The Golden Section

The golden section number is closely connected with the Fibonacci series and has a value of (√5 + 1)/2 or:

1·61803 39887 49894 84820 45868 34365 63811 77203 09179 80576 ..More..

which we call Phi (note the capital **P**) on these pages. The other number also called the golden section is Phi-1 or 0·61803... with exactly the same decimal fraction part as Phi. This value we call phi (with a small **p**) here. Phi and phi have some interesting and unique properties such as 1/phi is the same as 1+phi=Phi.
The third of Simon Singh's BBC Radio 4 Five Numbers programmes was all about the Golden Ratio. It is an excellent introduction to the golden section. I spoke on it about the occurrence in nature of the golden section and also the Change Puzzle.
Hear the whole programme (14 minutes) using the free RealOne Player.

- The Golden section and Geometry
  The golden section is also called the golden ratio, the golden mean and the divine proportion.

  Two more pages look at its applications in Geometry: first in flat (or two dimensional) geometry and then in the solid geometry of three dimensions.
    - Fantastic Flat Phi Facts
      See some of the unexpected places that the golden section (Phi) occurs in Geometry and in Trigonometry: pentagons and decagons, paper folding and Penrose Tilings where we phind phi phrequently!
    - The Golden Geometry of Solids or Phi in 3 dimensions
      The golden section occurs in the most symmetrical of all the three-dimensional solids - the Platonic solids. What are the best shapes for fair dice? Why are there only 5?

  The next pages are about the numbers Phi = 1·61803.. and phi = 0·61803... and their properties.

    - Phi's Fascinating Figures - the Golden Section number
      All the powers of Phi are just whole multiples of itself plus another whole number. Did you guess that these multiples *and*

the whole numbers are, of course, the Fibonacci numbers again? Each power of Phi is the sum of the previous two - just like the Fibonacci numbers too.

  - Introduction to Continued Fractions
    is an optional page that expands on the idea of a continued fraction introduced in the Phi's Fascinating Figures page.

- Phigits and Base Phi Representations
  We have seen that using a *base* of the Fibonacci Numbers we can represent all integers in a binary-like way. Here we show there is an interesting way of representing *all* integers in a binary-like fashion but using only powers of Phi instead of powers of 2 (binary) or 10 (decimal).

The Golden String

The golden string is also called the *Infinite Fibonacci Word* or the *Fibonacci Rabbit sequence*. There is another way to look at Fibonacci's Rabbits problem that gives an infinitely long sequence of 1s and 0s called the Golden String:-

1 0 1 1 0 1 0 1 1 0 1 1 0 1 0 1 1 0 1 ...

This string is a closely related to the golden section and the Fibonacci numbers.

- Fibonacci Rabbit Sequence
  See show how the golden string arises directly from the Rabbit problem and also is used by computers when they compute the Fibonacci numbers. You can hear the Golden sequence as a sound track too.
  The Fibonacci Rabbit sequence is an example of a **fractal** - a mathematical object that contains the whole of itself within itself infinitely many times over.

Fibonacci - the Man and His Times

- Who was Fibonacci?
  Here is a brief biography of Fibonacci and his historical achievements in mathematics, and how he helped Europe replace the Roman numeral system with the "algorithms" that we use today.
  Also there is a guide to some memorials to Fibonacci to see in Pisa, Italy.

More Applications of Fibonacci Numbers and Phi

- The Fibonacci numbers in a formula for Pi (π)
  There are several ways to compute pi (3·14159 26535 ..) accurately. One that has been used a lot is based on a nice formula for calculating which angle has a given tangent, discovered by James Gregory. His formula together with the Fibonacci numbers can be used to compute pi. This page introduces you to all these concepts from scratch.
- Fibonacci Forgeries
  Sometimes we find series that for quite a few terms look exactly like the Fibonacci numbers, but, when we look a bit more closely, they aren't - they are Fibonacci Forgeries.
  Since we would not be telling the truth if we said they *were* the Fibonacci numbers, perhaps we should call them **Fibonacci Fibs** !!
- The Lucas Numbers
  Here is a series that is very similar to the Fibonacci series, **the Lucas series**, but it starts with 2 and 1 instead of Fibonacci's 0 and 1. It sometimes pops up in the pages above so here we investigate it some more and discover its properties.
  It ends with a number trick which you can use "to impress your friends with your amazing calculating abilities" as the adverts say. It uses facts about the golden section and its relationship with the Fibonacci and Lucas numbers.
  - The first 200 Lucas numbers and their factors
    together with some suggestions for investigations you can do.

  2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843 .... 1·618

- General Fibonacci Series NEW
  The Lucas numbers change the two starting values of the Fibonacci series from 0 and 1 to 2 and 1. What if we changed these to any two values? These General Fibonacci series are called the G series but *the* Fibonacci series and Phi again play a prominent role in their mathematical properties. Also we look at two special arrays (tables) of numbers, the Wythoff array and the Stolarsky array and show how a these two collections of general Fibonacci series contain *each whole number exactly once*. The secret behind such clever arrays is ... the golden section number Phi!

Fibonacci and Phi in the Arts

- Fibonacci Numbers and The Golden Section In Art, Architecture and Music
  The golden section has been used in many designs, from the ancient **Parthenon** in Athens (400BC) to Stradivari's violins. It was known to

artists such as **Leonardo da Vinci** and musicians and composers, notably **Bartók** and **Debussy**. This is a different kind of page to those above, being concerned with speculations about where Fibonacci numbers and the golden section both do and do not occur in art, architecture and music. All the other pages are factual and verifiable - the material here is a often a matter of opinion. What do *you* think?

Reference

- Fibonacci and Phi Formulae
  A reference page of almost 200 formulae and equations showing the properties of the Fibonacci and Lucas series, the general Fibonacci G series and Phi. Also available in PDF format (13 pages, 340K) for which you will need the free Acrobat PDF Reader or plug-in for your browser.
- Links and Bibliography
  Links to other sites on Fibonacci numbers and the Golden section together with references to books and articles.