AP Calculus: Fall 2013 EXAM REVIEW

Name: _____ **Date:** _____ **Period:** _____

1. Evaluate the following limit.

$$\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\left(\frac{\pi}{4}\right)}{h} = \_\_\_\_\_$$

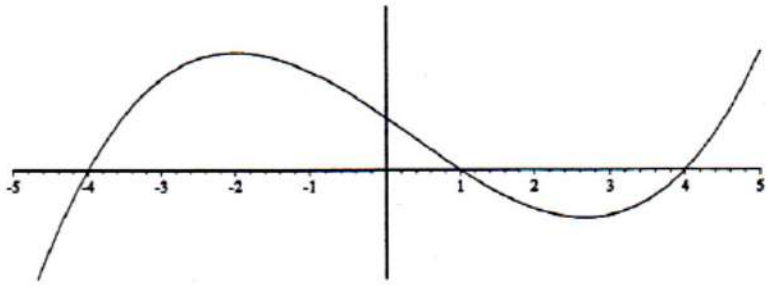
2. A ball is thrown from the top of a 180-foot building. Its position at time t is given by the function $s(t) = -16t^2 + 64t + 180$ feet.
   (a) What is the maximum height that the ball reaches?

   (b) What is the speed of the ball when it is 100 feet above the ground?

3. Find the equation of the line tangent to the curve $x^2 + 4xy - 5y^3 = 7$ at the point (2, 1). Put the answer in the form $y = mx + b$.

4. A two-piece extension ladder leaning against a wall is collapsing at a rate of 2 ft/sec at the same time as its base is moving away from the wall at a rate of 3 ft/sec. How fast is the top of the ladder moving down the wall when the top of the ladder is 8 feet above the ground and the base is 6 feet from the wall?

5. Let $H'(x)$, the derivative of a continuous function $H(x)$ be given by the graph shown below in the interval $-5, 5$.



a. Determine the intervals for which H(x) is increasing. Justify your answer.

b. Determine the intervals for which H(x) is decreasing. Justify your answer.

c. Determine the approximate intervals for which H(x) is concave up. Justify your answer.

d. Determine the approximate intervals for which H(x) is concave down. Justify your answer.

e. Determine the x-coordinates of all relative maximum of H(x). Justify your answer.

f. Determine the approximate x-coordinates of all inflection points of H(x). Justify your answer.

g. Determine the x-coordinates of all the points on the curve of H(x) for which there is a horizontal tangent.

6. A ball is moving along a horizontal line starting at time $t = 0$, so that its position function at any time $t \geq 0$ is given by $x\ t\ = (7 - 4t)(t - 2)^3$
   a. Find an expression for the velocity of the ball at any given time t. JYA.

   b. For what values of t is the ball moving to the right? JYA.

   c. Find all the values of t for which the acceleration of the ball is zero. JYA.

   d. Is the ball stationary at any value(s) of t found in part (c) ? JYA.

7. Use the table given to the right to find the derivatives of each of the following functions at the indicated point. JYA.
   a. $2h\ x\ - 3\ p\ x \qquad at\ x = -1$

   b. $2h\ x\ \cdot p\ x \qquad at\ x = 2$

   c. $\frac{4h(x)}{p(x)} \qquad at\ x = 5$

x	$h(x)$	$p(x)$	$h'(x)$	$p'(x)$
-1	-3	6	8	-10
2	$\frac{2}{5}$	-1	11	$\frac{3}{4}$
5	1	7	15	-9

8. Find the derivatives of each of the following. No need to simplify:

   a. $F(x) = \sin(4x^5 - 6x)$ $F'(x) =$

   b. $g(x) = \cos(8x^2 - 3)$ $g'(x) =$

   c. $g(x) = \tan(5x)$ $g'(x) =$

   d. $h(x) = \cot(2x - 3)$ $h'(x) =$

   e. $p(x) = \sec(3x^2 - 3)$ $p'(x) =$

9. Find $h'(x)$ for each of the following. Express your answer in simplest factored form:

   a. $h(x) = 2x^2 \sin^2(3x - 1) + 2x^2 \cos^2(3x - 1)$

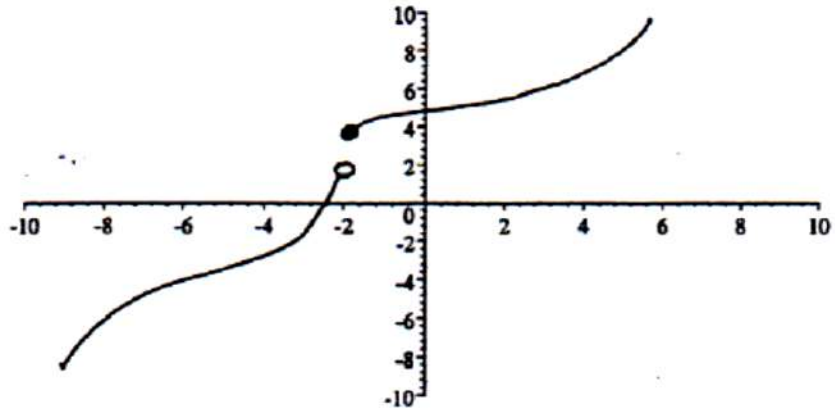
   b. $h(x) = \cos^2(3x - 1) - \sin^2(3x - 1)$

10. A spherical balloon is to be deflated so that the radius decreases at the rate of 3 ft^3/min. How fast is the ***diameter*** of the balloon increasing when the radius is 1 ft?

11. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 8 meters and its volume is 72 cubic meters. If building the tank costs $20 per square meter for the base and $10 per square meter for the sides, what is the cost of the least expensive tank? JYA.

12. Every day, a flight from Los Angeles to Atlanta flies directly over my home at a constant altitude of 4 miles. If I assume that the plan is flying at a constant speed of 400 mi/hr, at what rate is the angle of elevation of my line of sight changing with respect to time when the horizontal distance between the approaching plane and my location is exactly 3 mi?

13. The function G is shown below. Which of the following statements are true?



I. $\lim_{x \to -2^+} G(x) = 4$ II. $G(-2) \neq \lim_{x \to -2^+} G(x)$ III. $G(x)$ has a removable discontinuity at $x = 4$.

a. I only b. II only c. III only d. I and II only e. I, II, III.

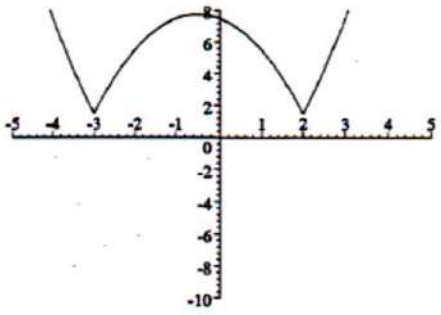
14. The number of inflection points of $h\ x\ = 3x - \sin\ x^2$ in the interval $[-1, 3]$ is:

a. 7 b. 6 c. 5 d. 4 e. 3

15. Consider the function $g\ x\ = \frac{5x}{d+x^2}$ for which $g'0) = 7$. The value of d is:

a. $\frac{7}{5}$ b. $\frac{7}{5}$ c. 0 d. $\frac{5}{7}$ e. 2

16. The graph of h', the derivative of a continuous function h, is shown below.



I. h is decreasing in the interval (-4,-2).
II. h has an inflection point at x=-3.
III. h is concave down in the interval (-3, 2).

a. I only b. II only c. III only d. I and III only e. I and II only

17. Let $V\ t\ = 4x^3 - 5t^2 + 3$ represent the velocity of a particle moving along a horizontal line. Find the average velocity over the time interval $0 \leq t \leq 3$.

a. 0 b. 21 c. 26 d. 78 e. 63

18. The intervals for which $f\ x\ = -3x^{\frac{7}{3}} + \frac{21}{4}x^{4/3}$

I. $x > 1$ **II.** $0 < x < 1$ **III.** $x < 0$

a. I only **b. II only** **c. III only** **d. I and II only** **e. I and III on**

19. $\lim_{h\to 0} \frac{\sin\ 2\ x-h\ \ -\sin 2x}{h} =$ _____

a. $sin(2x)$ b. $2sin(2x)$ c. $2\cos(2x)$ d. $-2\cos(2x)$ e. does not exist

20. On which of the following intervals is the graph of the curve $g\ x\ = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 3x^2 + 7$ concave down?

I. $x > 2$ II. $-3 < x < 2$ III. $x < -3$

a. I only b. II only c. III only d. I and II only e. I and III only

21. Find the limit: $\lim_{x\to 2} \frac{x-2}{x^2-4}$.

(a) 0 (b) $\frac{1}{4}$ (c) ∞

(d) 1 (e) None of these

22. Find the limit: $\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$.

(a) 0 (b) $\frac{1}{4}$ (c) ∞

(d) 1 (e) None of these

23. Find the limit: $\lim_{x\to 3} \frac{x-3}{|x-3|}$.

(a) 0 (b) 1 (c) 3

(d) The limit does not exist. (e) None of these

24. At which values of x is $f(x) = \dfrac{x^2 - 2x - 3}{x - 2}$ discontinuous?

(a) 2 (b) $-1, 2, 3$ (c) 1

(d) $-1, \dfrac{3}{2}, 2, 3$ (e) None of these

25. Find the limit: $\lim\limits_{x \to 0} \dfrac{1 - \cos^2 x}{x}$.

(a) 1 (b) 0 (c) ∞

(d) The limit does not exist. (e) None of these

EXTRA CREDIT:

A.) For what values of $x \neq 0$, the slope of the tangent line to $G\ x\ = 3x \cdot \sin(2x)$ equals zero whenever:

a. $-\frac{1}{2}\cot(2x) = x$ b. $-\frac{1}{2}\tan(2x) = x$

c. $-\tan(2x) = \frac{1}{x}$ d. $-\cot(2x) = \frac{1}{x}$

e. $\cos(2x) = \frac{3}{x}$

B.) Assume that $h\ x\ $ is an odd function, that is $h\ -x\ = -h\ x\ $. If $h'\ 3\ = 4$, find the value of $h'(-3)$.

a. Cannot be determined from the information given
b. 0
c. -4
d. ¼
e. 4