Academy of Dover
Math Curriculum Overview
2020

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Mathematics at Academy of Dover

At Academy of Dover, we have worked diligently to improve our mathematics instruction. In 2020, we adopted Eureka Math as our primary curricular resource for mathematics. The introduction of this high quality instructional material has had a direct impact on our students. AOD uses Eureka as designed by the publisher.

Our focus on creating hands-on learning experiences for children is centered around manipulating the concrete world around them and encourages students to develop strong numeracy skills that prepare them for high school. Mathematics at Academy of Dover is not a simple rote memorization of facts; it is an exploration into the mathematics that provide the foundation for scientific exploration.

We have designed our schedule to enable our Professional Learning Communities to flourish. Teachers meet weekly to analyze data. Teachers devote their time to best practices for use within the Eureka materials, incorporating manipulatives into their daily lessons, and focus on ways to differentiate and meet the needs of all of their learners. Teachers spend time in these PLCs workshopping to share innovative strategies, analyze date from Eureka exit tickets, and collaborate using a cycle of inquiry. The PLCs are facilitated by the Head of School and the Assistant Head of School. Also, teachers use the assessments in Eureka during whole and small group to provide additional data for forming RTI groups and addressing the needs of Tier 1- Tier 3 students.)

Additionally, Academy of Dover has Professional Development days. Whenever possible, teachers are encouraged to meet with their PLC team to further their data analysis and further their work in mathematics instruction. In order to address the weakness of the yellow rating in Gateway 3 the administration is ensuring professional development with grades 6 – 8 to provide acquisition and enhancement of the curriculum.

Academy of Dover's Mathematics RTI process

At Academy of Dover, we are transitioning from RTI to a more encompassing MTSS system and will closely follow any new regulations. Our current process is as follows:

1. Benchmark all students at the beginning of the school year per DE regulations. In school year 2021-2022 we are using Edmentum to regularly assess student progress. Benchmarks are given four times per year with scores communicated to both classroom teachers and parents. All students in all tiers, receive RTI time in the classroom 30 minutes a day.
2. Students performing below expectations are provided interventions in a Tier 2 setting based on the specific needs of the student. Interventions introduce students to the computerized and adaptive programs on Edmentum in a small group setting with heavy teacher support. Teachers incorporate intensive assistance on missing standards using manipulatives and Eureka Math material into these small group settings. Tier 2 interventions are primarily provided by the classroom teacher. Students receive the intervention a minimum of two times per week, with each child's schedule based on a combination of their math and reading needs.
3. Students who do not improve with Tier 2 interventions and who perform in the lowest 10% (nationally) or those who are not making any progress toward grade level expectations after multiple strategies have been attempted in the Tier 2 setting are provided more intense interventions via Tier 3 interventions. Special focus is given to hands-on learning activities, student discussion, and the development of numeracy at the developmental level. Students in 6 – 8 grades meet in a small group with their grade level math teacher.
4. Academy of Dover holds Student Support Team (SST) meetings per DE regulations to discuss strategies for working with the students. Meetings are facilitated by the Head of School and Assistant Head of School and always include the classroom teacher and a special educator. Parent communication is facilitated by the classroom teacher. Each meeting focuses on the needs of that child and analyzes the intervention strategies. New strategies are incorporated into that child's plan when and if needed.
5. Progress Monitoring: Teachers monitor the progress of the student in a variety of ways. First, is the child on trajectory to meet end of grade level expectations? This is the beginning question of each meeting. To analyze that question, teachers look at classroom work and benchmarks. Next, the team looks at the intervention provided, and ask if it is working for that child. Edmentum reports include growth, standard mastery, and time on task. Within the program, changes can be made to direct the student to focus on those standards that best align with the need of the child. The teacher also monitors the progress of the small group interaction.

6. Second Level Screening: For those students who require additional information in order to determine the best intervention, the math specialist uses the ongoing Edmentum reporting in grades 1 – 8. For kindergarten, the Edmentum reporting is used. Teachers monitor and map progress in all grades from their small groups during RTI.

P-5 AOD Adoption of Eureka

A Story of Units:

A Curriculum Overview for Grades KN–5

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Introduction

This document provides an overview of the academic year for Kindergarten through Grade 5, beginning with a curriculum map and followed by detailed grade-level descriptions.

The curriculum map is a chart that shows, at a glance, the sequence of modules comprising each grade of the entire elementary curriculum. The map also indicates the approximate number of instructional days designated for each module of each grade. Details that elaborate on the curriculum map are found in the grade-level descriptions. Each grade-level description begins with a list of the five to eight modules that comprise the instruction of that grade. That introductory component is followed by three sections: the **Summary of Year**, the **Rationale for Module Sequence**, and the **Alignment Chart** with the grade-level standards. The **Summary of Year** portion of each grade level includes four pieces of information:

- The critical instructional areas for the grade, as described in the Common Core State Standards for Mathematics[1] (CCSS-M)
- The Key Areas of Focus[2] for the grade band (Note that this information is not available for Pre-Kindergarten.)
- The Required Fluencies for the grade (Note that this information is not available for Pre-Kindergarten.)
- The Major Emphasis Clusters for the grade (Note that this information is not available for Pre-Kindergarten.)

The **Rationale for Module Sequence** portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The **Alignment Chart** for each grade lists the standards that are addressed in each module of the grade. Throughout the alignment charts, when a cluster is included without a footnote, it is taught in its entirety; there are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

[1] http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf

[2] http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

	Kindergarten	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
1st TRIMESTER	M1: Numbers to 10 (43 days)	M1: Sums and Differences to 10 (45 days)	M1: Sums and Differences to 100 (10 days)	M1: Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10 (25 days)	M1: Place Value, Rounding, and Algorithms for Addition and Subtraction (25 days)	M1: Place Value and Decimal Fractions (20 days)
			M2: Addition and Subtraction of Length Units (12 days)			
	**M2: 2D and 3D Shapes (12 days)	M2: Introduction to Place Value Through Addition and Subtraction Within 20 (35 days)	M3: Place Value, Counting, and Comparison of Numbers to 1,000 (25 days)	M2: Place Value and Problem Solving with Units of Measure (25 days)	**M2: Unit Conversions (7 days)	M2: Multi-Digit Whole Number and Decimal Fraction Operations (35 days)
2nd TRIMESTER	M3: Comparison of Length, Weight, Capacity, and Numbers to 10 (38 days)	M3: Ordering and Comparing Length Measurements as Numbers (15 days)	M4: Addition and Subtraction Within 200 with Word Problems to 100 (35 days)	M3: Multiplication and Division with Units of 0, 1, 6-9, and Multiples of 10 (25 days)	M3: Multi-Digit Multiplication and Division (43 days)	M3: Addition and Subtraction of Fractions (22 days)
			M5: Addition and Subtraction Within 1,000 with Word Problems to 100 (24 days)	M4: Multiplication and Area (20 days)	M4: Angle Measure and Plane Figures (20 days)	M4: Multiplication and Division of Fractions and Decimal Fractions (38 days)
	M4: Number Pairs, Addition and Subtraction to 10 (47 days)	M4: Place Value, Comparison, Addition and Subtraction to 40 (35 days)	M6: Foundations of Multiplication and Division (24 days)	M5: Fractions as Numbers on the Number Line (35 days)	M5: Fraction Equivalence, Ordering, and Operations (45 days)	
3rd TRIMESTER		M5: Identifying, Composing, and Partitioning Shapes (15 days)	M7: Problem Solving with Length, Money, and Data (30 days)	M6: Collecting and Displaying Data (10 days)		M5: Addition and Multiplication with Volume and Area (25 days)
	M5: Numbers 10-20 and Counting to 100 (30 days)	M6: Place Value, Comparison, Addition and Subtraction to 100 (35 days) M6: Analyzing, Comparing and Composing Shapes	M8: Time, Shapes, and Fractions as Equal Parts of Shapes (20 days)	M7: Geometry and Measurement Word Problems (40 days)	M6: Decimal Fractions (20 days) M7: Exploring Measurement with Multiplication (20 days)	M6: Problem Solving with the Coordinate Plane (40 days) – spiraled through other units, not completed as stand alone

Key:

Number	Geometry	Number and Geometry, Measurement	Fractions

Sequence of Kindergarten Modules Aligned with the Standards

Module 1: Numbers to 10
Module 2: Two-Dimensional and Three-Dimensional Shapes
Module 3: Comparison of Length, Weight, Capacity, and N umbers to 10
Module 4: Number Pairs, Addition and Subtraction to 10
Module 5: Numbers 10–20 and Counting to 100
Module 6: Analyzing, Comparing, and Composing Shapes

Summary of Year

Kindergarten mathematics is about (1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

Key Areas of Focus for K–2: Addition and subtraction—concepts, skills, and problem solving

Required Fluency: K.OA.5 Add and subtract within 5.

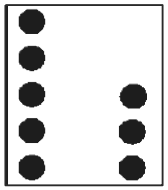
Major Emphasis Clusters
Counting and Cardinality • Know number names and count sequence. • Count to tell the number of objects. • Compare numbers. Operations and Algebraic Thinking • Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. Number and Operations in Base Ten • Work with numbers 11–19 to gain foundations for place value.

Rationale for Module Sequence in Kindergarten

A Story of Units continues in Kindergarten. Just like in Pre-K, ladybugs, fingers, and plastic bears are manipulated and counted in Kindergarten, with work consistently moving to the pictorial and abstract levels. The new, foundational unit introduced in Kindergarten's Module 5, is the supremely important unit of *one*. By the end of the Kindergarten year, students' first steps into place value are evidenced as they make precise statements such as, "12 is the same as 10 ones and 2 ones!" Notice how this sets the foundation for later work with decimal units (e.g., in Grade 1, "12 is the same as 1 ten and 2 ones;" in Grade 2, "12 tens is the same as 10 tens and 2 tens or 1 hundred 2 tens;" and in Grade 4, "12 tenths is the same as 10 tenths and 2 tenths or 1 one and 2 tenths").

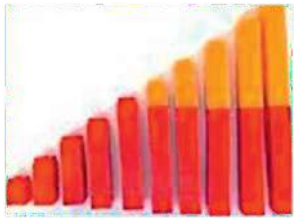
To begin the year, Kindergarten students start out classifying and categorizing objects, leading to making one group (e.g., "I made a group of 9 goldfish. Look how I can count them in a line, in rows, and in a circle"). Students learn the way each number from 0 to 10 relates to five using fingers,

cubes, drawings, 5-groups (pictured below) and the Rekenrek, an abacus with a color change after the fifth bead (pictured below). The materials support students in seeing all numbers to ten in relationship to five, as they also see them on their fingers, the best manipulative of all! This renders 6, 7, 8, 9, and 10 more friendly as they see, for example, the 3 and 5 embedded within 8. Notice how the distribution of 8 beads as 5 beads and 3 beads sets the stage for the distributive property in Grade 3 ("8 fours = 5 fours + 3 fours, so (5 × 4) + (3 × 4) = 20 + 12 = 32"). Students close the module by investigating patterns of *1 more* and *1 less* (excluding the word *than*) using models such as the number stairs (pictured below right) with a color change after the fifth cube.



5-Group Card

Rekenrek



Number Stairs

In Module 2, students take a needed break from numbers to analyze their environment and describe and identify squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres. During both Modules 2 and 3, students also practice their fluency by counting and manipulating numbers to 10 during their fluency practice, giving them ample time to prepare for the addition and subtraction of Module 4.

In Module 3, students directly compare two quantities, first learning to identify the attribute being compared. The use of the word *than* is carefully developed first in the context of length (e.g., *taller than*, *shorter than*), then weight (*heavier than*, *lighter than*), and finally capacity. Notice how *more than* and *less than* are used to compare capacities (e.g., "The bucket holds *more than* the cup"). This transitions students smoothly into comparing numbers (e.g., "9 chairs is *more than* 6 chairs"). This concrete foundation for comparison is essential to students' entire K–12 experience. Ask any Grade 5 teacher which of the two following word problems is more challenging for students:

a) *There are 34.6 kilograms of sand and 3 kilograms more gravel than sand. What is the total weight of the gravel and sand?*

b) *There are 34.6 kilograms of sand and 3 times as much gravel. What is the total weight of the gravel and the sand?*

Problem (a) is more challenging because of the language of *more than*. Students consistently struggle to reason about the relationship of quantities, often resorting to using ineffective tricks (e.g., "If the problem says *more than*, subtract," which is not correct in the sand and gravel problem). Module 3 in Kindergarten is intended to provide a solid foundation to future comparison work in the meaningful context of measurement.

In Module 4, comparison flows into addition and subtraction, as it does in all the elementary grades (e.g., "7 is more than 3" leads to, "7 = 3 + 4," and "3 + 4 = 7"). Students represent *add to*, *take away*, and *put together* stories with blocks, drawings, and equations. Toward the end of the module,

students start to reorient from 5 toward 10 ones with "How much more does 7 need to make ten?" These final lessons set the stage for Module 5 wherein 10 ones is the structure on which students build the teen numbers. They are also critical foundation standards for Grade 1. Students must know how much a number needs to make ten in order to use the *make ten* strategy in Grades 1 and 2, shown to be an important route to place value understanding as they master their sums and differences to 20 by the end of Grade 2.

In Module 5, after an extended experience of addition and subtraction with totals up to 10, students progress to investigating numbers 10–20. For example, thirteen beans are decomposed as 10 beans and 3 beans just as 8 beans are decomposed as 5 beans and 3 beans. Students record their decompositions of the teen numbers as equations, 13 = 10 + 3, and start to think, "10. 3 more is 13." As mentioned at the beginning of the story in Grade 1, the unit *one* is introduced as students learn to think of the teen numbers as 10 ones and some ones. For the first time, *one* is not an object but rather a noun! Notice how this sets the stage for expanded form in the upper grades (e.g., 36 = 30 + 6, or 13.6 = 10 + 3 + 0.6).

Module 6 rounds out the year with an exploration of shapes. Students build shapes from components, analyze and compare them, and discover that they can be composed of smaller shapes, just as larger numbers are composed of smaller numbers.

Alignment Chart[9]

Module and Approximate Number of Instructional Days	Standards Addressed in Kindergarten Modules
Module 1: **Numbers to 10** [10] (43 days)	**Know number names and the count sequence.**[11] **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). **Count to tell the number of objects.**[12] **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality. a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

[9] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[10] In this module, standards work is limited to within 10.

[11] The balance of this cluster is addressed in Module 5.

[12] K.CC.4d is addressed in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Kindergarten Modules
	b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **K.CC.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.**[13] **K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **Classify objects and count the number of objects in each category.** **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)
Module 2: **Two-Dimensional and Three-Dimensional Shapes** (12 days)	**Classify objects and count the number of objects in each category.** **K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.) **Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).** **K.G.1** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **K.G.2** Correctly name shapes regardless of their orientations or overall size.

[13] The balance of this cluster is addressed in Module 4.

Module and Approximate Number of Instructional Days	Standards Addressed in Kindergarten Modules
	K.G.3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **Analyze, compare, create, and compose shapes.**[14] **K.G.4** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
Module 3: **Comparison of Length, Weight, Capacity, and Numbers to 10** (38 days)	**Compare numbers.** **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (Include groups with up to ten objects.) **K.CC.7** Compare two numbers between 1 and 10 presented as written numerals. **Describe and compare measurable attributes.** **K.MD.1** Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. **K.MD.2** Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*
Module 4: **Number Pairs, Addition and Subtraction to 10** (47 days)	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. (Drawings need not show details, but should show the mathematics in the problem.) **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

[14] The balance of this cluster is addressed in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Kindergarten Modules
	K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). **K.OA.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings and record the answer with a drawing or equation. **K.OA.5** Fluently add and subtract within 5.[15]
Module 5: **Numbers 10–20 and Counting to 100** (30 days)	**Know number names and the count sequence.** **K.CC.1** Count to 100 by ones and by tens. **K.CC.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1). **K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects). **Count to tell the number of objects.**[16] **K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality. b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. c. Understand that each successive number name refers to a quantity that is one larger. **K.CC.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

[15] From this point forward, fluency practice is part of students' on-going experience.

[16] K.CC.4a, K.CC.4b, and K.CC.4c are addressed in Module 1; K.CC.4d is addressed in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Kindergarten Modules
	Work with numbers 11–19 to gain foundations for place value. **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two three, four, five, six, seven, eight or nine ones.
Module 6: **Analyzing, Comparing, and Composing Shapes** (10 days)	**Count to tell the number of objects.**[17] **K.CC.4** Understand the relationship between numbers and quantities: connect counting to cardinality. d. Develop understanding of ordinal numbers (first through tenth) to describe the relative position and magnitude of whole numbers.[18] **Analyze, compare, create and compose shapes.**[19] **K.G.5** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. ***K.G.6*** Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

[17] Ordinality is introduced in the context of constructing and manipulating shapes. The balance of this cluster is addressed in Modules 1 and 5.

[18] K.CC.4d originates from the New York State Common Core Learning Standards and is not part of the CCSS-M.

[19] K.G.4 is addressed in Module 2.

Sequence of Grade 1 Modules Aligned with the Standards

Module 1: Sums and Differences to 10
Module 2: Introduction to Place Value Through Addition and Subtraction Within 20
Module 3: Ordering and Comparing Length Measurements as Numbers
Module 4: Place Value, Comparison, Addition and Subtraction to 40
Module 5: Identifying, Composing, and Partitioning Shapes
Module 6: Place Value, Comparison, Addition and Subtraction to 100

Summary of Year

Grade 1 mathematics is about (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

Key Areas of Focus for K–2: Addition and subtraction—concepts, skills, and problem solving

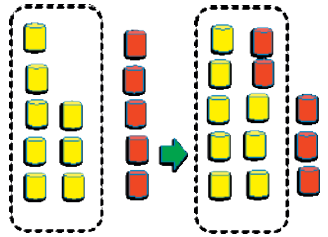
Required Fluency: 1.OA.6 Add and subtract within 10.

Major Emphasis Clusters
Operations and Algebraic Thinking • Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations. Number and Operations in Base Ten • Extend the counting sequence. • Understand place value. • Use place value understanding and properties of operations to add and subtract. Measurement and Data • Measure lengths indirectly and by iterating length units.

Rationale for Module Sequence in Grade 1

In Grade 1, work with numbers to 10 continues to be a major stepping-stone in learning the place value system. In Module 1, students work to further understand the meaning of addition and subtraction begun in Kindergarten, largely within the context of the Grade 1 word problem types. They begin intentionally and energetically building fluency with addition and subtraction facts—a major gateway to later grades.

In Module 2, students add and subtract within 20. Work begins by modeling *adding and subtracting across ten* in word problems and with equations. Solutions involving decomposition and composition like that shown to the right for 8 + 5 reinforce the need to *make 10*. In Module 1, students grouped 10 objects, saw numbers 0 to 9 in relationship to ten, added to make ten, and subtracted from ten. They now transition to conceptualizing that ten as a single unit (e.g., using 10 linking cubes stuck together). This is the next major stepping-stone in understanding place value, learning to group *10 ones* as a single unit: 1 ten. Learning to *complete a unit* empowers students in later grades to understand *renaming* in the addition algorithm, to add 298 and 35 mentally (i.e., 298 + 2 + 33), and to add measurements like 4 m, 80 cm, and 50 cm (i.e., 4 m + 80 cm + 20 cm + 30 cm = 4 m + 1 m + 30 cm = 5 m 30 cm).



8 + 5 = 8 + (2 + 3) = (8 + 2) + 3 = 10 + 3 = 13

Adding Across a Ten

Module 3, which focuses on measuring and comparing lengths indirectly and by iterating length units, gives students a few weeks to practice and internalize *making a 10* during daily fluency activities.

Module 4 returns to understanding place value. Addition and subtraction within 40 rest on firmly establishing a *ten* as a unit that can be counted, first introduced at the close of Module 2. Students begin to see a problem like 23 + 6 as an opportunity to separate the 2 tens in 23 and concentrate on the familiar addition problem 3 + 6. Adding 8 + 5 is related to solving 28 + 5; complete a unit of ten and add 3 more.

In Module 5, students think about attributes of shapes and practice composing and decomposing geometric shapes. They also practice working with addition and subtraction within 40 during daily fluency activities (from Module 4). Thus, this module provides important internalization time for students between two intense number-based modules. The module placement also gives more spatially-oriented students the opportunity to build their confidence before they return to arithmetic.

Although Module 6 focuses on *adding and subtracting within 100*, the learning goal differs from the *within 40* module. Here, the new level of complexity is to build off the place value understanding and mental math strategies that were introduced in earlier modules. Students explore by using simple examples and the familiar units of 10 made out of linking cubes, bundles, and drawings. Students also count to 120 and represent any number within that range with a numeral.

Alignment Chart[20]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
Module 1: **Sums and Differences to 10** [21] (45 days)	**Represent and solve problems involving addition and subtraction.**[22] **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Understand and apply properties of operations and the relationship between addition and subtraction.** **1.OA.3** Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **1.OA.4** Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.* **Add and subtract within 20.** **1.OA.5** Relate counting to addition and subtraction (e.g., by counting on 2 to add 2). **1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

[20] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[21] In this module, work is limited to within 10.

[22] 1.OA.2 is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	Work with addition and subtraction equations. **1.OA.7** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.* **1.OA.8** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = _ - 3, 6 + 6 = _.*
Module 2: **Introduction to Place Value Through Addition and Subtraction Within 20** (35 days)	**Represent and solve problems involving addition and subtraction.** **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **1.OA.2** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. **Understand and apply properties of operations and the relationship between addition and subtraction.** **1.OA.3** Apply properties of operations as strategies to add and subtract. (Students need not use formal terms for these properties.) *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* **1.OA.4** Understand subtraction as an unknown-addend problem. *For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.*

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	Add and subtract within 20.[23] **1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). **Understand place value.**[24] **1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
Module 3: **Ordering and Comparing Length Measurements as Numbers** (15 days)	**Represent and solve problems involving addition and subtraction.**[25] **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Measure lengths indirectly and by iterating length units.** **1.MD.1** Order three objects by length; compare the lengths of two objects indirectly by using a third object.

[23] From this point forward, fluency practice is part of students' on-going experience; the balance of this cluster is addressed in Module 1.

[24] Focus in this module is on numbers to 20. The balance of this cluster is addressed in Modules 4 and 6.

[25] The balance of this cluster is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	1.MD.2 Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Represent and interpret data.** **1.MD.4** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Module 4: **Place Value, Comparison, Addition and Subtraction to 40** [26] (35 days)	**Represent and solve problems involving addition and subtraction.**[27] **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Extend the counting sequence.**[28] **1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.**[29] **1.NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten."

[26] While pennies and dimes are used throughout the module, 1.MD.3 is not a focus grade level standard in Module 4. Instead, this standard becomes a focal standard in Module 6, when all coins are introduced and used.

[27] The balance of this cluster is addressed in Module 2.

[28] Focus on numbers to 40.

[29] Focus on numbers to 40; 1.NBT.2b is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **1.NBT.3** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.**[30] **1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **1.NBT.6** Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Module 5: **Identifying, Composing, and Partitioning Shapes** (15 days)	**Tell and write time and money.**[31] **1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks. Recognize and identify coins, their names, and their value. **Reason with shapes and their attributes.** **1.G.1** Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

[30] Focus on numbers to 40.

[31] Time alone is addressed in this module. Although money is not addressed until Grade 2 in the CCSS-M, it is addressed in Grade 1 Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	1.G.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.") **1.G.3** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.
Module 6: **Place Value, Comparison, Addition and Subtraction to 100** (35 days)	**Represent and solve problems involving addition and subtraction.**[32] **1.OA.1** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Extend the counting sequence.** **1.NBT.1** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.**[33] **1. NBT.2** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

[32] The balance of this cluster is addressed in Module 2.

[33] 1.NBT.2b is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 1 Modules
	1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.** **1.NBT.4** Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. **1.NBT.5** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count: explain the reasoning used. **1.NBT.6** Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Tell and write time and money.**[34] **1.MD.3** Tell and write time in hours and half-hours using analog and digital clocks. Recognize and identify coins, their names, and their value.

[34] Although money is not addressed until Grade 2 in the CCSS-M, money is addressed in this module. Time is addressed in Module 5.

Sequence of Grade 2 Modules Aligned with the Standards

Module 1: Sums and Differences to 100
Module 2: Addition and Subtraction of Length Units
Module 3: Place Value, Counting, and Comparison of Numbers to 1,000
Module 4: Addition and Subtraction Within 200 with Word Problems to 100
Module 5: Addition and Subtraction Within 1,000 with Word Problems to 100
Module 6: Foundations of Multiplication and Division
Module 7: Problem Solving with Length, Money, and Data
Module 8: Time, Shapes, and Fractions as Equal Parts of Shapes

Summary of Year

Grade 2 mathematics is about (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

Key Areas of Focus for K–2: Addition and subtraction—concepts, skills, and problem solving

Required Fluency:

2.OA.2	Add and subtract within 20.
2.NBT.5	Add and subtract within 100.

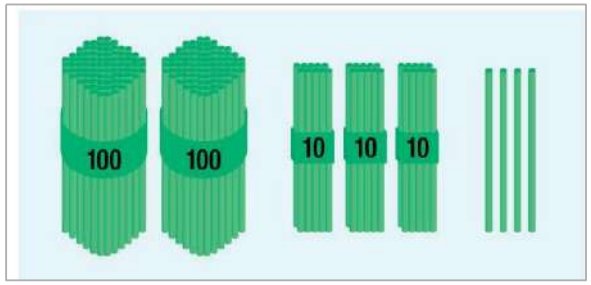
Major Emphasis Clusters
Operations and Algebraic Thinking • Represent and solve problems involving addition and subtraction. • Add and subtract within 20. Number and Operations in Base Ten • Understand place value. • Use place value understanding and properties of operations to add and subtract. Measurement and Data • Measure and estimate lengths in standard units. • Relate addition and subtraction to length.

Rationale for Module Sequence in Grade 2

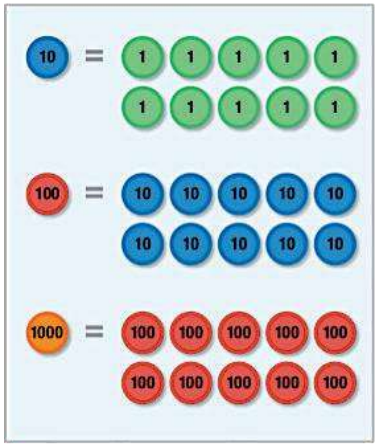
From Grade 1, students have fluency of addition and subtraction within 10 and extensive experience working with numbers to 100. Module 1 of Grade 2 establishes a motivating, differentiated fluency program in the first few weeks that will provide each student with enough practice to achieve mastery of the new required fluencies (i.e., adding and subtracting within 20 and within 100) by the end of the year. Students also solve all addition and subtraction word problem situations (See the Standards Glossary, Table 1) that do not involve comparison using the Read-Draw-Write process, a practice that will also continue throughout the year. Though encouraged to use math drawings that are intuitive for them, each situation is also modeled using the tape diagram, encouraging students to generalize and analyze part–whole relationships.

In Module 2, students learn to measure and estimate using standard units for length and solve measurement problems involving addition and subtraction of length, now encountering the word problem situations involving comparison. A major objective is for students to use measurement tools with the understanding that linear measure involves an iteration of units and that the smaller a unit, the more iterations are necessary to cover a given length. Students work exclusively with metric units (e.g., centimeters and meters) in this module to support upcoming work with place value concepts in Module 3. Units also play a central role in the addition and subtraction algorithms of Modules 4 and 5. An underlying goal for this module is for students to learn the meaning of a *unit* in a different context, that of length. This understanding serves as the foundation of arithmetic, measurement, and geometry in elementary school. Students also solve word problems involving all addition and subtraction comparison situations, so that by the end of Module 2, they have encountered the full set of situations.

All arithmetic algorithms are manipulations of place value units: ones, tens, hundreds, etc. In Module 3, students extend their understanding of base-ten notation and apply their understanding of place value to count and compare numbers to 1,000. In Grade 2, the place value units move from a proportional model to a non-proportional number disk model (see the pictures below). The place value table with number disks can be used through Grade 5 for modeling very large numbers and decimals, thus providing students greater facility with, and understanding of, mental math and algorithms.



Proportional Model for Place Value



Non-Proportional Model for Place Value

In Module 4, students apply their work with place value units to add and subtract within 200, moving from concrete to pictorial to abstract. This work deepens their understanding of base ten, place value, and the properties of operations. It also challenges them to apply their knowledge to one-step and two-step word problems. During this module, students also continue to develop one of the required fluencies of the grade: addition and subtraction within 100.

Module 5 builds upon the work of Module 4. Students again use place value strategies, manipulatives, and math drawings to extend their conceptual understanding of the addition and subtraction algorithms to numbers within 1,000. They maintain addition and subtraction fluency within 100 through daily application work to solve one- and two-step word problems of all types. A key component of Modules 4 and 5 is that students use place value reasoning to explain why their addition and subtraction strategies work.

In Module 6, students extend their understanding of a unit to build the foundation for multiplication and division wherein any number, not just powers of ten, can be a unit. Making equal groups of *four apples each* establishes the unit *four apples* (or just *four*) that can then be counted: *1 four, 2 fours, 3 fours*, etc. Relating the new unit to the one used to create it lays the foundation for multiplication: *3 groups of 4 apples equal 12 apples* (or *3 fours is 12*).

Module 7 provides another opportunity for students to practice their algorithms and problem-solving skills with perhaps the most well-known, interesting units of all: dollars, dimes, pennies, quarters, and nickels. Measuring and estimating length is revisited in this module in the context of units from both the customary system (e.g., inches and feet) and the metric system (e.g., centimeters and meters). As they study money and length, students represent data given by measurement and money data using picture graphs, bar graphs, and line plots.

Students finish Grade 2 by describing and analyzing shapes in terms of their sides and angles. In Module 8, students investigate, describe, and reason about the composition and decomposition of shapes to form other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Alignment Chart[35]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
Module 1: **Sums and Differences to 100** (10 days)	**Represent and solve problems involving addition and subtraction.**[36] **2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Add and subtract within 20.**[37] **2.OA.2** Fluently add and subtract within 20 using mental strategies. (See standard 1.OA.6 for a list of mental strategies.) By end of Grade 2, know from memory all sums of two one-digit numbers. **Use place value understanding and properties of operations to add and subtract.**[38] **2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
Module 2: **Addition and Subtraction of Length Units** (12 days)	**Measure and estimate lengths in standard units.**[39] **2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **2.MD.3** Estimate lengths using units of inches, feet, centimeters, and meters.

[35] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[36] In this module, word problems focus primarily on result unknown and change unknown situations.

[37] From this point forward, fluency practice with addition and subtraction to 20 is part of students' ongoing experience.

[38] This standard is addressed again in Modules 4 and 7; the balance of this cluster is addressed in Modules 4 and 5.

[39] Focus is on metric measurement in preparation for place value in Module 3. Customary measurement is addressed in Module 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
	2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** **2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram.
Module 3: **Place Value, Counting, and Comparison of Numbers to 1,000** (25 days)	**Understand place value.** **2.NBT.1** Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens—called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.NBT.2** Count within 1000; skip-count by 5s[40], 10s, and 100s. **2.NBT.3** Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. **2.NBT.4** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

[40] Use an analog clock to provide a context for skip-counting by fives.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
Module 4: **Addition and Subtraction Within 200 with Word Problems to 100** (35 days)	**Represent and solve problems involving addition and subtraction.** **2.OA.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 1.) **Use place value understanding and properties of operations to add and subtract.**[41] **2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **2.NBT.6** Add up to four two-digit numbers using strategies based on place value and properties of operations. **2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. **2.NBT.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. **2.NBT.9** Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)
Module 5: **Addition and Subtraction Within 1,000 with Word Problems to 100** (24 days)	**Use place value understanding and properties of operations to add and subtract.**[42] **2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is

[41] In this module, work is limited to within 200. This work is extended to numbers within 1,000 in the next module.

[42] The balance of this cluster is addressed in Modules 1, 4, and 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
	necessary to compose or decompose tens or hundreds. **2.NBT.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900. **2.NBT.9** Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)
Module 6: **Foundations of Multiplication and Division** (24 days)	**Work with equal groups of objects to gain foundations for multiplication.** **2.OA.3** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s: write an equation to express an even number as a sum of two equal addends. **2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends. **Reason with shapes and their attributes.**[43] **2.G.2** Partition a rectangle into rows and columns of same size squares and count to find the total number of them.
Module 7: **Problem Solving with Length, Money, and Data** (30 days)	**Use place value understanding and properties of operations to add and subtract.**[44] **2.NBT.5** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. **Measure and estimate lengths in standard units.** **2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

[43] 2.G.2 is included in this module because the array model is so important to the foundation for multiplication. The balance of this cluster is addressed in Module 8.

[44] This standard is also addressed in Modules 1 and 4; the balance of this cluster is addressed in Modules 4 and 5.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
	2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **2.MD.3** Estimate lengths using units of inches, feet, centimeters, and meters. **2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** **2.MD.5** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. **2.MD.6** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, …, and represent whole-number sums and differences within 100 on a number line diagram. **Work with time and money.**[45] **2.MD.8** Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?* **Represent and interpret data.** **2.MD.9** Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. **2.MD.10** Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems (See Standards Glossary, Table 1.) using information presented in a bar graph.

[45] Focus on money. Time is addressed in Module 8.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 2 Modules
Module 8: **Time, Shapes, and Fractions as Equal Parts of Shapes** (20 days)	**Work with time and money.**[46] **2.MD.7** Tell time and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **Reason with shapes and their attributes.**[47] **2.G.1** Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. (Sizes are compared directly or visually, not compared by measuring.) Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. **2.G.3** Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[46] Focus on time. Money is addressed in Module 7.

[47] 2.G.2 is addressed in Module 6.

Sequence of Grade 3 Modules Aligned with the Standards

Module 1: Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10
Module 2: Place Value and Problem Solving with Units of Measure
Module 3: Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10
Module 4: Multiplication and Area
Module 5: Fractions as Numbers on the Number Line
Module 6: Collecting and Displaying Data
Module 7: Geometry and Measurement Word Problems

Summary of Year

Grade 3 mathematics is about (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

Key Areas of Focus for 3–5:	Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving	
Required Fluency:	3.OA.7	Multiply and divide within 100.
	3.NBT.2	Add and subtract within 1000.

Major Emphasis Clusters

Operations and Algebraic Thinking
- Represent and solve problems involving multiplication and division.
- Understand the properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations and identify and explain patterns in arithmetic.

Number and Operations—Fractions
- Develop understanding of fractions as numbers.

Measurement and Data
- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

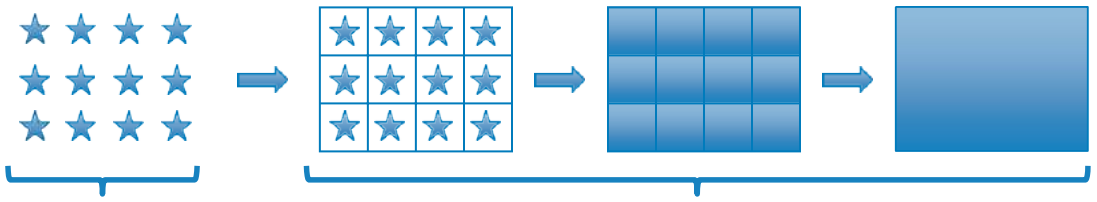
Rationale for Module Sequence in Grade 3

The first module builds upon the foundation of multiplicative thinking with units started in Grade 2. First, students concentrate on the meaning of multiplication and division and begin developing fluency for learning products involving factors of 2, 3, 4, 5, and 10 (see Key Areas of Focus and

Required Fluency above). The restricted set of facts keeps learning manageable, and also provides enough examples to do one- and two-step word problems and to start measurement problems involving weight, capacity, and time in the second module.

Module 2 focuses on measurement of time and metric weight and capacity. In exploratory lessons, students decompose a kilogram into 100 gram, 10 gram, and 1 gram weights and decompose a liter into analogous amounts of milliliters. Metric measurement thereby develops the concept of mixed units (e.g., 3 kilograms 400 grams is clearly related to 3 thousands, 4 hundreds). Students then apply their new understanding of number to place value, comparison and rounding, composing larger units when adding, decomposing into smaller units when subtracting. Students also draw proportional tape diagrams to solve word problems (e.g., "If this tape represents 62 kg, then a tape representing 35 kg needs to be slightly longer than half the 62 kg bar …"). Drawing the relative sizes of the lengths involved in the model prepares students to locate fractions on a number line in Module 5 (where they learn to locate points on the number line relative to each other and relative to the whole unit). Module 2 also provides students with internalization time for learning the 2, 3, 4, 5, and 10 facts as part of their fluency activities.

Students learn the remaining multiplication and division facts in Module 3 as they continue to develop their understanding of multiplication and division strategies within 100 and use those strategies to solve two-step word problems. The "2, 3, 4, 5, and 10 facts" module (Module 1) and the "0, 1, 6, 7, 8, 9, and multiples of 10 facts" module (Module 3) both provide important, sustained time for work in understanding the structure of rectangular arrays to prepare students for area in Module 4. This work is necessary because students initially find it difficult to distinguish the different units in a grid (the third array in the picture below), count them, and recognize that the count is related to multiplication. Tiling also supports a correct interpretation of the grid. Modules 1 and 3 slowly build up to the area model (the fourth model in the picture below), using rectangular arrays in the context of learning multiplication and division:



Progression from Rectangular Array to Area Model

By Module 4, students are ready to investigate area. They measure the area of a shape by finding the total number of same-size units of area (e.g., tiles) required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole number side lengths, it is easy to partition the rectangle into squares with equal areas (as in the third stage of the illustration above).

One goal of Module 5 is for students to transition from thinking of fractions as area or parts of a figure to points on a number line and finally, as numbers. To make that jump, students think of fractions as being constructed out of unit fractions: *1 fourth* is the length of a segment on the number line such that the length of four concatenated fourth segments on the line equals 1 (the whole). Once the unit *1 fourth* has been established, counting them is as easy as counting whole numbers: 1 fourth, 2 fourths, 3 fourths, 4 fourths, 5 fourths, etc. Students also compare fractions, find equivalent fractions in special cases, and solve problems that involve fractions. They realize that equivalent fractions share the same point on the number line.

In Module 6, by applying their knowledge of fractions from Module 5, students round lengths to the nearest halves and fourths of an inch and record that information on line plots. This module also prepares students for the multiplicative comparison problems of Grade 4 by asking students "how many more" and "how many less" questions about scaled bar graphs.

The year rounds out with plenty of time to solve two-step word problems involving the four operations and to improve fluency for concepts and skills initiated earlier in the year. In Module 7, students also describe, analyze, and compare properties of two-dimensional shapes. By now, students have done enough work with both linear and area measurement models to understand that there is no relationship in general between the area of a figure and its perimeter, which is one of the concepts taught in the last module.

Alignment Chart[48]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
Module 1: **Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10** (25 days)	**Represent and solve problems involving multiplication and division.[49]** **3.OA.1** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

[48] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[49] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 2.) **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = _ ÷ 3, 6 x 6 = ?* **Understand properties of multiplication and the relationship between multiplication and division.**[50] **3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*[51] **3.OA.6** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **Multiply and divide within 100.**[52] **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[53] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard

[50] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.

[51] The associative property is addressed in Module 3.

[52] In this module, work is limited to factors of 2–5 and 10 and the corresponding dividends.

[53] In this module, problem solving is limited to multiplication and division and limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
Module 2: **Place Value and Problem Solving with Units of Measure** (25 days)	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**[54] **3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100. **3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.** **3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. **3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems, i.e., problems involving notions of "times as much"; see Standards Glossary, Table 2.)

[54] From this point forward, fluency practice with addition and subtraction is part of students' on-going experience. 3.NBT.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
Module 3: **Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10** (25 days)	**Represent and solve problems involving multiplication and division.**[55] **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Standards Glossary, Table 2.) **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 =___÷ 3, 6 x 6 = ?* **Understand properties of multiplication and the relationship between multiplication and division.**[56] **3.OA.5** Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* **Multiply and divide within 100.**[57] **3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[58] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students

[55] The balance of this cluster is addressed in Module 1.

[56] The balance of this cluster is addressed in Module 1.

[57] From this point forward, fluency practice with multiplication and division facts is part of students' on-going experience.

[58] After being fully taught in Module 3, this standard (as well as 3.OA.3) continues being practiced throughout the remainder of the school year.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.) **3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* **Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)[59]** **3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 ×60) using strategies based on place value and properties of operations.
Module 4: **Multiplication and Area** (20 days)	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **3.MD.7** Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

[59] The balance of this cluster is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
Module 5: **Fractions as Numbers on the Number Line** (35 days)	**Develop understanding of fractions as numbers. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)** **3.NF.1** Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$. **3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. **3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and1 at the same point of a number line diagram*. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Reason with shapes and their attributes.**[60] **3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area and describe the area of each part as ¼ of the area of the shape.
Module 6: **Collecting and Displaying Data** (10 days)	**Represent and interpret data.** **3.MD.3** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two- step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.
Module 7: **Geometry and Measurement Word Problems**[61] (40 days)	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**[62] **3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard

[60] 3.G.1 is addressed in Module 7.

[61] The seemingly eclectic set of standards in Module 7 allows for a new level of word problems, including perimeter and measurement word problems.

[62] 3.OA.9 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 3 Modules
	is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.) **Represent and interpret data.**[63] **3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. **Reason with shapes and their attributes.**[64] **3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

[63] 3.MD.3 is addressed in Module 6.

[64] 3.G.2 is addressed in Module 5.

Sequence of Grade 4 Modules Aligned with the Standards

Module 1: Place Value, Rounding, and Algorithms for Addition and Subtraction
Module 2: Unit Conversions and Problem Solving with Metric Measurement
Module 3: Multi-Digit Multiplication and Division
Module 4: Angle Measure and Plane Figures
Module 5: Fraction Equivalence, Ordering, and Operations
Module 6: Decimal Fractions
Module 7: Exploring Measurement with Multiplication

Summary of Year

Grade 4 mathematics is about (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Key Areas of Focus for 3–5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving

Required Fluency: 4.NBT.4 Add and subtract within 1,000,000.

Major Emphasis Clusters
Operations and Algebraic Thinking • Use the four operations with whole numbers to solve problems. Number and Operations in Base Ten • Generalize place value understanding for multi-digit whole numbers. • Use place value understanding and properties of operations to perform multi-digit arithmetic. Number and Operations—Fractions • Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimal fractions.

Rationale for Module Sequence in Grade 4

In Grade 4, students extend their work with whole numbers. They begin with large numbers using familiar units (tens and hundreds) and develop their understanding of thousands by building knowledge of the pattern of *times ten* in the base-ten system on the place value chart (**4.NBT.1**). In

Grades 2 and 3, students focused on developing the concept of composing and decomposing place value units within the addition and subtraction algorithms. Now, in Grade 4, those (de)compositions are seen through the lens of multiplicative comparison (e.g., 1 thousand is 10 times as much as 1 hundred). They next apply their broadened understanding of patterns on the place value chart to compare, round, add, and subtract. The addition and subtraction algorithms are then efficient and useful applications of students' knowledge of and skill with composing and decomposing higher value units. The module culminates with solving multi-step word problems involving addition and subtraction modeled with tape diagrams that focus on numerical relationships.

The algorithms continue to play a part in Module 2 as students relate place value units to metric units. This module helps students draw similarities between:

1 ten	= 10 ones
1 hundred	= 10 tens
1 hundred	= 100 ones
1 meter	= 100 centimeters
1 thousand	= 1,000 ones
1 kilometer	= 1,000 meters
1 kilogram	= 1,000 grams
1 liter	= 1,000 milliliters

Students work with metric measurement in the context of the addition and subtraction algorithms, mental math, place value, and word problems. Customary units are used as a context for fractions in Modules 5 and 7.

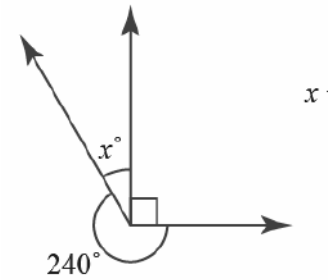
In Module 3, measurement of perimeter and area provide the concrete foundation behind the distributive property in the multiplication algorithm: 4 × (1 m 2 cm) can be modeled concretely using ribbon, since it is easy to see the 4 copies of 1 meter and the 4 copies of 2 centimeters. Likewise, 4 × (1 ten 2 ones) = 4 tens 8 ones. Students next use place value disks to develop efficient procedures and the algorithms for multiplying and dividing one-digit whole numbers. They understand and explain why the procedures work, and connections are made between the area model and work on the place value chart. Two-digit by two-digit multiplication is then modeled using the area model, extending students' earlier experiences with measurement and the distributive property. Students also solve word problems throughout the module where they select and accurately apply appropriate methods to estimate, mentally calculate, or use written strategies to compute products and quotients.

Module 4 focuses as much on solving unknown angle problems using letters and equations as it does on building, drawing, and analyzing two-dimensional shapes in geometry. Students have already used letters and equations to solve word problems in earlier grades. They continue to do so in Grade 4, and now they also learn to solve unknown angle problems: work that challenges students to build and solve equations to find unknown angle measures. First, students learn the definition of degree and learn how to measure angles in degrees using a circular protractor. From the definition of degree and the fact that angle measures are additive, the following rudimentary facts about angles naturally follow:

1. The sum of angle measurements around a point is 360 degrees.
2. The sum of angle measurements on a line is 180 degrees.

Hence, from 1 and 2, students see that vertical angles are equal. Armed only with these facts, students are able to generate and solve equations as in the following problem:

Find the unknown angle x.



$xx + 240 + 90 = 360$

$xx + 330 = 360$

$xx = 30$

Unknown angle problems help to unlock algebraic concepts for students because such problems are visual. The clearly stands for a specific number. If a student wished, he could place a protractor down on that angle and measure it to find . But doing so destroys the joy of deducing the answer and solving the puzzle on his own.

Module 5 centers on equivalent fractions and operations with fractions. We use fractions when there is a given unit, the *whole unit*, but we want to measure using a smaller unit, called the *fractional unit*. To prepare students to explore the relationship between a fractional unit and its whole unit, examples of such relationships in different contexts were already carefully established earlier in the year:

360 degrees in	1 complete turn
100 centimeters in	1 meter
1000 grams in	1 kilogram
1000 milliliters in	1 liter

The beauty of fractional units, once defined and understood, is that they behave just as all other units do:

- "3 fourths + 5 fourths = 8 fourths" just as "3 meters + 5 meters = 8 meters"
- "4 × 3 fourths = 12 fourths" just as "4 × 3 meters = 12 meters"

Students add and subtract fractions with like units using the area model and the number line. They multiply a fraction by a whole number where the interpretation is as repeated addition (e.g., 3 fourths + 3 fourths = 2 × 3 fourths). Through this introduction to fraction arithmetic they gradually come to understand fractions as units they can manipulate, just like whole numbers. Throughout the module, customary units of measurement provide a relevant context for the arithmetic.

Module 6, on decimal fractions, starts with the realization that decimal place value units are simply special fractional units: 1 tenth = 1/10, 1 hundredth = 1/100, etc. Fluency plays an important role in this topic as students learn to relate 3/10 = 0.3 = 3 tenths. They also recognize that 3 tenths is equal to 30 hundredths and subsequently have their first experience adding and subtracting fractions with unlike units (e.g., 3 tenths + 4 hundredths = 30 hundredths + 4 hundredths).

The year ends with a module focused on multiplication and measurement, as they solve multi-step word problems. Exploratory lessons support conceptual understanding of the relative sizes of measurement units. Students explore conversion in hands-on settings and subsequently apply those conversions to solve multi-step word problems involving all operations and multiplicative comparison.

Alignment Chart[65]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
Module 1: **Place Value, Rounding, and Algorithms for Addition and Subtraction** (25 days)	**Use the four operations with whole numbers to solve problems.[66]** **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

[65] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[66] The balance of this cluster is addressed in Modules 3 and 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Generalize place value understanding for multi-digit whole numbers. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.) **4.NBT.1** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.* **4.NBT.2** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.NBT.3** Use place value understanding to round multi-digit whole numbers to any place. **Use place value understanding and properties of operations to perform multi-digit arithmetic.**[67] **4.NBT.4** Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Module 2: **Unit Conversions and Problem Solving with Metric Measurement** (7 days)	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[68] **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3,36), …* **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

[67] From this point forward, fluency practice is part of students' on-going experience. The balance of this cluster is addressed in Module 3.

[68] The focus of this module is on the metric system to reinforce place value, mixed units, and word problems with unit conversions. Decimal and fraction word problems wait until Modules 6 and 7. 4.MD.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
Module 3: **Multi-Digit Multiplication and Division** (43 days)	**Use the four operations with whole numbers to solve problems.** **4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Standards Glossary, Table 2.) **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiples.** **4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite. **Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)**[69] **4.NBT.5** Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **4.NBT.6** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the

[69] 4.NBT.4 is addressed in Module 1 and is then reinforced throughout the year.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[70] **4.MD.3** Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*
Module 4: **Angle Measure and Plane Figures** (20 days)	**Geometric measurement: understand concepts of angle and measure angles.** **4.MD.5** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles. b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **4.MD.6** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **4.MD.7** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

[70] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.2 is addressed in Modules 2, 6, and 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Draw and identify lines and angles, and classify shapes by properties of their lines and angles. **4.G.1** Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **4.G.2** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **4.G.3** Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.
Module 5: **Fraction Equivalence, Ordering, and Operations**[71] (45 days)	**Generate and analyze patterns.** **4.OA.5** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.* **Extend understanding of fraction equivalence and ordering. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)** **4.NF.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole.

[71] Tenths and hundredths are important fractions in this module, represented in decimal form in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Build fractions from unit fractions by applying and extending previous understanding of operations on whole numbers.** **4.NF.3** Understand a fraction *a/b* with *a* > 1 as a sum of fractions 1/*b*. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. *b.* Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. **4.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. *a.* Understand a fraction *a/b* as a multiple of *1/b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* *b.* Understand a multiple of *a/b* as a multiple of *1/b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Represent and interpret data.** **4.MD.4** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*
Module 6: **Decimal Fractions** (20 days)	**Understand decimal notation for fractions, and compare decimal fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)**[72] **4.NF.5** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* **4.NF.6** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **4.NF.7** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

[72] In this module, we continue to work with fractions, now including decimal form.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules
	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[73] **4.MD.2** Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Module 7: **Exploring Measurement with Multiplication** (20 days)	**Use the four operations with whole numbers to solve problems.** **4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **4.OA.2** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See Standards Glossary, Table 2.) **4.OA.3** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**[74] **4.MD.1** Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a

[73] 4.MD.1 is addressed in Modules 2 and 7; 4.MD.3 is addressed in Module 3.

[74] The focus now is on customary units in word problems for application of fraction concepts. 4.MD.3 is addressed in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 4 Modules	
		larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs* $(1, 12)$, $(2, 24)$, $(3, 36)$, *…*
	4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Sequence of Grade 5 Modules Aligned with the Standards

Module 1: Place Value and Decimal Fractions
Module 2: Multi-Digit Whole Number and Decimal Fraction Operations
Module 3: Addition and Subtraction of Fractions
Module 4: Multiplication and Division of Fractions and Decimal Fractions
Module 5: Addition and Multiplication with Volume and Area
Module 6: Problem Solving with the Coordinate Plane

Summary of Year

Grade 5 mathematics is about (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

Key Areas of Focus for 3–5: Multiplication and division of whole numbers and fractions—concepts, skills, and problem solving

Required Fluency: 5.NBT.5 Multi-digit multiplication.

Major Emphasis Clusters
Number and Operations in Base Ten • Understand the place value system. • Perform operations with multi-digit whole numbers and with decimals to hundredths. Number and Operations—Fractions • Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions. Measurement and Data • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Rationale for Module Sequence in Grade 5

Students' experiences with the algorithms as ways to manipulate place value units in Grades 2–4 really begin to pay dividends in Grade 5. In Module 1, whole number patterns with number disks on the place value chart are easily generalized to decimal numbers. As students work word problems with measurements in the metric system, where the same patterns occur, they begin to appreciate the value and the meaning of decimals. Students apply their work with place value to adding, subtracting, multiplying, and dividing decimal numbers with tenths and hundredths.

Module 2 begins by using place value patterns and the distributive and associative properties to multiply multi-digit numbers by multiples of 10 and leads to fluency with multi-digit whole number multiplication.[75] For multiplication, students must grapple with and fully understand the distributive property (one of the key reasons for teaching the multi-digit algorithm). While the multi-digit multiplication algorithm is a straightforward generalization of the one-digit multiplication algorithm, the division algorithm with two-digit divisors requires far more care to teach because students have to also learn estimation strategies, error correction strategies, and the idea of successive approximation (all of which are central concepts in math, science, and engineering).

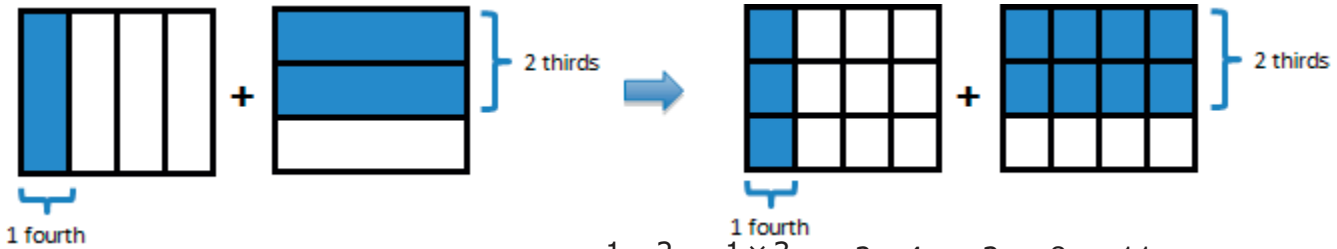
Work with place value units paves the path toward fraction arithmetic in Module 3 as elementary math's place value emphasis shifts to the larger set of fractional units for algebra. Like units are added to and subtracted from like units:

$1.5 + 0.8 = 1\frac{5}{10} + \frac{8}{10}$ = 15 tenths + 8 tenths = 23 tenths = 2 and 3 tenths = $2\frac{3}{10}$ = 2.3

$1\frac{5}{9} + \frac{8}{9}$ = 14 ninths + 8 ninths = 22 ninths = 2 and 4 ninths = $2\frac{4}{9}$

The new complexity is that when units are not equivalent, they must be changed for smaller equal units so that they can be added or subtracted. Probably the best model for showing this is the rectangular fraction model pictured below. The equivalence is then represented symbolically as students engage in active meaning-making rather than obeying the perhaps mysterious command to "multiply the top and bottom by the same number."

1 boy + 2 girls = 1 child + 2 children = 3 children
1 fourth + 2 thirds = 3 twelfths + 8 twelfths = 11 twelfths

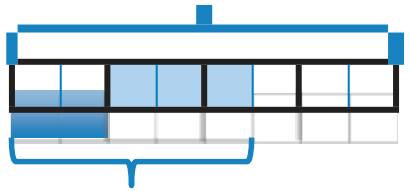


$$\frac{1}{4} + \frac{2}{3} = \frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

[75] Multi-digit decimal multiplication such as 4.1 × 3.4 and division such as 4.5 ÷ 1.5 are studied in Module 4.

Relating different fractional units to one another requires extensive work with area and number line diagrams whereas tape diagrams are used often in word problems. Tape diagrams, which students began using in the early grades and which become increasingly useful as students applied them to a greater variety of word problems, hit their full strength as a model when applied to fraction word problems. At the heart of a tape diagram is the now-familiar idea of forming units. In fact, forming units to solve word problems is one of the most powerful examples of the unit theme and is particularly helpful for understanding fraction arithmetic, as in the following example:

Jill had \$32. She gave $\frac{1}{4}$ of her money to charity and $\frac{3}{8}$ of her money to her brother. How much did she give altogether?



Solution with units:

8 units = \$32

1 unit = \$4

Solution with arithmetic:

$\frac{1}{4}$ $\frac{3}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{5}{8}$

$\frac{5}{8} \times 32 = 20$

Jill gave \$20 altogether.

Near the end of Module 4, students know enough about fractions and whole number operations to begin to explore multi-digit decimal multiplication and division. In multiplying 2.1×3.8, for example, students now have multiple skills and strategies that they can use to locate the decimal point in the final answer, including:

- Unit awareness: $2.1 \times 3.8 = 21$ tenths $\times$ 38 tenths = 798 hundredths
- Estimation (through rounding): $2.1 \times 3.8 \approx 2 \times 4 = 8$, so $2.1 \times 3.8 = 7.98$
- Fraction multiplication: $\frac{21}{10} \times \frac{38}{10} = 21 \times \frac{1}{10} \times 38 \times \frac{1}{10} = \frac{21}{10} \times 38 \times \frac{1}{100} = \frac{798}{100}$

Similar strategies enrich students' understanding of division and help them to see multi-digit decimal division as whole number division in a different unit. For example, we divide to find, "How many groups of 3 apples are there in 45 apples?" and write *45 apples ÷ 3 apples = 15*. Similarly, *4.5 ÷ 0.3* can be written as *45 tenths ÷ 3 tenths* with the same answer: *There are 15 groups of 0.3 in 4.5*. This idea was used to introduce fraction division earlier in the module, thus gluing division to whole numbers, fractions, and decimals together through an understanding of units.

Frequent use of the area model in Modules 3 and 4 prepares students for an in-depth discussion of area and volume in Module 5. But the module on area and volume also reinforces work done in the fraction module. Now, questions about how the area changes when a rectangle is scaled by a

whole or fractional scale factor may be asked, and missing fractional sides may be found. Measuring volume once again highlights the unit theme, as a unit cube is chosen to represent a volume unit and used to measure the volume of simple shape composed of regular prisms .

A

In this final module of *A Story of Units*, students connect plane geometry with numerical work to investigate relationships. They construct the coordinate plane, plot points and draw lines. For points on a given line, students discover a common relationship between the *x* and *y* coordinates, foreshadowing the proportional reasoning of Grade 6, and later, the slope of a line.

Alignment Chart[76]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
Module 1: **Place Value and Decimal Fractions** (20 days)	**Understand the place value system.** **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **5.NBT.3** Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **5.NBT.4** Use place value understanding to round decimals to any place. **Perform operations with multi-digit whole numbers and with decimals to hundredths.**[77] **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

[76] When a cluster is referred to in this chart without a footnote, the cluster is addressed in its entirety.

[77] This standard is addressed again in Modules 2 and 4; the balance of this cluster is addressed in Module 2.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	Convert like measurement units within a given measurement system.[78] **5.MD.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Module 2: **Multi-Digit Whole Number and Decimal Fraction Operations** (35 days)	**Write and interpret numerical expressions.**[79] **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols. **5.OA.2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.* **Understand the place value system.**[80] **5.NBT.1** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. **5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.NBT.5** Fluently multiply multi-digit whole numbers using the standard algorithm.[81] **5.NBT.6** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using

[78] The focus of this module is on the metric system to reinforce place value and writing measurements using mixed units.

[79] These skills are also applied to fractions in this module.

[80] The balance of this cluster is addressed in Module 1.

[81] From this point forward, fluency practice is part of students' on-going experience.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	equations, rectangular arrays, and/or area models. **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. [82] **Convert like measurement units within a given measurement system.** **5.MD.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.
Module 3: **Addition and Subtraction of Fractions** (22 days)	**Use equivalent fractions as a strategy to add and subtract fractions.**[83] **5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
Module 4: **Multiplication and Division of Fractions and Decimal Fractions** (38 days)	**Write and interpret numerical expressions.** **5.OA.1** Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

[82] Focus on decimal multiplication of a single-digit, whole number factor times a multi-digit number with up to 2 decimal places (e.g., 3 x 64.98). Restrict decimal division to a single-digit whole number divisor with a multi-digit dividend with up to 2 decimal places (e.g., 64.98 ÷ 3). The balance of the standard is addressed in Module 4.

[83] Examples in this module also include tenths and hundredths in fraction and decimal form.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Perform operations with multi-digit whole numbers and with decimals to hundredths.**[84] **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**[85] **5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?* **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. *a.* Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

[84] The balance of this cluster is addressed in Module 2. Teach problems such as 2.7 x 2.1 and 4.5 ÷ 1.5. See the Progression Document "K–5, Number and Operations in Base Ten" pp. 17–18 (http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf).

[85] 5.NF.4b is addressed in Module 5. Include problems involving decimal fractions throughout the cluster.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	5.NF.5 Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) *a.* Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* *b.* Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* *c.* Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	Convert like measurement units within a given measurement system.[86] **5.MD.1** Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. **Represent and interpret data.** **5.MD.2** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
Module 5: **Addition and Multiplication with Volume and Area** (25 days)	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**[87] **5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas. **5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

[86] The focus of 5.MD.1 in this module is on the customary system of units as a means of introducing fractions (e.g., 1 inch is 1/12 foot, 1 foot is 1/3 yard).

[87] The balance of this cluster is addressed in Module 4. In this module, 5.NF.4b is applied to multiplying to find volume and area. 5.NF.4b includes decimal fraction side lengths of sides of a rectangle (in both fraction and decimal form).

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **5.MD.3** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **5.MD.4** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.MD.5** Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems. **Classify two-dimensional figures into categories based on their properties.** **5.G.3** Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 5 Modules
	5.G.4 Classify two-dimensional figures in a hierarchy based on properties.
Module 6: **Problem Solving with the Coordinate Plane** (40 days)	**Write and interpret numerical expressions.** **5.OA,2** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships.** **5.OA.3** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.* **Graph points on the coordinate plane to solve real-world and mathematical problems.** **5.G.1** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate). **5.G.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

6-8 AOD Math adoption of Eureka

A Story of Ratios:

A Curriculum Overview for Grades 6–8

Table of Contents

Introduction

This document provides an overview of the academic year for Grades 6 through 8, beginning with a curriculum map and followed by detailed grade-level descriptions.

The curriculum map is a chart that shows, at a glance, the sequence of modules comprising each grade of the Grades 6 through 8 curricula. The map also indicates the approximate number of instructional days designated for each module of each grade. Details that elaborate on the curriculum map are found in the grade-level descriptions.

Each grade-level description begins with a list of the six to seven modules that comprise the instruction of that grade. That introductory component is followed by three sections: the Summary of Year, the Rationale for Module Sequence, and the alignment chart with the grade-level standards.

The Summary of Year portion of each grade level includes four pieces of information:

- The critical instructional areas for the grade, as described in the Common Core State Standards for Mathematics[1] (CCSS-M)
- The Key Areas of Focus[2] for the grade
- The Required Fluencies for the grade
- The Major Emphasis Clusters[3] for the grade

The Rationale for Module Sequence portion of each grade level provides a brief description of the instructional focus of each module for that grade and explains the developmental sequence of the mathematics.

The alignment chart for each grade lists the standards that are addressed in each module of the grade. Note that when a cluster is referred to without a footnote, it is taught in its entirety. There are also times when footnotes are relevant to particular standards within a cluster. All standards for each grade have been carefully included in the module sequence. Some standards are deliberately included in more than one module so that a strong foundation can be built over time.

[1] http://www.corestandards.org/wp-content/uploads/Math_Standards1.pdf

[2] http://www.achievethecore.org/downloads/E0702_Description_of_the_Common_Core_Shifts.pdf

[3] http://www.parcconline.org/resources/educator-resources/model-content-frameworks/mathematics-model-content-framework

	Grade 6	Grade 7	Grade 8	
1st TRIMESTER	M1: Ratios and Unit Rates (35 days)	M1: Ratios & Proportional Relationships (30 days	M1: Integer Exponents and Scientific Notation (20 da ys)	
	M2: Arithmetic Operations Including Di vis ion of Fractions (25 days)	M2: Rational Numbers (30 days)	M2: The Concept of Congruence (25 days)	
			M3: Similarity (25 days)	
2nd TRIMESTER	M3: Rational Numbers (25 days)	M3: Expressions and Equations (35 days)	M4: Linear Equations (40 days)	
	M4: Expressions and Equations (45 days)	M4: Percent and Proportional Relationships (25 days)		
			M5: Examples of Functions from Geometry (15 da ys)	
3rd TRIMESTER		M5: Statistics and Probability (25 days)	M6: Linear Functions (20 days)	
	M5: Area, Surface Area, and Volume Problems (25 days)			
	M6: Statistics (25 days)	M6: Geometry (35 days)	M7: Introduction to Irrational Numbers Using Geometry (35 days)	

K y:					
Number	Geometry	Ratios and Proportions	Expressions and Equations	Statistics and Probability	Functions

*The columns indicating trimesters and quarters are provided to give you a rough guideline. Please use this additional column for your own pacing considerations based on the specific dates of your academic calendar.

Sequence of Grade 6 Modules Aligned with the Standards

Module 1: Ratios and Unit Rates
Module 2: Arithmetic Operations Including Division of Fractions
Module 3: Rational Numbers
Module 4: Expressions and Equations
Module 5: Area, Surface Area, and Volume Problems
Module 6: Statistics

Summary of Year

Grade 6 mathematics is about (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

Key Areas of Focus for Grade 6: Ratios and proportional reasoning; early expressions and equations

Required Fluency: 6.NS.B.2 Multi-digit division
6.NS.B.3 Multi-digit decimal operations

Major Emphasis Clusters
Ratios and Proportional Relationships • Understand ratio concepts and use ratio reasoning to solve problems.
The Number System • Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Apply and extend previous understandings of numbers to the system of rational numbers.
Expressions and Equations • Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.

Rationale for Module Sequence in Grade 6

In Module 1, students build on their prior work in measurement and in multiplication and division as they study the concepts and language of ratios and unit rates. They use proportional reasoning to solve problems. In particular, students solve ratio and rate problems using tape diagrams, tables of equivalent ratios, double number line diagrams, and equations. They plot pairs of values generated from a ratio or rate on the first quadrant of the coordinate plane.

Students expand their understanding of the number system and build their fluency in arithmetic operations in Module 2. Students learned in Grade 5 to divide whole numbers by unit fractions and unit fractions by whole numbers. Now, they apply and extend their understanding of multiplication and division to divide fractions by fractions. The meaning of this operation is connected to real-world problems as students are asked to create and solve fraction division word problems. Students continue (from Grade 5) to build fluency with adding, subtracting, multiplying, and dividing multi-digit decimal numbers using the standard algorithms.

Major themes of Module 3 are to understand rational numbers as points on the number line and to extend previous understandings of numbers to the system of rational numbers, which now include negative numbers. Students extend coordinate axes to represent points in the plane with negative number coordinates and, as part of doing so, see that negative numbers can represent quantities in real-world contexts. They use the number line to order numbers and to understand the absolute value of a number. They begin to solve real-world and mathematical problems by graphing points in all four quadrants, a concept that continues throughout to be used into high school and beyond.

With their sense of number expanded to include negative numbers, in Module 4 students begin formal study of algebraic expressions and equations. Students learn equivalent expressions by continuously relating algebraic expressions back to arithmetic and the properties of arithmetic (commutative, associative, and distributive). They write, interpret, and use expressions and equations as they reason about and solve one-variable equations and inequalities and analyze quantitative relationships between two variables.

Module 5 is an opportunity to practice the material learned in Module 4 in the context of geometry; students apply their newly acquired capabilities with expressions and equations to solve for unknowns in area, surface area, and volume problems. They find the area of triangles and other two-dimensional figures and use the formulas to find the volumes of right rectangular prisms with fractional edge lengths. Students use negative numbers in coordinates as they draw lines and polygons in the coordinate plane. They also find the lengths of sides of figures, joining points with the same first coordinate or the same second coordinate, and apply these techniques to solve real-world and mathematical problems.

In Module 6, students develop an understanding of statistical variability and apply that understanding as they summarize, describe, and display distributions. In particular, careful attention is given to measures of center and variability.

Alignment Chart[4]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
Module 1: **Ratios and Unit Rates** (35 days)	**Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.1** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* **6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[5] **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. *b.* Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?* c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

[4] When a cluster is referred to in this chart without a footnote, the cluster is taught in its entirety.

[5] Expectations for unit rates in this grade are limited to non-complex fractions.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
Module 2: **Arithmetic Operations Including Division of Fractions** (25 days)	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that(2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* **Compute fluently with multi-digit numbers and find common factors and multiples.** **6.NS.B.2** Fluently divide multi-digit numbers using the standard algorithm.[6] **6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.[7] **6.NS.B.4** Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 +2).*
Module 3: **Rational Numbers** (25 days)	**Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane

[6] This fluency standard begins in this module and is practiced throughout the remainder of the year.

[7] This fluency standard begins in this module and is practiced throughout the remainder of the year.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. **6.NS.C.7** Understand ordering and absolute value of rational numbers. *a.* Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret -3 > -7 as a statement that -3 is located to the right of -7 on a number line oriented from left to right.* *b.* Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write -3°C > -7°C to express the fact that -3°C is warmer than-7°C.* *c.* Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars.* *d.* Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.* **6.NS.C.8** Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
Module 4: **Expressions and Equations** (45 days)	**Apply and extend previous understandings of arithmetic to algebraic expressions.**[8] **6.EE.A.1** Write and evaluate numerical expressions involving whole-number exponents. **6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers. *a* Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 - y.* *b* Identify parts of an expression using mathematical terms (sum, term, product, factor quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.* *c* Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length s = 1/2.* **6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.* **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

[8] 6.EE.A.2c is also taught in Module 4 in the context of geometry.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	Reason about and solve one-variable equations and inequalities.[9] **6.EE.B.5** Understand solving an equation or inequality as a process of answering a question: Which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. **6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q, and x are all nonnegative rational numbers. **6.EE.B.8** Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams. **Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*
Module 5: **Area, Surface Area, and Volume Problems** (25 days)	**Solve real-world and mathematical problems involving area, surface area, and volume.** **6.G.A.1** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

[9] Except for 6.EE.B.8, this cluster is also taught in Module 4 in the context of geometry.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. **6.G.A.3** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. **6.G.A.4** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.
Module 6: **Statistics** (25 days)	**Develop understanding of statistical variability.** **6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.* **6.SP.A.2** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. **6.SP.A.3** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. **Summarize and describe distributions.** **6.SP.B.4** Display numerical data in plots on a number line, including dot plots, histograms, and box plots. **6.SP.B.5** Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 6 Modules
	b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Sequence of Grade 7 Modules Aligned with the Standards

Module 1: Ratios and Proportional Relationships

Module 2: Rational Numbers

Module 3: Expressions and Equations

Module 4: Percent and Proportional Relationships

Module 5: Statistics and Probability

Module 6: Geometry

Summary of Year

Grade 7 mathematics is about (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

Key Areas of Focus for Grade 7: Ratios and proportional reasoning; arithmetic of rational numbers

Major Emphasis Clusters
Ratios and Proportional Relationships • Analyze proportional relationships and use them to solve real-world and mathematical problems. The Number System • Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Expressions and Equations • Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Rationale for Module Sequence in Grade 7

In Module 1, students build on their Grade 6 experiences with ratios, unit rates, and fraction division to analyze proportional relationships. They decide whether two quantities are in a proportional relationship, identify constants of proportionality, and represent the relationship by equations. These skills are then applied to real-world problems including scale drawings.

In Module 2 students continue to build on understanding of the number line.. They learn to add, subtract, multiply, and divide rational numbers. Module 2 includes rational numbers as they appear in expressions and equations—work that is continued in Module 3.

Module 3 consolidates and expands students' previous work with generating equivalent expressions and solving equations. Students solve real-life and mathematical problems using numerical and algebraic expressions and equations. Their work with expressions and equations is applied to finding unknown angles and problems involving area, volume, and surface area.

Module 4 parallels Module 1's coverage of ratio and proportion but this time with a concentration on percent. Problems in this module include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error. Additionally, this module includes percent problems about populations, which prepare students for probability models about populations covered in the next module.

In Module 5, students learn to draw inferences about populations based on random samples. Through the study of chance processes, students learn to develop, use, and evaluate probability models.

The year concludes with students drawing and constructing geometrical figures in Module 6. They also revisit unknown angle, area, volume, and surface area problems, which now include problems involving percentages of areas or volumes.

Alignment Chart[10]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
Module 1: Ratios and Proportional Relationships (30 days)	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**[11] **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

[10] When a cluster is referred to in this chart without a footnote, the cluster is taught in its entirety.

[11] Percent and proportional relationships are covered in Module 4.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.* d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**[12] **7.EE.B.4**[13] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. *a.* Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* **Draw, construct, and describe geometrical figures and describe the relationships between them.**[14] **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

[12] The balance of this cluster is taught in Modules 2, 3, and 4.

[13] In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Module 2 and Module 3.

[14] 7.G.A.1 is also covered in Module 4. The balance of this cluster is taught in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
Module 2: **Rational Numbers** (30 days)	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. *a.* Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.* *b.* Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. *c.* Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (–q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. *d.* Apply properties of operations as strategies to add and subtract rational numbers. **7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(–1)(–1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats. **7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.[15] **Use properties of operations to generate equivalent expressions.**[16] **7.EE.A.2**[17] Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** [18] **7.EE.B.4**[19] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. *a.* Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*
Module 3: **Expressions and Equations** (35 days)	**Use properties of operations to generate equivalent expressions.** **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, a + 0.05a = 1.05a means*

[15] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

[16] The balance of this cluster is taught in Module 3.

[17] In this module, this standard is applied to expressions with rational numbers in them.

[18] The balance of this cluster is taught in Module 3.

[19] In this module the equations include negative rational numbers.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	that "increase by 5%" is the same as "multiply by 1.05." **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.** **7.EE.B.3**[20] Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. *a.* Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?* *b.* Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

[20] Problems in this module take on any form but percent, which is included in Module 4.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. [21] **7.G.B.4** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
Module 4: **Percent and Proportional Relationships**[22] (25 days)	**Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.1** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.* **7.RP.A.2** Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. *c.* Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.*

[21] Emphasis of 7.G.B.5 and 7.G.B.6 in this module is on solving equations. The standards are returned to in Module 6.

[22] The emphasis in this module is on percent.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where *r* is the unit rate. **7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.* **Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**[23] **7.EE.B.3** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.* **Draw, construct, and describe geometrical figures and describe the relationships between them.**[24] **7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
Module 5: **Statistics and Probability** (25 days)	**Use random sampling to draw inferences about a population.** **7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

[23] 7.EE.B.3 is introduced in Module 3. The balance of this cluster was taught in the first three modules.

[24] 7.G.A.1 is introduced in Module 1. The balance of this cluster is taught in Module 6.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*
	Draw informal comparative inferences about two populations.
	7.SP.B.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
	7.SP.B.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*
	Investigate chance processes and develop, use, and evaluate probability models.
	7.SP.C.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
	7.SP.C.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
	7.SP.C.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.* *b.* Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?* **7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event. *c.* Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*
Module 6: Geometry (35 days)	**Draw, construct, and describe geometrical figures and describe the relationships between them.**[25] **7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

[25] The balance of this cluster is taught in Modules 1 and 4.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 7 Modules
	7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. **Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**[26] **7.G.B.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **7.G.B.6** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

[26] 7.G.B.4 is taught in Module 3; 7.G.B.5 and 7.G.B.6 are introduced in Module 3.

Sequence of Grade 8 Modules Aligned with the Standards

Module 1: Integer Exponents and Scientific Notation
Module 2: The Concept of Congruence
Module 3: Similarity
Module 4: Linear Equations
Module 5: Examples of Functions from Geometry
Module 6: Linear Functions
Module 7: Introduction to Irrational Numbers Using Geometry

Summary of Year

Grade 8 mathematics is about (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean theorem.

Key Area of Focus for Grade 8: Linear algebra

Major Emphasis Clusters
Expressions and Equations • Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations. Functions • Define, evaluate, and compare functions. Geometry • Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean theorem.

Rationale for Module Sequence in Grade 8

This year begins with students extending the properties of exponents to integer exponents in Module 1. They use the number line model to support their understanding of the rational numbers and the number system. The number system is revisited at the end of the year (in Module 7) to develop the *real* number line through a detailed study of irrational numbers.

In Module 2, students study congruence by experimenting with rotations, reflections, and translations of geometrical figures. Their study of congruence culminates with an introduction to the Pythagorean theorem in which the teacher guides students through the "square-within-a-square" proof of the theorem. Students practice the theorem in real-world applications and mathematical problems throughout the year. (In Module 7, students learn to prove the Pythagorean theorem on their own and are assessed on that knowledge in that module.)

The experimental study of rotations, reflections, and translations in Module 2 prepares students for the more complex work of understanding the effects of dilations on geometrical figures in their study of similarity in Module 3. They use similar triangles to solve unknown angle, side length and area problems. Module 3 concludes with revisiting a proof of the Pythagorean theorem from the perspective of similar triangles.

In Module 4, students use similar triangles learned in Module 3 to explain why the slope of a line is well-defined. Students learn the connection between proportional relationships, lines, and linear equations as they develop ways to represent a line by different equations (e.g., $y = mx + b$, $y - y_1 = m\,(x - x_1)$). They analyze and solve linear equations and pairs of simultaneous linear equations. The equation of a line provides a natural transition into the idea of a function explored in the next two modules.

Students are introduced to functions in the context of linear equations and area/volume formulas in Module 5. They define, evaluate, and compare functions using equations of lines as a source of linear functions and area and volume formulas as a source of non-linear functions.

In Module 6, students return to linear functions in the context of statistics and probability as bivariate data provides support in the use of linear functions.

By Module 7, students have been using the Pythagorean theorem for several months. They are sufficiently prepared to learn and explain a proof of the theorem on their own. The Pythagorean theorem is also used to motivate a discussion of irrational square roots (irrational cube roots are introduced via volume of a sphere). Thus, as the year began with looking at the number system, so it concludes with students understanding irrational numbers and ways to represent them (radicals, non-repeating decimal expansions) on the real number line.

Alignment Chart[27]

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
Module 1: **Integer Exponents and Scientific Notation** (20 days)	**Work with radicals and integer exponents.[28]** **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.* **8.EE.A.3** Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* **8.EE.A.4** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Module 2: **The Concept of Congruence** (25 days)	**Understand congruence and similarity using physical models, transparencies, or geometry software.[29]** **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines. **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[27] When a cluster is referred to in this chart without a footnote, the cluster is taught in its entirety.

[28] 8.EE.A.2 is covered in Module 7.

[29] 8.G.A.3, 8.G.A.4, and the balance of 8.G.A.5 are taught in Module 3.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	8.G.A.5[30] Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.* **Understand and apply the Pythagorean Theorem.**[31] **8.G.B.6**[32] Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7**[33] Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Module 3: **Similarity** (25 days)	**Understand congruence and similarity using physical models, transparencies, or geometry software.**[34] **8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. ***8.G.A.5*** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

[30] Congruence is addressed in this module. The balance of this standard (similarity) is taught in Module 3.

[31] 8.G.B.6 and 8.G.B.7 are also taught in Module 3. The balance of 8.G.B.6 and 8.G.B.7 are covered in Module 7, along with standard 8.G.B.8.

[32] The Pythagorean theorem is proved in this module guided by the teacher (square within a square proof). Students are not responsible for explaining a proof until Module 7.

[33] This standard is started in this module and practiced during the year. No solutions that involve irrational numbers are introduced until Module 7.

[34] The balance of this cluster is taught in Module 1.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	Understand and apply the Pythagorean Theorem.[35] **8.G.B.6**[36] Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7**[37] Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Module 4: **Linear Equations** (40 days)	**Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.C.7** Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

[35] 8.G.B.6 and 8.G.B.7 are also taught in Module 2. The balance of standards 8.G.B.6 and 8.G.B.7 are covered in Module 7, along with standard 8.G.B.8.

[36] The Pythagorean theorem is proved in this module with guidance by the teacher (proof using similar triangles). Students are not responsible for explaining a proof until Module 7.

[37] This standard is started in this module and practiced during the year. No solutions that involve irrational numbers are introduced until Module 7.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	8.EE.C.8 Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. *b.* Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* *c.* Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Module 5: **Examples of Functions from Geometry** (15 days)	**Define, evaluate, and compare functions.**[38] **8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[39] ***8.F.A.2*** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.* ***8.F.A.3*** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

[38] Linear and non-linear functions are compared in this module using linear equations and area/volume formulas as examples.

[39] Function notation is not required in Grade 8.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. **8.G.C.9**[40] Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
Module 6: **Linear Functions** (20 days)	**Use functions to model relationships between quantities.** **8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two *(x, y)* values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **Investigate patterns of association in bivariate data.**[41] **8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

[40] Solutions that introduce irrational numbers are not introduced until Module 7.

[41] 8.SP standards are used as applications to the work done with 8.F standards.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*
Module 7: **Introduction to Irrational Numbers Using Geometry** (35 days)	**Know that there are numbers that are not rational, and approximate them by rational numbers.** **8.NS.A.1** Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **8.NS.A.2** Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **Work with radicals and integer exponents.**[42] **8.EE.A.2** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

[42] The balance of this cluster is taught in Module 1.

Module and Approximate Number of Instructional Days	Standards Addressed in Grade 8 Modules
	Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. **8.G.C.9** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.[43]

[43] Solutions that introduce irrational numbers are allowed in this module.

Eureka Math is available to the public free of charge, and is available here. All units from K – 8 are the open source material formerly known as Engage NY. Academy of Dover classrooms have the full complement of manipulatives to accompany each unit.