, the line $y = m(x - 3) + 4$ passes through the point $(3,4)$.

Is it certain, then, that the system of equations:

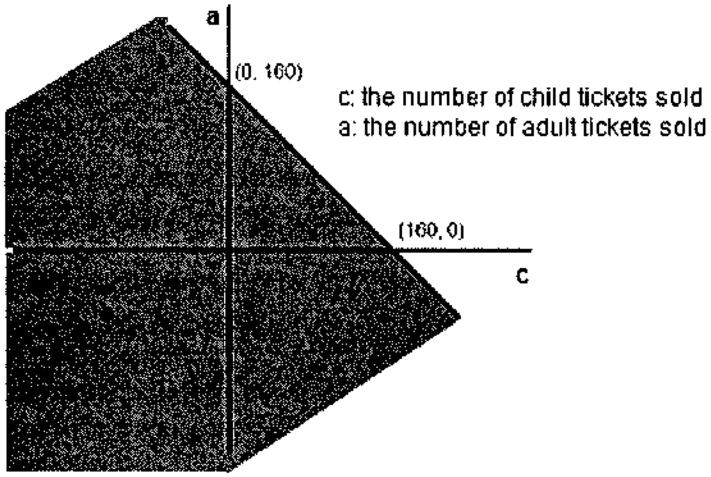
Equation D1: $y = x + 1$

Equation D2: $y = m(x - 3) + 4$

has only the solution $x = 3$, $y = 4$? Explain.

No. If m = 1, then the two lines have the same slope. Since both lines pass through the point (3, 4), and the lines are parallel, they, therefore, coincide. There are infinite solutions. The solution set is all the points on the line. Any other non-zero value of m would create a system with the only solution of (3, 4).

12. The local theater in Jamie's home town has a maximum capacity of 160 people. Jamie shared with Venus the following graph and said that the shaded region represented all the possible combinations of adult and child tickets that could be sold for one show.

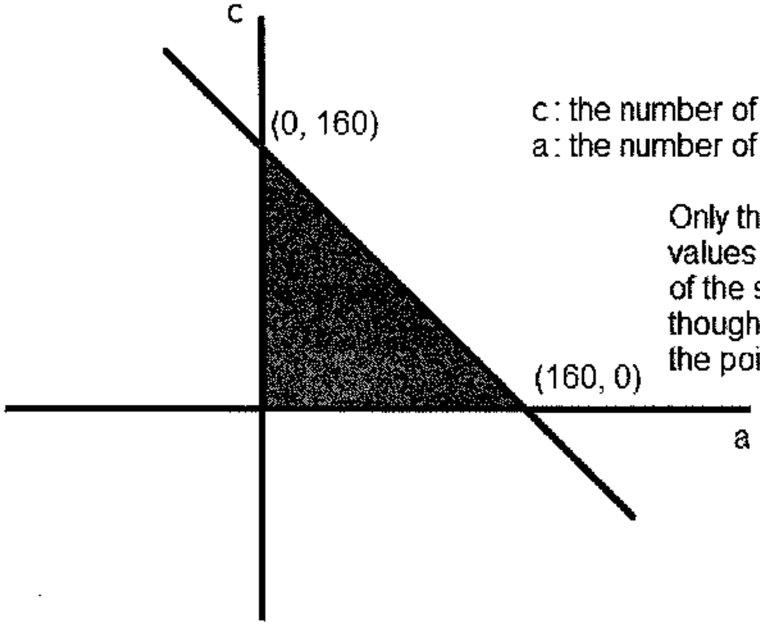


a. Venus objected and said there was more than one reason that Jamie's thinking was flawed. What reasons could Venus be thinking of?

1. *The graph implies that the number of tickets sold could be a fractional amount, but really it only makes sense to sell whole number tickets. X and y must be whole numbers.*
2. *The graph also shows that negative ticket amounts could be sold which does not make sense.*

b. Use equations, graphs and/or words to describe for Jamie the set of all possible combinations of adult and child tickets that could be sold for one show.

The system would be $\begin{cases} a + c \le 160 \\ a \ge 0 \\ c \ge 0 \end{cases}$ where a and c are whole numbers.



c : the number of child tikcets sold
a : the number of adult tickets sold

Only the whole number values of a and c are part of the solution set (even though it appears that all the points are shaded).

c. The theater charges $9 for each adult ticket and $6 for each child ticket. The theater sold 144 tickets for the first showing of the new release. The total money collected from ticket sales for that show was $1164. Write a system of equations that could be used to find the number of child tickets and the number of adult tickets sold, and solve the system algebraically. Summarize your findings using the context of the problem.

a: the number of adult tickets sold (must be a whole number)

c: the number of child tickets sold (must be a whole number)

$\begin{cases} 9a + 6c = 1164 \\ a + c = 144 \end{cases}$

$9a + 6c = 1164$

$-6a - 6c = -864$

$3a = 300 \quad a = 100, \quad c = 44.$

In all, 100 adult tickets and 44 child tickets were sold.

UNIT TWO

for

Content Area of

MATHEMATICS

HS Band
Algebra I

Mathematics Curriculum

Table of Contents[1]

Linear and Exponential Functions

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[1] Each lesson is ONE day, and ONE day is considered a 45 minute period.

Algebra I • Module 3

Linear and Exponential Functions

OVERVIEW

In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities (**8.F.A.1, 8.F.A.2, 8.F.A.3, 8.F.B.4, 8.F.B.5**). In this module, students extend their study of functions to include function notation and the concepts of domain and range. They explore many examples of functions and their graphs, focusing on the contrast between linear and exponential functions. They interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations.

In Topic A, students explore arithmetic and geometric sequences as an introduction to the formal notation of functions (**F-IF.A.1, F-IF.A.2**). They interpret arithmetic sequences as linear functions with integer domains and geometric sequences as exponential functions with integer domains (**F-IF.A.3, F-BF.A.1a**). Students compare and contrast the rates of change of linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change (**F-IF.B.6, F-LE.A.1, F-LE.A.2, F-LE.A.3**).

In Topic B, students connect their understanding of functions to their knowledge of graphing from Grade 8. They learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations (**F-IF.A.1, F-IF.A.2**). Students examine the graphs of a variety of functions and learn to interpret those graphs using precise terminology to describe such key features as domain and range, intercepts, intervals where function is increasing or decreasing, and intervals where the function is positive or negative. (**F-IF.A.1, F-IF.B.4, F-IF.B.5, F-IF.C.7a**).

In Topic C, students extend their understanding of piecewise functions and their graphs including the absolute value and step functions. They learn a graphical approach to circumventing complex algebraic solutions to equations in one variable, seeing them as $f(x) = g(x)$ and recognizing that the intersection of the graphs of $f(x)$ and $g(x)$ are solutions to the original equation (**A-REI.D.11**). Students use the absolute value function and other piecewise functions to investigate transformations of functions and draw formal conclusions about the effects of a transformation on the function's graph (**F-IF.C.7, F-BF.B.3**).

Finally, in Topic D students apply and reinforce the concepts of the module as they examine and compare exponential, piecewise, and step functions in a real-world context (**F-IF.C.9**). They create equations and functions to model situations (**A-CED.A.1, F-BF.A.1, F-LE.A.2**), rewrite exponential expressions to reveal and relate elements of an expression to the context of the problem (**A-SSE.B.3c, F-LE.B.5**), and examine the key features of graphs of functions, relating those features to the context of the problem (**F-IF.B.4, F-IF.B.6**).

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*[2]

Create equations that describe numbers or relationships.

A-CED.A.1[3] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★

Represent and solve equations and inequalities graphically.

A-REI.D.11[4] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3[5] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1)$

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[2] Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra I, tasks are limited to exponential expressions with integer exponents.

[3] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

[4] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.

[5] This standard is part of the Major Content in Algebra I and will be assessed accordingly.

$= 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[6] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[7] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.C.9[8] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build a function that models a relationship between two quantities.

F-BF.A.1[9] Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

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[6] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.

[7] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.

[8] In Algebra I, tasks are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

[9] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

Build new functions from existing functions.

F-BF.B.3[10] Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2[11] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Interpret expressions for functions in terms of the situation they model.

F-LE.B.5[12] Interpret the parameters in a linear or exponential function in terms of a context.★

Foundational Standards

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical

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[10] In Algebra I, identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions. The focus in this module is on linear and exponential functions.

[11] In Algebra I, tasks are limited to constructing linear and exponential functions in simple (e.g., not multi-step) context.

[12] Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.

expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[13]

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function* $A = s^2$ *giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two *(x, y)* values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Reason quantitatively and use units to solve problems.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2[14] Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

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[13] Function notation is not required in Grade 8.

[14] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*

Create equations that describe numbers or relationships.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance R.*★

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A-REI.C.6[15] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

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[15] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (e.gi.e., less-defined tasks, more of the modeling cycle, etc.).

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain insight into the problem.

MP.2 **Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and apply reason (from the shape of the graphs) to infer the quantities being displayed and consider possible units to represent those quantities.

MP.4 **Model with mathematics.** Students have numerous opportunities to solve problems that arise in everyday life, society, and the workplace (e.g., modeling bacteria growth, and understanding the federal progressive income tax system).

MP.7 **Look for and make use of structure.** Students reason with and analyze collections of equivalent expressions to see how they are linked through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves. (e.g., $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$)

MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters $(e.g., ax + b = cx + d$). They pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequalities, to find equivalent expressions and solve equations, while recognizing common ways to solve different types of equations.

Terminology

New or Recently Introduced Terms

- **Function** (A *function* is a correspondence between two sets, X and Y , in which each element of X is matched[16] to one and only one element of Y . The set X is called the *domain*; the set Y is called the *range*.)
- **Domain** (Refer to the definition of *function*.)
- **Range** (Refer to the definition of *function*.)
- **Linear Function** (A *linear function* is a polynomial function of degree 1.)
- **Average Rate of Change** (Given a function f whose domain includes the closed interval of real numbers $[a, b]$ and whose range is a subset of the real numbers, the *average rate of change on the*

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[16] "Matched" can be replaced with "assigned" after students understand that each element of x is matched to exactly one element of y .

interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.)

- **Piecewise Linear Function** (Given non-overlapping intervals on the real number line, a (real) *piecewise linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)

Familiar Terms and Symbols[17]

- Numerical Symbol
- Variable Symbol
- Constant
- Numerical Expression
- Algebraic Expression
- Number Sentence
- Truth Values of a Number Sentence
- Equation
- Solution
- Solution Set
- Simple Expression
- Factored Expression
- Equivalent Expressions
- Polynomial Expression
- Equivalent Polynomial Expressions
- Monomial
- Coefficient of a Monomial
- Terms of a Polynomial

Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities
- Graphing Calculator

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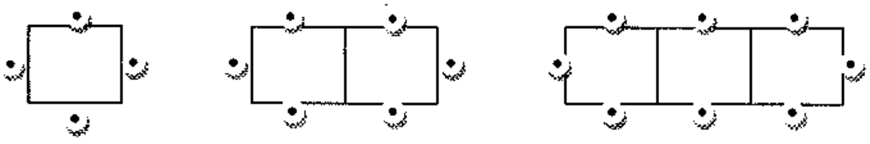
[17] These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a, F-BF.A.1a, F-LE.A.1, F-LE.A.2, F-LE.A.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	A-CED.A.1, A-REI.D.11, A-SSE.B.3c, F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.9, F-BF.A.1a, F-BF.B.3, F-LE.A.1, F-LE.A.2, F-LE.A.3, F-LE.B.5

Name ______________________________ Date ________________

1. The diagram below shows how tables and chairs are arranged in the school cafeteria. One table can seat 4 people, and tables can be pushed together. When two tables are pushed together, 6 people can sit around the table.



1 Table 2 Tables 3 Tables

a. Complete this table to show the relationship between the number of tables, n, and the number of students, S, that can be seated around the table.

n (tables)						
S (students)						

b. If we made a sequence where the first term of the sequence was the number of students that can fit at 1 table, the 2nd term where the number that could fit at 2 tables, etc, would the sequence be arithmetic, geometric, or neither? Explain your reasoning.

c. Create an explicit formula for a sequence that models this situation. Use $n = 1$ as the first term, representing how many students can sit at 1 table. How do the constants in your formula relate to the situation?

d. Using this seating arrangement, how many students could fit around 15 tables pushed together in a row?

The cafeteria needs to provide seating for 189 students. They can fit up to 15 rows of tables in the cafeteria. Each row can contain at most 9 tables but could contain less than that. The tables on each row must be pushed together. Students will still be seated around the tables as described earlier.

e. If they use exactly 9 tables pushed together to make each row, how many rows will they need to seat 189 students, and how many tables will they have used to make those rows?

f. Is it possible to seat the 189 students with fewer total tables? If so, what is the fewest number of tables needed? How many tables would be used in each row? (Remember that the tables on each row must be pushed together.) Explain your thinking.

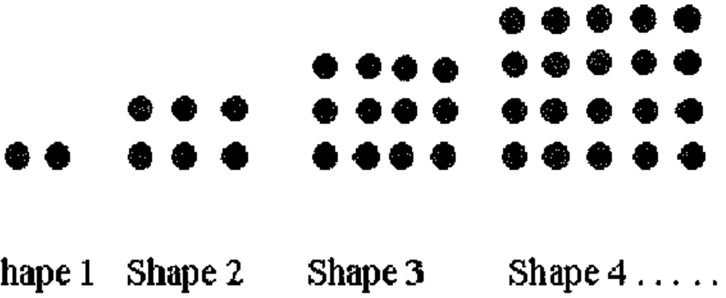
2. Sydney was studying the following functions:

$$f(x) = 2x + 4 \text{ and } g(x) = 2(2^x) + 4$$

She said that linear functions and exponential functions are basically the same. She made her statement based on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and constrasting f and g. Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

3. Dots can be arranged in rectangular shapes like the one shown below.



a. Assuming the trend continues, draw the next three shapes in this particular sequence of rectangles. How many dots are in each of the shapes you drew?

The numbers that represent the number of dots in this sequence of rectangular shapes are called rectangular numbers. For example, 2 is the first rectangular number and 6 is the 2nd rectangular number.

b. What is the 50th rectangular number? Explain how you arrived at your answer.

c. Write a recursive formula for the rectangular numbers.

d. Write an explicit formula for the rectangular numbers.

e. Could an explicit formula for the n th rectangular number be considered a function? Explain why or why not. If yes, what would be the domain and range of the function?

4. Stephen is assigning parts for the school musical.

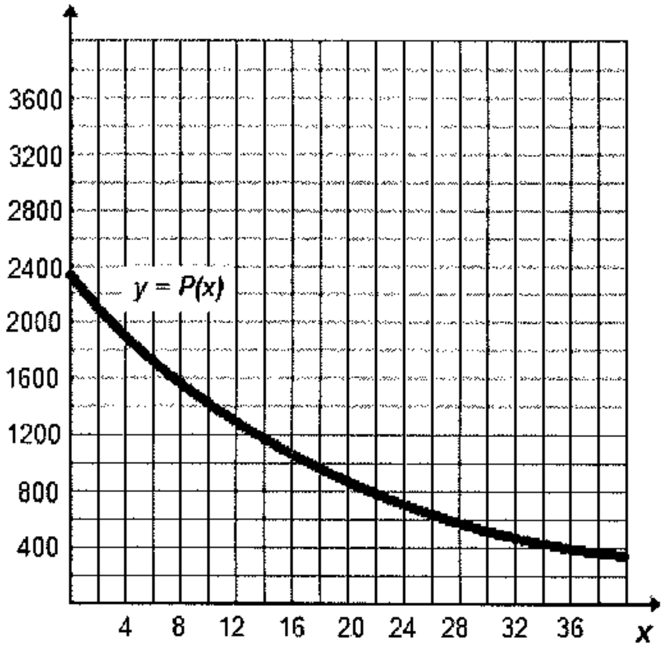
a. Suppose there are 20 students participating, and he has 20 roles available. If each of the 20 students will be assigned to exactly one role in the play, and each role will be played by only one student, is the assignment of the roles to the students in this way certain to be an example of a function? Explain why or why not. If yes, state the domain and range of the function.

The school musical also has a pit orchestra.

b. Suppose there are 10 instrumental parts but only 7 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some musicians will have to cover two instrumental parts, but no two musicians will have the same instrumental part. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

c. Suppose there are 10 instrumental parts but 13 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some instrumental parts will have two musicians assigned so that all the musicians have instrumental parts. When two musicians are assigned to one part, they alternate who plays at each performance of the play. If the instrumental parts are the domain, and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

5. The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2350 . In 1962 the population was 1270 . They chose an exponential decay model and arrived at the function: $p(x) = 2350(0.95)^x, x \geq 0$, where x is the number of years since 1950. The graph of this function is given below.



a. What is the y -intercept of the graph? Interpret its meaning in the context of the problem.

b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?

c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

d. Write the linear function that this second group of scientists would have used.

e. What is an appropriate domain for the function? Explain your choice within the context of the problem.

f. Graph the function on the coordinate plane.

g. What is the x -intercept of the function? Interpret its meaning in the context of the problem.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem
1	a – d F-BF.A.1a F-LE.A.1 F-LE.A.2 F-LE.A.3 F-IF.A.2	Student is unable to identify the sequence as arithmetic and unable to create an explicit formula for finding the number of students that can be seated at n tables.	Student is able to recognize the sequence as arithmetic, but displays a logic error in the explicit formula for the sequence. Student may also have errors in the table, the relating of constants in the formula to context, or in the computation of how many students can sit at 15 tables pushed together.	Student is able to recognize the sequence as arithmetic, and create an explicit formula for the sequence. Student has one or more errors in the table, the relating of constants in the formula to context, or in the computation of how many students can sit at 15 tables pushed together.	Student provides correct entries for the table and explains that the sequence is arithmetic because each successive term is two more than the last term. Student writes an explicit formula for the sequence using an appropriate sequence notation, such as $f(n)$ or a_n, and correctly relates the constants of their formula to the context of the problem. Student correctly determines that 32 students can sit at 15 tables pushed together in a row.
	e F-IF.A.2 MP 1 MP 4	Student is not able to demonstrate how to determine the number of students that can sit at 9 tables pushed together.	Student makes calculation errors that lead to an incorrect answer, but demonstrates some understanding of how to determine the number of students that can sit at 9 tables pushed	Student articulates that 10 rows of 9 tables are needed to seat 189 students, but fails to answer the question of how many tables that would be.	Student articulates that 10 rows of 9 tables are needed to seat 189 students, and that 10 rows of 9 tables would be 90 tables.

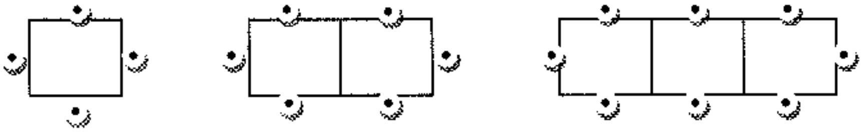
			together, and some understanding of the need to divide 189 by 20 to see that it takes more than 9 rows or 10 rows.		
	f F-IF.A.2 MP 1 MP 4	Student demonstrates very little reasoning skills in their attempted answer.	Student demonstrates some correct reasoning but is unable to arrive at one of the possible arrangements that use only 80 tables distributed across 15 rows.	Student chooses an arrangement of 80 tables distributed across 15 rows in one of any of the many possible configurations.	Student articulates that any arrangement involving 80 tables distributed among all 15 rows would use the minimum number of tables.
2	F-IF.B.5 F-IF.B.6 F-BF.A.1a F-LE.A.1 F-LE.A.2 F-LE.A.3	Student provides incorrect or insufficient tables, graphs, and written explanation.	Student provides a table and/or a graph that may have minor errors but does not provide a correct written explanation, OR student provides a limited written explanation but does not support the answer with tables or graphs.	Student demonstrates that the functions are not the same with accurate tables or graphs and provides a limited written explanation that does not thoroughly describe the differences in the rates of change of the functions, OR student has errors in his or her table or graphs but provides a thorough written explanation that references the rates of change of the functions.	Student demonstrates that the functions are not the same with accurate tables or graphs and provides a thorough written explanation that includes accurate references to the rates of change of the functions.
3	a F-IF.A.3 F-BF.A.1a	Student does not demonstrate any understanding of the pattern of the sequence described in the problem.	Student has a significant error or omission in the task but demonstrates some understanding of the pattern of the sequence described in the problem.	Student has a minor error or omission in the task but demonstrates clear understanding of the pattern of the sequence described in the problem.	Sequence is continued correctly three times AND correct number of dots is given.
	b F-IF.A.3 F-BF.A.1a	Student demonstrates no understanding of using the pattern to find the 50th term.	Student attempts to use the pattern to find the 50th term but has a flaw in this or her reasoning and therefore arrives at an incorrect answer.	Student correctly answers 2550 but does not provide an explanation based on sound reasoning, or makes a calculation error, but provides an explanation based on sound reasoning.	Student correctly answers 2550 and displays sound reasoning as they explain using the pattern of the sequence to arrive at the answer.
	c-d F-IF.A.3 F-BF.A.1a	Student work demonstrates no or very little understanding needed to write recursive or	Student is only able to provide either the recursive or the explicit formula and may have provided an explicit	Student provides correct formulas but uses an explicit formula where it asked for a recursive one, and a recursive one	Student provides a correct recursive formula for part (c) and a correct explicit formula for part (d), each using either

		explicit formulas for this sequence.	formula when it asked for a recursive formula or vice versa, OR, student demonstrates that he or she understands the difference between a recursive formula and an explicit one by providing formulas for both, but the formulas provided are incorrect. Student may or may not have provided the necessary declaration of the initial term number in each case, and for the initial term value in the recursive case.	where it asked for an explicit one, OR the student provides the correct formulas in each case, but neglects to provide the necessary declaration of the initial term number in each case, and for the initial term value in the recursive case.	function notation such as f(n) or subscript notation such as an. Student also provides the necessary declaration of the initial term number in each case, and for the initial term value in the recursive case.
	e **F-IF.A.2** **F-IF.A.3**	Student leaves the question blank or answers that it is not a function.	Student answers that it is a function but gives insufficient reasoning. Or is unable to identify the domain and range.	Student answers that it is a function but makes minor errors or omissions in his or her reasoning, or makes errors in naming of the domain and range.	Student answers that it is a function and gives sufficient reasoning, stating the domain and the range accurately.
4	**a** **F-IF.A.1** **F-IF.A.2**	Student indicates it is not a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.	Student reasoning indicates he or she understands what is required for a relation to be a function, but is not able to discern that this relation is a function, OR, student indicates it is a function but does not provide sufficient explanation and/or omits the naming of the domain and range.	Student identifies the relation as a function but provides an explanation and/or identification of the domain and range that contains minor errors or omissions.	Student identifies the relation as a function and provides a thorough explanation of how this situation meets the criteria for a function: that every input is assigned to one and only one output and student chooses either the list of students or the list of roles as the domain and the other list as the range.
	b **F-IF.A.1** **F-IF.A.2**	Student indicates it is not a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.	Student reasoning indicates he or she understands what is required for a relation to be a function but is not able to discern that this relation is a function, OR, student indicates it is a function but does not provide sufficient explanation and/or	Student identifies the relation as a function but provides an explanation and/or interpretation of $A(piano) = Scott$ that contains minor errors or omissions.	Student identifies the relation as a function and provides a thorough explanation of how this situation meets the criteria for a function: that every input is assigned to one and only one output and student interprets $A(piano) = Scott$ to

			omits or incorrectly interprets the meaning of $A(piano) = Scott$.		mean that the part of the piano is being played by Scott.
	c **F-IF.A.1** **F-IF.A.2**	Student indicates it is a function and provides no reasoning or incorrect reasoning, indicating he or she does not have sufficient understanding of what is required for a relation to be a function.	Student reasoning indicates he or she understands what is required for a relation to be a function but is not able to discern that this relation is not a function, OR, student indicates it is not a function but does not provide sufficient explanation.	Student identifies the relation is not a function but provides an explanation that contains minor errors or omissions.	Student identifies the relation is not a function and provides a thorough explanation of how this situation does not meet the criteria for a function: that every input is assigned to one and only one output.
5	**a** **F-IF.B.4**	Student is unable to correctly identify the y-intercept.	Student identifies that the y-intercept is the point (0, 2350) but fails to correctly relate the point to the context of the problem.	Student identifies that the y-intercept is the point (0, 2350) and relates 2350 to the population when $x = 0$, but fails to relate $x = 0$ to the year 1950.	Student identifies that the y-intercept is the point (0, 2350) and relates the point to the context that in 1950 the population was 2350.
	b – c **F-IF.B.4**	Student is unable to correctly identify that the function is always decreasing, never increasing.	Student identifies that there are no intervals for which the function is increasing that it is decreasing over its entire domain, but student does not correctly interpret this answer in the context of the problem.	Student identifies that there are no intervals for which the function is increasing, that it is decreasing over its entire domain, and student interprets this answer in the context of the problem, but has minor errors or omissions in the language used to answer the questions.	Student identifies that there are no intervals for which the function is increasing, that it is decreasing over its entire domain, and student interprets this answer in the context of the problem, using mathematically correct language and sound reasoning.
	d, f, g **F-BF.A.1a** **F-IF.B.4** **F-IF.C.7a**	Student does not use the two data points, and/or does not create a linear equation using the two points; therefore, the graph and/or x-intercept of parts (f) and (g) are likely incorrect.	Student attempts to use the two data points to write a linear function but makes a significant error in arriving at the equation of the line. Student graphs the equation he or she created and attempts to identify an x-intercept but may have identified the y-intercept instead.	Student uses the two data points to write the linear function correctly using either function notation or an equation in two variables but may have made a computational error in arriving at the slope (it is evident that the student understands how to compute slope). Student graphs the function created but may have	Student uses the two data points to write the linear function correctly using either function notation or an equation in two variables. Student graphs the function correctly depicting only the domain values identified in part e, and correctly identifies the x-intercept, relating it to the context of the problem.

				extended the graph beyond the domain identified in part (e). Student identifies the x-intercept, relating it to the context of the problem.	
	e **F-IF.B.5**	Student does not demonstrate sound reasoning in restricting the domain given the context of the problem.	Student restricts the domain and explains the answer by either considering that we only wish to start in the year 1950, or considering that it does not make sense to continue the model when the population has fallen below zero, but does not consider both factors.	Student restricts the domain and explains the answer by considering that we only wish to start in the year 1950, or considering that it does not make sense to continue the model when the population has fallen below zero, but student makes an error in calculating the end points of the interval for the domain.	Student restricts the domain and explains the answer by considering that we only wish to start in the year 1950, or considering that it does not make sense to continue the model when the population has fallen below zero, and student and correctly calculates the end points of the interval for the domain.

1. The diagram below shows how tables and chairs are arranged in the school cafeteria. One table can seat 4 people, and tables can be pushed together. When two tables are pushed together, 6 people can sit around the table.



1 Table 2 Tables 3 Tables

a. Complete this table to show the relationship between the number of tables, *n*, and the number of students, *S*, that can be seated around the table.

n (tables)	1	2	3	4	5	6
S (students)	4	6	8	10	12	14

a. If we made a sequence where the first term of the sequence was the number of students that can fit at 1 table, the 2nd term where the number that could fit at 2 tables, etc, would the sequence be arithmetic, geometric, or neither? Explain your reasoning.

It would be an arithmetic sequence because every term is 2 more than the previous term.

b. Create an explicit formula for a sequence that models this situation. How do the constants in your equation relate to the situation?

$f(n) = 4 + 2(n - 1)$

4 is the number of students that can be seated at one table by itself.

2 is the number of additional students that can be seated each time a table is added.

c. Using this seating arrangement, how many students could fit around 15 tables pushed together in a row?

$f(15) = 4 + 2(15 - 1) = 32$

The cafeteria needs to provide seating for 189 students. They can fit up to 15 rows of tables in the cafeteria. Each row can contain at most 9 tables but could contain less than that. The tables on each row must be pushed together. Students will still be seated around the tables as described earlier.

d. If they use exactly 9 tables pushed together to make each row, how many rows will they need to seat 189 students, and how many tables will they have used to make those rows?

$f(9) = 4 + 2(9 - 1) = 20$

9 tables pushed together seats 20 students.

It will take 10 rows to get enough rows to seat 189 students.

10 rows of 9 tables each is 90 tables.

e. Is it possible to seat the 189 students with fewer total tables? If so, what is the fewest number of tables needed? How many tables would be used in each row? (Remember that the tables on each row must be pushed together.) Explain your thinking.

Yes, they would use the fewest tables to seat the 189 students if they used all of the 15 rows, because with each new row, you get the added benefit of the 2 students that sit on each end of the row.

Any arrangement that uses 80 total tables spread among all 15 rows will be the best. There will be 1 extra seat, but no extra tables.

One solution that evens out the rows pretty well but still uses as few tables as possible would be **5 rows of 6 tables and 10 rows of 5 tables.**

Another example that has very uneven rows would be **8 rows of 9 tables, 1 row of 2 tables, and 6 rows of 1 table.**

2. Sydney was studying the following functions:

$$f(x) = 2x + 4 \quad \text{and} \quad g(x) = 2(2^x) + 4$$

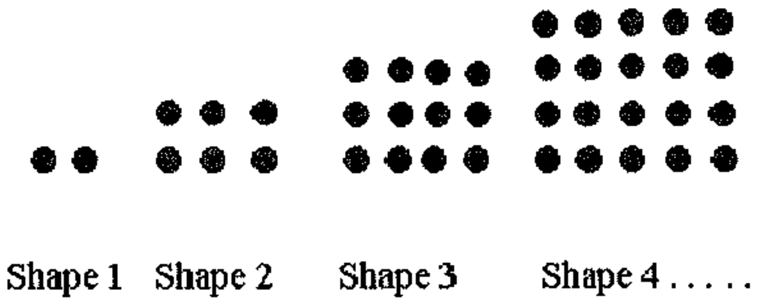
She said that linear functions and exponential functions are basically the same. She made her statement based on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and constrasting f and g. Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

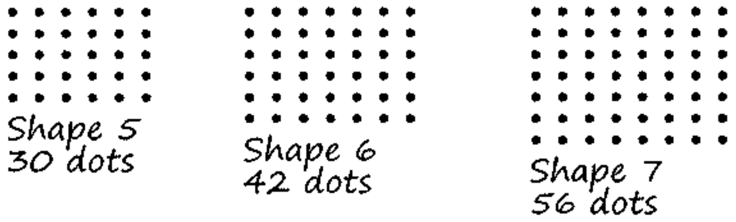
x	f(x)	Avg rate of change of f(x) from previous x-value to this one	g(x)	Avg rate of change of g(x) from previous x-value to this one
0	4		6	
1	6	2	8	2
2	8	2	12	4
3	10	2	20	8
4	12	2	36	16
5	14	2	68	32

Linear functions have a constant rate of change. f(x) increases by 2 units for every 1 unit that x increases. Exponential functions do not have a constant rate of change. The rate of change of g(x) is increasing as x increases. The average rate of change across an x interval of length 1 doubles for each successive x interval of length 1. No matter how large the rate of change is for the linear function, there is a x-value at which the rate of change for the exponential function will exceed the rate of change for the linear function.

3. Dots can be arranged in rectangular shapes like the one shown below.



a. Assuming the trend continues, draw the next three shapes in this sequence of rectangles. How many dots are in each shape?



The numbers that represent the number of dots in each rectangular shape are called rectangular numbers. For example, 2 is the first rectangular number and 6 is the 2nd rectangular number.

b. What is the 50th rectangular number? Explain how you arrived at your answer.

$50(51) = 2550$

The 1st figure had 1 row and 2 columns, giving $1(2)$ dots. The 2nd figure had 2 rows and 3 columns, giving $2(3)$ dots. The pattern for the nth figure is n rows and $n+1$ columns. So, the 50th figure will have $50(51)$ dots.

c. Write a recursive formula for the rectangular numbers.

$f(1) = 2 = 1 \cdot 2$ $\quad f(n) = f(n-1) + 2n$; natural number $n>1$, and $f(1) = 2$

$f(2) = 6 = 2 \cdot 3 = f(1) + 4$

$f(3) = 12 = 3 \cdot 4 = f(2) + 6$

$f(4) = 20 = 4 \cdot 5 = f(3) + 8$

d. Write an explicit formula for the rectangular numbers.

$f(n) = n(n+1)$; natural number $n > 0$.

e. Could an explicit formula for the n^{th} rectangular number be considered a function? Explain why or why not. If yes, what would be the domain and range of the function?

Yes, consider the domain to be all the integers greater than or equal to 1, and the range to all the rectangular numbers. Then every element in the domain corresponds to exactly one element in the range.

4. Stephen is assigning parts for the school musical.

a. Suppose there are 20 students participating, and he has 20 roles available. If each of the 20 students will be assigned to exactly one role in the play, and each role will be played by only one student, is the assignment of the roles to the students in this way certain to be an example of a function? Explain why or why not. If yes, state the domain and range of the function.

Yes, since every student gets a role and every role gets a student, and there are exactly 20 roles and 20 students, there is no possibility that a student is given more than one role, or that a role is given to more than one student. Therefore, the domain could be the list of students with the range being the list of roles, or we could consider the domain to be the list of roles and the range to be the list of students. Either way you would have an example of a function.

The school musical also has a pit orchestra.

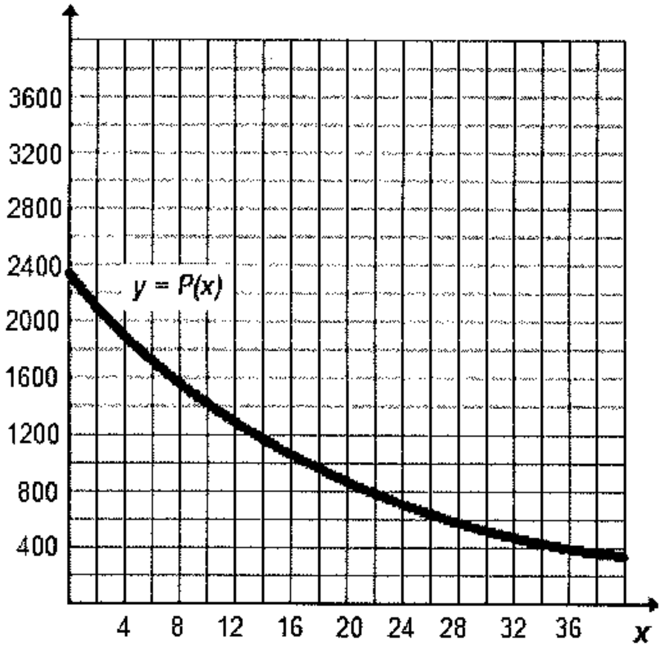
a. Suppose there are 10 instrumental parts but only 7 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some musicians will have to cover two instrumental parts, but no two musicians will have the same instrumental part. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

Yes, each element of the domain (the instrumental parts) are assigned to one and only one element in the range (the musicians).
A(Piano) = Scott means that the part of the piano is being played by Scott.

b. Suppose there are 10 instrumental parts but 13 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some instrumental parts will have two musicians assigned, so that all the musicians have instrumental parts. When two musicians are assigned to one part, they alternate who plays at each performance of the play. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

No, if the instrumental parts are the domain, then it cannot be an example of a function because there are 3 cases where one element in the domain (the instrumental parts) will be assigned to more than one element of the range (the musicians).

5. The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2350 . In 1962 the population was 1270 . They chose an exponential decay model and arrived at the function, $p(x) = 2350(.95)^x, x \geq 0,$ where x is the number of years since 1950. The graph of this function is given below.



a. What is the y-intercept of the graph? Interpret its meaning in the context of the problem.

The y-intercept is the point (0, 2350). When x is 0, there have been 0 years since 1950, so in the year 1950, the population was 2350.

b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?

There are no intervals in the domain where it is increasing. This means that the population is always decreasing, never increasing.

c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

The function is decreasing over its entire domain: $[0, \infty)$. This means that the population will continue to decline, except eventually when the function value is close to zero; then essentially the population will be zero from that point forward.

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

d. Write the linear function that this second group of scientists would have used.

$$L(x) = \frac{(1270 - 2350)}{12}x + 2350$$

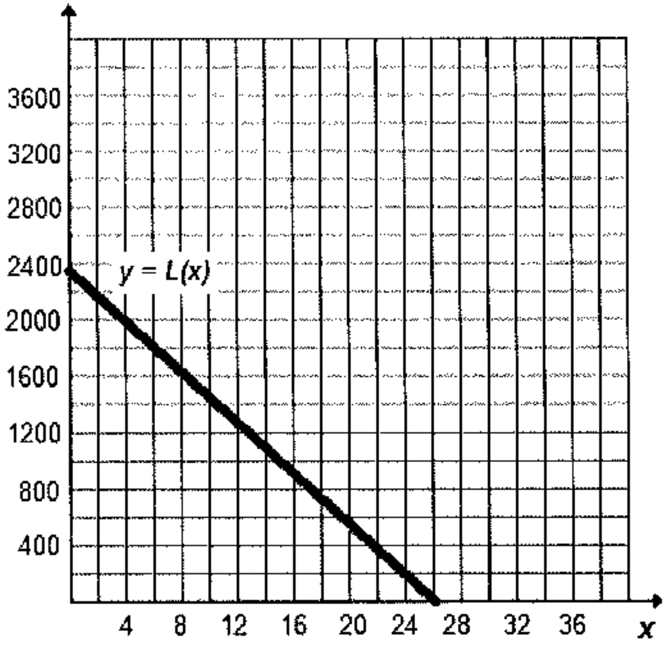
$$L(x) = -90x + 2350$$

e. What is an appropriate domain for the function? Explain your choice within the context of the problem.

D: $0 \le x \le 26\frac{1}{9}$

We are only modeling the decline of the population which scientist say started in 1950, so that means x starts at 0 years past 1950. Once the population hits zero, which occurs $26\frac{1}{9}$ years past 1950, the model no longer makes sense because population cannot be a negative number

f. Graph the function on the coordinate plane.



g. What is the x-intercept of the function? Interpret its meaning in the context of the problem.

$\left(26\frac{1}{9}, 0\right)$

At $26\frac{1}{9}$ years past 1950, in the year 1976, the population will be zero.

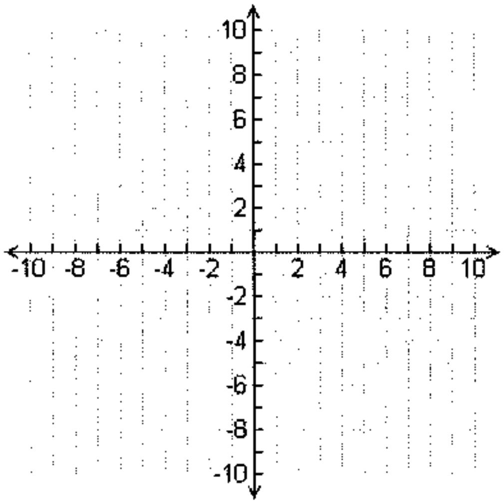
Name ____________________ Date ____________

1. Given $h(x) = |x + 2| - 3$ and $g(x) = -|x| + 4$.

   a. Describe how to obtain the graph of g from the graph of $a(x) = |x|$ using transformations.

   b. Describe how to obtain the graph of h from the graph of $a(x) = |x|$ using transformations.

   c. Sketch the graphs of $h(x)$ and $g(x)$ on the same coordinate plane.



   d. Use your graphs to estimate the solutions to the equation:
   $|x + 2| - 3 = -|x| + 4$

   Explain how you got your answer.

   e. Were your estimations you made in part (d) correct? If yes, explain how you know. If not explain why not.

2. Let f and g be the functions given by $f(x) = x^2$ and $g(x) = x|x|$.

   a. Find $f\left(\frac{1}{3}\right)$, $g(4)$, and $g(-\sqrt{3})$.

   b. What is the domain of f ?

   c. What is the range of g ?

   d. Evaluate $f(-67) + g(-67)$.

   e. Compare and contrast f and g . How are they alike? How are they different?

   f. Is there a value of x , such that $f(x) + g(x) = -100$? If so, find x . If not, explain why no such value exists.

   g. Is there a value of x such that $(x) + g(x) = 50$? If so, find x . If not, explain why no such value exists.

3. A boy bought 6 guppies at the beginning of the month. One month later the number of guppies in his tank had doubled. His guppy population continued to grow in this same manner. His sister bought some tetras at the same time. The table below shows the number of tetras, t , after n months have passed since they bought the fish.

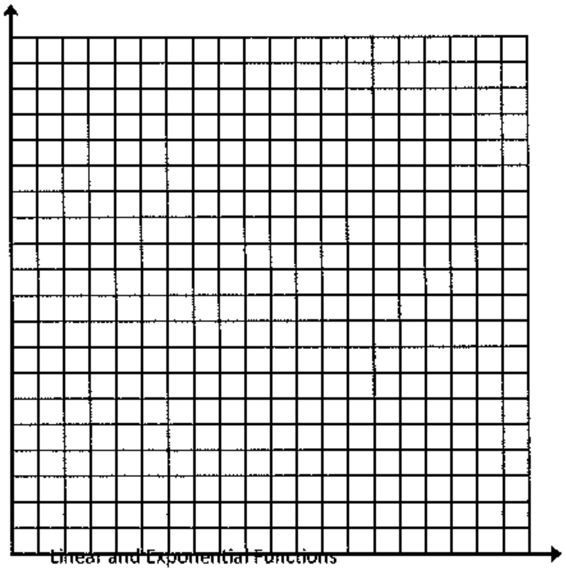
n , months	0	1	2	3
t , tetras	8	16	24	32

a. Create a function g to model the growth of the boy's guppy population, where $g(n)$ is the number of guppies at the beginning of each month, and n is the number of months that have passed since he bought the 6 guppies. What is a reasonable domain for g in this situation?

b. How many guppies will there be one year after he bought the 6 guppies?

c. Create an equation that could be solved to determine how many months after he bought the guppies there will be 100 guppies.

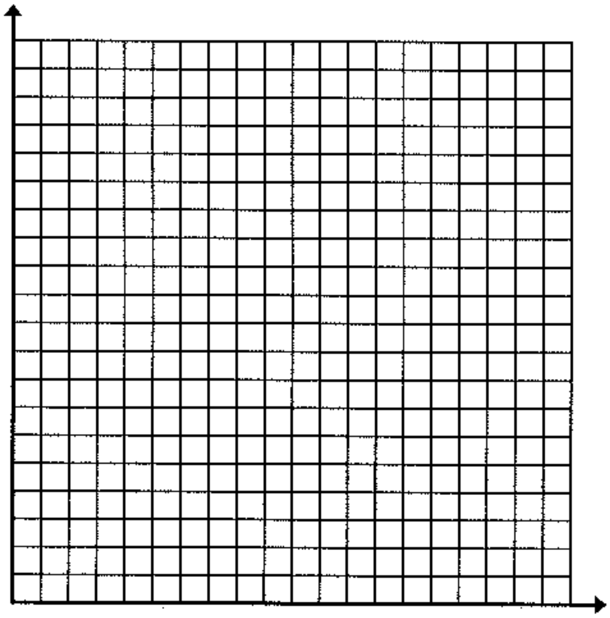
d. Use graphs or tables to approximate a solution to the equation from part (c). Explain how you arrived at your estimate.



e. Create a function, t, to model the growth of the sister's tetra population, where $t(n)$ is the number of tetras after n months have passed since she bought the tetras.

f. Compare the growth of the sister's tetra population to the growth of the guppy population. Include a comparison of the average rate of change for the functions that model each population's growth over time.

g. Use graphs to estimate the number of months that will have passed when the population of guppies and tetras will be the same.



h. Use graphs or tables to explain why the guppy population will eventually exceed the tetra population even though there were more tetras to start with.

i. Write the function $g(n)$ in such a way that the percent increase in the number of fish per month can be identified. Circle or underline the expression representing percent increase in number of fish per month.

4. Regard the solid dark equilateral triangle as figure 0. Then, the first figure in this sequence is the one composed of three dark triangles, the second figure is the one composed of nine dark triangles, and so on.


Figure 0


Figure 1


Figure 2

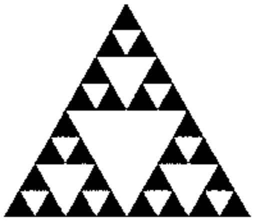


Figure 3

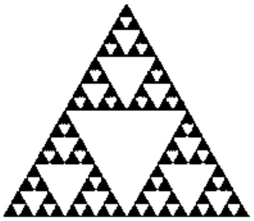


Figure 4

a. How many dark triangles are in each figure? Make a table to show this data.

n (Figure Number)					
T (# of dark triangles)					

b. Describe in words how, given the number of dark triangles in a figure, to determine the number of dark triangles in the next figure.

c. Create a function that models this sequence. What is the domain of this function?

d. Suppose the area of the solid dark triangle in Figure 0 is 1 square meter. The areas of one dark triangle in each figure form a sequence. Create an explicit formula that gives the area of just one of the dark triangles in the n^{th} figure in the sequence?

e. The sum of the areas of all the dark triangles in Figure 0 is 1 m^2; there is only one triangle in this case.

The sum of the areas of all the dark triangles in Figure 1 is $\frac{3}{4}$ m^2. What is the sum of the areas of all the dark triangles in the n^{th} figure in the sequence? Is this total area increasing or decreasing as n increases?

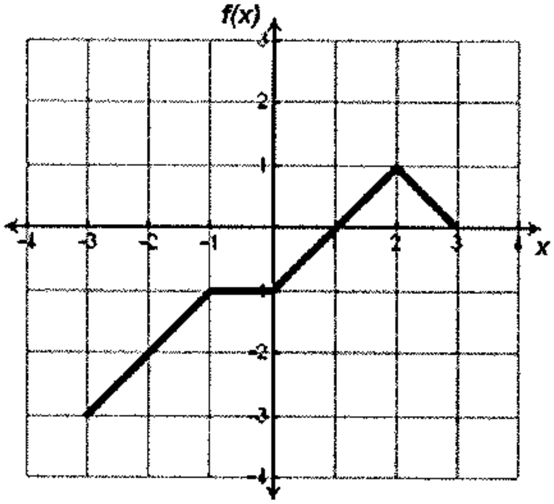
f. Let $P(n)$ be the sum of the perimeters of the all dark triangles in the n^{th} figure in the sequence of figures. There is a real number k so that:

$$P(n+1) = kP(n)$$

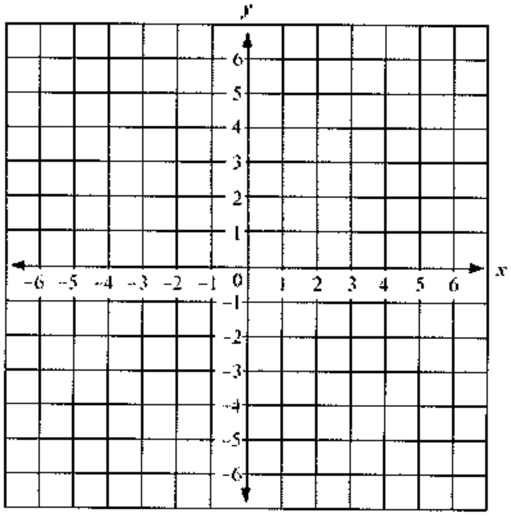
is true for each positive whole number n . What is the value of k ?

5. The graph of a piecewise function f is shown to the right. The domain of f is $-3 \le x \le 3$.

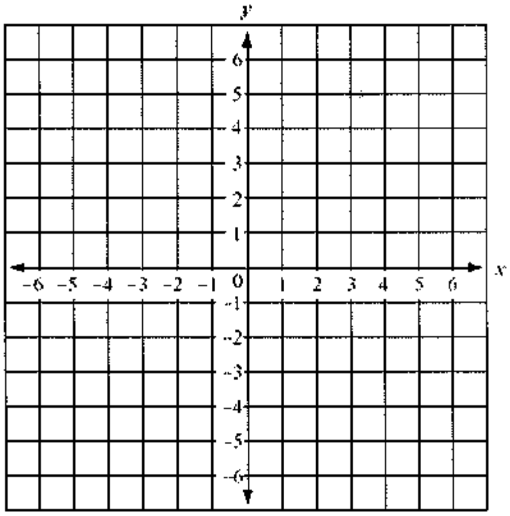
   a. Create an algebraic representation for f. Assume that the graph of f is composed of straight line segments.



   b. Sketch the graph of $y = 2f(x)$ and state the domain and range.



c. Sketch the graph of $y = f(2x)$ and state the domain and range.



d. How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

e. How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?

6.

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem
1	a F-BF.B.3	Student answer is missing or entirely incorrect.	Student's descriptions of transformations are partially correct.	Student's descriptions of the transformations are correct, but there may be some minor misuse or omission of appropriate vocabulary.	Student's descriptions of the transformations are clear and correct and use appropriate vocabulary.
	b F-BF.B.3	Student answer is missing or entirely incorrect.	Student's descriptions of transformations are partially correct.	Student's descriptions of the transformations are correct, but there may be some minor misuse or omission of appropriate vocabulary.	Student's descriptions of the transformations are clear and correct and use appropriate vocabulary.
	c-e A-REI.D.11 F-BF.B.3	Student's sketches do not resemble absolute value functions, and/or student is unable to use the graphs to estimate the solutions to the equation. Student may or may not have arrived at correct solutions of the equation via another method such as trial and error.	Student's sketches resemble the graph of an absolute value function, but are inaccurate. Student shows evidence of using the intersection point of the graphs to find the solution but is unable to confirm his or her solution points; therefore, the conclusion in (e) is inconsistent with the intersection points.	Student sketches are accurate with no more than one minor error; the student shows evidence of using the intersection points to find the solutions to the equation. The conclusion in (e) is consistent with their estimated solutions but may have one error. Student's communication is clear but could include more appropriate use of vocabulary or more detail.	Student sketches are accurate and solutions in part (d) match the x -coordinates of the intersection points. The student's explanation for part (d) reflects an understanding that the process is analogous to solving the system $y = h(x)$ and $y = g(x)$. The work shown in part (e) supports his or her conclusion that estimates were or were not solutions and

					includes supporting explanation using appropriate vocabulary.
2	a F-IF.A.2	Student provides no correct answers.	Student provides only one correct answer.	Student provides two correct answers.	Student provides correct answers for all three items.
	b – c F-IF.A.1	Neither domain nor range is correct.	One of the two is identified correctly, or the student has reversed the ideas, giving the range of f when asked for domain of f, and the domain of g when asked for the range of g.	Both domain and range are correct but notation may contain minor errors.	Both domain and range are correct and use appropriate notation.
	d F-IF.A.2	Student makes a major error or omission in evaluating the expression (e.g., doesn't substitute -67 into f or g)	Student makes one or more errors in evaluating the expression.	Student evaluates the expression correctly, but work to support the answer is limited, or there is one minor error present.	Student evaluates the expression correctly and shows the work to support their answer.
	e F-IF.A.1 F-IF.A.2 F-IF.C.7a	Student makes little or no attempt to compare the two functions.	Student's comparison does not note the similarity of the two functions yielding identical outputs for positive inputs and opposite outputs for negative inputs; it may be limited to superficial features, such as one involves squaring and the other contains an absolute value.	Student recognizes that they are equal for $x = 0$ and positive x -values but may not clearly articulate that the two functions are opposites when x is negative.	Student clearly describes when that the two functions yielding identical outputs for positive inputs, and for an input of $x = 0$, and opposite outputs for negative inputs.
	f F-IF.A.1 F-IF.A.2	Student provides an incorrect conclusion, <u>OR</u> makes little or no attempt to answer.	Student identifies that there is no solution but provides little or no supporting work or explanation.	Student identifies that there is no solution and provides an explanation, but the explanation is limited or contains minor inconsistencies or errors.	Student identifies that there is no solution and provides an explanation and/or work that clearly supports valid reasoning.
	g F-IF.A.1 F-IF.A.2	Student provides an incorrect conclusion, <u>OR</u> makes little or no attempt to answer.	Student identifies that $x = 5$ is a solution but provides little or no supporting work or explanation.	Student identifies that $x = 5$ is a solution and provides an explanation, but the explanation is limited or contains minor	Student identifies that $x = 5$ is a solution and provides an explanation and/or work that clearly supports valid reasoning.

				inconsistencies/ errors.	
3	**a** **A-CED.A.1** **F-BF.A.1a** **F-IF.B.5**	Student does not provide an exponential function, OR student provides an exponential function that does not model the data the domain is incorrect or omitted.	Student provides a correct exponential function, but the domain is incorrect or omitted, OR student provides an exponential function that does not model the data but correctly identifies the domain in this situation.	Student has made only minor errors in providing an exponential function that models the data and a domain that fits the situation.	Student provides a correct exponential function and identifies the domain to fit the situation.
	b **F-IF.A.2**	Student gives an incorrect answer with no supporting calculations.	Student gives an incorrect answer, but the answer is supported with the student's function from part (a).	Student has a minor calculation error in arriving at the answer. Student provides supporting work.	Student provides a correct answer with proper supporting work.
	c **F-BF.A.1a**	Student provides no equation or gives an equation that does not demonstrate understanding of what is required to solve the problem described.	Student sets up an incorrect equation that demonstrates limited understanding of what is required to solve the problem described.	Student provides a correct answer but then simplifies it into an incorrect equation, OR student has a minor error in the equation given but demonstrates substantial understanding of what is required to solve the problem.	Student provides a correct equation that demonstrates understanding of what is required to solve the problem.
	d **F-IF.A.2**	Student provides an equation or graph that does not reflect the correct data, OR student fails to provide an equation or graph.	Student provides a correct graph or table, but the answer to the question is either not given or incorrect.	Student provides a correct table or graph, but the answer is 4 months with an explanation that the 100 mark occurs during the 4th month.	Student provides a correct table or graph, AND the answer is correct (5 months) with a valid explanation.
	e **F-BF.A.1a**	Student does not provide a linear function.	Student provides a function is linear but does not reflect data.	Student provides a correct linear function, but the function was either simplified incorrectly or does not use the notation, $t(n)$.	Student provides a correct linear function using the notation, $t(n)$.
	f **A-CED.A.2** **F-IF.B.6**	Student does not demonstrate an ability to recognize and distinguish between linear and exponential growth or to compare	Student makes a partially correct but incomplete comparison of growth rates that does not include or incorrectly applies the	Student makes a correct comparison of growth rates that includes an analysis of the rate of change of each function. However, student's	Student identifies that the Guppies' population will increase at a faster rate and provides a valid explanation that includes an analysis of the rate of

	F-LE.A.3	growth rates or average rate of change of functions.	concept of average rate of change.	communication contains minor errors or misuse of mathematical terms.	change of each function.
	g A-REI.D.11 F-IF.A.2 F-IF.C.9	Student does not provide correct graphs of the functions and is unable to provide an answer that is based on reasoning.	Student provides correct graphs but is unable to arrive at a correct answer from the graphs, OR student's graphs are incomplete or incorrect, but the student arrives at an answer based on sound reasoning.	Student provides graphs that contain minor imprecisions and therefore arrives at an answer that is supportable by the graphs but incorrect.	Student provides correct graphs and arrives at an answer that is supportable by the graphs and correct.
	h F-IF.B.6 F-LE.A.1 F-LE.A.3	Student does not provide tables or graphs that are accurate enough to support an answer, and shows little reasoning in an explanation.	Student provides tables or graphs that are correct but provides limited or incorrect explanation of results.	Student provides tables or graphs that are correct and gives an explanation that is predominantly correct but contains minor errors or omissions in the explanation.	Student provides tables or graphs that are correct and gives a complete explanation that uses mathematical vocabulary correctly.
	i A-SSE.B.3c	Student does not provide an exponential function that shows percent increase.	Student writes an exponential function that uses an incorrect version of the growth factor, such as 0.02, 2%, 20%, or 0.20.	Student creates a correct version of the function using a growth factor expressed as 200% or expressed as 2 with a note that 2 is equivalent to 200%. Student has a minor error in notation, or in the domain, or does not specify the domain.	Student creates a correct version of the function using a growth factor expressed as 200% or expressed as 2 with a note that 2 is equivalent to 200%. Student specifies the domain correctly.
4	a – c F-BF.A.1a F-IF.A.3 F-LE.A.1 F-LE.A.2	Student does not fill in the table correctly and does not describe the relationship correctly. Student does not provide an exponential function.	Student completes the table correctly and describes the sequence correctly but gives an incorrect function. Student may or may not have given a correct domain.	Student completes the table correctly, and describes the sequence correctly, but has a minor error in either his or her function or domain. The function provided is exponential with a growth factor of 3. Description or notation may contain minor errors.	Student completes the table correctly, describes the sequence correctly, and provides a correct exponential function including the declaration of the domain. Student uses precise language and proper notation (either function or subscript notation) for the function.

	d **F-BF.A.1a** **F-LE.A.1** **F-LE.A.2**	Student fails to provide an explicit exponential formula.	Student provides an explicit formula that is exponential but incorrect; supporting work is missing or reflects limited reasoning about the problem.	Student provides a correct explicit exponential formula. Notation or supporting work may contain minor errors.	Student provides a correct explicit exponential formula using function or subscript notation; formula and supporting work are free of errors.
	e **F-BF.A.1a** **F-LE.A.1** **F-LE.A.2**	Student fails to provide an explicit exponential formula.	Student provides an explicit formula that is exponential but incorrect; supporting work is missing or reflects limited reasoning about the problem.	Student provides a correct explicit exponential formula. Notation or supporting work may contain minor errors.	Student provides a correct explicit exponential formula using function or subscript notation; formula and supporting work are free of errors.
	f **F-BF.A.1a** **F-LE.A.1**	Student provides little or no evidence of understanding how to determine the perimeter of the dark triangles nor how to recognize the common factor between two successive figures' perimeter.	Student's value of k is incorrect or not provided, but solution shows some understanding of how to determine the perimeter of the dark triangles.	Student's solution shows significant progress towards identifying that k is 3/2 but contains minor errors or is not complete. <u>OR</u> student computes an incorrect k value due to a minor error but otherwise demonstrates a way to determine k either by recognizing that the given equation is a recursive form of a geometric sequence or by approaching the problem algebraically.	Student identifies the correct value of k with enough supporting evidence of student thinking (correct table, graph, marking on diagram, or calculations) that shows how her or she arrived at the solution.
5	**a** **F-BF.A.1a**	Student does not provide a piecewise definition of the function and/or more than two expressions in the answer are incorrect.	Student provides a piecewise function in which at least one of the expressions is correct, the solution may contain errors with the intervals or notation.	Student provides a piecewise function with correct expressions, but the answer may contain minor errors with the intervals or use of function notation. OR one expression is incorrect, but intervals and use of function notation is correct.	Student provides a correctly defined piecewise function with correct intervals.
	b – c **F-BF.B.3** **F-IF.A.1**	Student's graphs contain major errors; domain and range are missing or are inconsistent with the graphs.	Student's graph for (b) would be correct for (c) and vice versa, OR student answers either (b) or (c) correctly. Minor errors may exist in	Student's graphs contain one minor error. The domain and range are consistent with the graphs.	Student provides correct graphs for both (b) and (c) and provides a domain and range for each that are consistent with student graphs.

			the domain and range.		
	d – e **F-BF.B.3** **F-IF.A.1**	Both explanations and solutions are incorrect or have major conceptual errors (e.g., confusing domain and range).	Student answers contain more than one minor error, OR student answers only one of (d) and (e) correctly.	Student answer only explains how the domain/range changes; it may contain one minor error.	Student answer not only explains how the domain/range changes, but also explains how knowing $k > 1$ aids in finding the new domain/range.

Name ______________________________ Date ____________________

1. Given $h(x) = |x + 2| - 3$ and $g(x) = -|x| + 4$.

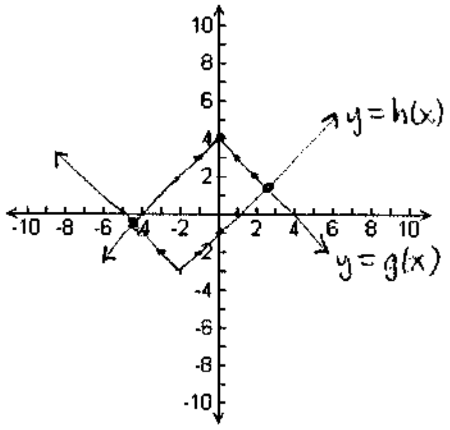
   a. Describe how to obtain the graph of g from the graph of $a(x) = |x|$ using transformations.

   To obtain the graph of g, reflect the graph of a about the x-axis and translate this graph up 4 units.

   b. Describe how to obtain the graph of h from the graph of $a(x) = |x|$ using transformations.

   To obtain the graph of h, translate the graph of a 2 units to the left and 3 units down.

   c. Sketch the graphs of $h(x)$ and $g(x)$ on the same coordinate plane.



d. Use your graphs to estimate the solutions to the equation:
$|x + 2| - 3 = -|x| + 4$

Explain how you got your answer.

Solution: $x \approx 2.5$ or $x \approx -4.5$

The solutions are the x-coordinates of the intersection points of the graphs of g and h.

e. Were your estimations you made in part (d) correct? If yes, how do you know? If not explain why not.

Let $x = 2.5$	Let $x = -4.5$
Is $\|2.5 + 2\| - 3 = -\|2.5\| + 4$ true?	Is $\|-4.5 + 2\| - 3 = -\|4.5\| + 4$ true?
Yes, $4.5 - 3 = -2.5 + 4$ is true.	Yes, $2.5 - 3 = -4.5 + 4$ is true.

Yes, the estimates are correct. They each make the equation true.

2. Let f and g be the functions given by $f(x) = x^2$ and $g(x) = x|x|$.

a. Find $f\left(\frac{1}{3}\right)$, $g(4)$, and $g(-\sqrt{3})$.

$f(1/3) = 1/9$, $g(4) = 16$, $g(-\sqrt{3}) = -3$

b. What is the domain of f ?

D: all real numbers.

c. What is the range of g ?

R: all real numbers.

d. Evaluate $f(-67) + g(-67)$.

$(-67)^2 + -67|-67| = 0.$

e. Compare and contrast f and g. How are they alike? How are they different?

When x is positive, both functions give the same value. But when x is negative, f gives the always positive value of x^2, whereas g gives a value that is the opposite of what f gives.

f. Is there a value of x, such that $f(x) + g(x) = -100$? If so, find x. If not, explain why no such value exists.

No, f and g are either both zero, giving a sum of zero, both positive, giving a positive sum, or the opposite of each other, giving a sum of zero. So, there is no way to get a negative sum.

g. Is there a value of x such that $(x) + g(x) = 50$? If so, find x. If not, explain why no such value exists.

Yes, if $x = 5$, $f(x) = g(x) = 25$, thus $f(x) + g(x) = 50$.

3. A boy bought 6 guppies at the beginning of the month. One month later the number of guppies in his tank had doubled. His guppy population continued to grow in this same manner. His sister bought some tetras at the same time. The table below shows the number of tetras, t, after n months have passed since they bought the fish.

n, months	0	1	2	3
t, tetras	8	16	24	32

a. Create a function *g* to model the growth of the boy's guppy population, where *g*(*n*) is the number of guppies at the beginning of each month and *n* is the number of months that have passed since he bought the 6 guppies. What is a reasonable domain for *g* in this situation?

$g(n) = 6 \cdot 2^n$ *Domain: n is a whole number.*

b. How many guppies will there be one year after he bought the 6 guppies?

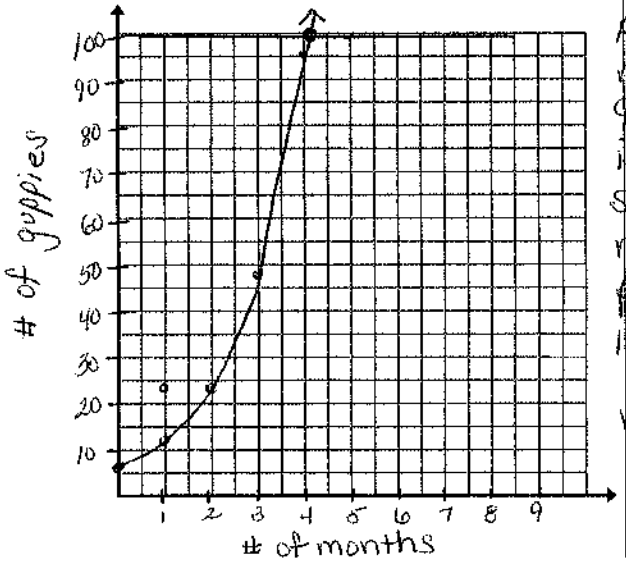
$g(12) = 6 \cdot 2^{12} = 24{,}576$ guppies

c. Create an equation that could be solved to determine how many months after he bought the guppies there will be 100 guppies.

$100 = 6 \cdot 2^n$

d. Use graphs or tables to approximate a solution to the equation from part c. Explain how you arrived at your estimate.

n	g(n)
0	6
1	12
2	24
3	48
4	96
5	192



At the end of 4 months, there are 96 guppies which is not quite 100, so during the 5th month, the guppie population reaches 100.

n = 5 months

e. Create a function, t, to model the growth of the sister's tetra population, where $t(n)$ is the number of tetras after n months have passed since she bought the tetras.

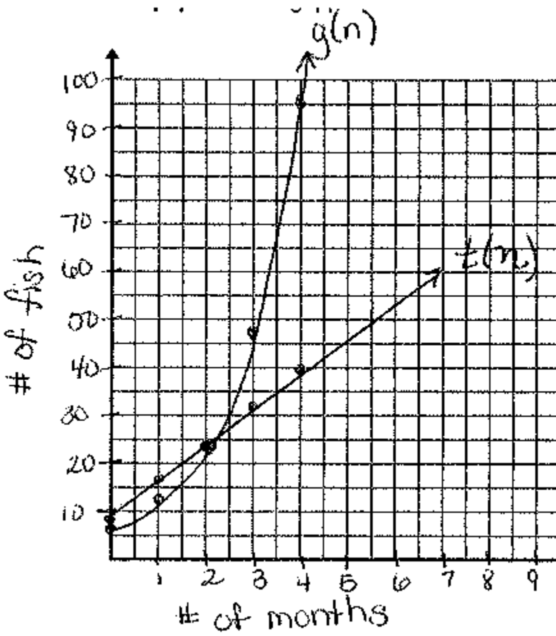
$t(n) = 8(n+1)$, n is a whole number.

Or, $t(n) = 8n + 8$, n is a whole number.

f. Compare the growth of the sister's tetra population to the growth of the guppy population. Include a comparison of the average rate of change for the functions that model each population's growth over time.

The guppies' population is increasing faster than the tetras' population. Each month, the number of guppies doubles, while the number of tetra's increases by 8. The rate of change for the tetras is constant, but the rate of change for the guppies is always increasing.

g. Use graphs to estimate the number of months that will have passed when the population of guppies and tetras will be the same.



g(2) = t(2) = 24

The guppies and tetras populations will be the same, 24, after 2 months.

h. Use graphs or tables to explain why the guppy population will eventually exceed the tetra population even though there were more tetras to start with.

The guppy population's growth is exponential, and the tetra populations' growth is linear. The graph in part (g) shows how the population of the guppies eventually overtakes the population of the tetras. The table below shows that by the end of the 3rd month, there are more guppies than tetras.

n	0	1	2	3	4	5	
g(n)	6	12	24	48	96	192	The average rate of change is doubling.
t(n)	8	16	24	32	40	48	The rate of change is constant.

i. Write the function $g(n)$ in such a way that the percent increase in the number of fish per month can be identified. Circle or underline the expression representing percent increase in number of fish per month.

$g(n) = 6(\underline{200\%})^n$

4. Regard the solid dark equilateral triangle as figure 0. Then, the first figure in this sequence is the one composed of three dark triangles, the second figure is the one composed of nine dark triangles, and so on.


Figure 0


Figure 1


Figure 2

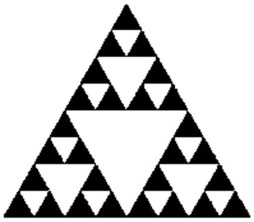


Figure 3

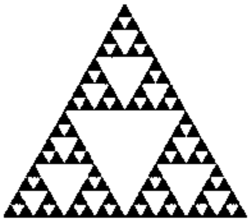


Figure 4

a. How many dark triangles are in each figure? Make a table to show this data.

n (Figure Number)	0	1	2	3	4
T (# of dark triangles)	1	3	9	27	81

b. Describe in words how, given the number of dark triangles in a figure, to determine the number of dark triangles in the next figure.

The number of triangles in a figure is 3 times the number of triangles in the previous figure.

c. Create a function that models this sequence. What is the domain of this function?

$T(n) = 3^n$, D: n is a whole number.

d. Suppose the area of the solid dark triangle in Figure 0 is 1 square meter. The areas of one dark triangle in each figure form a sequence. Create an explicit formula that gives the area of just one of the dark triangles in the n^{th} figure in the sequence?

Figure, n	Area of one dark triangle, $A(n)$
0	1
1	1/4
2	1/16
3	1/64

$A(n) = \left(\frac{1}{4}\right)^n$

e. The sum of the areas of all the dark triangles in Figure 0 is 1 m^2; there is only one triangle in this case. The sum of the areas of <u>all</u> the dark triangles in Figure 1 is ¾ m^2. What is the sum of the areas of <u>all</u> the dark triangles in the n^{th} figure in the sequence? Is this total area increasing or decreasing as n increases?

Figure	Area in m²
0	1
1	3/4
2	9/16
3	27/64

$$T(n) = \left(\frac{3}{4}\right)^n$$

The total area is decreasing as n increases

f. Let $P(n)$ be the sum of the perimeters of the <u>all</u> dark triangles in the n^{th} figure in the sequence of figures. There is a real number k so that:

$$P(n+1) = kP(n)$$

is true for each positive whole number n . What is the value of k ?

Let x represent the number of meters long of one side of the triangle in Figure 0.

Figure	P(n)
0	$3x$
1	$3x + \frac{3}{2}x = \frac{9}{2}x$
2	$\frac{9}{2}x + \frac{9}{4}x = \frac{27}{4}x$

P is a geometric sequence and k is the ratio between any term and the previous term, so k = P(n+1)/P(n).

So, for example, for n = 0, $k = \frac{P(1)}{P(0)} = \frac{\frac{9}{2}x}{3x} = \frac{3}{2}$

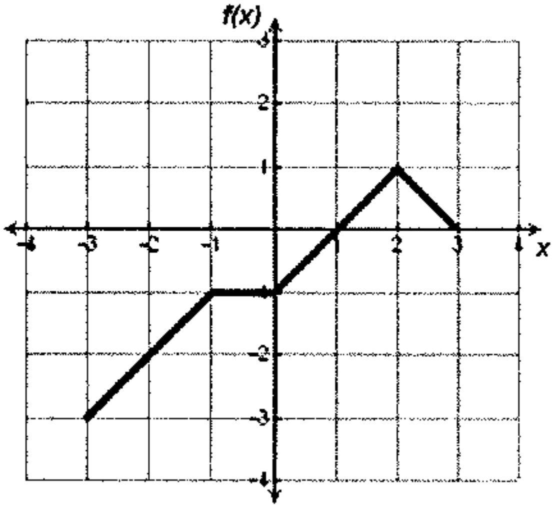
For n = 1, $k = \frac{P(2)}{P(1)} = \frac{\frac{27}{4}x}{\frac{9}{2}x} = \frac{3}{2}$

5. The graph of a piecewise –defined function f is shown to the right. The domain of f is $-3 \le x \le 3$.

a. Create an algebraic representation for f . Assume that the graph of f is composed of straight line segments.

$$f(x) = \begin{cases} x, & -3 \le x < -1 \\ -1, & -1 \le x < 0 \\ x-1, & 0 \le x < 2 \\ -x+3, & 2 \le x \le 3 \end{cases}$$

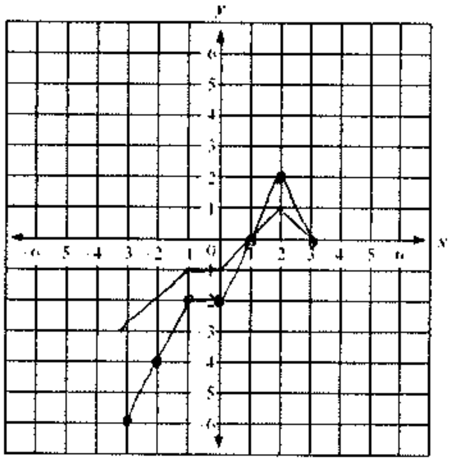
or $$f(x) = \begin{cases} x, & -3 \le x < -1 \\ -1, & -1 \le x < 0 \\ -|x-2|+1, & 0 \le x \le 3 \end{cases}$$



b. Sketch the graph of $y = 2f(x)$ and state the domain and range.

Domain: $-3 \le x \le 3$

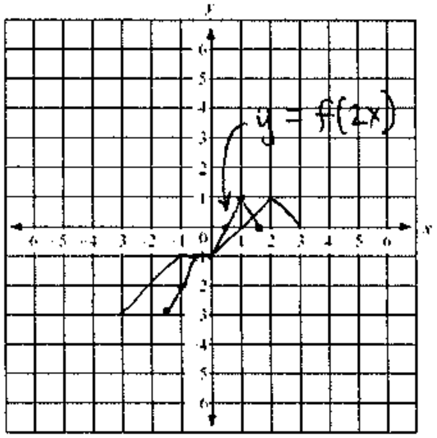
Range: $-6 \le y \le 2$



c. Sketch the graph of $y = f(2x)$ and state the domain and range.

Domain: $-1.5 \le x \le 1.5$

Range: $-3 \le y \le 1$



d. How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

Every value in the range of $y = f(x)$ would be multiplied by k. Since $k>1$ we can represent this by multiplying the compound inequality that gives the range of $y = f(x)$ by k, giving $-3k \le y \le k$.

e. How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?

Every value in the domain of $y = f(x)$ would be divided by k. Since $k>1$ we can represent this by multiplying the compound inequality that gives the domain of $y = f(x)$ by $1/k$, giving $-3/k \le x \le 3/k$.

UNIT THREE

for

Content Area of

MATHEMATICS

HS Band
Algebra I

New York State Common Core

Mathematics Curriculum

Table of Contents[1]

ALGEBRA I • MODULE 4

Polynomial and Quadratic Expressions, Equations, and Functions

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[1] Each lesson is ONE day and ONE day is considered a 45 minute period.

Algebra I • Module 4

Polynomial and Quadratic Expressions, Equations, and Functions

OVERVIEW

By the end of middle school, students are familiar with linear equations in one variable (**6.EE.B.5, 6.EE.B.6, 6.EE.B.7**) and have applied graphical and algebraic methods to analyze and manipulate equations in two variables (**7.EE.A.2**). They used expressions and equations to solve real-life problems (**7.EE.B.4**). They have experience with square and cube roots, irrational numbers (**8.NS.A.1**), and expressions with integer exponents (**8.EE.A.1**).

In Grade 9, students have been analyzing the process of solving equations and developing fluency in writing, interpreting, and translating between various forms of linear equations (Module 1) and linear and exponential functions (Module 3). These experiences combined with modeling with data (Module 2), set the stage for Module 4. Here students continue to interpret expressions, create equations, rewrite equations and functions in different but equivalent forms, and graph and interpret functions, but this time using polynomial functions, and more specifically quadratic functions, as well as square root and cube root functions.

Topic A introduces polynomial expressions. In Module 1, students learned the definition of a polynomial and how to add, subtract, and multiply polynomials. Here their work with multiplication is extended and then, connected to factoring of polynomial expressions and solving basic polynomial equations (**A-APR.A.1, A-REI.D.11**). They analyze, interpret, and use the structure of polynomial expressions to multiply and factor polynomial expressions (**A-SSE.A.2**). They understand factoring as the reverse process of multiplication. In this topic, students develop the factoring skills needed to solve quadratic equations and simple polynomial equations by using the zero-product property (**A-SSE.B.3a**). Students transform quadratic expressions from standard or extended form, $ax^2 + bx + c$, to factored form and then solve equations involving those expressions. They identify the solutions of the equation as the zeros of the related function. Students apply symmetry to create and interpret graphs of quadratic functions (**F-IF.B.4, F-IF.C.7a**). They use average rate of change on an interval to determine where the function is increasing/decreasing (**F-IF.B.6**). Using area models, students explore strategies for factoring more complicated quadratic expressions, including the product-sum method and rectangular arrays. They create one- and two-variable equations from tables, graphs, and contexts and use them to solve contextual problems represented by the quadratic function (**A-CED.A.1, A-CED.A.2**) and relate the domain and range for the function, to its graph, and the context (**F-IF.B.5**).

In Topic B, students apply their experiences from Topic A as they transform standard form quadratic functions into the completed square form $f(x) = a(x - h)^2 + k$ (sometimes referred to as the vertex form). Known as, *completing the square*, this strategy is used to solve quadratic equations when the quadratic expression cannot be factored (**A-SSE.B.3b**). Students recognize that this form reveals specific features of quadratic functions and their graphs, namely the *minimum* or *minimum of the function* (the vertex of the graph) and the line of symmetry of the graph (**A-APR.B.3, F-IF.B.4, F-IF.C.7a**). Students derive the quadratic formula by completing the square for a general quadratic equation in standard form ($y = ax^2 + bx + c$) and use it to determine the nature and number of solutions for equations when y equals zero (**A-SSE.A.2, A-REI.B.4**). For quadratics with irrational roots students use the quadratic formula and explore the properties of irrational numbers (**N-RN.B.3**). With the added technique of completing the square in their toolboxes, students come to see the structure of the equations in their various forms as useful for gaining insight into the features of the graphs of equations (**A-SSE.B.3**). Students study business applications of quadratic functions as they create quadratic equations and/or graphs from tables and contexts and use them to solve problems involving profit, loss, revenue, cost, etc. (**A-CED.A.1, A-CED.A.2, F-IF.B.6, F-IF.C.8a**). In addition to applications in business, they also solve physics-based problems involving objects in motion. In doing so, students also interpret expressions and parts of expressions, in context and recognize when a single entity of an expression is dependent or independent of a given quantity (**A-SSE.A.1**).

In Topic C, students explore the families of functions that are related to the parent functions, specifically for quadratic ($f(x) = x^2$), square root ($f(x) = \sqrt{x}$), and cube root ($f(x) = \sqrt[3]{x}$), to perform first horizontal and vertical translations and shrinking and stretching the functions (**F-IF.C.7b, F-BF.B.3**). They recognize the application of transformations in the vertex form for the quadratic function and use it to expand their ability to efficiently sketch graphs of square and cube root functions. Students compare quadratic, square root, or cube root functions in context, and each represented in different ways (verbally with a description, as a table of values, algebraically, or graphically). In the final two lessons, students are given real-world problems of quadratic relationships that may be given as a data set, a graph, described relationship, and/or an equation. They choose the most useful form for writing the function and apply the techniques learned throughout the module to analyze and solve a given problem (**A-CED.A.2**), including calculating and interpreting the rate of change for the function over an interval (**F-IF.B.6**).

Focus Standards

Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.*

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.*

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.[2]

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.[3]

Create equations that describe numbers or relationships.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★[4]

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

Solve equations and inequalities in one variable.

A-REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in x into

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[2] In Algebra I, tasks are limited to numerical expressions and polynomial expressions in one variable. Examples: Recognize that $53^2 - 47^2$ is the difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. In preparation for Regents Exams, does not include factoring by grouping and factoring the sum and difference of cubes.

[3] In Algebra I, tasks are limited to quadratic and cubic polynomials, in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.

[4] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b[5].

Represent and solve equations and inequalities graphically.

A-REI.D.11[6] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[7] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[8] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

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[5] Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require that students recognize cases in which a quadratic equation has no real solutions.

[6] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.

[7] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

[8] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9[9] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build new functions from existing functions.

F-BF.B.3[10] Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Foundational Standards

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Reason quantitatively and use units to solve problems.

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[9] In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

[10] In Algebra I, identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions. The focus in this module is on linear and exponential functions.

N-Q.A.2[11] Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Create equations that describe numbers or relationships.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*★

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Build a function that models a relationship between two quantities.

F-BF.A.1[12] Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

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[11] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6–8) require the student to create a quantity of interest in the situation being described.

[12] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 4, students make sense of problems by analyzing the critical components of the problem, a verbal description, data set, or graph and persevere in writing the appropriate function to describe the relationship between two quantities.

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpreting the quantities in the relationship in terms of the context. Students must be able to *decontextualize* – to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents; and then to *contextualize* – to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations.

MP.4 **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic C, students use the full modeling cycle. They model quadratic functions presented mathematically or in a context. They explain the reasoning used in their writing and/or by use of appropriate tools, such as graphing paper, graphing calculator, or computer software.

MP.5 **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module, students must decide whether to use a tool to help find a solution. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root functions), and are sometimes required to perform procedures that, when performed without technology, can be tedious and, thus, distract them from thinking mathematically. (e.g., completing the square with non-integer coefficients) In such cases, students must decide when to use a tool to help with the calculation or graph so they can better analyze the model.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Module 4, students must be precise in choose the appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing, they must select an appropriate scale.

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. They can see complicated things, such as some algebraic expressions, single objects, or being composed of several objects. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression $x^2 + 9x + 14$, students must see the 14 as 2 × 7 and the 9 as 2 + 7 to find the factors of the quadratic. In relating an equation to a graph, they can see $y = -3(x - 1)^2 + 5$ as 5 added to a negative number times a square and thus realize that its value cannot be more than 5 for any real domain value.

Terminology

New or Recently Introduced Terms

- **Degree of a monomial term** (the sum of the exponents of the variables that appear in a term of a polynomial)
- **Degree of a polynomial** (the highest degree of the terms of a polynomial gives its degree to the polynomial)
- **Leading coefficient** (the coefficient on the term of highest degree in a polynomial expression, which effects the shape and end behavior of the graph of the function represented by the expression)
- **Parent function** (a basic function used as a building block for more complicated functions)
- **Quadratic function** (a polynomial function of degree 2)
- **Cubic function** (a polynomial function of degree 3)
- **Square root function** (the parent function: $f(x) = \sqrt{x}$)
- **Cube root function** (the parent function: $f(x) = \sqrt[3]{x}$)
- **Standard form for a quadratic function** (e.g., $f(x) = ax^2 + bx + c$)
- **Factored form for a quadratic function** (e.g., $f(x) = (x - n)(x - m)$)
- **Vertex form** (completed-square form) for a quadratic function (e.g., $f(x) = a(x - h)^2 + k$)
- **Roots of a polynomial function** (the domain values for a polynomial function that, when substituted for the variable, make the value of the polynomial function equal zero)
- **Axis of symmetry** (Given a quadratic function in standard form, $f(x) = ax^2 + bx + c$, the vertical line given by the graph of the equation, $x = -\frac{b}{2a}$, is called the *axis of symmetry* of the graph of the

quadratic function.)

- **Vertex** (The point where the graph of a quadratic function and its axis of symmetry intersect is called the *vertex*.)
- **End behavior** (Given a quadratic function in the form $f(x) = ax^2 + bx + c$ (or $f(x) = a(x - h)^2 + k$), the quadratic function is said to *open up* if $a > 0$ and *open down* if $a < 0$.)
- **Quadratic formula** (the formula that emerges from solving the general form of a quadratic equation by completing the square; it can be used to solve any quadratic equation)
- **Discriminant** (An expression defined using the coefficients of a quadratic equation in the form $ax^2 + bx + c = 0$ that can be used to identify the nature of the roots of a quadratic equation)

Familiar Terms and Symbols[13]

- Solutions (solution set) of an equation
- Zeros of a function
- Integers
- Rational numbers
- Irrational numbers
- Real numbers
- Solution set
- Coefficient
- Term
- Factor
- Power
- Square root
- Cube root
- Closed
- Closure
- Monomial
- Binomial
- Trinomial
- Degree of a polynomial
- Quadratic
- Cubic
- Average rate of change
- Explicit expression
- Parabola

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[13] These are terms and symbols students have seen previously.

- Domain and range
- Recursive process

Suggested Tools and Representations

- Graphing calculator
- Graph paper
- Coordinate Plane
- Equations

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-APR.A.1, A-REI.B.4b, A-REI.D.11, A-CED.A.1, A-CED.A.2, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	N-RN.B.3, A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-SSE.B.3b, A-APR.B.3, A-REI.B.4, A-CED.A.1, A-CED.A.2, F-IF.4, F-IF.6, F-IF.C.7, F-IF.C.8a, F-IF.C.9, F-BF.B.3

Name ______________________________ Date ________________

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of x for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.

   a. Perimeter:

   b. Area:

   c. Are both your answers polynomials? Explain why or why not for each.

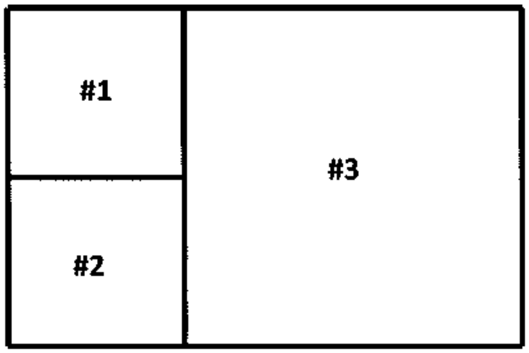
   d. Is it possible for the perimeter of the rectangle to be 16 units? If so what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

e. For what value(s) of the domain will the area equal zero?

f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares #1 and #2 in the figure below. All three shapes are squares. The area of square #1 equals that of square #2 and each can be represented by the expression $4x^2 - 8x + 4$.



   a. Find the side length of the father's plot, square #3, and show or explain how you found it.

   b. Find the area of the father's plot and show or explain how you found it.

   c. Find the total area of all three plots by adding the three areas and verify your answer by multiplying the outside dimensions. Show your work.

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle, so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from the time it was thrown could be modeled closely by the function,

$$h(t) = -16t^2 + 64t + 80$$

where $h(t)$ represents the height of the ball in feet after t seconds.

a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

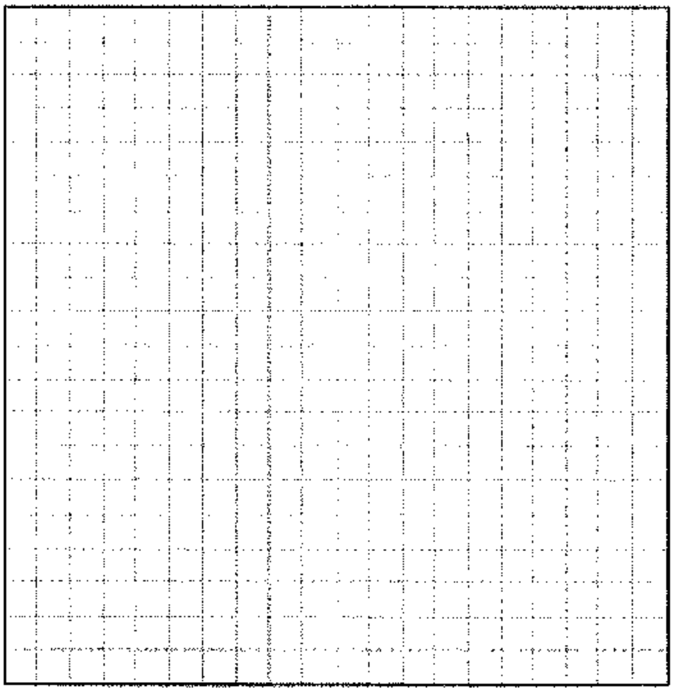
c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

e. How long is the ball in the air? Explain your answer.

f. State the domain of the function and explain the restrictions on the domain based on the context of the problem.

g. Graph the function indicating the vertex, axis of symmetry, intercepts and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.



h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, or an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a–b** **A-APR.A.1**	There is little evidence of understanding of the properties of polynomial operations. Little or no attempt made or attempt aborted before calculations completed.	There is some evidence of understanding of operations with polynomials but there are errors in the calculations (e.g., side lengths are not doubled for perimeter or the product is missing terms). The final answers are not given in standard polynomial form.	The expressions are treated accurately and appropriately with operations that relate to perimeter and area that are carried out correctly. The final answers are not given in standard polynomial form.	The expressions are treated accurately and appropriately with operations that relate to perimeter and area that are carried out correctly. Final answers are in simplest and standard polynomial form.
	c **A-APR.A.1**	No attempt is made.	There is an attempt to explain but the explanation shows little understanding of the definition of a polynomial and of the concept of closure for	The explanation shows some understanding of the definition of a polynomial or of the concept of closure for polynomial operations.	The explanation is correct and includes understanding of the definition of a polynomial and of closure for polynomial operations.

			polynomial operations.		
	d **A-SSE.B.3a** **A-CED.A.1** **A-REI.D.11**	There is an answer (yes or no) with no supporting explanation. No attempt is made.	An equation is created and solved with a logical explanation for the process and solution. However, the equation may be incorrect or there are errors in calculation that lead to an incorrect final answer and incorrect values for x .	An equation is created and solved correctly with a logical explanation for the process and solution. However both values of x are given as the final solution, with some explanation offered as justification.	An equation is created and solved correctly with a logical explanation for the process and solution, which includes only the correct value of x that works in the equation, with the extraneous solution noted in the explanation.
	e **A-SSE.B.3a** **A-REI.D.11**	There is an attempt to answer this question but the original equation representing the area is not factored correctly and no correct results were found. OR No attempt was made.	There is an attempt to factor the original form of the equation representing the area and it is set equal to zero. There is one (or no) correct result given.	There is an attempt to factor the original form of the equation representing the area and it is set equal to zero. Two correct results are given.	The equation is accurately factored into its four linear factors and set equal to zero. It is correctly solved for the four values of x that make the product equal to zero.
	f **A-SSE.B.3a** **A-CED.A.1** **A-REI.D.11**	No attempt was made to find the two values or the attempt was aborted before a conclusion could be reached.	Only one correct value is given and checked effectively. Two values are given but only one is checked effectively. Two logically selected values are given but the checks attempted are ineffective for both.	Two values are correctly selected and substituted into the equation. There are calculation errors in the check that do not affect the final outcome.	Two values are correctly selected and substituted into the equation to check whether the x -value produces a positive area. (NOTE: The zeros found in part (e) might be used as boundaries for the correct values in this part.)

	g A-SSE.B.3a A-APR.A.1 A-REI.D.11	Little or no attempt is made to answer this question.	There is an attempt made to answer the question. However, the explanation is missing important parts: there are no references to the dimensions being positive or to the requirement that there must be an even number of negative factors for area. There might be specific examples of negative values that produce a positive area given but without explanation.	The question is answered correctly but is based on a partially correct explanation: The explanation is missing a reference to the need for an even number of negative factors in the area expression or that both dimensions must be positive (if two factors are negative, they must both represent the same dimension).	The question is answered correctly and completely, including the following: There are references in the explanation to the need for positive dimensions, and that if an x -value makes any of the factors negative, there must be an even number of negative factors. This means that both negative factors must be for the same dimension.
2	a–b A-SSE.A.1 A-SSE.A.2 A-SSE.B.3 A-APR.A.1	There is no evidence to indicate a connection is made between the information given in the prompt and the side lengths of squares 1 and 2.	There is evidence to indicate a connection is made between the information in the prompt and the side lengths of squares 1 and 2. However, there is no evidence that a connection is made to the side length of square 3 and the operations needed to answer the questions. Calculations contain errors and the explanation is missing or inadequate.	There is evidence of understanding of the connection between the information given in the prompt and the side length and area of square 3. However, calculations are completed accurately but the explanations are incomplete. OR Calculations contain errors but the explanation is adequate and is not dependent on errors in the calculations.	There is evidence of understanding of the connection between the information given in the prompt and the side length and area of square 3. Calculations are completed accurately and the explanations complete. (NOTE: equivalent forms of the solution are acceptable, e.g., $2(2x-2)=4x-4$; $⟦(4x-4)⟧^{\uparrow}2 = 16$

	c A-SSE.A.1 A-SSE.A.2 A-SSE.B.3 A-APR.A.1	Little or no attempt is made to find the area using either method.	There is an attempt made to find the total area using adding the three smaller areas but there are errors and verification is impossible. Work is shown.	The total area is correctly determined by adding the three smaller areas but either there was no attempt to check by multiplying or there are errors in the attempt to check by multiplying. Work is shown and supports the correct results.	The total area is correctly determined by adding the three smaller areas and is correctly verified by multiplying the total length by total width. All work is shown and supports the results.
3	a F-IF.B.4	No attempt was made.	There is an attempt to find the maximum value for the function, but no connection is made to the leading coefficient being negative and calculations performed are ineffective. OR The connection between the sign on the leading coefficient and the direction of the opening of the graph is understood but incorrectly applied (e.g., the graph is said to have a minimum because the leading coefficient is negative).	There is an attempt to find the maximum value for the function and a connection between the sign of the leading coefficient is apparently understood but the explanation does not make it clear that the negative leading coefficient indicates that the graph opens down.	There is a clear understanding of the connection between the sign on the leading coefficient and the direction the graph opens. The explanation provided is clear and logical.

	b–e **A-APR.B.3** **F-IF.B.4** **F-IF.B.6**	There is no evidence of understanding of the properties of key features of the quadratic function. Calculations are ineffective and/or incorrect. Explanations are missing or ineffective. No attempt made.	There is some evidence of understanding of the properties of the key features of the quadratic function. However, calculations are incorrect and explanations are missing or inadequate.	There are accurate interpretations of the key features of the quadratic function, but some calculations are incorrectly performed. Explanations are based on calculations present and are logical and complete.	There are accurate interpretations of the key feature of the quadratic function and all calculations are performed correctly. Explanations are logical and complete.
	f **A-APR.B.3** **F-IF.B.4** **F-IF.B.5**	No attempt was made.	The domain is given incorrectly as all real numbers (i.e., the domain of the function with no consideration of the context).	The domain is described with no consideration given to the context; only as positive or greater than zero OR as less than 5 (partial consideration of the context).	Consideration is given to the beginning of the experiment (0 seconds) and the end (5 seconds). The domain is given as a set or described accurately.
	g **A-APR.B.3** **F-IF.B.4** **F-IF.B.5** **F-IF.7a**	There is little indication of understanding of the graphic representation of the function. The graph is incorrectly drawn and the key features are missing or incorrectly identified. Little or no attempt was made.	There is an attempt to graph the function but key features are not indicated on the graph. The axes are not labeled clearly with a scale that fits the graph or allows for visual verification of the key features (y -intercept at $(0, 80)$, vertex $(2, 144)$, and x -intercept $(5, 0)$).	The graph of the function is clearly and correctly drawn but key features are not indicated on the graph. The axes are labeled clearly with a scale that fits the graph and allows for visual verification of the key features (even though they are not marked).	The graph of the function is clearly and correctly drawn with the y -intercept $(0, 80)$, the vertex $(2, 144)$, and the x -intercept $(5, 0)$, identified correctly. The axes are labeled clearly with a scale that fits the graph.

	h **F-IF.B.4** **F-IF.7a**	An answer is given, but there is no explanation provided. No attempt was made.	There is an attempt to use the laws of physics in the explanation for this question (the horizontal axis represents the change in time rather than forward motion). However the answer to the question is given incorrectly.	The question is answered correctly and has an explanation attached. However, the explanation is based only partially on the physics addressed in this problem (the horizontal axis represents the change in time rather than forward motion).	The question is answered correctly and includes an explanation that shows an understanding of the physics addressed in this problem (the horizontal axis represents the change in time rather than forward motion).

Name ______________________________ Date ________________

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of x for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.

   a. Perimeter:

   $2(3x^2 + 5x - 8) + 2(2x^2 + 6x) =$

   $6x^2 + 10x - 16 + 4x^2 + 12x =$

   $10x^2 + 22x - 16$

   b. Area:

   $(3x^2 + 5x - 8)(2x^2 + 6x) =$

   $6x^4 + 18x^3$

   $10x^3 + 30x^2$

$- 16x^2 - 48x$

$6x^4 + 28x^3 + 14x^2 - 48x$

c. Are both your answers polynomials? Explain why or why not for each.

Yes, both have terms with only whole number exponents (greater than or equal to 0 *), the coefficients are real numbers, and the leading coefficient is not* 0 *.*

d. Is it possible for the perimeter of the rectangle to be 16 units? If so what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

$10x^2 + 22x - 16 = 16$ *If* $x = 1$ *the length would be:* $3(1) + 5(1) - 8 = 0$ *so not possible for* $x = 1$

$10x^2 + 22x - 32 = 0$ *If* $x = -3.2$ *the length would be:* $3(-3.2)^2 + 5(-3.2) - 8 =$

$2(5x^2 + 11x - 16) = 0$ $= 3(10.24) - 16 - 8 = 30.72 - 24 = 6.72$

$2(5x + 16)(x - 1) = 0$ *and the width would be:* $2(-3.2)^2 + 6(-3.2) = 20.48 - 19.2$

So $x = -\frac{16}{5}$ *or* 1 *OR* -3.2 *or* 1 $= 1.28$

Check: $2(length) + 2(width) = 2(6.72) + 2(1.28) = 13.44 + 2.56 = 16$

Yes, the perimeter could be 16 *units with length* 6.72 *and width* 1.28 *.*

e. For what value(s) of the domain will the area equal zero?

In factored form: $(3x^2 + 5x - 8)(2x^2 + 6x) = (3x + 8)(x - 1)(2x)(x + 3) = 0$

The $Area = 0$ *when* $x = 1\frac{8}{3}, 1, 0$ *, or*

f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

Check the values around those we found in part (e), since on either side of the zeros there is likely to be either positive or negative values.

Try substituting $x = 2$ *into the factored form. The factors will then be* $(+)(+)(+)(+) > 0$

So all numbers greater than 1 *will give positive results.* $x = 3$ *, etc.*

NOTE: If there are any, there must be an even number of negative factors and any pair of negative factors must be for the same dimension.

g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

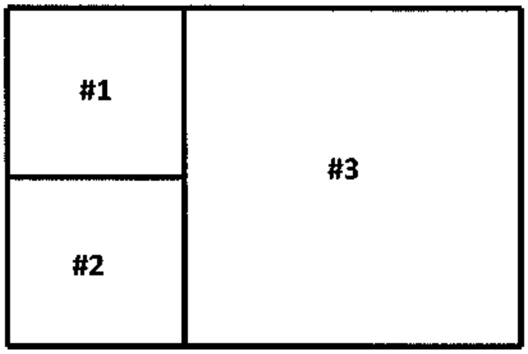
As long as the dimensions are positive, it is possible that the value of x *is negative. That means that only two of the four factors may be negative and the negative factors must both be from the same dimension (length or width). Using the logic in part (f) it is possible that numbers less than* $-\frac{8}{3}$ *or possibly between* 0 *and* $-\frac{8}{3}$ *might work.*

Let's try $x = -4$ *: The factors would be* $(-)(-)(-)(-)$ *this one works since both dimensions will be positive.*

Try $x = -1$ *: The factors would be* $(+)(-)$*... I can stop now because the length is negative which is impossible in the context of the problem.*

So the answer is YES. There are negative values for x *that produce positive area. They are less than and they result in both positive dimensions and positive area.*

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares #1 and #2 in the figure below. All three shapes are squares. The area of square #1 equals that of square #2 and each can be represented by the expression $4x^2 - 8x + 4$.



a. Find the side length of the father's plot, square #3, and show or explain how you found it.

$4x^2 - 8x + 4$ *is a perfect square that factors to*

$(2x - 2)^2$

The side length is the square root of that $= (2x - 2)$

The father's plot is twice the length of one of the smaller squares or the sum of the two.

The side length for plot #3 is $2(2x - 2) = 4x - 4$

b. Find the area of the father's plot and show or explain how you found it.

The area of the father's plot is the square of the side length:

$(4x - 4)^2 = 16x^2 - 32x + 16$

c. Find the total area of all three plots by adding the three areas and verify your answer by multiplying the outside dimensions. Show your work.

By adding the areas of the three squares:

$(4x^2 - 8x + 4) + (4x^2 - 8x + 4) + (16x^2 - 32x + 16) =$

$24x^2 - 48x + 24$

By multiplying total length by total width:

Total length $= (2x - 2) + (4x - 4) = 6x - 6$

Total width $= (2x - 2) + (2x - 2) = 4x - 4$

Area $= (6x - 6)(4x - 4) = 24x^2 - 48x + 24$

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle, so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from

the time it was thrown could be modeled closely by the function,

$$h(t) = -16t^2 + 64t + 80$$

where $h(t)$ represents the height of the ball in feet after t seconds.

a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

The function has a maximum because the leading coefficient is negative, making the graph of the function open down.

b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

To find the zeros of the function we factor as follows:

$$-16(t^2 - 4t - 5) = 0 = -16(t-5)(t+1)$$

So $t = -1$ *or* 5. *There are* 6 *units between and* 5 *so using symmetry we find the vertex at half that distance between the two:* $-1 + 3 = 2$ *so the* x *-coordinate of the vertex is* $x = 2$.

If we substitute 2 *for* x *into the original function we find that the vertex is at* $(2, 144)$ *and tells us that the maximum height is* 144 *ft., which occurs after* 2 *seconds.*

c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

The function is increasing from 0 *to* 2 *seconds and decreasing from* 2 *to* 5 *seconds. The rate of change over* $[0, 2]$ *is positive. The rate of change over* $[2, 5]$ *is negative. For an answer based on the graph: The graph has positive slope from* 0 *to* 2 *seconds and negative from* 2 *to* 5 *seconds.*

d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

$h(0) = 80$, this is the initial height: The height at which the ball was when it was thrown upward. The roof was 80 ft. high.

e. How long is the ball in the air? Explain your answer.

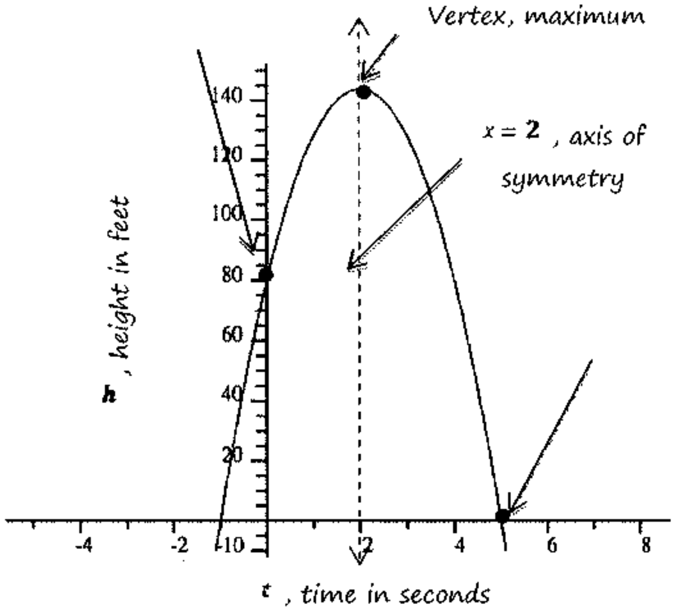
The ball is in the air for 5 seconds. When $t = 0$ the ball is released and when $t = 5$, the height is 0 which means the ball hits the ground 5 seconds after it is thrown.

f. State the domain of the function and explain the restrictions on the domain based on the context of the problem.

If we consider the experiment over at the time the ball reaches the ground, the values for t, as it is described in this context, must be greater than 0 (since time was measurement began when the ball was thrown) and will be less than 5. $t : \{0 \le t \le 5\}$

g. Graph the function indicating the vertex, axis of symmetry, intercepts and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.

The graph shows the function crossing the y-axis at $(0, 80)$, the height at which the ball was thrown. Then it travels to a height of 144 ft. after 2 seconds and hits the ground at 5 seconds.



h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

No, because movement along the horizontal axis represents changes in time, not horizontal distance. The ball could be going straight up and then straight down with very little change in horizontal position and the graph would be the same.

Name ______________________________ Date ________________

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   a. Perimeter:

   b. Area:

   c. Are both your answers polynomials? Explain why or why not for each.

   d. Is it possible for the perimeter of the rectangle to be 16 units? If so what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

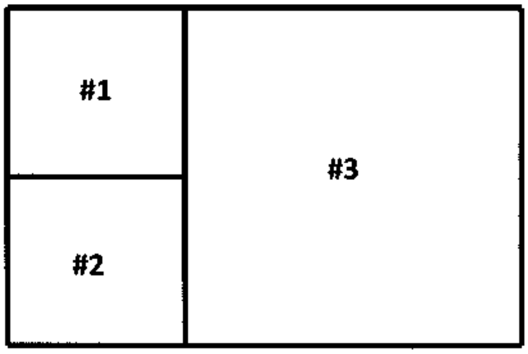
e. For what value(s) of the domain will the area equal zero?

f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares #1 and #2 in the figure below. All three shapes are squares. The area of square #1 equals that of square #2 and each can be represented by the expression $4x^2 - 8x + 4$.

   a. Find the side length of the father's plot, square #3, and show or explain how you found it.



   b. Find the area of the father's plot and show or explain how you found it.

   c. Find the total area of all three plots by adding the three areas and verify your answer by multiplying the outside dimensions. Show your work.

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle, so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from the time it was thrown could be modeled closely by the function,

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where $h(t)$ represents the height of the ball in feet after t seconds.

a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

e. How long is the ball in the air? Explain your answer.

f. State the domain of the function and explain the restrictions on the domain based on the context of the problem.

g. Graph the function indicating the vertex, axis of symmetry, intercepts and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.

h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

A Progression Toward Mastery

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	c A-APR.A.1	No attempt is made.	There is an attempt to explain but the explanation shows little understanding of the definition of a polynomial and of the concept of closure for	The explanation shows some understanding of the definition of a polynomial or of the concept of closure for polynomial operations.	The explanation is correct and includes understanding of the definition of a polynomial and of closure for polynomial operations.

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	g **A-SSE.B.3a** **A-APR.A.1** **A-REI.D.11**	Little or no attempt is made to answer this question.	There is an attempt made to answer the question. However, the explanation is missing important parts: there are no references to the dimensions being positive or to the requirement that there must be an even number of negative factors for area. There might be specific examples of negative values that produce a positive area given but without explanation.	The question is answered correctly but is based on a partially correct explanation: The explanation is missing a reference to the need for an even number of negative factors in the area expression or that both dimensions must be positive (if two factors are negative, they must both represent the same dimension).	The question is answered correctly and completely, including the following: There are references in the explanation to the need for positive dimensions, and that if an x -value makes any of the factors negative, there must be an even number of negative factors. This means that both negative factors must be for the same dimension.
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	c A-SSE.A.1 A-SSE.A.2 A-SSE.B.3 A-APR.A.1	Little or no attempt is made to find the area using either method.	There is an attempt made to find the total area using adding the three smaller areas but there are errors and verification is impossible. Work is shown.	The total area is correctly determined by adding the three smaller areas but either there was no attempt to check by multiplying or there are errors in the attempt to check by multiplying. Work is shown and supports the correct results.	The total area is correctly determined by adding the three smaller areas and is correctly verified by multiplying the total length by total width. All work is shown and supports the results.
3	a F-IF.B.4	No attempt was made.	There is an attempt to find the maximum value for the function, but no connection is made to the leading coefficient being negative and calculations performed are ineffective. OR The connection between the sign on the leading coefficient and the direction of the opening of the graph is understood but incorrectly applied (e.g., the graph is said to have a minimum because the leading coefficient is negative).	There is an attempt to find the maximum value for the function and a connection between the sign of the leading coefficient is apparently understood but the explanation does not make it clear that the negative leading coefficient indicates that the graph opens down.	There is a clear understanding of the connection between the sign on the leading coefficient and the direction the graph opens. The explanation provided is clear and logical.

	b–e **A-APR.B.3** **F-IF.B.4** **F-IF.B.6**	There is no evidence of understanding of the properties of key features of the quadratic function. Calculations are ineffective and/or incorrect. Explanations are missing or ineffective. No attempt made.	There is some evidence of understanding of the properties of the key features of the quadratic function. However, calculations are incorrect and explanations are missing or inadequate.	There are accurate interpretations of the key features of the quadratic function, but some calculations are incorrectly performed. Explanations are based on calculations present and are logical and complete.	There are accurate interpretations of the key feature of the quadratic function and all calculations are performed correctly. Explanations are logical and complete.
	f **A-APR.B.3** **F-IF.B.4** **F-IF.B.5**	No attempt was made.	The domain is given incorrectly as all real numbers (i.e., the domain of the function with no consideration of the context).	The domain is described with no consideration given to the context; only as positive or greater than zero OR as less than 5 (partial consideration of the context).	Consideration is given to the beginning of the experiment (0 seconds) and the end (5 seconds). The domain is given as a set or described accurately.
	g **A-APR.B.3** **F-IF.B.4** **F-IF.B.5** **F-IF.7a**	There is little indication of understanding of the graphic representation of the function. The graph is incorrectly drawn and the key features are missing or incorrectly identified. Little or no attempt was made.	There is an attempt to graph the function but key features are not indicated on the graph. The axes are not labeled clearly with a scale that fits the graph or allows for visual verification of the key features (y-intercept at $(0, 80)$, vertex $(2, 144)$, and x-intercept $(5, 0)$).	The graph of the function is clearly and correctly drawn but key features are not indicated on the graph. The axes are labeled clearly with a scale that fits the graph and allows for visual verification of the key features (even though they are not marked).	The graph of the function is clearly and correctly drawn with the y-intercept $(0, 80)$, the vertex $(2, 144)$, and the x-intercept $(5, 0)$, identified correctly. The axes are labeled clearly with a scale that fits the graph.

	h **F-IF.B.4** **F-IF.7a**	An answer is given, but there is no explanation provided. No attempt was made.	There is an attempt to use the laws of physics in the explanation for this question (the horizontal axis represents the change in time rather than forward motion). However the answer to the question is given incorrectly.	The question is answered correctly and has an explanation attached. However, the explanation is based only partially on the physics addressed in this problem (the horizontal axis represents the change in time rather than forward motion).	The question is answered correctly and includes an explanation that shows an understanding of the physics addressed in this problem (the horizontal axis represents the change in time rather than forward motion).

Name ______________________________ Date ________________

1. A rectangle with positive area has length represented by the expression $3x^2 + 5x - 8$ and width by $2x^2 + 6x$. Write expressions in terms of x for the perimeter and area of the rectangle. Give your answers in standard polynomial form and show your work.

   a. Perimeter:

   $2(3x^2 + 5x - 8) + 2(2x^2 + 6x) =$

   $6x^2 + 10x - 16 + 4x^2 + 12x =$

   $10x^2 + 22x - 16$

   b. Area:

   $(3x^2 + 5x - 8)(2x^2 + 6x) =$

   $6x^4 + 18x^3$

   $10x^3 + 30x^2$

$-16x^2 - 48x$

$6x^4 + 28x^3 + 14x^2 - 48x$

c. Are both your answers polynomials? Explain why or why not for each.

Yes, both have terms with only whole number exponents (greater than or equal to 0 *), the coefficients are real numbers, and the leading coefficient is not* 0 *.*

d. Is it possible for the perimeter of the rectangle to be 16 units? If so what value(s) of x will work? Use mathematical reasoning to explain how you know you are correct.

$10x^2 + 22x - 16 = 16$ *If* $x = 1$ *the length would be:* $3(1) + 5(1) - 8 = 0$ *so not possible for* $x = 1$

$10x^2 + 22x - 32 = 0$ *If* $x = -3.2$ *the length would be:* $3(-3.2)^2 + 5(-3.2) - 8 =$

$2(5x^2 + 11x - 16) = 0$ $= 3(10.24) - 16 - 8 = 30.72 - 24 = 6.72$

$2(5x + 16)(x - 1) = 0$ *and the width would be:* $2(-3.2)^2 + 6(-3.2) = 20.48 - 19.2$

So $x = -\frac{16}{5}$ *or* 1 *OR* -3.2 *or* 1 $= 1.28$

Check: $2(length) + 2(width) = 2(6.72) + 2(1.28) = 13.44 + 2.56 = 16$

Yes, the perimeter could be 16 *units with length* 6.72 *and width* 1.28 *.*

e. For what value(s) of the domain will the area equal zero?

In factored form: $(3x^2 + 5x - 8)(2x^2 + 6x) = (3x + 8)(x - 1)(2x)(x + 3) = 0$

The $Area = 0$ *when* $x = \mathrm{I}\frac{8}{3}, 1 , 0$ *, or*

f. The problem states that the area of the rectangle is positive. Find and check two positive domain values that will produce a positive area.

Check the values around those we found in part (e), since on either side of the zeros there is likely to be either positive or negative values.

Try substituting $x = 2$ *into the factored form. The factors will then be* $(+)(+)(+)(+) > 0$

So all numbers greater than 1 *will give positive results.* $x = 3$ *, etc.*

NOTE: If there are any, there must be an even number of negative factors and any pair of negative factors must be for the same dimension.

g. Is it possible that negative domain values could produce a positive function value (area)? Explain why or why not in the context of the problem.

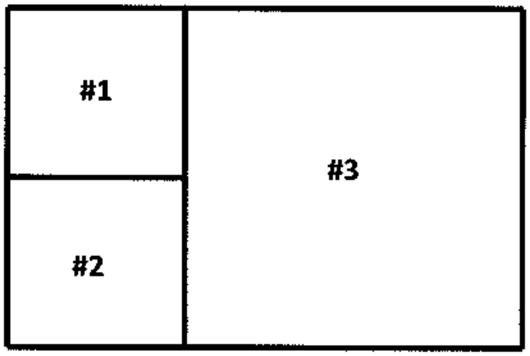
As long as the dimensions are positive, it is possible that the value of x *is negative. That means that only two of the four factors may be negative and the negative factors must both be from the same dimension (length or width). Using the logic in part (f) it is possible that numbers less than* $-\frac{8}{3}$ *or possibly between* 0 *and* $-\frac{8}{3}$ *might work.*

Let's try $x = -4$ *: The factors would be* $(-)(-)(-)(-)$ *this one works since both dimensions will be positive.*

Try $x = -1$ *: The factors would be* $(+)(-)$*... I can stop now because the length is negative which is impossible in the context of the problem.*

So the answer is YES. There are negative values for x *that produce positive area. They are less than and they result in both positive dimensions and positive area.*

2. A father divided his land so that he could give each of his two sons a plot of his own and keep a larger plot for himself. The sons' plots are represented by squares #1 and #2 in the figure below. All three shapes are squares. The area of square #1 equals that of square #2 and each can be represented by the expression $4x^2 - 8x + 4$.



a. Find the side length of the father's plot, square #3, and show or explain how you found it.

$4x^2 - 8x + 4$ *is a perfect square that factors to*

$(2x - 2)^2$

The side length is the square root of that $= (2x - 2)$

The father's plot is twice the length of one of the smaller squares or the sum of the two.

The side length for plot #3 is $2(2x - 2) = 4x - 4$

b. Find the area of the father's plot and show or explain how you found it.

The area of the father's plot is the square of the side length:

$(4x - 4)^2 = 16x^2 - 32x + 16$

c. Find the total area of all three plots by adding the three areas and verify your answer by multiplying the outside dimensions. Show your work.

By adding the areas of the three squares:

$(4x^2 - 8x + 4) + (4x^2 - 8x + 4) + (16x^2 - 32x + 16) =$

$24x^2 - 48x + 24$

By multiplying total length by total width:

Total length $= (2x - 2) + (4x - 4) = 6x - 6$

Total width $= (2x - 2) + (2x - 2) = 4x - 4$

Area $= (6x - 6)(4x - 4) = 24x^2 - 48x + 24$

3. The baseball team pitcher was asked to participate in a demonstration for his math class. He took a baseball to the edge of the roof of the school building and threw it up into the air at a slight angle, so that the ball eventually fell all the way to the ground. The class determined that the motion of the ball from

the time it was thrown could be modeled closely by the function,

$$h(t) = -16t^2 + 64t + 80$$

where $h(t)$ represents the height of the ball in feet after t seconds.

a. Determine whether the function has a maximum value or a minimum value. Explain your answer mathematically.

The function has a maximum because the leading coefficient is negative, making the graph of the function open down.

b. Find the maximum or minimum value of the function. After how many seconds did the ball reach this value? Show how you found your answers.

To find the zeros of the function we factor as follows:

$$-16(t^2 - 4t - 5) = 0 = -16(t - 5)(t + 1)$$

So $t = -1$ *or* 5 *. There are* 6 *units between and* 5 *so using symmetry we find the vertex at half that distance between the two:* $-1 + 3 = 2$ *so the* x *-coordinate of the vertex is* $x = 2$ *.*

If we substitute 2 *for* x *into the original function we find that the vertex is at* $(2, 144)$ *and tells us that the maximum height is* 144 *ft., which occurs after* 2 *seconds.*

c. For what interval of the domain is the function increasing (i.e., ball going up)? For what interval of the domain is the function decreasing (i.e., ball going down)? Explain how you know.

The function is increasing from 0 *to* 2 *seconds and decreasing from* 2 *to* 5 *seconds. The rate of change over* $[0, 2]$ *is positive. The rate of change over* $[2, 5]$ *is negative. For an answer based on the graph: The graph has positive slope from* 0 *to* 2 *seconds and negative from* 2 *to* 5 *seconds.*

d. Evaluate $h(0)$. What does this value tell you? Explain in the context of the problem.

$h(0) = 80$, this is the initial height: The height at which the ball was when it was thrown upward. The roof was 80 ft. high.

e. How long is the ball in the air? Explain your answer.

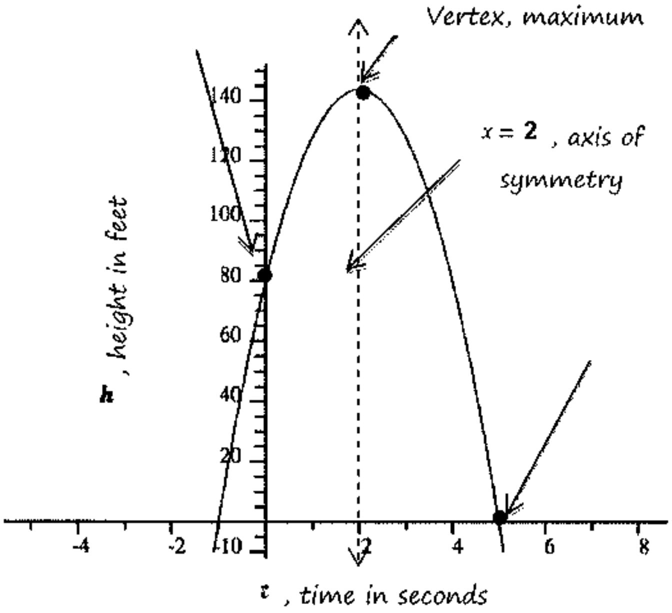
The ball is in the air for 5 seconds. When $t = 0$ the ball is released and when $t = 5$, the height is 0 which means the ball hits the ground 5 seconds after it is thrown.

f. State the domain of the function and explain the restrictions on the domain based on the context of the problem.

If we consider the experiment over at the time the ball reaches the ground, the values for t, as it is described in this context, must be greater than 0 (since time was measurement began when the ball was thrown) and will be less than 5. $t : \{0 \leq t \leq 5\}$

g. Graph the function indicating the vertex, axis of symmetry, intercepts and the point representing the ball's maximum or minimum height. Label your axes using appropriate scales. Explain how your answer to part (d) is demonstrated in your graph.

The graph shows the function crossing the Y-axis at $(0, 80)$, the height at which the ball was thrown. Then it travels to a height of 144 ft. after 2 seconds and hits the ground at 5 seconds.



h. Does your graph illustrate the actual trajectory of the ball through the air as we see it?

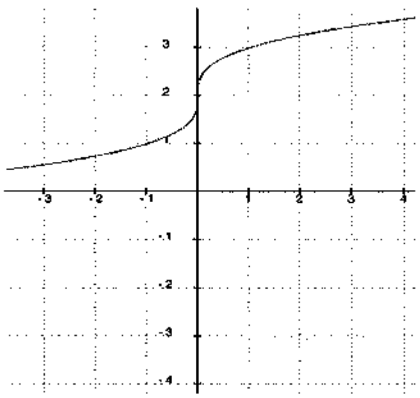
No, because movement along the horizontal axis represents changes in time, not horizontal distance. The ball could be going straight up and then straight down with very little change in horizontal position and the graph would be the same.

Name ______________________________ Date ____________________

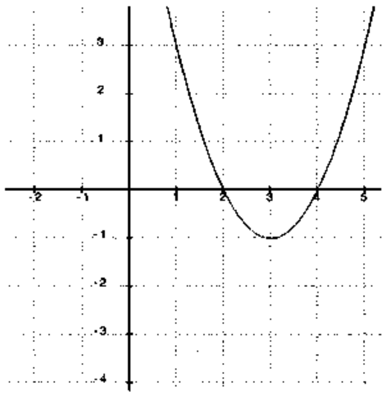
1. Label each graph with the function it represents; choose from those listed below:

$f(x) = 3\sqrt{x}$ $g(x) = \frac{1}{2}\sqrt[3]{x}$ $h(x) = -5x^2$

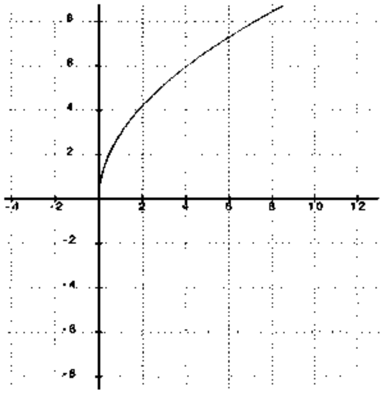
$k(x) = \sqrt{x+2} - 1$ $m(x) = \sqrt[3]{x} + 2$ $n(x) = (x-3)^2 - 1$



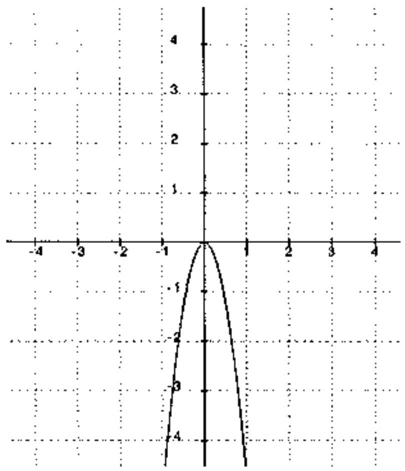
Function ______________



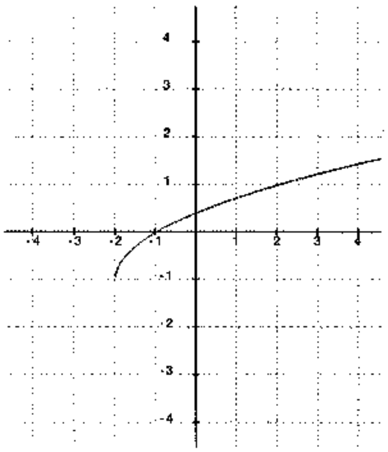
Function ______________



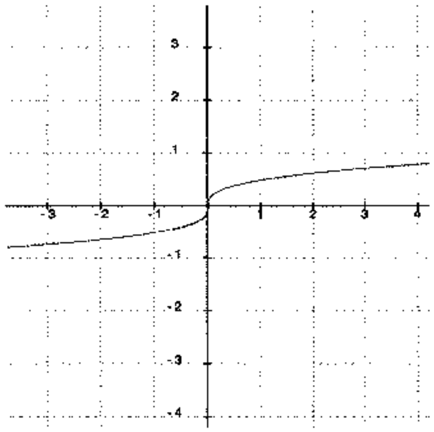
Function ______________



Function ______________

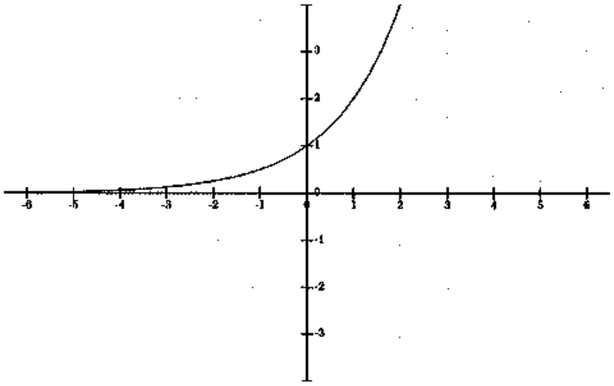


Function ______________



Function ______________

2. Compare the following three functions:
   i. A function f is represented by the graph below:



   ii. A function g is represented by the following equation:

$$g(x) = (x-6)^2 - 36$$

   iii. A linear function h is represented by the following table:

x	-1	1	3	5	7
$h(x)$	10	14	18	22	26

For each of the following, evaluate the three expressions given and identify which expression has the largest value and which has the smallest value. Show your work.

a. $f(0)$, $g(0)$, $h(0)$

b. $\frac{f(4)-f(2)}{4-2}$, $\frac{g(4)-g(2)}{4-2}$, $\frac{h(4)-h(2)}{4-2}$

c. $f(1000)$, $g(1000)$, $h(1000)$

3. An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t , and the height of the arrow in meters, h , is given by:

$$h(t) = -4.9t^2 + 29.4t + 2.5$$

a. Complete the square for this function.

b. What is the maximum height of the arrow? Explain how you know.

c. How long does it take the arrow to reach its maximum height? Explain how you know.

d. What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds? Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds and explain the difference in the context of the problem.

e. How long does it take the arrow to hit the ground? Show your work or explain your answer.

f. What does the constant term in the original equation tell you about the arrow's flight?

g. What do the coefficients on the second- and first-degree terms in the original equation tell you about the arrow's flight?

4. Rewrite each expression below in expanded (standard) form:

a. $(x+\sqrt{3})^2$

b. $(x-2\sqrt{5})(x-3\sqrt{5})$

c. Explain why, in these two examples, the coefficients of the linear terms are irrational and why the constants are rational.

Factor each expression below by treating it as the difference of squares:

d. q^2-8

e. $t-16$

5. Solve the following equations for r. Show your method and work. If no solution is possible, explain how you know.

    a. $r^2 + 12r + 18 = 7$

    b. $r^2 + 2r - 3 = 4$

    c. $r^2 + 18r + 73 = -9$

6. Consider the equation: $x^2 - 2x - 6 = y + 2x + 15$ and the function: $f(x) = 4x^2 + 16x - 84$ in the following questions:

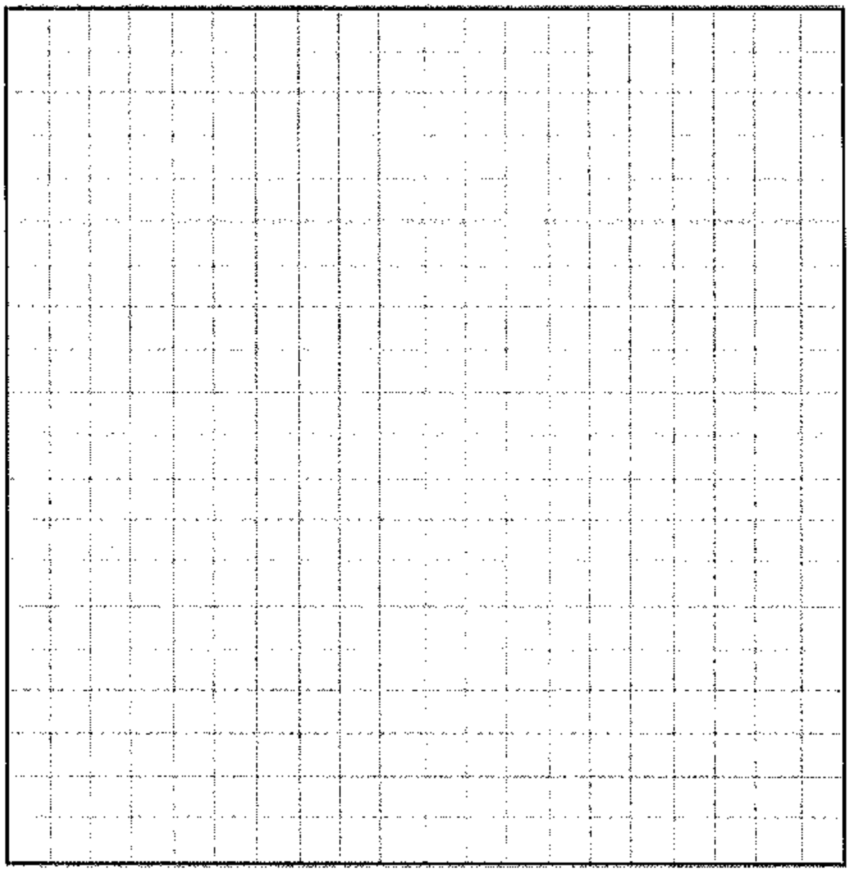
    a. Show that the graph of the equation, $x^2 - 2x - 6 = y + 2x + 15$, has x-intercepts at $x = -3$ and 7.

b. Show that the zeros of the function, $f(x) = 4x^2 | 16x - 84$, are the same as the x-values of the x-intercepts for the graph of the equation in part (a). (i.e., $x = -3$ and 7)

c. Explain how this function is different from the equation in part (a)?

d. Identify the vertex of the graphs of each by rewriting the equation and function in the completed-square form, $a(x - h)^2 + k$. Show your work. What is the same about the two vertices? How are they different? Explain why there is a difference.

e. Write a new quadratic function, with the same zeros, but with a maximum rather than a minimum. Sketch a graph of your function, indicating the scale on the axes and the key features of the graph.



A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, *or* an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**F-IC.C.7a** **F.IF.C.7b** **F-BF.B.3**	Only one or two of the six graphs and functions are matched accurately.	Three of the six graphs and functions are matched accurately.	Four or five of the six graphs and functions are matched accurately.	All six graphs and functions are matched accurately.
2	**a–c** **F-IF.B.6** **F-IF.C.9**	Mark each part separately, using the same scoring criteria for each: The function values for each part are not in the correct order and there is no evidence to show an understanding of the three representations as being exponential, quadratic, and linear or of how to determine the functions' values at the indicated x-values.	Mark each part separately, using the same scoring criteria for each: There is evidence in the work that shows understanding of the graphic representation as being exponential, the tabular as linear, and the symbolic as quadratic. Only one of the three function values is correctly determined, for the indicated values of x, and the function values are not in the correct order.	Mark each part separately, using the same scoring criteria for each: There is evidence in the work that shows understanding of the graphic representation as being exponential, the tabular as linear, and the symbolic as quadratic. The values for x are substituted correctly but there are one or more errors in calculating the values of the expressions. The function values are not ordered correctly.	Mark each part separately, using the same scoring criteria for each: There is evidence in the work that shows understanding of the graphic representation as being exponential, the tabular as linear, and the symbolic as quadratic. The values for x are substituted correctly and the expressions are ordered correctly.
3	**a**	Shows little or no understanding of the	There is an attempt to rewrite in completed	The function is written in completed square	The function is correctly rewritten in

	A-SSE.A.2 **F-IF.B.8a**	process required to complete the square.	square form, using a correct process. However there are errors in calculations and steps missing in the process.	form but critical steps in the work are missing or there are errors in the calculations.	completed square form and the correct steps for the process are included.
	b–g **A-SSE.A.1** **A-SSE.B.3b** **F-IF.B.4**	Shows little evidence of interpreting the function. Little or no attempt at these questions.	Uses the function form found in part (a) but shows only some understanding of interpreting the key features of the function. Errors are made in calculations and there is limited explanation.	Explanation indicates an understanding of the key features of the function. Uses the function form found in part (a) but incorrectly interprets the function or makes minor errors in calculation. Explains process somewhat but leaves gaps in the explanation.	Explanation indicates an understanding of the key features of the function. Uses the function form found in part (a) and correctly interprets the function features. Provides evidence of process and accurate explanation of the reasoning used.
4	**a–b** **A-SSE.A.2**	There is little evidence of understanding multiplication of binomials that include radicals. Little or no attempt is made.	There is some evidence of understanding multiplication of binomials that include radicals. There are errors in the calculations and work.	There is evidence of understanding multiplication of binomials that include radicals. There are no errors in the calculations but radical calculations are left unfinished, such as $(\sqrt{3})^2$ or .	Expansions are accurately performed, terms are in simplest radical form, and work supports the solutions.
	c **N-RN.B.3**	No attempt is made to provide an explanation.	There is little evidence of understanding of the properties of irrational numbers. The explanation uses numerical examples for both questions but provides no further explanation. Or an explanation is provided for only one part of the question.	There is some evidence in the explanation of understanding of the properties of irrational numbers. The explanation is partially correct and is attempted for both parts of the question but is missing one or more aspects.	There is clear evidence in the explanation of understanding of the properties of irrational numbers.
	d–e **A-SSE.A.2**	There is no evidence of understanding factoring the difference of squares. Little or no attempt is made.	There is some evidence of understanding factoring the difference of squares. There are errors in the calculations that lead to incorrect solutions.	There is evidence of understanding factoring the difference of squares. There are no errors in the calculations but the radical is left off (e.g.,	Factors are accurately determined, terms are in simplest radical form, and work supports the solutions. (NOTE: In part (d) full credit is given for either

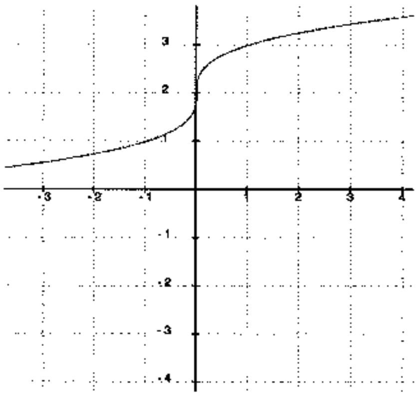
				√t) or irrational calculations are left unfinished (e.g., $\sqrt{16}$).	form of the radical: $\sqrt{8}$ or $2\sqrt{2}$. However in part (e) $\sqrt{16}$ must be changed to 4.)
5	**a–b** **A-REI.B.4** **A-REI.B.4a** **A-REI.4b**	There is no evidence of understanding the process of solving a quadratic equation. Little or no attempt is made.	There is some evidence of understanding the process of solving a quadratic equation in the work shown. There are errors in calculations and incorrect method used to find the solutions, or the process was aborted before completion.	The equation solving process is carried to completion using an appropriate method for each part. There are errors in calculations (e.g., factored incorrectly) or set up incorrectly) (e.g., failed to set the expression equal to 0) and/or there is only one solution found.	Equations are correctly solved, using an efficient method, and with accurate and supportive work shown.
	c **A-REI.B.4** **A-REI.B.4a** **A-REI.4b**	There is no evidence of understanding the process of solving a quadratic equation. Little or no attempt is made.	There is some evidence of understanding the process of solving a quadratic equation in the work shown. There are errors in calculations and incorrect method used to find the solutions, or the process was aborted before completion.	There is evidence of understanding the nature of this quadratic equation as having no real solutions. However the explanation does not include using the discriminant or a graphic representation to justify the reasoning.	Equation is correctly set up for solution. Discriminant values shows there are no real solutions. Accurate explanation is included. (NOTE: The explanation may include references to the graphic representation.)
6	**a–b** **A-APR.B.3**	There is no evidence of understanding the concept of verifying the zeros of a function. Little or no attempt is made.	An attempt is made to determine whether x = –3 and 7 are x-intercepts for the function. Errors are made in method selection and/or in calculations that lead to an inconsistency.	A valid method is used to show that the function has x-intercepts at x = –3 and 7 and includes an explanation to "show" the solutions are correct. Errors are made in the calculations that do not affect the final result.	A valid method is used to show that the function has x-intercepts at x = –3 and 7 and includes an explanation to "show" the solutions are correct.
	c **A-SSE.B.3a** **A-SSE.B.3b**	There is no evidence of understanding the relationship between the function and the equation and how that relationship is manifested in the relationship of then graphs.	There is understanding of how the graphs relate but no mention of how the expression used for the formula for f is 4 times the expression that defines y based on the given equation.	There is understanding of how the graphs relate and of how the expression used for the formula for f is 4 times the expression that defines y based on the given equation, but there are minor errors	There is understanding of how the graphs relate and of how the expression used for the formula for f is 4 times the expression that defines y based on the given equation. The ideas are

				of misuse of vocabulary.	communicated with accurate use of vocabulary.
	d **A-CED.A.2** **A-SSE.B.3a** **A-SSE.B.3b**	There is no evidence of understanding the process of completing the square. Little or no attempt is made.	There is some evidence of understanding the process of completing the square but attempts contain errors that lead to incorrect solutions. A valid explanation of the difference between the two vertices is not included.	There is evidence of understanding the process of completing the square. The attempt contains errors that lead to incorrect coordinates for the vertices. However the explanation of the differences is based on the vertices found and is logical.	The process of completing the square in accurately performed, the coordinates of the vertices are correct and the explanation of their differences is accurate and logical.
	e **A-CED.A.2** **A-APR.B.3**	There is little evidence of understanding the concept of creating an equation with a maximum. Little or no attempt is made at creating the equation or sketching its graph.	There is evidence of understanding that the leading coefficient for the function must be negative. However, the function does not have the same zeros as those in parts (a) and (b) *and* graph does not match the function created. The scale is not indicated on the graph *and* the key features are not identified.	There is evidence of understanding that the leading coefficient for the function must be negative. However, the function does not have the same zeros as those in parts (a) and (b) *or* graph does not match the function created. The scale is not indicated on the graph *or* the key features are not identified.	There is evidence of understanding that the leading coefficient for the function must be negative. The function created has the same zeros as those in parts (a) and (b) and graph matches the function created. The scale is indicated on the graph and the key features are identified.

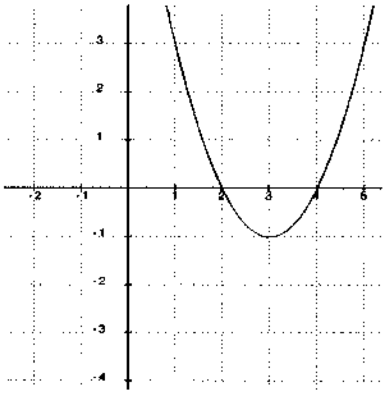
1. Label each graph with the function it represents; choose from those listed below:

$f(x) = 3\sqrt{x}$ $g(x) = \frac{1}{2}\sqrt[3]{x}$ $h(x) = -5x^2$

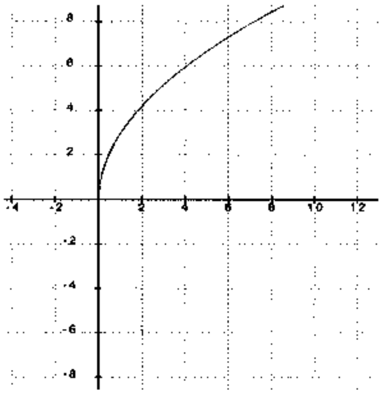
$k(x) = \sqrt{x+2} - 1$ $m(x) = \sqrt[3]{x} + 2$ $n(x) = (x-3)^2 - 1$



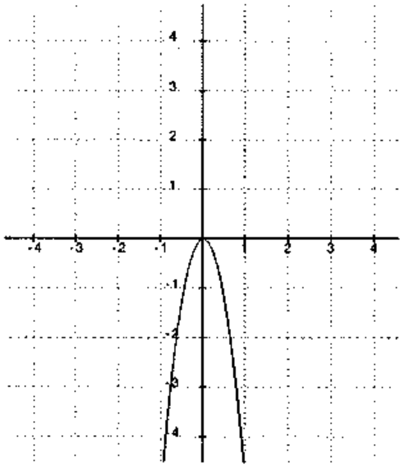
Function ______ m ________



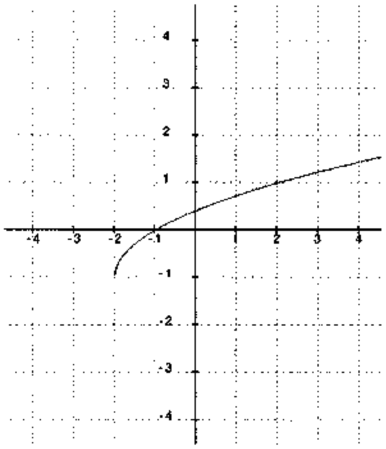
Function ______ n _______



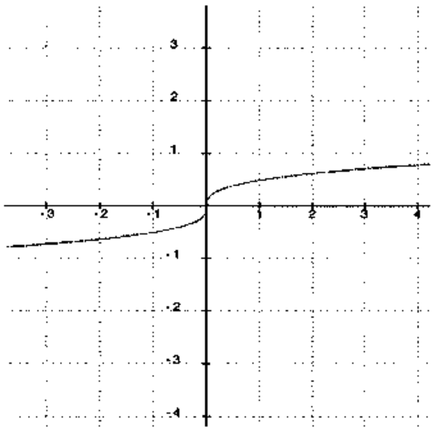
Function ______ f ________



Function ______ h ________

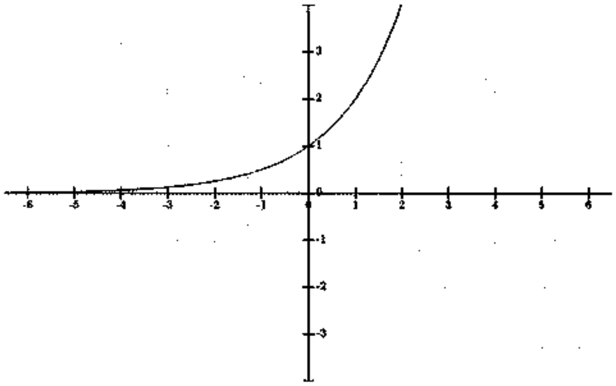


Function _____ k ________



Function ______ g ________

2. Compare the following three functions:
   i. A function f is represented by the graph below:



Note: $f(x) = 2^x$

   ii. Function g is represented by the following equation:

$$g(x) = (x - 6)^2 - 36$$

Note: $h(x) = 2x + 12$

   iii. Linear function h is represented by the following table:

x	-1	1	3	5	7
$h(x)$	10	14	18	22	26

For each of the following, evaluate the three expressions given and identify which expression has the largest value and which has the smallest value. Show your work.

a. $f(0)$, $g(0)$, $h(0)$

$f(0) = 1$, $g(0) = 0$, $h(0) = 12$,

$g(0)$ has the smallest value and $h(0)$ has the largest value.

b. $\dfrac{f(4) - f(2)}{4 - 2}$, $\dfrac{g(4) - g(2)}{4 - 2}$, $\dfrac{h(4) - h(2)}{4 - 2}$

$f(4) = 16$; $f(2) = 4$: $g(4) = -32$ $g(2) = -20$: $h(4) = 20$ $h(2) = 16$

So the values for the average rate of change over the interval [2, 4] for each function are as follows:

f: 6 g: −6 h: 2

g's rate of change is the smallest on this interval and f's is the largest.

c. $f(1000)$, $g(1000)$, $h(1000)$

$f(1000) = 2^{1000} = 1.1 \times 10^{301}$; $g(1000) = 994^2 - 36 = 988{,}000$; $h(1000) = 2012$

$h(1000)$ has the smallest value and $f(1000)$ has the largest value

3. An arrow is shot into the air. A function representing the relationship between the number of seconds it is in the air, t and the height of the arrow in meters, h, is given by:

$$h(t) = -4.9t^2 + 29.4t + 2.5$$

a. Complete the square for this function.

$h(t) = -4.9t^2 + 29.4t + 2.5 = -4.9(t^2 + 6t + \quad) + 2.5$ (factoring out the -4.9 from the two t terms and leaving the constant outside the parentheses)

$= -4.9(t^2 - 6t + 9) + 2.5 + 44.1$ (completing the square inside the parentheses)

$= -4.9(t - 3)^2 + 46.6$

b. What is the maximum height of the arrow? Explain how you know.

46.6 m. – this is the value of the function at its vertex so it is the highest the arrow will reach before it begins its descent.

c. How long does it take the arrow to reach its maximum height? Explain how you know.

3 sec., because that is the t-value (time in seconds) at which the arrow reached its highest point.

d. What is the average rate of change for the interval from $t = 1$ to $t = 2$ seconds? Compare your answer to the average rate of change for the interval from $t = 2$ to $t = 3$ seconds and explain the difference in the context of the problem.

$h(2) = -4.9(4) + 29.4(2) + 2.5 = 41.7$
$h(1) = -4.9(1) + 29.4(1) + 2.5 = 27$
The average rate of change for the interval [1, 2], is (41.7 − 27)/(2 − 1) = 14.7 meters per second.
$h(2) = 41.7$
$h(3) = -4.9(9) + 29.4(3) + 2.5 = 46.6$
The average rate of change for the interval [2, 3] is (46.6 − 41.7)/(3 − 2) = 4.9
The average rate for [1, 2] shows that the arrow is moving faster in the first interval than during the second (from 2 to 3 sec.). As the arrow moves upward the rate slows until it finally turns and begins its downward motion.

e. How long does it take the arrow to hit the ground? Show your work or explain your answer.

Since the zeros for the function are at −0.08 and 6.08 seconds the arrow was in flight from 0−6.08 seconds.

f. What does the constant term in the original equation tell you about the arrow's flight?

The constant (2.5) represents the height when t = 0 or h(0). That is the initial height of the arrow when it was shot, 2.5 m. (NOTE: 2.5 m is approximately 8 ft. 2 in. Since a bow and arrow at the ready is held a full arm's length above the head, this would suggest that the person shooting the arrow was around 6 ft. tall.)

g. What do the coefficients on the second and first degree terms in the original equation tell you about the arrow's flight?

−4.9: This is half of the local gravitational constant, -9.8 m/sec^2
29.4: The initial velocity of the arrow as it was shot upward was 29.4 m per sec. (approximately 66 mph).

4. Rewrite each expression below in expanded (standard) form:

a. $(x+\sqrt{3})^2$

$x^2 + 2\sqrt{3}\,x + (\sqrt{3})^2$

$= x^2 + 2\sqrt{3}\,x + 3$

b. $(x-2\sqrt{5})(x-3\sqrt{5})$

$x^2 - 3\sqrt{5}\,x - 2\sqrt{5}\,x + (2\sqrt{5})(3\sqrt{5})$

$= x^2 - 5\sqrt{5}\,x + 30$

c. Explain why, in these two examples, the coefficients of the linear terms are irrational and why the constants are rational.

When two irrational numbers are added (unless they are additive inverses) the result is irrational. Therefore the linear term will be irrational in both of these cases. When a square root is squared or multiplied by itself, the result is rational.

Factor each expression below by treating it as the difference of squares:

d. $q^2 - 8$

$(q + \sqrt{8})(q - \sqrt{8})$

$(q + 2\sqrt{2})(q - 2\sqrt{2})$

e. $t - 16$

$(\sqrt{t} + 4)(\sqrt{t} - 4)$ or

$(-\sqrt{t} + 4)(-\sqrt{t} - 4)$

5. Solve the following equations for r. Show your method and work. If no solution is possible, explain how you know.

a. $r^2 + 12r + 18 = 7$

$r^2 + 12r + 18 = 7$

$r^2 + 12r + 11 = 0$

$(r + 1)(r + 11) = 0$

$r = -1$ or -11

b. $r^2 + 2r - 3 = 4$

$r^2 + 2r - 3 = 4$

$r^2 + 2r - 7 = 0$

Completing the square:

$r^2 + 2r + 1 = 7 + 1$

$(r + 1)^2 = 8$

$r + 1 = \pm 2\sqrt{2}$

$r = -1 \pm 2\sqrt{2}$

NOTE: Students may opt to use the quadratic formula to solve this equation.

c. $r^2 + 18r + 73 = -9$

$r^2 + 18r + 73 = -9$

$r^2 + 18r + 82 = 0$

Discriminant:

$18^2 - 4(1)(82) =$

$324 - 328 = -4$

There are no real solutions since the discriminate is negative.

6. Consider the equation: $x^2 - 2x - 6 = y + 2x + 15$ and the function: $f(x) = 4x^2 \mid 16x - 84$ in the following questions:

a. Show that the graph of the equation, $x^2 - 2x - 6 = y + 2x + 15$, has x-intercepts at $x = -3$

Substituting -3 for x and 0 for y:

$9 + 6 - 6 = 0 - 6 + 15$

$9 = 9$

This true statement shows that (−3, 0) is an x-intercept on the graph of this equation.

Substituting 7 for x and 0 for y:

$49 - 14 - 6 = 0 + 14 + 15$

$29 = 29$

This true statement shows that (7, 0) is an x-intercept on the graph of this equation.

and 7.

b. Show that the zeros of the function, $f(x) = 4x^2$ I $16x - 84$, are the same as the x-values of the x-intercepts for the graph of the equation in part (a) (i.e., $x = -3$ and 7).

Substituting $x = -3$ and $f(x) = 0$:

$4(9) - 16(-3) - 84 = 0$

$36 + 48 - 84 = 0$

$0 = 0$

Substituting $x = -3$ and $f(x) = 0$:

$4(49) - 16(7) - 84 = 0$

$196 - 112 - 84 = 0$

$0 = 0$

If the equation and the function were both graphed on the same coordinate plate, explain how their graphs would relate and explain how you can determine this just by examining the equation and the formula of the function.

The graph of the function would be a vertical stretch, with a scale factor of 4, of the graph of the equation. Or, you could say that the graph of the equation is a vertical shrink, with a scale factor of ¼ of the graph of f. You know this because the formula for the function f is related to the equation. Solving the equation for y, you get $y = x^2 - 4x - 21$. So $y = ¼\, f(x)$.

c. Identify the vertex of the graphs of each by rewriting the equation and function in the completed-square form, $a(x - h)^2 + k$. Show your work. What is the same about the two vertices? How are they different? Explain why there is a difference.

$x^2 - 2x - 6 = y + 2x + 15$

$y = x^2 - 4x - 21$

$y = (x^2 - 4x \quad + 4\ ) - 21 - 4$

$y = (x - 2)^2 - 25$

Vertex $(2, -25)$

$4x^2 - 16x - 84 = f(x)$

$f(x) = 4(x^2 - 4x + \ 4\ ) - 84 \ - 16$

$f(x) = 4(x - 2)^2 - 100$

Vertex $(2, -100)$

d. Write a new quadratic function, with the same zeros, but with a maximum rather than a minimum.

Sketch a graph of your function, indicating the scale on the axes and the key features of the graph.

The two vertices have the same x coordinate (the same axis of symmetry) but the y coordinate for vertex of the graph of the function is 4 times the y-coordinate of the vertex of the graph of the two-variable equation, because the graph of the function would be a vertical stretch (with a scale factor of 4) of the graph of the equation.

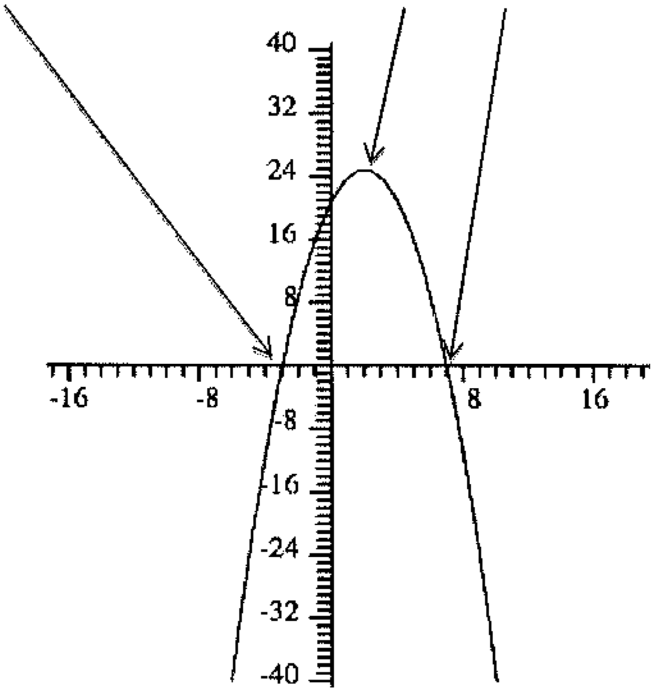
(0, -3) (2, 25) (7, 0)

NOTES: Factored form is easiest to use with the zeros as the given information. We just need to have a negative leading coefficient. Any negative number will work. This example uses z = −1:

$f(x) = -(x + 3)(x - 7)$

To graph this function plot the zeros/intercepts (−3, 0) and (7, 0). The vertex will be on the axis of symmetry (x = 2). Evaluate the equation for x = 2 to find the vertex, (2, 25), and sketch.

Results for the graphs may be wider or narrower and the vertex may be higher or lower. However all should open down, pass through the points (−3, 0) an (7, 0), and the x-coordinate for the vertex should be 2.



UNIT ONE

for

Content Area of

ENGLISH

HS Band
9th Grade English

Name .... **Text** ....

FINDING DETAILS I find interesting details that are <u>related</u> and that stand out to me from reading the text closely.	**Detail 1 (Ref.:)**	**Detail 2 (Ref.:)**	**Detail 3 (Ref.:)**

CONNECTING THE DETAILS I re-read and think about the details, and <u>explain</u> the connections I find among them.	**What I think about detail 1:**	**What I think about detail 2:**	**What I think about detail 3:**
	How I connect the details:		

MAKING A CLAIM I state a conclusion that I have come to and can support with <u>evidence</u> from the text after reading and thinking about it closely.	**My claim about the text:**

Name .... **Text** ....

CLAIM:

Supporting Evidence	Supporting Evidence	Supporting Evidence
(Reference:)	(Reference:)	(Reference:)

CLAIM:

Supporting Evidence	Supporting Evidence	Supporting Evidence
(Reference:)	(Reference:)	(Reference:)

Name .... **Text** ....

CLAIM:

Point 1		**Point 2**	
A Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)	**A** Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)
C Supporting Evidence (Reference:)	**D** Supporting Evidence (Reference:)	**C** Supporting Evidence (Reference:)	**D** Supporting Evidence (Reference:)

Name .... **Text** ....

CLAIM:

Point 1	Point 2	Point 3
A Supporting Evidence **(Reference:)**	**A** Supporting Evidence **(Reference:)**	**A** Supporting Evidence **(Reference:)**
B Supporting Evidence **(Reference:)**	**B** Supporting Evidence **(Reference:)**	**B** Supporting Evidence **(Reference:)**
C Supporting Evidence **(Reference:)**	**C** Supporting Evidence **(Reference:)**	**C** Supporting Evidence **(Reference:)**

MAKING
EVIDENCE-BASED CLAIMS

DEVELOPING CORE PROFICIENCIES
ENGLISH LANGUAGE ARTS / LITERACY UNIT

GRADE 9

"Apology"
Plato

ODELL
EDUCATION
www.odelleducation.com

DEVELOPING CORE PROFICIENCIES SERIES

This unit is part of the Odell Education Literacy Instruction: Developing Core Proficiencies program, an integrated set of ELA units spanning grades 6-12. Funded by USNY Regents Research Fund, the program (under development) is comprised of a series of four 3-week units at each grade level that provide direct instruction on a set of literacy proficiencies at the heart of the CCSS.

Unit I: Reading Closely for Textual Details
Unit II: Making Evidence-Based Claims
Unit III: Researching to Deepen Understanding
Unit IV: Building Evidence-Based Arguments

The Core Proficiencies units have been designed to be used in a variety of ways. They can be taught as short stand-alone units to introduce or develop key student proficiencies. Teachers can also integrate them into larger modules that build up to and around these proficiencies. Teachers can also apply the activity sequences and unit materials to different texts and topics. The materials have been intentionally designed for easy adaptation to new texts.

Unit materials available at
www.odelleducation.com/resources

MAKING EVIDENCE-BASED CLAIMS

Making evidence-based claims about texts is a core literacy and critical thinking proficiency that lies at the heart of the CCSS. The skill consists of two parts. The first part is the ability to extract detailed information from texts and grasp how it is conveyed. Education and personal growth require real exposure to new information from a variety of media. Instruction should push students beyond general thematic understanding of texts into deep engagement with textual content and authorial craft.

The second half of the skill is the ability to make valid claims about the new information thus gleaned. This involves developing the capacity to analyze texts, connecting information in literal, inferential, and sometimes novel ways. Instruction should lead students to do more than simply restate the information they take in through close reading. Students should come to see themselves as creators of meaning as they engage with texts.

It is essential that students understand the importance and purpose of making evidence-based claims, which are at the center of many fields of study and productive civic life. We must help students become invested in developing their ability to explore the meaning of texts. Part of instruction should focus on teaching students how to understand and talk about their skills.

It is also important that students view claims as their own. They should see their interaction with texts as a personal investment in their learning. They are not simply reading texts to report information expected by their teachers, but should approach texts with their own authority and confidence to support their analysis

This unit is designed to cultivate in students the ability to make evidence-based claims about texts. Students perform a sequence of activities centered on a close reading of text throughout the unit.

HOW THIS UNIT IS STRUCTURED

The unit activities are organized into five parts, each associated with sequential portions of text. The parts build on each other and can each span a range of instructional time depending on scheduling and student ability.

The unit intentionally separates the development of critical reading skills from their full expression in writing. A sequence of tools isolates and supports the progressive development of the critical reading skills. Parts 1-2 focus on making evidence-based claims as readers. Part 3 focuses on preparing to express evidence-based claims by organizing evidence and thinking. Parts 4 and 5 focus on expressing evidence-based claims in writing.

This organization is designed to strengthen the precision of instruction and assessment, as well as to give teachers flexibility in their use of the unit.

The first activities of Parts 2-5 – which involve independently reading sections of the text – are designed as independent reading assignments. If scheduling and student ability do not support independent reading outside of class, these activities can be done in class at the beginning of each Part. Accordingly, they are listed both as an independent reading activity at the end of each part and as an activity beginning the sequence of the next part.

Alternate configurations of Part 5 are given in the detailed unit plan to provide multiple ways of structuring a summative assessment.

HOW THIS UNIT ALIGNS WITH CCSS FOR ELA/LITERACY

The primary CCSS alignment of the unit instruction is with **RI.1** and **W.9b** (*cite evidence to support analysis of explicit and inferential textual meaning*).

The evidence-based analysis of the text, including the text-dependent questions and the focus of the claims, involve **RI.2** and **RI.3** (*determine a central idea and analyze how it is conveyed and elaborated with details over the course of a text*).

The numerous paired activities and structured class discussions develop **SL.1** (*engage effectively in a range of collaborative discussions building on others' ideas and expressing their own clearly*).

The evidence-based writing pieces involve **W.4** (*produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience*).

HOW THIS UNIT ASSESSES STUDENT LEARNING

The unit's primary instructional focus is on making evidence-based claims as readers and writers. Parts 1-3 develop the reading skill. Activities are sequenced to build the skill from the ground up. A series of tools supports students in their progressive development of the skill. These tools structure and capture students' critical thinking at each developmental stage and are the primary method of formative assessment. They are specifically designed to give teachers the ability to assess student development of the reading skill without the influence of their writing abilities.

From the first activity on, students are introduced to and then use a set of criteria that describes the characteristics of an evidence-based claim. In pair work and class discussions, students use the first five of these criteria to discuss and evaluate evidence-based claims made by the teacher and their peers. Teachers use these same criteria to assess student claims presented on the tools from Parts 1-3.

As the instructional focus shifts to writing in Parts 4 and 5 so does the nature of the assessment. In these parts, teachers assess the student writing pieces. Students continue using tools as well, giving teachers clear and distinct evidence of both their reading and writing skills for evaluation. In Parts 4-5, students learn about and use six additional criteria for writing claims. Teachers apply these criteria in the formative assessment of students' written work, as well as the evaluation of their final evidence-based writing pieces.

In addition to reading and writing, the unit incorporates many structured collaborative activities to develop key speaking and listening proficiencies. Students and teachers use the Text-Centered Discussion Checklist to structure and evaluate participation in those discussions. Opportunities are also given for teachers to directly observe and evaluate student speaking and listening skills using the checklist.

Part 5 can be configured in multiple ways giving teachers the flexibility to structure a summative assessment suitable for their students.

HOW THIS UNIT TEACHES VOCABULARY

This unit draws on several strategies for teaching academic and disciplinary vocabulary. The primary strategy is the way critical disciplinary vocabulary and concepts are built into the instruction. Students are taught words like "claim," "evidence," "reasoning," and "inference" through their explicit use in the activities. Students come to understand and use these words as they think about and evaluate their textual analysis and that of their peers.

The EBC Checklist plays a key role in this process. By the end of the unit, students will have developed deep conceptual knowledge of key vocabulary that they can transfer to a variety of academic and public contexts.

The texts and activities also provide many opportunities for text-based academic vocabulary instruction. Many activities focus directly on analyzing the way authors use language and key words to develop ideas and achieve specific purposes. The process of developing and evaluating claims supports the acquisition of these words and content knowledge.

The texts are formatted with integrated tools for vocabulary development. Each page includes editable glossaries where teachers and students can choose various words to define. Some words have been pre-selected and glossed. Teachers may choose to differentiate vocabulary support by student.

HOW THIS UNIT MIGHT BE EMBEDDED IN CONTENT-BASED CURRICULUM

The unit is explicitly and intentionally framed as *skills-based instruction*. It is critical for students to understand that they are developing core literacy proficiencies that will enrich their academic and civic lives. The unit and activities should be framed for them as such. Nonetheless, the texts have been chosen, in part, for their rich content and cultural significance. They contain many important historical and contemporary ideas and themes. Moreover, they have been selected to connect with topics and events typically addressed in the grade's social studies classrooms. Teachers are encouraged to sequence the unit strategically within their curriculum and instructional plans, and to establish content connections that will be meaningful for students. This might involve connecting the unit to the study of topics or eras in social studies, related genres or voices in literature, or themes and guiding questions.

Teachers can also adapt the unit activities and materials to other fiction and non-fiction texts. The materials have been intentionally designed for easy adaptation to a variety of texts.

Whatever the curricular context established by the teacher, the central emphasis of the unit should, however, be on evidence-based, text-focused instruction.

HOW TO USE THESE MATERIALS

This unit is in the format of a **Compressed File.** Files are organized so you can easily browse through the materials and find everything you need to print or e-mail for each day.

The materials are organized into three folders:

UNIT PLAN AND TEXTS

- Unit Plan
- Models
- Text(s)

The ***model claims and tools*** are meant only to illustrate the process, NOT to shape textual analysis. ***It is essential that both teachers and students develop claims based on their own analysis and class discussion.*** Teachers are encouraged to develop their own claims in the blank tools to use with students when modeling the process.

HANDOUTS

- Forming Evidence-Based Claims Handout
- Writing Evidence-Based Claims Handout
- Evidence-Based Claims Criteria Checklists I and II
- Evidence-Based Writing Rubric
- Text-Centered Discussion Checklist

TOOLS

- Forming Evidence-Based Claims
- Making Evidence-Based Claims
- Organizing Evidence-Based Claims
- Written Evidence-Based Claim

TEXTS are formatted with spacing and margins to support ***teacher and student annotation***. Students should be encouraged to mark up their texts (electronically or in print) as they search for details. ***Paragraphs and lines are numbered*** for referencing in writing and discussion. ***Editable glossaries*** are at the bottom of each page. While some words have already been bolded and glossed, teachers are encouraged to use the editable features for choosing words they wish to focus on or gloss, and to differentiate vocabulary support for their students.

TOOLS and **CHECKLISTS** have been created as ***editable PDF forms.*** With the free version of Adobe Reader, students and teachers are able to type in them and save their work for recording and e-mailing. This allows students and teachers to work either with paper and pencil or electronically according to their strengths and needs. It also allows teachers to collect and organize student work for evaluation and formative assessment.

If you decide to **PRINT** materials, please note that you can print them at ***actual size***, without enabling the auto-fit function. All materials can be printed either in color or in black and white.

UNIT OUTLINE

PART 1: UNDERSTANDING EVIDENCE-BASED CLAIMS

- The teacher presents the purpose of the unit and explains the skill of making EBCs.
- Students independently read part of the text with a text-dependent question to guide them.
- Students follow along as they listen to the text being read aloud and discuss a series of text-dependent questions.
- The teacher models a critical reading and thinking process for forming EBCs about texts.

PART 2: MAKING EVIDENCE-BASED CLAIMS

- Students independently read part of the text and look for evidence to support a claim made by the teacher.
- Students follow along as they listen to the text being read aloud and discuss a series of text-dependent questions.
- In pairs, students look for evidence to support claims made by the teacher.
- The class discusses evidence in support of claims found by student pairs.
- In pairs, students make an EBC of their own and present it to the class.

PART 3: ORGANIZING EVIDENCE-BASED CLAIMS

- Students independently read part of the text and make an EBC.
- Students follow along as they listen to part of the text being read aloud.
- The teacher models organizing evidence to develop and explain claims using student EBCs.
- In pairs, students develop a claim with multiple points and organize supporting evidence.
- The class discusses the EBCs developed by student pairs.

PART 4: WRITING EVIDENCE-BASED CLAIMS

- Students independently review the text and develop an EBC.
- The teacher introduces and models writing EBCs using a claim from Part 3.
- In pairs, students write EBCs using one of their claims from Part 3.
- The class discusses the written EBCs of volunteer student pairs.
- The class discusses their new EBCs and students read aloud portions of the text.
- Students independently write EBCs.

PART 5: DEVELOPING EVIDENCE-BASED WRITING

- Students review the entire text and make a new EBC.
- The teacher analyzes volunteer student evidence-based writing from Part 4 and discusses developing global EBCs.
- Students discuss their new claims in pairs and then with the class.
- Students independently write a final evidence-based writing piece.
- The class discusses final evidence-based writing pieces of student volunteers.

PART 1

UNDERSTANDING
EVIDENCE-BASED CLAIMS

"just a human sort of wisdom"

OBJECTIVE: Students learn the importance and elements of making evidence-based claims through a close reading of part of the text.

ESTIMATED TIME: 2-3 days

MATERIALS:
Forming EBC Handout
Forming EBC Tool
EBC Criteria Checklist I
Making EBC Tool

ACTIVITIES

1- INTRODUCTION TO UNIT
The teacher presents the purpose of the unit and explains the proficiency of making EBCs.

2- INDEPENDENT READING
Students independently read part of the text with a text-dependent question to guide them.

3- READ ALOUD AND CLASS DISCUSSION
Students follow along as they listen to the text being read aloud, and the teacher leads a discussion guided by a series of text-dependent questions.

4- MODEL FORMING EBCs
The teacher models a critical reading and thinking process for forming EBCs about texts.

ALIGNMENT TO CCSS

TARGETED STANDARD(S): RI.9-10.1
RI.9-10.1: Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.

SUPPORTING STANDARD(S): RI.9-10.2 RI.9-10.3 SL.9-10.1

RI.9-10.2: Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
RI.9-10.3: Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.
SL.9-10.1: Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9-10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

ACTIVITY 1: INTRODUCTION TO UNIT

The teacher presents the purpose of the unit and explains the proficiency of making evidence-based claims, making reference to the first five criteria from the EBC Checklist I.

INSTRUCTIONAL NOTES

Introduce the central purpose of the unit and the idea of a "claim" someone might make. The following is a possible approach:

Introduce the first characteristic of an evidence-based claim: "States a conclusion you have come to… and that you want others to think about." Pick a subject that is familiar to students, such as "school lunches" and ask them to brainstorm some claim statements they might make about the subject. Introduce the fourth characteristic: "All parts of the claim are supported by specific evidence you can point to" and distinguish claims that can be supported by evidence from those that are unsupported opinions, using the students' brainstorm list as a reference.

Move from experience-based claims to claims in a field like science. Start with more familiar, fact-based claims (For example, the claim "It is cold outside" is supported by evidence like "The outside thermometer reads 13 degrees F" but is not supported with statements like "It feels that way to me"). Then discuss a claim such as "Smoking has been shown to be hazardous to your health" and talk about how this claim was once considered to be an opinion, until a weight of scientific evidence over time led us to accept this claim as fact. Introduce the third characteristic/criterion: "Demonstrates knowledge of and sound thinking about a topic" and with it the idea that a claim becomes stronger as we expand our knowledge about a subject and find more and better evidence to support the claim.

Move from scientific claims to claims that are based in text that has been read closely. Use an example of a text read recently in class or one students are likely to be familiar with. Highlight that textual claims can start as statements about what a text tells us directly (literal comprehension) such as "Tom Sawyer gets the other boys to paint the fence" and then move to simple conclusions we draw from thinking about the text, like: "Tom Sawyer is a clever boy" because (evidence) "He tricks the other boys into doing his work and painting the fence." Then explain how text-based claims can also be more complex and require more evidence (e.g., "Mark Twain presents Tom Sawyer as a 'good bad boy' who tricks others and gets into trouble but also stands up for his friend Jim."), sometimes – as in this example – requiring evidence from more than one text or sections of text.

Explain that the class will be practicing the skill of making evidence-based claims that are based in the words, sentences, and ideas of a text by closely reading and analyzing the text (or texts) selected for this unit.

In the activities that follow, students will learn to make a text-based claim by moving from literal understanding of its details, to simple supported conclusions or inferences, to claims that arise from and are supported by close examination of textual evidence. This inductive process mirrors what effective readers do and is intended to help students develop a method for moving from comprehension to claim. In addition, the guiding questions, model claims, and movement through the text over the course of the unit are sequenced to transition students from an initial, literal understanding of textual details to:

- Claims about fairly concrete ideas presented in short sections of the text;
- Claims about more abstract ideas implied across sections of the text;
- More global claims about the entire text and its meaning.

ACTIVITY 2: INDEPENDENT READING

Students independently read part of the text with a text-dependent question to guide them.

INSTRUCTIONAL NOTES

Students independently read the first two paragraphs of the Apology text and answer a text-dependent question: What is Socrates being accused of?
Briefly introduce students to the text. The introduction should be kept to naming the author, the book and the year of publication. You might also read the introductory lines at the beginning of the text to make sure students understand the context of Socrates' apology.

Students should be allowed to approach the text freshly and to make their own inferences based on textual content. Plenty of instruction and support will follow to ensure comprehension for all students. The question helps orient students to the text and begins the focus on searching for textual evidence.

ACTIVITY 3: READ ALOUD AND CLASS DISCUSSION

Students follow along as they listen to the text being read aloud, and the teacher leads a discussion guided by a series of text-dependent questions.

INSTRUCTIONAL NOTES

Read the first three paragraphs aloud or play the audio file following the link in the text. (Note that while this recording is the same translation as the text provided, it is not excerpted.) Lead a discussion guided by three text-dependent questions:

1- What is Socrates being accused of?
2- How does Socrates make it clear that he is innocent?
3- In paragraph 3, Socrates says he is on trial because of a "certain kind of wisdom" he has. What kind of wisdom is this?

A Spanish translation is provided to support students whose first language is Spanish. These students should be encouraged to read from both translations, while using the English one to develop their evidence-based claims.

The close reading of these paragraphs serves three primary purposes: to ensure comprehension of an important part of the text, to orient students to the practice of close reading, and to guide students in searching for textual evidence.

Use the discussion about the questions to help students learn the essential skills of selecting interesting and significant textual details and connecting them inferentially. This process links directly to the forming of evidence-based claims they will begin in Activity Four.

ACTIVITY 3: READ ALOUD AND CLASS DISCUSSION (CONT'D)

INSTRUCTIONAL NOTES

1- What is Socrates being accused of?

Discuss with students how beginning with a statement of his accusation sets a foundation for the purpose and meaning of the text. Socrates begins by giving a summary of the charges against him. He cites an affidavit that accuses him: "Socrates is a criminal and meddles in matters where he has no business. He's always poking under the earth and up in the sky. He makes the worse case look better; and he teaches this sort of stuff to others." At this point in the text, it is unclear what it means to be accused of making "the worse case look better", or why Socrates is being put to trial for his curiosity. There are, however, a few concrete accusations that can be extracted from the affidavit; namely, that Socrates is being charged with unlawfully teaching doctrines that are not acceptable.

2- How does Socrates make it clear that he is innocent?

One of the pleasures and challenges of this text is figuring out Socrates' various purposes with his speech. This can be a recurrent theme for discussion. Throughout the speech, guide students through Socrates' subtle and ironic language by referring to the text. It is clear that these accusations against him are longstanding, and that he intends to refute them. He immediately gives the example of a play by Aristophanes that portrays Socrates as claiming he can "walk on air." Socrates states he knows nothing about Aristophanes' accusations and asks the crowd if they have ever heard him claiming he can do such things. The crowd agrees they never have." Talk about the importance of the title of the text here. Though the text is titled "The Apology," the Greek word apologia is better translated into English as defense. The text is meant to be Socrates' defense of himself, not his apology for something he admits he has done wrong.

3-What kind of wisdom does Socrates say he has?

In paragraph 3, Socrates introduces his strange account of how he is wiser than everyone else because he admits he knows nothing. As the students will soon read, the Delphi oracle confirms that he is the wisest. Before reading about the oracle, however, draw in students to the seemingly contradictory lines 25-28. At first, Socrates says he has a "human wisdom" for which he may really be wise. But then he goes on to say that he might not understand the "superhuman wisdom" of his accusers, bringing the reader to question Socrates' reasoning. Ask the students to reflect on this paragraph and see how Socrates might actually be joking. This form of talking forms one of the defining aspects of the Apology: Socrates' use of irony to defend himself and mock his accusers. In order for students to appreciate Socrates' argument, review the meaning of irony and how it is created in a text.

ACTIVITY 4: MODEL FORMING EBCs

INSTRUCTIONAL NOTES

Based on the class discussion of the text, the teacher models a critical reading and thinking process for forming EBCs: from comprehension of textual details that stand out, to an inference that arises from examining the details, to a basic EBC that is supported by specific references back to the text.

Once the class has reached an understanding of the text, use the Forming EBC Handout to introduce a three-step process for making a claim that arises from the text.

Exemplify the process by making a claim with the Forming EBC Tool.The tool is organized so that students first take note of "interesting" details that they also see as "related" to each other. The second section asks them to think about and explain a connection they have made among those details.
Such "text-to-text" connections should be distinguished from "text-to-self" connections readers make between what they have read and their own experiences. These "text-to-text" connections can then lead them to a "claim" they can make and record in the third section of the tool – a conclusion they have drawn about the text that can be referenced back to textual details and text-to-text connections. Have students follow along as you talk through the process with your claim.

To provide structured practice for the first two steps, you might give students a textual detail on a blank tool. In pairs, have students use the tool to find other details/quotations that could be related to the one you have provided, and then make/explain connections among those details. Use the EBC Checklist 1 to discuss the claim, asking students to explain how it meets (or doesn't yet meet) the criteria.

[Note: Here and throughout the entire unit, you are encouraged to develop claims based on your own analysis and class discussion. The provided models are possibilities meant more to illustrate the process than to shape textual analysis. Instruction will be most effective if the claims used in modeling flow naturally from the textual ideas and details you and the students find significant and interesting. Also, while the tools have three or four places for supporting evidence, students should know that not all claims require three pieces of evidence. Places on the tools can be left blank.]

INDEPENDENT READING ACTIVITY

Students read paragraphs 4-9 and use the Making EBC Tool to find evidence to support the teacher-provided claim. This activity overlaps with the first activity of Part 2 and can be given as homework or done at the beginning of the next class.

ASSESSMENT OPPORTUNITIES

The Forming EBC Tool should be evaluated to get an initial assessment of students' grasp of the relationship between claims and textual evidence. Even though the work was done together with the class, filling in the tool helps them get a sense of the critical reading and thinking process and the relationships among the ideas. Also make sure that students are developing the habit of using quotation marks and recording the reference.

PART 2

MAKING
EVIDENCE-BASED CLAIMS

"I neither know nor think I know."

OBJECTIVE: Students develop the ability to make evidence-based claims through a close reading of the text.

ESTIMATED TIME: 1-3 days

MATERIALS:
Making EBC Tool
Forming EBC Handout
Forming EBC Tool
EBC Criteria Checklist I
TCD Checklist

ACTIVITIES

1- INDEPENDENT READING AND FINDING SUPPORTING EVIDENCE
Students independently read part of the text and use the Making EBC Tool to look for evidence to support a claim made by the teacher.

2- READ ALOUD AND CLASS DISCUSSION
Students follow along as they listen to the same part of the text being read aloud and discuss a series of text-dependent questions.

3- FIND SUPPORTING EVIDENCE IN PAIRS
In pairs, students use the Making EBC Tool to look for evidence to support additional claims about the text made by the teacher.

4- CLASS DISCUSSION OF EBCs
The class discusses evidence in support of claims found by student pairs.

5- FORMING EBCs IN PAIRS
In pairs, students use the Forming EBC Tool to make an evidence-based claim of their own and present it to the class.

ALIGNMENT TO CCSS

TARGETED STANDARD(S): RI.9-10.1
RI.9-10.1: Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.

SUPPORTING STANDARD(S): RI.9-10.2 RI.9-10.3 SL.9-10.1
RI.9-10.2: Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
RI.9-10.3: Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.
SL.9-10.1: Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9-10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

ACTIVITY 1: INDEPENDENT READING AND FINDING SUPPORTING EVIDENCE

Students independently read part of the text and use the Making EBC Tool to look for evidence to support a claim made by the teacher.

INSTRUCTIONAL NOTES

Students independently work on paragraphs 4-9 of Plato's "Apology." Depending on scheduling and student ability, students can be assigned to read and complete the tool for homework. Teachers should decide what works best for their students. It's essential that students have opportunity to read the text independently. All students must develop the habit of perseverance in reading. Assigning the reading as homework potentially gives them more time with the text. Either way, it might be a good idea to provide some time at the beginning of class for students to read the section quietly by themselves. This ensures that all students have had at least some independent reading time.

Also depending on scheduling and student ability, some students might choose (or be encouraged) to read ahead. Instructional focus should follow the pacing outlined in the activities, but students will only benefit from reading and re-reading the text throughout the duration of the unit.

ACTIVITY 2: READ ALOUD AND CLASS DISCUSSION

Students follow along as they listen to the same part of the text being read aloud and discuss a series of text-dependent questions.

INSTRUCTIONAL NOTES

Students follow along as they listen to paragraphs 4-9 of the text being read aloud and discuss three text-dependent questions:

1- What does Socrates think about the oracle's message?

2- What does Socrates do in an attempt to test the truth of the oracle's prophesy?

3- Why do Socrates' actions incite the anger of his peers?

Read the text aloud to the class while students follow along. Alternatively, students could be asked to read aloud to the class. Work through the text using the following three text-dependent questions.

INSTRUCTIONAL NOTES

1- What does Socrates think about the oracle's message?

Socrates explains that a close friend of his, Chaerephon, went to the oracle and asked if there was anyone wiser than Socrates. The priestess answered that there is no one wiser than Socrates. When Chaerephon relayed this to Socrates, Socrates was sconfused and wondered, "what ever does the god mean?". Convinced of his own ignorance, but equally convinced of the infallibility of the oracle, Socrates concluded that the statement must be a riddle and set off to solve it. Emphasize that in ancient Greece, the oracle was thought to be a portal through which the gods spoke directly to people. The statements of the oracle were understood to be the word of god, and therefore never doubted. "He can't be telling a lie. That just wouldn't be right," Socrates reasons. If a statement was confusing or seemed incorrect, it was assumed to be a riddle. Therefore, when Socrates heard the oracle's statement that he was the wisest man in Greece, he took it as his life's calling to figure out the truth behind that statement.

2- What realization does Socrates come to while trying to prove the oracle wrong?

In order to "prove the oracle wrong," Socrates sought out Athenian citizens who were typically thought of as wise men. When he began to question their wisdom, Socrates found that not only were they not wise, but they were incapable of admitting their ignorance. Socrates comes to the conclusion that the wisdom he has lies in his ability to recognize what he does not know, which no one else seems willing to do: "I neither know nor think I know." This idea is a central theme of the text, and is worth emphasizing. Socrates points out that people simply assume he knows what he is talking about, when in fact he only reveals the others' own ignorance. Still on course to solving the oracle's riddle, Socrates states that the god must have meant that he is wise because he knows nothing (line 70). Discuss Socrates' irony with these statements, reminding students of what Socrates is accused of and how it compares to his revelation.

ACTIVITY 2: READ ALOUD AND CLASS DISCUSSION (CONT'D)

INSTRUCTIONAL NOTES

3- Even though he knows he has angered many people with his interrogations, how does Socrates turn the argument around to his own benefit?

In his "task of helping god," Socrates exposes the wise as unwise, which he does with apparent lack of satisfaction – he is, after all, simply trying to figure out the god's riddle. As he was doing this, Socrates was aware of the fact that his peers were angry with him, but he felt that he was responsible to try to understand the oracle's message. Ask students to focus on the specific words and phrases Socrates chooses to build his irony and innocence. Socrates says he is "sad and fearful" because he has to do this unpleasant work for the god – it is not his fault that he must reveal these peoples' lack of wisdom, but the fault of the oracle. In fact, he has gone so far to accept a "poverty-stricken" life due to his sense of obligation to help the god. Socrates paints himself as a victim of the oracle, rather than benefiting from it. Have students focus on other areas in the Apology where Socrates does the same: turns apparently negative aspects of the trial into positive ones for him, or "makes the worse case look better" as his accusers put it.

ACTIVITY 3: FIND SUPPORTING EVIDENCE

In pairs, students use the Making EBC Tool to look for evidence to support additional claims about the text made by the teacher.

INSTRUCTIONAL NOTES

Once the class has reached a solid understanding of the text, connect it to the skill of making claims and supporting them with evidence by presenting a few main claims. Pass out the tools and have students work in pairs to find evidence to support the claims.

Collect each student's Making EBC Tool with the evidence they found for the first claim. These should be evaluated to get an assessment of where each student is in the skill development. Students should use their tools for their work in pairs—repeating the first claim and refining their evidence based on the read aloud and class discussion. Even though students are not finding the evidence independently, they should each fill in the tools to reinforce their acquisition of the logical structure among the ideas. Students should get into the habit of using quotation marks when recording direct quotes and including the line numbers of the evidence.

The instructional focus here is developing familiarity with claims about texts and the use of textual evidence to support them. Students should still not be expected to develop complete sentences to express supporting evidence. The pieces of evidence should be as focused as possible. The idea is for students to identify the precise points in the text that support the claim. This focus is lost if the pieces of evidence become too large. The tools are constructed to elicit a type of "pointing" at the evidence.

One approach for ensuring a close examination of claims and evidence is to provide erroneous claims that contradict textual evidence and ask students to find the places that disprove the claim. Students could then be asked to modify it to account for the evidence.

ACTIVITY 4: CLASS DISCUSSION OF EBCs

The class discusses evidence in support of claims found by student pairs.

INSTRUCTIONAL NOTES

After students have finished their work in pairs, regroup for a class discussion. Have pairs volunteer to present their evidence to the rest of the class. Discuss the evidence, evaluating how each piece supports the claims. Begin by modeling the evaluation, referring to the checklist, and then call on students to evaluate the evidence shared by the other pairs.

They can offer their own evidence to expand the discussion. Carefully guide the exchanges, explicitly asking students to support their evaluations with reference to the text.

These constructive discussions are essential for the skill development. Listening to and evaluating the evidence of others and providing text-based criticism expands students' capacity to reason through the relationship between claims and evidence. Paying close attention to and providing instructional guidance on the student comments is as important to the process as evaluating the tools and creates a class culture of supporting all claims (including oral critiques) with evidence.

Using the Text-Centered Discussion Checklist is one way of talking about and supporting student participation in class and pair discussions, especially if students are already familiar with the TCD checklist from previous units. If not, time can be taken (if desired) to introduce them to some or all of the criteria of effective text-centered discussions.

ACTIVITY 5: FORMING EBCs IN PAIRS

In pairs, students use the Forming EBC Tool to make an evidence-based claim of their own and present it to the class.

INSTRUCTIONAL NOTES

Once the claims and evidence have been discussed, students return to the pairs and use the tool to make an evidence-based claim of their own. Pairs should make a single claim, but each student should fill in his or her own tool.

Regroup and discuss the claims and evidence as a class. Pairs can use their tool to present their claims and evidence orally.

Talk through the process modeled in the tool, including the nature of the details that stood out to students, the reasoning they used to group and relate them, and the claim they developed from the textual evidence.

Draw upon the Forming EBC Handout and EBC Criteria Checklist I to help guide discussion.

INDEPENDENT READING ACTIVITY

Students read paragraphs 10-17 of Plato's "Apology" and use the Forming EBC Tool to make a claim and support it with evidence. This activity overlaps with the first activity of Part 3 and can be given as homework or done at the beginning of the next class.

ASSESSMENT OPPORTUNITIES

The Making EBC Tools should be evaluated to assess the development of the student's grasp of the relationship between claims and textual evidence. They should show progress in the relevance and focus of the evidence. The Forming EBC Tools are students' first attempts at making their own claims with the help of a peer. Basic claims are fine at this point. Use the EBC Criteria Checklist to structure the evaluation and feedback to students. Evaluation should focus on the validity and clarity of the claim and the relevance of the evidence. Recording the "thinking" part of the tool is important in order to strengthen the student's reasoning skills as well as provide them with the academic vocabulary to talk about them.

Evidence should be in quotation marks and the reference recorded. Using quotation marks helps students make the distinction between quotes and paraphrases. It also helps them to eventually incorporate quotes properly into their writing. Recording references is critical not only for proper incorporation in writing, but also because it helps students return to text for re-evaluating evidence and making appropriate selections.

The Text-Centered Discussion Checklist can be used to evaluate student participation in discussions for formative and diagnostic information. Teachers and students can get a sense of areas where development in speaking and listening skills are needed.

PART 3

ORGANIZING
EVIDENCE-BASED CLAIMS

"You're not likely to get another gadfly like me."

OBJECTIVE: Students expand their ability into organizing evidence to develop and explain claims through a close reading of the text.

ESTIMATED TIME: 1-3 days

MATERIALS:
Organizing EBC Tool
Forming EBC Tool
EBC Criteria Checklist I

ACTIVITIES

1- INDEPENDENT READING AND FORMING EBCs
Students independently read part of the text and use the Forming EBC Tool to make an evidence-based claim.

2- READ ALOUD
Students follow along as they listen to part of the text being read aloud.

3- MODEL ORGANIZING EBCs
The teacher models organizing evidence to develop and explain claims using student evidence-based claims and the Organizing EBC Tool.

4- ORGANIZING EBCs IN PAIRS
In pairs, students develop a claim with multiple points using the Organizing EBC Tool.

5- CLASS DISCUSSION OF STUDENT EBCs
The class discusses the evidence-based claims developed by student pairs.

ALIGNMENT TO CCSS

TARGETED STANDARD(S): RI.9-10.1
RI.9-10.1: Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.

SUPPORTING STANDARD(S): RI.9-10.2 RI.9-10.3 SL.9-10.1

RI.9-10.2: Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
RI.9-10.3: Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.
SL.9-10.1: Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9-10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

ACTIVITY 1: INDEPENDENT READING AND FORMING EBCs

Students independently read part of the text and use the Forming EBC Tool to make an evidence-based claim.

INSTRUCTIONAL NOTES

Students independently work on paragraphs 10-17 of Plato's "Apology." Depending on scheduling and student ability, students can be assigned to read and complete the tool for homework. Teachers should decide what works best for their students. It's essential that students have an opportunity to read the text independently. All students must develop the habit of perseverance in reading. Assigning the reading as homework potentially gives them more time with the text. Either way, it might be a good idea to provide some time at the beginning of class for students to read quietly by themselves. This ensures that all students have had least some independent reading time.

ACTIVITY 2: READ ALOUD

Students follow along as they listen to part of the text being read aloud.

INSTRUCTIONAL NOTES

Read paragraphs 10-17 aloud to the class while students follow along. Alternatively, students could be asked to read aloud to the class.

ACTIVITY 3: MODEL ORGANIZING EBCs

The teacher models organizing evidence to develop and explain claims using student evidence-based claims and the Organizing EBC Tool.

INSTRUCTIONAL NOTES

The central focus of Part 3 is learning the thinking processes associated with developing an evidence-based claim: reflecting on how one has arrived at the claim; breaking the claim into parts; organizing supporting evidence in a logical sequence; anticipating what an audience will need to know in order to understand the claim; and, eventually, planning a line of reasoning that will substantiate the claim. This is a complex set of cognitive skills, challenging for most students , but essential so that students can move from the close reading process of arriving at a claim (Parts 1-2 of the unit) to the purposeful writing process of explaining and substantiating that claim (Parts 4-5).

How a reader develops and organizes a claim is dependent upon the nature of the claim itself – and the nature of the text (or texts) from which it arises. In some cases – simple claims involving literal interpretation of the text – indicating

ACTIVITY 3:
MODEL ORGANIZING EBCs (CONT'D)

INSTRUCTIONAL NOTES

where the claim comes from in the text and explaining how the reader arrived at it is sufficient. This suggests a more straightforward, explanatory organization. More complex claims, however, often involve multiple parts, points, or premises, each of which needs to be explained and developed, then linked in a logical order into a coherent development.

Students only learn how to develop and organize a claim through practice, ideally moving over time from simpler claims and more familiar organizational patterns to more complex claims and organizations.

Students can be helped in learning how to develop a claim by using a set of developmental guiding questions such as the following: [Note: the first few questions might be used with younger or less experiences readers, the latter questions with students who are developing more sophisticated claims.]

- What do I mean when I state this claim? What am I trying to communicate?
- How did I arrive at this claim? Can I "tell the story" of how I moved as a reader from the literal details of the text to a supported claim about the text?
- Can I point to the specific words and sentences in the text from which the claim arises?
- What do I need to explain so that an audience can understand what I mean and where my claim comes from?
- What evidence (quotations) might I use to illustrate my claim? In what order?
- If my claim contains several parts (or premises), how can I break it down, organize the parts, and organize the evidence that goes with them?
- If my claim involves a comparison or a relationship, how might I present, clarify, and organize my discussion of the relationship between parts or texts?

Students who are learning how to develop a claim, at any level, can benefit from graphic organizers or instructional scaffolding that helps them work out, organize, and record their thinking. While such models or templates should not be presented formulaically as a "how to" for developing a claim, they can be used to support the learning process. The Organizing EBC Tool can be used to provide some structure for student planning – or you can substitute another model or graphic organizer that fits well with the text , the types of claims being developed, and the needs of the students.

Begin by orienting students to the new tool and the idea of breaking down a claim into parts and organizing the evidence accordingly.

Ask for a volunteer to present his or her claim and supporting evidence. Use the example as a basis for a discussion. Based on the flow of discussion, bring in other volunteers to present their claims and evidence to build and help clarify the points. Work with students to hone and develop a claim. As a class, express the organized claim in Organizing EBC Tool The provided teacher version is one possible way a claim could be expressed and organized.

ACTIVITY 4: ORGANIZING EBCs IN PAIRS

In pairs, students develop and organize a claim using the Organizing EBC Tool.

INSTRUCTIONAL NOTES

When the class has reached a solid expression of an organized evidence-based claim, have students work in pairs, using the tool to develop and organize another claim. You might want to give students some general guidance by directing their focus to a specific section of the text.

ACTIVITY 5: CLASS DISCUSSION OF STUDENT EBCs

After students have finished their work in pairs, regroup for a class discussion about their EBCs.

INSTRUCTIONAL NOTES

Have pairs volunteer to present their claims and evidence to the rest of the class. Discuss the evidence and organization, evaluating how each piece supports and develops the claims. Repeat the process from activity two, using student work to explain how evidence is organized to develop aspects of claims. The teacher version of the Organizing EBC Tool is one possible way a claim could be expressed and organized.

INDEPENDENT READING ACTIVITY

Students review the text and use the Forming EBC Tool to make any claim and support it with evidence. This activity overlaps with the first activity of Part 4 and can be given as homework or done at the beginning of the next class.

ASSESSMENT OPPORTUNITIES

Students are now beginning to develop more complex claims about challenging portions of the text. Their Forming EBC Tool should demonstrate a solid grasp of the claim-evidence relationship, but do not expect precision in the wording of their claims. Using the Organizing EBC Tool will help them clarify their claims as they break it into parts and organize their evidence. How they have transferred their information will demonstrate their grasp of the concept of organizing. Their second Organizing EBC Tool should show progress in all dimensions including the clarity of the claim and the selection and organization of evidence. Use the EBC criteria checklist to structure the evaluation and feedback to students.

PART 4

WRITING
EVIDENCE-BASED CLAIMS

"The unexamined life is not worth living."

OBJECTIVE: Students develop the ability to express evidence-based claims in writing through a close reading of the text.

ESTIMATED TIME: 1-3 days

MATERIALS:
Writing EBC Handout
Forming EBC Tool
Organizing EBC Tool
EBC Criteria Checklist II
TCD Checklist

ACTIVITIES

1- INDEPENDENT READING AND MAKING EBCs
Students independently review the text and use the Forming EBC Tool to develop an evidence-based claim.

2- MODEL WRITING EBCs
The teacher introduces and models writing evidence-based claims using a claim developed in Part 3.

3- WRITING EBCs IN PAIRS
In pairs, students write evidence-based claims using one of their claims from Part 3.

4- CLASS DISCUSSION OF WRITTEN EBCs
The class discusses the written evidence-based claims of volunteer student pairs.

5- READ ALOUD AND CLASS DISCUSSION
The class discusses their new evidence-based claims and students read aloud portions of the text.

6- INDEPENDENT WRITING OF EBCs
Students independently write their new evidence-based claims.

ALIGNMENT TO CCSS

TARGETED STANDARD(S): RI.9-10.1 W.9-10.9b
RI.9-10.1: Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.
W.9-10.9b: Draw evidence from literary or informational texts to support analysis, reflection, and research.

SUPPORTING STANDARD(S): RI.9-10.2 RI.9-10.3 SL.9-10.1 W.8.4
RI.9-10.2: Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
RI.9-10.3: Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.
SL.9-10.1: Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9-10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.

ACTIVITY 1: INDEPENDENT READING AND MAKING EBCs

Students independently read paragraphs 18-23 in the text and use the Forming EBC Tool to develop an evidence-based claim.

INSTRUCTIONAL NOTES

Depending on scheduling and student ability, students can be assigned to read and complete the tool for homework. Teachers should decide what works best for their students. It's essential that students have an opportunity to read the text independently. All students must develop the habit of perseverance in reading. Assigning the reading as homework potentially gives them more time with the text. Either way, it might be a good idea to provide some time at the beginning of class for students to read the text quietly by themselves. This ensures that all students have had at least some independent reading time.

ACTIVITY 2: MODEL WRITING EBCs

The teacher introduces and models writing evidence-based claims using a claim developed in Part 3.

INSTRUCTIONAL NOTES

Parts 1-3 have built a solid foundation of critical thinking and reading skills for developing and organizing evidence-based claims. Parts 4 and 5 focus on expressing evidence-based claims in writing. Class discussions and pair work have given students significant practice expressing and defending their claims orally. The tools have given them practice selecting and organizing evidence. Expressing evidence-based claims in writing should now be a natural transition from this foundation.

Begin by explaining that expressing evidence-based claims in writing follows the same basic structure that they have been using with the tools; one states a claim and develops it with evidence. Discuss the additional considerations when writing evidence-based claims like establishing a clear context and using proper techniques for incorporating textual evidence. Introduce the EBC Criteria Checklist II with the additional writing-related criteria The Writing EBC Handout gives one approach to explaining writing evidence-based claims. Model example written evidence-based claims are provided with the materials.

Explain that the simplest structure for writing evidence-based claims is beginning with a paragraph stating the claim and its context and then using subsequent paragraphs logically linked together to develop the necessary points of the claim with appropriate evidence. (More advanced writers can organize the expression differently, like establishing a context, building points with evidence, and stating the claim at the end for a more dramatic effect. It's good to let students know that the simplest structure is not the only effective way).

Incorporating textual evidence into writing is difficult and takes practice. Expect all students to need a lot of guidance deciding on what precise evidence to use, how to order it, and deciding when to paraphrase or to quote. They will also need guidance structuring sentence syntax and grammar to smoothly and effectively incorporate textual details, while maintaining their own voice and style.

ACTIVITY 2: MODEL WRITING EBCs (CONT'D)

INSTRUCTIONAL NOTES

Three things to consider when teaching this difficult skill:

- A "think-aloud" approach can be extremely effective here. When modeling the writing process, explain the choices you make. For example, "I'm paraphrasing this piece of evidence because it takes the author four sentences to express what I can do in one." Or, "I'm quoting this piece directly because the author's phrase is so powerful, I want to use the original words."
- Making choices when writing evidence-based claims is easiest when the writer has "lived with the claims." Thinking about a claim—personalizing the analysis—gives a writer an intuitive sense of how she wants to express it. Spending time with the tools selecting and organizing evidence will start students on this process.
- Students need to know that this is a process—that it can't be done in one draft. Revision is fundamental to honing written evidence-based claims.

ACTIVITY 3: WRITING EBCs IN PAIRS

In pairs, students write evidence-based claims using their claims from Part 3.

INSTRUCTIONAL NOTES

Students return to the same pairs they had in Part 3 and use their Organizing EBC Tools as guidelines for their writing. Teachers should roam, supporting pairs by answering questions and helping them get comfortable with the techniques for incorporating evidence. Use questions from pairs as opportunities to instruct the entire class.

ACTIVITY 4: CLASS DISCUSSION OF WRITTEN EBCs

The class discusses the written evidence-based claims of volunteer student pairs.

INSTRUCTIONAL NOTES

Have a pair volunteer to write their evidence-based claim on the board. The class together should evaluate the way the writing sets the context, expresses the claim, effectively organizes the evidence, and incorporates the evidence properly. Use the EBC Criteria Checklist II to guide evaluation. The Text-Centered Discussion Checklist (if being used) is helpful here to guide effective participation in discussion. Of course, it's also a good opportunity to talk about grammatical structure and word choice. Let other students lead the evaluation, reserving guidance when needed and appropriate. It is likely and ideal that other students will draw on their own versions in when evaluating the volunteer pair's. Make sure that class discussion maintains a constructive collegial tone and all critiques are backed with evidence.

Model written evidence-based claims are provided in the materials.

ACTIVITY 5: READ ALOUD AND CLASS DISCUSSION

The class discusses their new evidence-based claims from Activity 1 and students read aloud portions of the text.

INSTRUCTIONAL NOTES

At this stage, this activity is reversed from earlier similar ones. Students should present their evidence-based claims and allow discussion to determine areas of the text to be read aloud. Students read aloud relevant portions to help the class analyze claims and selected evidence. Have students transfer their claims from the Forming EBC Tool to the Organizing EBC Tool to help them organize and refine their evidence in preparation for writing.

ACTIVITY 6: INDEPENDENT WRITING OF EBCs

Students independently write their evidence-based claims from their Organizing EBC Tools.

INSTRUCTIONAL NOTES

Students should have refined their claims and developed an Organizing EBC Tool based on class discussion. Now they independently write their claims based on their tools.

INDEPENDENT READING ACTIVITY

Students review the entire text and use an Organizing EBC Tool to make a new claim of their choice and develop it with evidence. This activity overlaps with the first activity of Part 5 and can be given as homework or done at the beginning of the next class.

ASSESSMENT OPPORTUNITIES

At this stage teachers can assess students' reading and writing skills. Students should be comfortable making claims and supporting them with organized evidence. Their tools should demonstrate evidence of mastery of the reading skill. Student writing should demonstrate the same qualities of organization. Make sure they have properly established the context; that the claim is clearly expressed; and that each paragraph develops a coherent point. Evaluate the writing for an understanding of the difference between paraphrase and quotation. All evidence should be properly referenced. Use the EBC Criteria Checklist II to structure the evaluation and feedback to students.

PART 5

DEVELOPING
EVIDENCE-BASED WRITING

"The easiest and finest escape is not by doing people in."

OBJECTIVE: Students develop the ability to express global evidence-based claims in writing through a close reading of the text.

ESTIMATED TIME: 1-2 days

MATERIALS:
Forming EBC Tool
Organizing EBC Tool
Writing EBC Handout
EBC Criteria Checklist II
Evidence-Based Writing Rubric

ACTIVITIES

1- INDEPENDENT READING AND MAKING EBCs
Students independently review the entire text and use the Forming EBC Tool to make a new evidence-based claim.

2- CLASS DISCUSSION OF GLOBAL EBCs
The teacher analyzes volunteer students' written evidence-based claims from Part 4 and discusses developing global EBCs.

3- PAIRS DISCUSS THEIR EBCs
Students discuss their new claims in pairs and then with the class.

4- INDEPENDENT WRITING OF FINAL PIECE
Students independently write a final evidence-based writing piece using their new claims.

5- CLASS DISCUSSION OF FINAL WRITING PIECES
The class discusses final evidence-based writing pieces of student volunteers.

ALIGNMENT TO CCSS

TARGETED STANDARD(S): RI.9-10.1 W.9-10.9b
RI.9-10.1: Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.
W.9-10.9b: Draw evidence from literary or informational texts to support analysis, reflection, and research.

SUPPORTING STANDARD(S): RI.9-10.2 RI.9-10.3 SL.9-10.1 W.9-10.4
RI.9-10.2: Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.
RI.9-10.3: Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.
SL.9-10.1: Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on grades 9-10 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
W.9-10.4: Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

ACTIVITY 1: INDEPENDENT READING AND MAKING EBCs

Students independently review the entire text and use the Forming EBC Tool to make a new evidence-based claim.

INSTRUCTIONAL NOTES

Depending on scheduling and student ability, students can be assigned to read and complete the tool for homework. Teachers should decide what works best for their students. It's essential that students have an opportunity to read the text independently. All students must develop the habit of perseverance in reading. Assigning the reading as homework potentially gives them more time with the text. Either way, it might be a good idea to provide some time at the beginning of class for students to read the text quietly by themselves. This ensures that all students have had at least some independent reading time.

ACTIVITY 2: CLASS DISCUSSION OF GLOBAL EBCs

The teacher analyzes volunteer students' written evidence-based claims from Part 4 and discusses developing global EBCs.

INSTRUCTIONAL NOTES

This activity should be seen as an expansion of the skills developed in Part 4. Begin by analyzing volunteer student-written claims to review the critical aspects of writing. These claims will vary in the amount of text they span and the global nature of the ideas. Use various examples to demonstrate the differences, moving to a discussion of how claims build on each other to produce more global analysis of entire texts.

Throughout the unit the text has been chunked into gradually larger sections, and now students have been asked to consider the entire text for their final claim. Model making a more global claim, discussing its relationship to smaller local claims. Demonstrate how claims can become sub-points for other claims.

Some students can be asked to present the claims they have developed as further models. The Writing EBC Handout could aid discussion on how various claims require various ways of establishing their context and relevance.

ACTIVITY 3: PAIRS DISCUSS THEIR EBCs

Students discuss their new claims from Activity 1 in pairs and then with the class.

INSTRUCTIONAL NOTES

Once the class has a general understanding of the nature of more global claims, break them into pairs to work on the claims they have begun to develop in Activity 1. Have the pairs discuss if their claims contain sub-claims and how best they would be organized. It may be helpful to provide students with both the two-point and three-point organizational tools to best fit their claims.

Volunteer pairs should be asked to discuss the work they did on their claims. At this point they should be able to talk about the nature of their claims and why they have chosen to organize evidence in particular ways.

ACTIVITY 4: INDEPENDENT WRITING OF FINAL PIECE

Students independently write a final evidence-based writing piece using their new claims.

INSTRUCTIONAL NOTES

This evidence-based writing piece should be used as a summative assessment to evaluate acquisition of the reading and writing skills. Evaluating the claims and discussing ways of improving their organization breaks the summative assessment into two parts: making an evidence-based claim, and writing an evidence-based claim.

ACTIVITY 5: CLASS DISCUSSION OF FINAL WRITING PIECES

The class discusses the final evidence-based writing piece of student volunteers. If the Text-Centered Discussion Checklist has been used throughout the unit, this activity can be used for formative assessment on student discussion skills. In this case, the activity can be structured more formally, either as small group discussions where each student reads, receives constructive evidence-based feedback from other group members, and then responds orally with possible modifications.

ASSESSMENT

At this stage teachers can assess students' reading and writing skills. Students should be comfortable making claims and supporting them with organized evidence. Their tools should demonstrate mastery of the reading skill. Their final evidence-based writing piece can be seen as a summative assessment of both the reading and writing skills. Use the Evidence-Based Writing Rubric to evaluate their pieces.

If activity 5 is used for assessment of discussion skills, use the Text-Centered Discussion Checklist to structure evaluation and feedback.

ALTERNATIVE ORGANIZATION OF PART 5

The activities of Part 5 can be re-ordered to provide a slightly different summative assessment. Teachers could choose not to give Activity 1 as an initial homework assignment or begin the part with it. Instead they can begin with the analysis of student writing from Part 4 and the discussion of global claims. Then students can be assigned to review the entire speech, use a tool to make a global evidence-based claim ,and move directly to developing the final evidence-based writing piece. This configuration of the activities provides a complete integrated reading and writing assessment. Depending on scheduling, this activity could be done in class or given partially or entirely as a homework assignment. Even with this configuration, ELL students or those reading below grade level can be supported by having their claims evaluated before they begin writing their pieces.

ACTIVITY 1- CLASS DISCUSSION OF GLOBAL EBCs
The teacher analyzes volunteer students' written evidence-based claims from Part 4 and discusses developing global claims.

ACTIVITY 2- INDEPENDENT READING AND MAKING EBCs
Students review the entire text and use an Organizing EBC Tool to make a global EBC.

ACTIVITY 3- INDEPENDENT WRITING OF FINAL PIECE
Students independently write a final evidence-based writing piece using their global claims.

ACTIVITY 4- CLASS DISCUSSION OF FINAL WRITING PIECES
The class discusses final evidence-based writing pieces of student volunteers.

HOW TO USE THESE MATERIALS

This unit is in the format of a **Compressed File**. Files are organized so you can easily browse through the materials and find everything you need to print or e-mail for each day.

The materials are organized into three folders:

UNIT PLAN AND TEXTS

- Unit Plan
- Models
- Text(s)

The ***model claims and tools*** are meant only to illustrate the process, NOT to shape textual analysis. ***It is essential that both teachers and students develop claims based on their own analysis and class discussion***. Teachers are encouraged to develop their own claims in the blank tools to use with students when modeling the process.

HANDOUTS

- Forming Evidence-Based Claims Handout
- Writing Evidence-Based Claims Handout
- Evidence-Based Claims Criteria Checklists I and II
- Evidence-Based Writing Rubric
- Text-Centered Discussion Checklist

TOOLS

- Forming Evidence-Based Claims
- Making Evidence-Based Claims
- Organizing Evidence-Based Claims
- Written Evidence-Based Claim

TEXTS are formatted with spacing and margins to support ***teacher and student annotation.*** Students should be encouraged to mark up their texts (electronically or in print) as they search for details. ***Paragraphs and lines are numbered*** for referencing in writing and discussion. ***Editable glossaries*** are at the bottom of each page. While some words have already been bolded and glossed, teachers are encouraged to use the editable features for choosing words they wish to focus on or gloss, and to differentiate vocabulary support for their students.

TOOLS and **CHECKLISTS** have been created as ***editable PDF forms.*** With the free version of Adobe Reader, students and teachers are able to type in them and save their work for recording and e-mailing. This allows students and teachers to work either with paper and pencil or electronically according to their strengths and needs. It also allows teachers to collect and organize student work for evaluation and formative assessment.

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UNIT TWO

for

Content Area of

ENGLISH

HS Band
9th Grade English

READING CLOSELY FOR DETAILS: GUIDING QUESTIONS

APPROACHING TEXTS

Reading closely begins by considering my specific purposes for reading and important information about a text.

I am aware of my purposes for reading:

- Why am I reading this text?
- In my reading, should I focus on:

⇒ The content and information about the topic?
⇒ The structure and language of the text?
⇒ The author's view?

I take note of information about the text:

- What is the title?
- Who is the author?
- What type of text is it?
- Who published the text?
- When was the text published?

QUESTIONING TEXTS

Reading closely involves:
1) initially questioning a text to focus my attention on its structure, ideas, language and perspective
then
2) questioning further as I read to sharpen my focus on the specific details in the text.

I begin my reading with questions to help me understand the text and
I pose new questions while reading that help me deepen my understanding:

Structure:

- How is the text organized?
- How has the author structured the sentences, lines, paragraphs, scenes or stanzas?

Topic, Information and Ideas:

- What information/ideas are presented at the beginning of the text?
- What stands out to me as I first examine this text?
- What information/ideas are described in detail?
- What do I learn about the topic as I read?
- How do the ideas relate to what I already know?
- What do I think this text is mainly about?

Language:

- What words or phrases stand out to me as I read?
- What words and phrases are powerful or unique?
- What do the author's words cause me to see or feel?
- What words do I need to define to better understand the text?
- What words and phrases are repeated?

Perspective:

- Who is the intended audience of the text?
- What is the author's/narrator's stance or attitude about the topic or theme?
- How does the author's language show his/her perspective?
- What is the author's personal relationship to the topic or themes?

ANALYZING DETAILS

Reading closely involves analyzing and connecting the details I have found through my questioning to determine their meaning, importance, and the ways they help develop ideas across a text.

I analyze the details I find through my questioning:

Patterns across the text:

- What does the repetition of words or phrases in the text suggest?
- How do details, information, characters or ideas change across the text?
- How do the text's structure and features influence my reading?

Meaning of Language:

- How do specific words or phrases impact the meaning of the text?
- What words or phrases are critical for my understanding of the text?

Importance:

- Which details are most important to the overall meaning of the text?
- Which sections are most challenging and require closer reading?

Relationships among details:

- How are details in the text related in a way that develops themes or ideas?
- What does the text leave uncertain or unstated? Why?

READING CLOSELY FOR DETAILS CHECKLIST		✓	COMMENTS
I. APPROACHING TEXTS ***Reading closely begins by considering my specific purposes for reading and important information about a text.***	I am aware of my purposes for reading.	☐	
	I take note of key information about the text.	☐	
II. QUESTIONING TEXTS ***Reading closely involves:*** ***1) initially questioning a text to focus my attention on its structure, ideas, language and perspective*** ***then*** ***2) questioning further as I read to sharpen my focus on the specific details in the text.***	I begin my reading with questions to help me understand the text.	☐	
	I annotate the text marking details that relate to my guiding questions.	☐	
	I pose new questions while reading that help me deepen my understanding.	☐	
III. ANALYZING DETAILS ***Reading closely involves thinking deeply about the details I have found through my questioning to determine their meaning, importance, and the ways they help develop ideas across a text.***	I analyze the details I find through my questioning.	☐	
	I pose further *text-specific* questions based on my analysis that cause me to re-read more deeply.	☐	
IV. COMMUNICATING UNDERSTANDING ***Reading closely involves explaining what I have come to understand about texts and topics to clarify and share my ideas.***	I explain my ideas clearly in a manner appropriate for my task and audience.	☐	
	I cite details and evidence to support my explanations.	☐	

TEXT-CENTERED DISCUSSIONS CHECKLIST		✓	COMMENTS
I. PREPARING	**Reading & Research**: I come to the discussion prepared, having read the text and/or researched the topic we are studying.	☐	
II. ENGAGING AND PARTICIPATING	**Engaging Actively**: I pay attention to, respect, and work with all other participants in the discussion.	☐	
	Participating Responsibly: I take a variety of roles in the discussion, and I follow the guidelines or agreements we have set for the conversation.	☐	
	Recognizing Purpose & Goals: I understand the purpose and goals of our discussion or work, and I contribute to our progress.	☐	
III. COMMUNICATING IDEAS, CLAIMS AND EVIDENCE	**Presenting Ideas Coherently**: I present my ideas and claims clearly, using relevant evidence and well-chosen details from the text.	☐	
	Communicating Clearly: When I talk with others, I make eye contact and speak in a clear, respectful voice so they can understand me.	☐	
IV. QUESTIONING	**Posing Questions**: I pose good questions that are centered on the text or topic and that help us think more deeply.	☐	
	Responding to Questions: I respond to others' questions or comments by citing specific, relevant evidence and ideas.	☐	
	Making Connections: I make valid and thoughtful connections and comparisons among my ideas and those of others.	☐	
V. LISTENING RESPECTFULLY	**Acknowledging Others**: I pay attention to, acknowledge, and consider thoughtfully new information and ideas from others.	☐	
	Qualifying or Justifying Views: I modify or further justify my ideas in response to evidence and ideas I have heard from others.	☐	

Note: This checklist supports instruction of CCSS SL.1.

Name .... **Text** ....

Reading Purpose:

A question I have about the text:

SEARCHING FOR DETAILS	**I read the text closely and mark words and phrases that help me answer my question.**		
SELECTING DETAILS I select words or phrases from my search that I think are the <u>most important</u> for answering my question.	**Detail 1 (Ref.:)**	**Detail 2 (Ref.:)**	**Detail 3 (Ref.:)**
ANALYZING DETAILS I re-read parts of the text and <u>think about the meaning of the details</u> and what they tell me about my question.	**What I think about detail 1:**	**What I think about detail 2:**	**What I think about detail 3:**
CONNECTING DETAILS I compare the details and explain <u>the connections</u> I see among them.	**How I connect the details:**		

Name .... **Text** ....

APPROACHING THE TEXT Before reading, I consider what my specific purposes for reading are.	What are my reading purposes?		
I also take note of key information about the text.	Title:	Author: Text Type:	Source/Publisher: Publication Date:
	What do I already understand about the text based on this information?		
QUESTIONING THE TEXT As I read the text for the first time, I use guiding questions that relate to my reading purpose and focus. *(Can be taken from the Guiding Questions handout).*	Guiding questions for *my first reading* of the text:		
AS I READ I MARK DETAILS ON THE TEXT THAT RELATE TO MY GUIDING QUESTIONS.			
As I re-read, I use questions I have about specific details that have emerged in my reading to focus my analysis and deepen my understanding.	Text-specific questions to help focus *my re-reading* of the text:		

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HANDOUTS

- Guiding Questions Handout
- Reading Closely Checklist
- Text-Centered Discussion Checklist

TOOLS

- Analyzing Details
- Questioning Texts

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UNIT THREE

for

Content Area of

ENGLISH

HS Band
11th Grade English

EVIDENCE-BASED CLAIMS CRITERIA CHECKLIST I - G11-12		✓	COMMENTS
I. CONTENT AND ANALYSIS ***An EBC is a clearly stated inference that arises from close reading of a text.***	**Clarity of the Claim:** Purposefully states a valid, text-based inference or conclusion that an identified audience should consider.	☐	
	Conformity to the Text: Arises from (and sometimes comments on) the main ideas, supporting details, language and form of one or more texts.	☐	
	Understanding of the Topic: Presents a knowledgeable analysis, interpretation, commentary, and/or conclusion about a substantive text or topic.	☐	
II. COMMAND OF EVIDENCE ***An EBC is supported by specific textual evidence and developed through valid reasoning.***	**Reasoning:** Communicates a supported inference and valid reasoning based directly on relevant and sufficient textual evidence.	☐	
	Use and Integration of Evidence: Is developed through a logical sequence of quotations, references, or citations that are thoughtfully selected and coherently integrated.	☐	

EVIDENCE-BASED CLAIMS CRITERIA CHECKLIST II - G11-12		✓	COMMENTS
I. CONTENT AND ANALYSIS ***An EBC is a clearly stated inference that arises from close reading of a text.***	**Clarity of the Claim:** Purposefully states a valid, text-based inference or conclusion that an identified audience should consider.	☐	
	Conformity to the Text: Arises from (and sometimes comments on) the main ideas, supporting details, language and form of one or more texts.	☐	
	Understanding of the Topic: Presents a knowledgeable analysis, interpretation, commentary, and/or conclusion about a substantive text or topic.	☐	
II. COMMAND OF EVIDENCE ***An EBC is supported by specific textual evidence and developed through valid reasoning.***	**Reasoning :** Communicates a supported inference and valid reasoning based directly on relevant and sufficient textual evidence.	☐	
	Use and Integration of Evidence: Is developed through a logical sequence of quotations, references, or citations that are thoughtfully selected and coherently integrated.	☐	
	Thoroughness and Objectivity: Is explained fairly and thoroughly, and distinguished objectively from relevant counterclaims and/or conflicting evidence.	☐	
III. COHERENCE AND ORGANIZATION ***An EBC and its support are coherently organized into a unified explanation.***	**Relationship to Context:** Is linked to a clearly identified context that helps establish its relevance.	☐	
	Relationships among Parts: Establishes clear and logical relationships among the claim and the various pieces of evidence that have led to it and support it.	☐	
	Relationship to Other Claims: Can be linked in a logical sequence that indicates relationships among multiple claims and evidence and that coheres into a unified line of reasoning or argument.	☐	
IV. CONTROL OF LANGUAGE AND CONVENTIONS ***An EBC is communicated clearly and precisely, with responsible use/citation of supporting evidence.***	**Clarity of Communication:** Is communicated clearly, coherently, precisely, and objectively, using appropriate language, syntax and writing conventions.	☐	
	Responsible Use of Evidence: Cites textual evidence in a responsible manner, quoting exactly or paraphrasing accurately and referencing precisely where the evidence can be found.	☐	

EVIDENCE-BASED WRITING RUBRIC

	HIGH PROFICIENCY	BASIC PROFICIENCY	APPROACHING PROFICIENCY	NOT PROFICIENT
CONTENT AND ANALYSIS	• Contains a clear, compelling claim. • Claim demonstrates insightful comprehension and valid precise inferences. • Overall analysis follows logically from the text.	• Contains a clear claim. • Claim demonstrates sufficient comprehension and valid basic inferences. • Overall analysis follows logically from the text.	• Contains a claim, but it is not fully articulated. • Claim demonstrates basic literal comprehension and significant misinterpretation. • Major points of textual analysis are missing or irrelevant to accomplish purpose.	• Contains a minimal claim that is not beyond correct literal repetition. • Minimal inferential analysis serving no clear purpose.
COMMAND OF EVIDENCE	• Central claim is well-supported by textual evidence. • Use of relevant evidence is sustained throughout the entire analysis. • The core reasoning follows from evidence.	• Central claim is well-supported by textual evidence. • Use of relevant evidence is generally sustained with some gaps. • The core reasoning follows from evidence.	• Central claim is only partially supported by textual evidence. • Analysis is occasionally supported with significant gaps or misinterpretation. • The core reasoning is tangential or invalid with respect to the evidence.	• Demonstrates some comprehension of the idea of evidence, but only supports the claim with minimal evidence which is generally invalid or irrelevant.
COHERENCE AND ORGANIZATION	• The organization strengthens the exposition. The introduction establishes context ; the organizational strategies are appropriate for the content and purpose. • There is a smooth progression of ideas enhanced by proper integration of quotes and paraphrase, effective transitions, sentence variety, and consistent formatting. •	• The organization supports the exposition. The introduction establishes the context; the organizational strategies are appropriate for the content and purpose. • The ideas progress smoothly with appropriate transitions, but evidence is not always integrated properly. Sentences relate relevant information and formatting is consistent.	• Some attempt has been made at a sustained organization, but major pieces are missing or inadequate. The introduction does not establish the context; The organizational strategy is unclear and impedes exposition. • Paragraphs do contain separate ideas, but the relationships among them are not indicated with transitions. Quotes and paraphrases may be present, but no distinction is made between the two and they are not effectively integrated into the exposition. Sentences are repetitive and fail to develop ideas from one to the next.	• There is no sustained organization for the exposition. Organization does not rise above the paragraph level. The essay does contain discrete paragraphs, but the relationships among them are unclear. • Ideas do not flow across paragraphs and are often impeded by erroneous sentence structure and paragraph development.
CONTROL OF LANGUAGE AND GRAMMAR	• Contains precise and vivid vocabulary, which may include imagery or figurative language and appropriate academic vocabulary. The sentence structure draws attention to key ideas and reinforces relationships among ideas. • Successful and consistent stylistic choices have been made that serve the writing purpose. • Illustrates consistent command of standard, grade-level-appropriate writing conventions. Errors are so few and so minor that they do not disrupt readability or affect the force of the writing.	• Contains appropriate vocabulary that may lack some specificity, including some imagery or figurative language and appropriate academic vocabulary. The sentence structure supports key ideas and relationships among ideas, but may lack some variety and clarity. • There is some evidence of stylistic choices that serve the purpose of the essay. • Illustrates consistent command of standard, grade-level-appropriate writing conventions. Minor errors do not disrupt readability, but may slightly reduce the force of the writing.	• Contains vague, repetitive and often incorrect word choice. Sentence structure is repetitive, simplistic and often incorrect, disrupting the presentation of ideas. • There are few or no attempts to develop an appropriate style. • Illustrates consistent errors of standard, grade-level-appropriate writing conventions. Errors disrupt readability and undermine the force of the writing.	• Contains very limited and often incorrect word choice. Sentence structure is repetitive, simplistic and often incorrect, resulting in a minimal expression of a few simplistic ideas. • Illustrates consistent errors of standard, grade-level-appropriate writing conventions. Errors impede readability and comprehension of the writing.

FORMING EVIDENCE-BASED CLAIMS - LITERATURE

FINDING DETAILS

I find interesting details that are related and that stand out to me from reading the text closely.

As I read, I notice authors use a lot of details and strategies to develop a lot of details and techniques to develop their ideas and characters. I might then ask myself: What details should I look for? How do I know they are important? Below are examples of types of details authors often use in important ways.

Author's Facts and Ideas
- Examples
- Vivid Description
- Characters/Actors
- Events

Author's Words and Organization
- Repeated words
- Strong Language
- Figurative language
- Tone
- Organizational Structure/Phrases

Opinions and Point of View
- Interpretations
- Explanation of ideas or events
- Narration
- Dialogue

CONNECTING THE DETAILS

I re-read and think about the details, and explain the connections I find among them.

I can draw inferences about the effects the author's use of details has on my experience as a reader. Below are some techniques authors use to create effects.

- Authors follow and/or modify established genres.
- Authors build and develop characters across the story.
- Authors sequence events to express a plot.
- Authors use description to establish a setting for the action.
- Authors use description, dialogue and events to create foreshadowing and irony.

- Authors use description, dialogue, and structures to establish a tone and mood.
- Authors use figurative language to infer emotion and embellish meaning.
- Authors organize lines, paragraphs, stanzas, and scenes to enhance a point or add meaning.
- Authors use rhythm, meter, and rhyme to build and emphasize meaning.
- Authors use words, objects, events and characters to build symbolism.

- Authors use different types of point of view and narration to shape meaning.
- Authors use explanation of ideas, events and characters to convey perspectives.
- Authors use dialogue to develop characters and points of view.
- Authors develop characters and events to express a perspective or feeling about a topic.

MAKING A CLAIM

I state a conclusion that I have come to and can support with evidence from the text after reading and thinking about it closely.

As I group and connect my details, I can come to a conclusion and form a statement about the text.

TEXT-CENTERED DISCUSSIONS CHECKLIST		✓	COMMENTS
I. PREPARING	**Reading & Research**: I come to the discussion prepared, having read the text and/or researched the topic we are studying.	☐	
II. ENGAGING AND PARTICIPATING	**Engaging Actively**: I pay attention to, respect, and work with all other participants in the discussion.	☐	
	Participating Responsibly: I take a variety of roles in the discussion, and I follow the guidelines or agreements we have set for the conversation.	☐	
	Recognizing Purpose & Goals: I understand the purpose and goals of our discussion or work, and I contribute to our progress.	☐	
III. COMMUNICATING IDEAS, CLAIMS AND EVIDENCE	**Presenting Ideas Coherently**: I present my ideas and claims clearly, using relevant evidence and well-chosen details from the text.	☐	
	Communicating Clearly: When I talk with others, I make eye contact and speak in a clear, respectful voice so they can understand me.	☐	
IV. QUESTIONING	**Posing Questions**: I pose good questions that are centered on the text or topic and that help us think more deeply.	☐	
	Responding to Questions: I respond to others' questions or comments by citing specific, relevant evidence and ideas.	☐	
	Making Connections: I make valid and thoughtful connections and comparisons among my ideas and those of others.	☐	
V. LISTENING RESPECTFULLY	**Acknowledging Others**: I pay attention to, acknowledge, and consider thoughtfully new information and ideas from others.	☐	
	Qualifying or Justifying Views: I modify or further justify my ideas in response to evidence and ideas I have heard from others.	☐	

Note: This checklist supports instruction of CCSS SL.1.

WRITING EVIDENCE-BASED CLAIMS

Writing evidence-based claims is a little different from writing stories or just writing about something. You need to **follow a few steps** as you write.

1. ESTABLISH THE CONTEXT

Your readers must know **where your claim is coming from** and **why it's relevant.**

Depending on the scope of your piece and claim, the context differs.

If your whole piece is one claim or if you're introducing the first major claim of your piece, the entire context must be established:

> In "The Red Convertible," Louise Erdrich develops...

Purposes of evidence-based writing vary. In some cases, naming the book and author might be enough to establish the relevance of your claim. In other cases, you might want to supply additional information:

> In order to develop dramatic irony in fiction, authors will often play with time in their narrative. In the short story "The Red Convertible," Louise Erdrich develops...

If your claim is part of a larger piece with multiple claims, then the context might be simpler:

> To create this effect, Erdrich... or In paragraph 5, Erdrich...

2. STATE YOUR CLAIM CLEARLY

How you state your claim is important; it must **precisely and comprehensively express your analysis.** Figuring out how to state claims is a **process**; writers revise them continually as they write their supporting evidence. Here's a claim about how Erdrich's use of a particular style of narration achieves a specific effect in the story:

> In "The Red Convertible," Louise Erdrich uses a first person narrator, Lyman, to recount a narrative with a poignant and ironic resolution.

When writing claims, it is often useful to describe parts of the claim before providing the supporting evidence. In this case, the writer might want to briefly identify and describe the specific technique Erdrich uses to achieve the ironic resolution:

> In "The Red Convertible," Louise Erdrich uses a first person narrator, Lyman, to recount a narrative with a poignant and ironic resolution. To heighten the mystery of the story as it unfolds and to foreshadow the dramatic irony of its ending, Erdrich plays with time within her episodic narrative structure.

The explanation in the second sentence about how Erdrich creates foreshadowing and irony by playing with time is relevant to the claim. It also begins connecting the claim to ideas that will be used as evidence.

Remember, you should continually return to and re-phrase your claim as you write the supporting evidence to make sure you are capturing exactly what you want to say. Writing out the evidence always helps you figure out what you really think.

3. ORGANIZE YOUR SUPPORTING EVIDENCE

Many claims contain multiple aspects that require different evidence that can be expressed in separate paragraphs. This claim can be **broken down into two parts:**

NARRATION AND PLAY WITH TIME and

CRYPTIC AND PUZZLING PHRASES/WORDS.

3. ORGANIZE YOUR SUPPORTING EVIDENCE (CONT'D)

This paragraph supports the claim with evidence by describing Erdrich's use of **NARRATION AND PLAY WITH TIME:**

> The reader experiences the story of "The Red Convertible" through the eyes of Lyman, one of its chief characters and the brother of Henry Junior. Lyman recounts the narrative as a series of (sometimes disconnected) episodes - moving backward in time following the opening paragraph to tell a story that unfolds in a sequence of scenes: the buying of the car, the brothers' unexpected trip to Alaska, Henry's induction into the Marines and then return from Vietnam, Lyman's intentional destruction of the red convertible and Henry's subsequent restoration of the car, Bonita taking the photo of the two of them before their final drive, and the moments before and after Henry's drowning. While these incidents are mostly presented in chronological sequence, there are many leaps forward in time, and at three key points the narration actually shifts in its time frame. The first of these occurs in paragraph 1, when Lyman initially says, "We owned it together..." (past tense) and then, "Now Henry owns the whole car" (present tense). This shift mirrors one that introduces its final, climactic sequence of events, when Lyman first poetically describes Henry using the past tense: "His face was totally white and hard. Then it broke, like stones break all of a sudden when water boils up inside of them" then immediately shifts to present when Henry replies: "'I know it,' he says. 'I know it. I can't help it. It's no use.'" (paragraphs 49-50)

The second paragraph analyzes Erdrich's use of **CRYPTIC AND PUZZLING WORDS/PHRASES**:

> Erdrich's manipulation of time in her first person narrative is underscored by her presentation, through Lyman's eyes and voice, of images and words which are at first cryptic and puzzling and finally chilling and ironic - creating a circular structure to both the narrative and its language. In paragraph 1, Lyman tells the reader, "We owned it together until his boots filled with water on a windy night and he bought out my share.," a mysterious early detail in the story. At the story's climactic moment, Lyman then sparsely reports Henry's last words: "'My boots are filling,' he says. He says this in a normal voice, like he just noticed and he doesn't know what to think of it. Then he's gone. A branch comes by." (paragraphs 67-8). Moments later, Lyman sends the car into the river, with its headlights "reach[ing] in as they go down, searching..." He then ends the story with a final stark description: "It is all finally dark. And then there is only the water, the sound of it going and running and going and running and running," with the repetition of the word "running" ironically circling back to the final sentence of the narrative's first paragraph: "Lyman walks everywhere he goes" -the only time he refers to himself in the third person.

Notice how the phrase, "Erdrich's manipulation of time in her first person narrative," furthers the discussion on time and narrative from first paragraph. **Transitional phrases** like this one aid the organization by showing how the ideas relate to each other or are further developed.

4. PARAPHRASE AND QUOTE

Written evidence from texts can be paraphrased or quoted. It's up to the writer to decide which works better for each piece of evidence. Paraphrasing is **putting the author's words into your own.** This works well when the author originally expresses the idea you want to include across many sentences. You might write it more briefly.

The second sentence of paragraph 1 paraphrases each episode of the action:

> "... tell a story that unfolds in a sequence of scenes: the buying of the car, the brothers' unexpected trip to Alaska, Henry's induction into the Marines and then return from Vietnam... Bonita taking the photo of the two of them before their final drive, and the moments before and after Henry's drowning."

Some evidence is better quoted than paraphrased. If an author has found the quickest way to phrase the idea or the words are especially strong, you might want to **use the author's words**.

The second sentence in paragraph 2 quotes Erdrich exactly:

> In paragraph 1, Lyman tells the reader, "We owned it together until his boots filled with water on a windy night and he bought out my share," a mysterious early detail in the story.

5. REFERENCE YOUR EVIDENCE

Whether you paraphrase or quote the author's words, you must include **the exact location where the ideas come from.** Direct quotes are written in quotation marks. How writers include the reference can vary depending on the piece and the original text. Here the writer puts the paragraph numbers from the original text in parentheses at the end of the sentence.

Name .... **Text** ....

FINDING DETAILS I find interesting details that are <u>related</u> and that stand out to me from reading the text closely.	Detail 1 (Ref.:)	Detail 2 (Ref.:)	Detail 3 (Ref.:)
CONNECTING THE DETAILS I re-read and think about the details, and <u>explain</u> the connections I find among them.	What I think about detail 1:	What I think about detail 2:	What I think about detail 3:
	How I connect the details:		
MAKING A CLAIM I state a conclusion that I have come to and can support with <u>evidence</u> from the text after reading and thinking about it closely.	My claim about the text:		

ODELL EDUCATION

FORMING EVIDENCE-BASED CLAIMS

Name .... **Text** ....

CLAIM:

Supporting Evidence	Supporting Evidence	Supporting Evidence
(Reference:)	**(Reference:)**	**(Reference:)**

CLAIM:

Supporting Evidence	Supporting Evidence	Supporting Evidence
(Reference:)	**(Reference:)**	**(Reference:)**

Name .... **Text** ....

CLAIM:

Point 1		**Point 2**	
A Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)	**A** Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)
C Supporting Evidence (Reference:)	**D** Supporting Evidence (Reference:)	**C** Supporting Evidence (Reference:)	**D** Supporting Evidence (Reference:)

ORGANIZING EVIDENCE-BASED CLAIMS

Name .... **Text** ....

CLAIM:

Point 1	Point 2	Point 3
A Supporting Evidence (Reference:)	**A** Supporting Evidence (Reference:)	**A** Supporting Evidence (Reference:)
B Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)	**B** Supporting Evidence (Reference:)
C Supporting Evidence (Reference:)	**C** Supporting Evidence (Reference:)	**C** Supporting Evidence (Reference:)

ODELL EDUCATION

ORGANIZING EVIDENCE-BASED CLAIMS

Name ....

Text ....

ODELL EDUCATION

WRITTEN
EVIDENCE-BASED CLAIMS

HOW TO USE THESE MATERIALS

This unit is in the format of a **Compressed File.** Files are organized so you can easily browse through the materials and find everything you need to print or e-mail for each day.

The materials are organized into three folders:

UNIT PLAN

- Unit Plan
- Model Tools

HANDOUTS

- Forming Evidence-Based Claims Handout
- Writing Evidence-Based Claims Handout
- Evidence-Based Claims Criteria Checklists I and II
- Evidence-Based Writing Rubric
- Text-Centered Discussion Checklist

TOOLS

- Forming Evidence-Based Claims
- Making Evidence-Based Claims
- Organizing Evidence-Based Claims
- Written Evidence-Based Claim

The ***model claims and tools*** are meant only to illustrate the process, NOT to shape textual analysis. ***It is essential that both teachers and students develop claims based on their own analysis and class discussion.*** Teachers are encouraged to develop their own claims in the blank tools to use with students when modeling the process.

TOOLS and **CHECKLISTS** have been created as ***editable PDF forms.*** With the free version of Adobe Reader, students and teachers are able to type in them and save their work for recording and e-mailing. This allows students and teachers to work either with paper and pencil or electronically according to their strengths and needs. It also allows teachers to collect and organize student work for evaluation and formative assessment.

If you decide to **PRINT** materials, please note that you can print them at ***actual size,*** without enabling the auto-fit function. All materials can be printed either in color or in black and white.