Warm Up

Determine whether the following are perfect squares. If so, find the square root.

1. 64 yes; 8

2. 36 yes; 6

3. 45 no

4. x^2 yes; x

5. y^8 yes; y^4

6. $4x^6$ yes; $2x^3$

7. $9y^7$ no

8. $49p^{10}$ yes; $7p^5$

Objectives

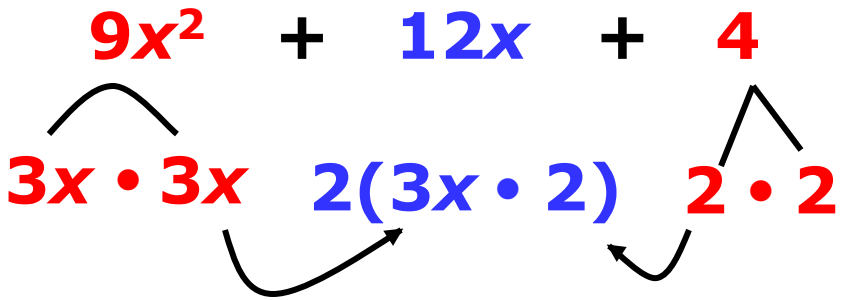
Factor perfect-square trinomials.

Factor the difference of two squares.

A trinomial is a perfect square if:

- The first and last terms are perfect squares.

- The middle term is two times one factor from the first term and one factor from the last term.



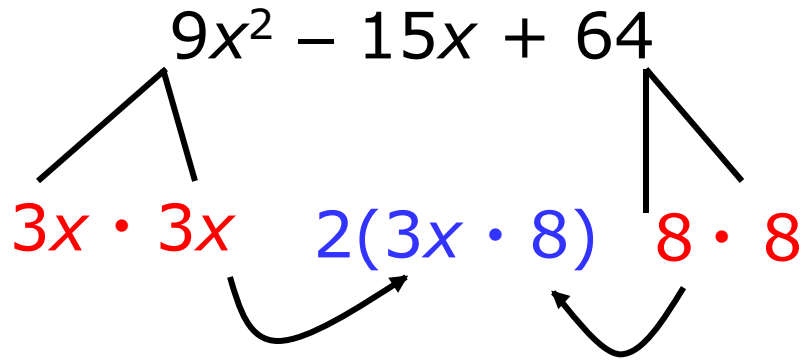
Perfect-Square Trinomials

PERFECT-SQUARE TRINOMIAL	EXAMPLES
$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$	$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$
$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$	$x^2 - 2x + 1 = (x - 1)(x - 1) = (x - 1)^2$

Example 1A: Recognizing and Factoring Perfect-Square Trinomials

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$9x^2 - 15x + 64$



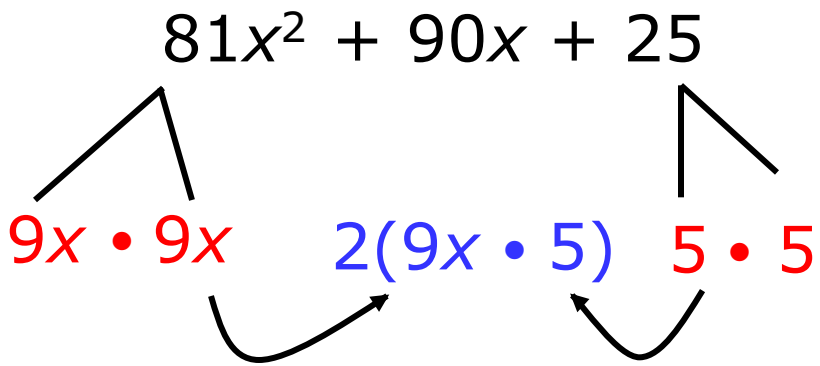
2(3x • 8) ≠ –15x.

$9x^2 - 15x + 64$ is not a perfect-square trinomial because $-15x \neq 2(3x \cdot 8)$.

Example 1B: Recognizing and Factoring Perfect-Square Trinomials

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$81x^2 + 90x + 25$



The trinomial is a perfect square. Factor.

Example 1B Continued

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

Method 2 Use the rule.

$81x^2 + 90x + 25$ — *$a = 9x$, $b = 5$*

$(9x)^2 + 2(9x)(5) + 5^2$ — *Write the trinomial as $a^2 + 2ab + b^2$.*

$(9x + 5)^2$ — *Write the trinomial as $(a + b)^2$.*

Example 1C: Recognizing and Factoring Perfect-Square Trinomials

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$36x^2 - 10x + 14$

$36x^2 - 10x + 14$

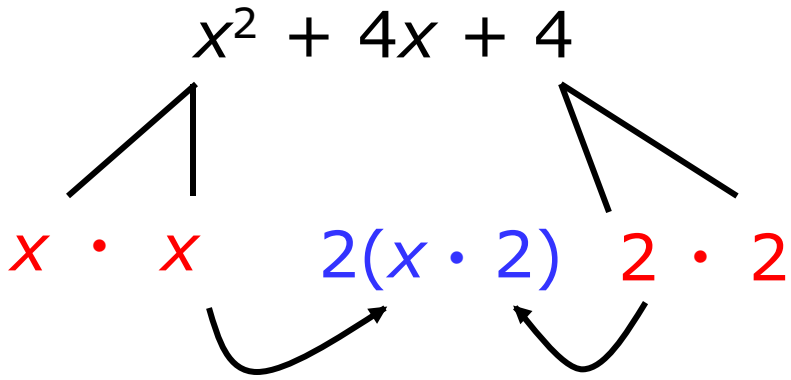
The trinomial is not a perfect-square because 14 is not a perfect square.

$36x^2 - 10x + 14$ is not a perfect-square trinomial.

Check It Out! Example 1a

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$x^2 + 4x + 4$



The trinomial is a perfect square. Factor.

Check It Out! Example 1a Continued

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

Method 1 Factor.

$x^2 + 4x + 4$

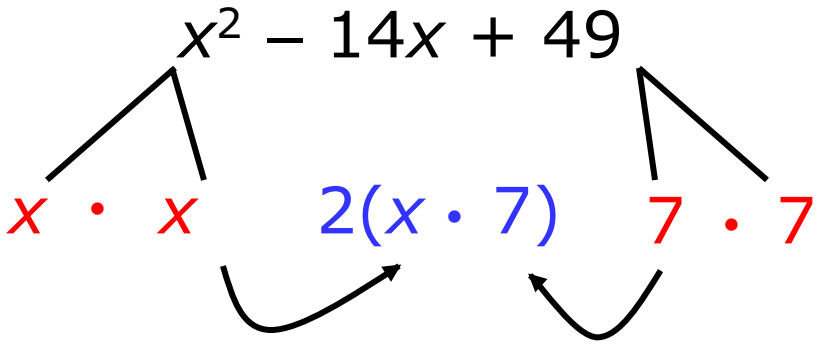
Factors of 4	Sum
(1 and 4)	5 ✗
(2 and 2)	4 ✓

$(x + 2)(x + 2) = (x + 2)^2$

Check It Out! Example 1b

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$x^2 - 14x + 49$



The trinomial is a perfect square. Factor.

Check It Out! Example 1b Continued

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

Method 2 Use the rule.

$x^2 - 14x + 49$ *a = 1, b = 7*

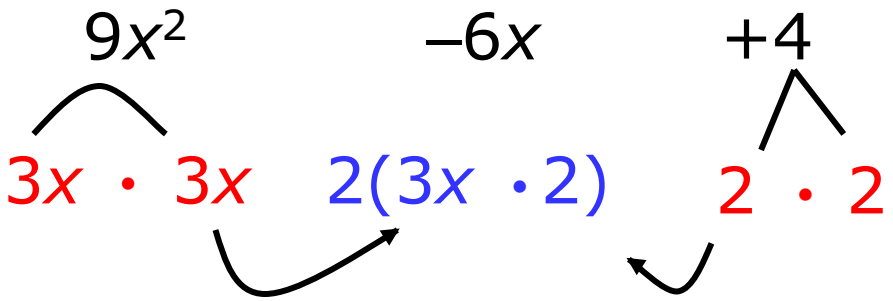
$(x)^2 - 2(x)(7) + 7^2$ *Write the trinomial as $a^2 - 2ab + b^2$.*

$(x - 7)^2$ *Write the trinomial as $(a - b)^2$.*

Check It Out! Example 1c

Determine whether each trinomial is a perfect square. If so, factor. If not explain.

$9x^2 - 6x + 4$



$2(3x)(4) \neq -6x$

$9x^2 - 6x + 4$ is not a perfect-square trinomial because $-6x \neq 2(3x \cdot 2)$

Example 2: *Problem-Solving Application*

A square piece of cloth must be cut to make a tablecloth. The area needed is $(16x^2 - 24x + 9)$ in^2. The dimensions of the cloth are of the form $cx - d$, where c and d are whole numbers. Find an expression for the perimeter of the cloth. Find the perimeter when $x = 11$ inches.

Example 2 Continued

Understand the Problem

The **answer** will be an expression for the perimeter of the cloth and the value of the expression when $x = 11$.

List the **important information:**

- The tablecloth is a square with area $(16x^2 - 24x + 9)$ in^2.
- The side length of the tablecloth is in the form $cx - d$, where c and d are whole numbers.

Example 2 Continued

Make a Plan

The formula for the area of a square is area = (side)^2.

Factor $16x^2 - 24x + 9$ to find the side length of the tablecloth. Write a formula for the perimeter of the tablecloth, and evaluate the expression for $x = 11$.

Example 2 Continued

3 Solve

$16x^2 - 24x + 9$ *$a = 4x, b = 3$*

$(4x)^2 - 2(4x)(3) + 3^2$ *Write the trinomial as $a^2 - 2ab + b^2$.*

$(4x - 3)^2$ *Write the trinomial as $(a - b)^2$.*

$16x^2 - 24x + 9 = (4x - 3)(4x - 3)$

The side length of the tablecloth is $(4x - 3)$ in.

Example 2 Continued

Write a formula for the perimeter of the tablecloth.

$P = 4s$ *Write the formula for the perimeter of a square.*

$= 4(4x - 3)$ *Substitute the side length for s.*

$= 16x - 12$ *Distribute 4.*

An expression for the perimeter of the tablecloth in inches is $16x - 12$.

Example 2 Continued

Evaluate the expression when $x = 11$.

$$P = 16x - 12$$

$$= 16(11) - 12 \qquad \textit{Substitute 11 for x.}$$

$$= 164$$

When $x = 11$ in. the perimeter of the tablecloth is 164 in.

Example 2 Continued

4 Look Back

For a square with a perimeter of 164, the side length is $\frac{164}{4} = 41$ in. and the area is $41^2 = 1681$ in^2.

Evaluate $16x^2 - 24x + 9$ for $x = 11$.

$16(11)^2 - 24(11) + 9$

$1936 - 264 + 9$

1681 ✓

Check It Out! Example 2

What if …? A company produces square sheets of aluminum, each of which has an area of $(9x^2 + 6x + 1)$ m^2. The side length of each sheet is in the form $cx + d$, where c and d are whole numbers. Find an expression in terms of x for the perimeter of a sheet. Find the perimeter when $x = 3$ m.

Check It Out! Example 2 Continued

Understand the Problem

The **answer** will be an expression for the perimeter of a sheet and the value of the expression when $x = 3$.

List the **important information:**

- A sheet is a square with area $(9x^2 + 6x + 1)$ m^2.
- The side length of a sheet is in the form $cx + d$, where c and d are whole numbers.

Check It Out! Example 2 Continued

Make a Plan

The formula for the area of a sheet is area = (side)^2

Factor $9x^2 + 6x + 1$ to find the side length of a sheet. Write a formula for the perimeter of the sheet, and evaluate the expression for $x = 3$.

Check It Out! Example 2 Continued

Solve

$9x^2 + 6x + 1$ *a = 3x, b = 1*

$(3x)^2 + 2(3x)(1) + 1^2$ *Write the trinomial as $a^2 + 2ab + b^2$.*

$(3x + 1)^2$ *Write the trinomial as $(a + b)^2$.*

$9x^2 + 6x + 1 = (3x + 1)(3x + 1)$

The side length of a sheet is $(3x + 1)$ m.

Check It Out! Example 2 Continued

Write a formula for the perimeter of the aluminum sheet.

$P = 4s$ *Write the formula for the perimeter of a square.*

$= 4(3x + 1)$ *Substitute the side length for s.*

$= 12x + 4$ *Distribute 4.*

An expression for the perimeter of the sheet in meters is $12x + 4$.

Check It Out! Example 2 Continued

Evaluate the expression when $x = 3$.

$P = 12x + 4$

$= 12(3) + 4$ *Substitute 3 for x.*

$= 40$

When $x = 3$ m. the perimeter of the sheet is 40 m.

Check It Out! Example 2 Continued

Look Back

For a square with a perimeter of 40, the side length is $\frac{40}{4} = 10$ m and the area is $10^2 = 100$ m^2.

Evaluate $9x^2 + 6x + 1$ for $x = 3$

$9(3)^2 + 6(3) + 1$

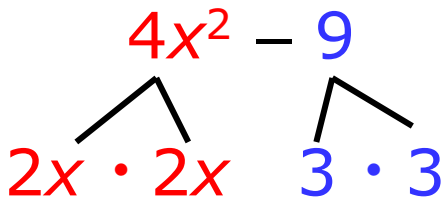
$81 + 18 + 1$

100 ✓

In Chapter 7 you learned that the difference of two squares has the form $a^2 - b^2$. The difference of two squares can be written as the product $(a + b)(a - b)$. You can use this pattern to factor some polynomials.

A polynomial is a difference of two squares if:

- There are two terms, one subtracted from the other.
- Both terms are perfect squares.



Difference of Two Squares

DIFFERENCE OF TWO SQUARES	EXAMPLE
$a^2 - b^2 = (a + b)(a - b)$	$x^2 - 9 = (x + 3)(x - 3)$

Reading Math

Recognize a difference of two squares: the coefficients of variable terms are perfect squares, powers on variable terms are even, and constants are perfect squares.

Example 3A: Recognizing and Factoring the Difference of Two Squares

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$3p^2 - 9q^4$

$3p^2 - 9q^4$

$3q^2 \cdot 3q^2$

$3p^2$ is not a perfect square.

$3p^2 - 9q^4$ is not the difference of two squares because $3p^2$ is not a perfect square.

Example 3B: Recognizing and Factoring the Difference of Two Squares

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$100x^2 - 4y^2$

$100x^2 - 4y^2$

$10x \cdot 10x \quad 2y \cdot 2y$

$(10x)^2 - (2y)^2$

$(10x + 2y)(10x - 2y)$

The polynomial is a difference of two squares.

$a = 10x$, $b = 2y$

Write the polynomial as $(a + b)(a - b)$.

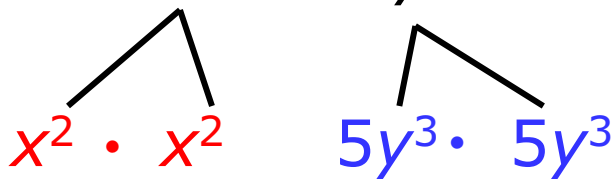
$100x^2 - 4y^2 = (10x + 2y)(10x - 2y)$

Example 3C: Recognizing and Factoring the Difference of Two Squares

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$x^4 - 25y^6$

$x^4 - 25y^6$



$(x^2)^2 - (5y^3)^2$

The polynomial is a difference of two squares.

$a = x^2, b = 5y^3$

$(x^2 + 5y^3)(x^2 - 5y^3)$

Write the polynomial as $(a + b)(a - b)$.

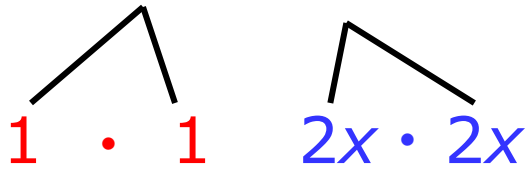
$x^4 - 25y^6 = (x^2 + 5y^3)(x^2 - 5y^3)$

Check It Out! Example 3a

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$1 - 4x^2$

$1 - 4x^2$



$(1) - (2x)^2$

$(1 + 2x)(1 - 2x)$

The polynomial is a difference of two squares.

a = 1, *b* = 2*x*

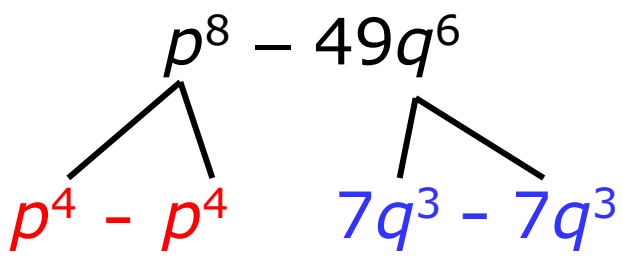
Write the polynomial as (a + b)(a – b).

$1 - 4x^2 = (1 + 2x)(1 - 2x)$

Check It Out! Example 3b

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$$p^8 - 49q^6$$



$$(p^4)^2 - (7q^3)^2$$

$$(p^4 + 7q^3)(p^4 - 7q^3)$$

The polynomial is a difference of two squares.

$a = p^4$, $b = 7q^3$

Write the polynomial as $(a + b)(a - b)$.

$$p^8 - 49q^6 = (p^4 + 7q^3)(p^4 - 7q^3)$$

Check It Out! Example 3c

Determine whether each binomial is a difference of two squares. If so, factor. If not, explain.

$16x^2 - 4y^5$

$16x^2 - 4y^5$

$4x \cdot 4x$

$4y^5$ is not a perfect square.

$16x^2 - 4y^5$ is not the difference of two squares because $4y^5$ is not a perfect square.

Lesson Quiz: Part I

Determine whether each trinomial is a perfect square. If so factor. If not, explain.

1. $64x^2 - 40x + 25$ Not a perfect-square trinomial because $-40x \neq 2(8x \cdot 5)$.

2. $121x^2 - 44x + 4$ $(11x - 2)^2$

3. $49x^2 + 140x + 100$ $(7x^2 + 10)^2$

4. A fence will be built around a garden with an area of $(49x^2 + 56x + 16)$ ft^2. The dimensions of the garden are $cx + d$, where c and d are whole numbers. Find an expression for the perimeter when $x = 5$. $P = 28x + 16$; 156 ft

Lesson Quiz: Part II

Determine whether the binomial is a difference of two squares. If so, factor. If not, explain.

5. $9x^2 - 144y^4$ $(3x + 12y^2)(3x - 12y^2)$

6. $30x^2 - 64y^2$ Not a difference of two squares; $30x^2$ is not a perfect square

7. $121x^2 - 4y^8$ $(11x + 2y^4)(11x - 2y^4)$