Exponential Functions

Adapted from www.MAthXTC.com

Exponential Growth Functions

If a quantity increases by the same proportion *r* in each unit of time, then the quantity displays exponential growth and can be modeled by the equation

$$y = C(1 + r)^t$$

Where

C = initial amount

r = growth rate (percent written as a decimal)

t = time   where $t \geq 0$

(1+r) = growth factor where $1 + r > 1$

GOAL 1

WRITING EXPONENTIAL GROWTH MODELS

A quantity is **growing exponentially** if it increases by the same percent in each time period.

EXPONENTIAL GROWTH MODEL

C is the **initial amount.**

t is the **time period.**

$$y = C(1 + r)^t$$

$(1 + r)$ is the **growth factor,** r is the **growth rate.**

The **percent of increase** is $100r$.

Example: Compound Interest

You deposit \$1500 in an account that pays 2.3% interest compounded yearly,

1) What was the initial principal (**P**) invested?
2) What is the growth rate (**r**)? The growth factor?
3) Using the equation **A = P(1+r)^t**, how much money would you have after 2 years if you didn't deposit any more money?

1) The initial principal (P) is \$1500.
2) The growth rate (r) is 0.023. The growth factor is 1.023.
3) $A = P(1+r)^t$

$$A = 1500(1+0.023)^2$$

$$A = \$1569.79$$

Exponential Decay Functions

If a quantity decreases by the same proportion *r* in each unit of time, then the quantity displays exponential decay and can be modeled by the equation

$$y = C(1 - r)^t$$

Where

C = initial amount

r = growth rate (percent written as a decimal)

t = time   where $t \geq 0$

(1 - r) = decay factor where $1 - r < 1$

GOAL 1

WRITING EXPONENTIAL DECAY MODELS

A quantity is **decreasing exponentially** if it decreases by the same percent in each time period.

EXPONENTIAL DECAY MODEL

C is the **initial amount.**

t is the **time period.**

$$y = C(1 - r)^t$$

$(1 - r)$ is the **decay factor,** r is the **decay rate.**

The **percent of decrease** is $100r$.

Example: Exponential Decay

You buy a new car for $22,500. The car depreciates at the rate of 7% per year,

1) **What was the initial amount invested?**
2) **What is the decay rate? The decay factor?**
3) **What will the car be worth after the first year? The second year?**

1) The initial investment was $22,500.

2) The decay rate is 0.07. The decay factor is 0.93.

3)

$$y = C(1-r)^t$$

$$y = 22{,}500(1-0.07)^1$$

$$y = \$20{,}925$$

$$y = C(1-r)^t$$

$$y = 22{,}500(1-0.07)^2$$

$$y = \$19460.25$$

EXAMPLE **Writing an Exponential Growth Model**

EXAMPLE Writing an Exponential Growth Model

A population of 20 rabbits is released into a wildlife region. The population triples each year for 5 years.

b. What is the population after 5 years?

Help

SOLUTION

After 5 years, the population is

$P = C(1 + r)^t$	Exponential growth model
$= 20(1 + 2)^5$	Substitute C, r, and t.
$= 20 \cdot 3^5$	Simplify.
$= 4860$	Evaluate.

There will be about 4860 rabbits after 5 years.

EXAMPLE

Writing an Exponential Decay Model

COMPOUND INTEREST From 1982 through 1997, the purchasing power of a dollar decreased by about **3.5%** per year. Using 1982 as the base for comparison, what was the purchasing power of a dollar in 1997?

SOLUTION Let y represent the purchasing power and let **t = 0** represent the year 1982. The initial amount is $1. Use an exponential decay model.

$y = C(1 - r)^t$ Exponential decay model

$= (1)(1 - 0.035)^t$ Substitute 1 for C, 0.035 for r.

$= 0.965^t$ Simplify.

Because 1997 is 15 years after 1982, substitute 15 for t.

$y = 0.965^{15}$ Substitute 15 for t.

≈ 0.59

The purchasing power of a dollar in 1997 compared to 1982 was $0.59.

You Try It

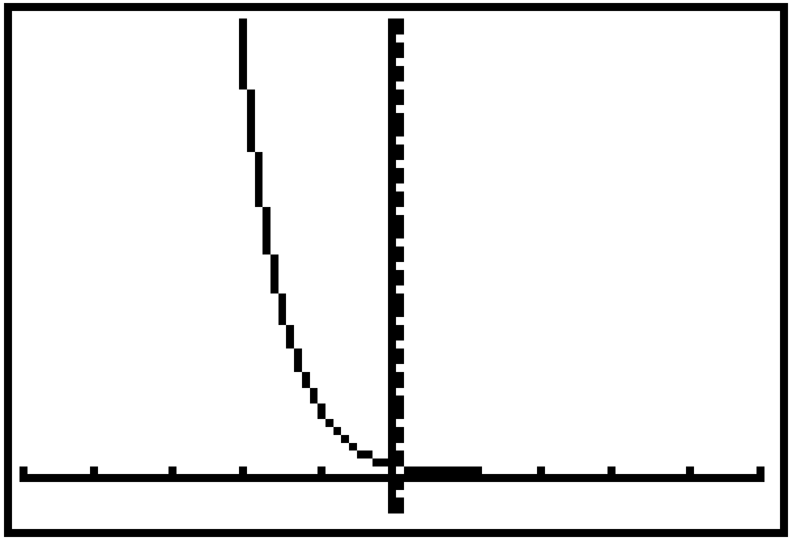
1) Make a table of values for the function

$$y = \left(\frac{1}{6}\right)^{x}$$

using x-values of –2, -1, 0, 1, and Graph the function. Does this function represent exponential growth or exponential decay?

Problem 1

x	$y = \left(\frac{1}{6}\right)^x$	y
-2	$\left(\frac{1}{6}\right)^{-2} = 6^2$	36
-1	$\left(\frac{1}{6}\right)^{-1} = 6^1$	6
0	$\left(\frac{1}{6}\right)^{0}$	1
1	$\left(\frac{1}{6}\right)^{1}$	$\frac{1}{6}$
2	$\left(\frac{1}{6}\right)^{2}$	$\frac{1}{36}$



This function represents exponential decay.

You Try It

2) Your business had a profit of $25,000 in 1998. If the profit increased by 12% each year, what would your expected profit be in the year 2010? Identify ***C***, ***t***, ***r***, and *the growth factor*. Write down the equation you would use and solve.

Problem 2

$C = \$25{,}000$

$T = 12$

$R = 0.12$

$\textit{Growth factor} = 1.12$

$$y = C(1 + r)^{t}$$

$$y = \$25{,}000(1 + 0.12)^{12}$$

$$y = \$25{,}000(1.12)^{12}$$

$$y = \$97{,}399.40$$

You Try It

3) Iodine-131 is a radioactive isotope used in medicine. Its half-life or decay rate of 50% is 8 days. If a patient is given 25mg of iodine-131, how much would be left after 32 days or 4 half-lives. Identify C, t, r, and the *decay factor*. Write down the equation you would use and solve.

Problem 3

$$C = 25 \text{ mg}$$

$$T = 4$$

$$R = 0.5$$

$$Decay\ factor = 0.5$$

$$y = C(1 - r)^t$$

$$y = 25mg(1 - 0.5)^4$$

$$y = 25mg(0.5)^4$$

$$y = 1.56mg$$

Helpful Videos

Exponential Growth & Decay

l.earnzillion.com/lessonsets/36

Exponential Growth & Decay Models

learnzillion.com/lessons/256-model-exponential-growth-drawing-graphs-and-writing-equations

Exponential Growth

https://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_func/exp_growth_decay/v/exponential-growth-functions