Foundations of Algebra

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics coherent. The implementation of the Georgia Standards of Excellence in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, modeling, representation, connections, and communication.

Foundations of Algebra

Foundations of Algebra is a first year high school mathematics course option for students who have completed mathematics in grades 6 – 8 yet will need substantial support to bolster success in high school mathematics. The course is aimed at students who have reported low standardized test performance in prior grades and/or have demonstrated significant difficulties in previous mathematics classes.

Foundations of Algebra will provide many opportunities to revisit and expand the understanding of foundational algebra concepts, will employ diagnostic means to offer focused interventions, and will incorporate varied instructional strategies to prepare students for required high school mathematics courses. The course will emphasize both algebra and numeracy in a variety of contexts including number sense, proportional reasoning, quantitative reasoning with functions, and solving equations and inequalities.

Instruction and assessment should include the appropriate use of manipulatives and technology. Mathematics concepts should be represented in multiple ways, such as concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used, where appropriate, in the context of realistic experiences.

The Standards for Mathematical Practice will provide the foundation for instruction and assessment. The content standards are an amalgamation of mathematical standards addressed in grades 3 through high school. The standards from which the course standards are drawn are identified for reference.

Mathematics | Standards for Mathematical Practice

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

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Foundations of Algebra

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts.

Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a

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problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Foundations of Algebra | Content Standards

Number Sense and Quantity — NSQ

Students will compare different representations of numbers (i.e. fractions, decimals, radicals, etc.) and perform basic operations using these different representations.

MFA.NSQ.1 Students will analyze number relationships.

a. Solve multi-step real world problems, analyzing the relationships between all four operations. *For example, understand division as an unknown-factor problem in order to solve problems. Knowing that 50 x 40 = 2000 helps students determine how many boxes of cupcakes they will need in order to ship 2000 cupcakes in boxes that hold 40 cupcakes each.* (MGSE3.OA.6, MGSE4.OA.3)
b. Understand a fraction *a/b* as a multiple of *1/b*. (MGSE4.NF.4)
c. Explain patterns in the placement of decimal points when multiplying or dividing by powers of ten. (MGSE5.NBT.2)
d. Compare fractions and decimals to the thousandths place. *For fractions, use strategies other than cross multiplication. For example, locating the fractions on a number line or using benchmark fractions to reason about relative size. For decimals, use place value.* (MGSE4.NF.2;MGSE5.NBT.3,4)

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MFA.NSQ.2 Students will conceptualize positive and negative numbers (including decimals and fractions).

a. Explain the meaning of zero. (MGSE6.NS.5)
b. Represent numbers on a number line. (MGSE6.NS.5,6)
c. Explain meanings of real numbers in a real world context. (MGSE6.NS.5)

MFA.NSQ.3 Students will recognize that there are numbers that are not rational, and approximate them with rational numbers.

a. Find an estimated decimal expansion of an irrational number locating the approximations on a number line. *For example, for $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue this pattern in order to obtain better approximations.* (MGSE8.NS.1,2)
b. Explain the results of adding and multiplying with rational and irrational numbers. (MGSE9-12.N.RN.3)

MFA.NSQ.4 Students will apply and extend previous understanding of addition, subtraction, multiplication, and division.

a. Find sums, differences, products, and quotients of multi-digit decimals using strategies based on place value, the properties of operations, and/or relationships between operations. (MGSE5.NBT.7; MGSE6.NS.3)
b. Find sums, differences, products, and quotients of all forms of rational numbers, stressing the conceptual understanding of these operations. (MGSE7.NS.1,2)
c. Interpret and solve contextual problems involving division of fractions by fractions. *For example, how many 3/4-cup servings are in 2/3 of a cup of yogurt?* (MGSE6.NS.1)
d. Illustrate and explain calculations using models and line diagrams. (MGSE7.NS.1,2)
e. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using estimation strategies and graphing technology. (MGSE7.NS.3, MGSE7.EE.3, MGSE9-12.N.Q.3)

Arithmetic to Algebra AA

Students will extend arithmetic operations to algebraic modeling.

MFA.AA.1 Students will generate and interpret equivalent numeric and algebraic expressions.

a. Apply properties of operations emphasizing when the commutative property applies. (MGSE7.EE.1)
b. Use area models to represent the distributive property and develop understandings of addition and multiplication (all positive rational numbers should be included in the models). (MGSE3.MD.7)
c. Model numerical expressions (arrays) leading to the modeling of algebraic expressions. (MGSE7.EE.1,2; MGSE9-12.A.SSE.1,3)
d. Add, subtract, and multiply algebraic expressions. (MGSE6.EE.3, MGSE6.EE.4, MC7.EE.1, MGSE9-12.A.SSE.3)
e. Generate equivalent expressions using properties of operations and understand various representations within context. *For example, distinguish multiplicative comparison from additive comparison. Students should be able to explain the difference between "3 more" and "3 times"*. (MGSE4.0A.2; MGSE6.EE.3, MGSE7.EE.1,2;MGSE9-12.A.SSE.3)
f. Evaluate formulas at specific values for variables. *For example, use formulas such as A = l x w and find the area given the values for the length and width*. (MGSE6.EE.2)

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MFA.AA.2 Students will interpret and use the properties of exponents.

a. Substitute numeric values into formulas containing exponents, interpreting units consistently. (MGSE6.EE.2, MGSE9-12.N.Q.1, MGSE9-12.A.SSE.1, MGSE9-12.N.RN.2)
b. Use properties of integer exponents to find equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. (MGSE8.EE.1)
c. Evaluate square roots of perfect squares and cube roots of perfect cubes (MGSE8.EE.2)
d. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. (MGSE8.EE.2)
e. Use the Pythagorean Theorem to solve triangles based on real-world contexts (Limit to finding the hypotenuse given two legs). (MGSE8.G.7)

Proportional Reasoning PR

Students will use ratios to solve real-world and mathematical problems.

MFA.PR.1 Students will explain equivalent ratios by using a variety of models. *For example, tables of values, tape diagrams, bar models, double number line diagrams, and equations*. (MGSE6.RP.3)

MFA.PR.2 Students will recognize and represent proportional relationships between quantities.

a. Relate proportionality to fraction equivalence and division. *For example,* $\frac{3}{6}$ *is equal to* $\frac{4}{8}$ *because both yield a quotient of ½ and, in both cases, the denominator is double the value of the numerator.* (MGSE4.NF.1)
b. Understand real-world rate/ratio/percent problems by finding the whole given a part and find a part given the whole. (MGSE6.RP.1,2,3;MGSE7.RP.1,2)
c. Use proportional relationships to solve multistep ratio and percent problems. (MGSE7.RP.2,3)

MFA.PR.3 Students will graph proportional relationships.

a. Interpret unit rates as slopes of graphs. (MGSE8.EE.5)
b. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane. (MGSE8.EE.6)
c. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (MGSE8.EE.5)

Equations and Inequalities EI

Students will solve, interpret, and create linear models using equations and inequalities.

MFA.EI.1 Students will create and solve equations and inequalities in one variable.

a. Use variables to represent an unknown number in a specified set. (MGSE.6.EE2,5,6)
b. Explain each step in solving simple equations and inequalities using the equality properties of numbers. (MGSE9-12.A.REI.1)
c. Construct viable arguments to justify the solutions and methods of solving equations and inequalities. (MGSE9-12.A.REI.1)
d. Represent and find solutions graphically.
e. Use variables to solve real-world and mathematical problems. (MGSE6.EE.7,MGSE7.EE.4)

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MFA.EI.2 Students will use units as a way to understand problems and guide the solutions of multi-step problems.

a. Choose and interpret units in formulas. (MGSE9-12.N.Q.1)
b. Choose and interpret graphs and data displays, including the scale and comparisons of data. (MGSE3.MD.3, MGSE9-12.N.Q.1)
c. Graph points in all four quadrants of the coordinate plane. (MGSE6.NS.8)

MFA.EI.3 Students will create algebraic models in two variables.

a. Create an algebraic model from a context using two-variable equations. (MGSE6.EE.6, MGSE8.EE.8, MGSE9-12.A.CED.2)
b. Find approximate solutions using technology to graph, construct tables of values, and find successive approximations. (MGSE9-12.A.REI.10,11)
c. Represent solutions to systems of equations graphically or by using a table of values. (MGSE6.EE.5; MGSE7.EE3; MGSE8.EE.8; MGSE9-12.A.CED.2)
d. Analyze the reasonableness of the solutions of systems of equations within a given context. (MGSE6.EE.5,6,MGSE7.EE4)

MFA.EI.4 Students will solve literal equations.

a. Solve for any variable in a multi-variable equation. (MGSE6.EE.9,MGSE9-12.A.REI.3)
b. Rearrange formulas to highlight a particular variable using the same reasoning as in solving equations. *For example, solve for the base in A = ½ bh*. (MGSE9-12.A.CED.4)

Quantitative Reasoning with Functions QR

Students will create function statements and analyze relationships among pairs of variables using graphs, tables, and equations.

MFA.QR.1 Students will understand characteristics of functions.

a. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.(MGSE9-12.F.IF.1)
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function*. (MGSE9-12.F.IF.5)
c. Graph functions using sets of ordered pairs consisting of an input and the corresponding output. (MGSE8.F.1, 2)

MFA.QR.2 Students will compare and graph functions.

a. Calculate rates of change of functions, comparing when rates increase, decrease, or stay constant. *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* (MGSE6.RP.2;MGSE7.RP.1,2,3;MGSE8.F.2,5; MGSE9-12.F.IF.6)
b. Graph by hand simple functions expressed symbolically (use all four quadrants). (MGSE9-12.F.IF.7)
c. Interpret the equation y = mx + b as defining a linear function whose graph is a straight line. (MGSE8.F.3)
d. Use technology to graph non-linear functions. (MGSE8.F.3, MGSE9-12.F.IF.7)

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e. Analyze graphs of functions for key features (intercepts, intervals of increase/decrease, maximums/minimums, symmetries, and end behavior) based on context. (MGSE9-12.F.IF.4,7)
f. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the great rate of change.* (MGSE8.F.2)

MFAQR3. Students will construct and interpret functions.

a. Write a function that describes a relationship between two quantities. (MGSE8.F.4, MGSE9-12.F.BF.1)
b. Use variables to represent two quantities in a real-world problem that change in relationship to one another (conceptual understanding of a variable). (MGSE6.EE.9)
c. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context. (MGSE9-12.F.IF.2)

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