Exponential Growth and Decay

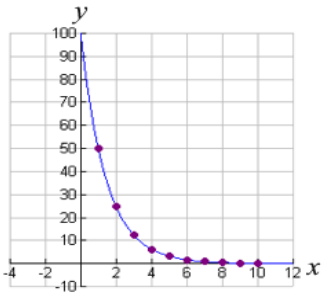
Observe how the graphs of exponential functions change based upon the values of a and b:

$$y = a \cdot b^x$$

Example: $y = 100 \cdot (0.5)^x$

when $a > 0$ and the b is between 0 and 1, the graph will be decreasing (decaying).

For this example, each time x is increased by 1, y decreases to one half of its previous value.



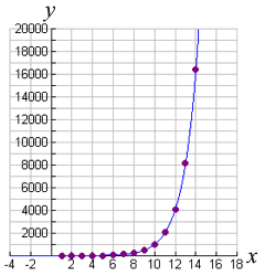
Such a situation is called **Exponential Decay.**

$$y = a \cdot b^x$$

Example: $y = 1 \cdot (2)^x$

when $a > 0$ and the b is greater than 1, the graph will be increasing (growing).

For this example, each time x is increased by 1, y increases by a factor of 2.



Such a situation is called **Exponential Growth.**

When a quantity grows by a fixed percent at regular intervals, the pattern can be represented by the functions,

Growth: $y = a(1+r)^x$

Decay: $y = a(1-r)^x$

a = initial **amount** before measuring growth/decay
r = growth/decay **rate** (often a percent)
x = number of **time** intervals that have passed

Example: A bank account balance, *b,* for an account starting with *s* dollars, earning an annual interest rate, *r,* and left untouched for *n* years can be calculated as $b = s(1+r)^n$ *(an exponential growth formula).* Find a bank account balance to the *nearest dollar*, if the account starts with $100, has an annual rate of 4%, and the money left in the account for 12 years.

$$b = s(1+r)^n$$
$$b = 100(1+.04)^{12}$$
$$b = \$160$$

Growth by doubling:

Bacteria

One of the most common examples of exponential growth deals with bacteria. Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. For example, if we start with only one bacteria which can double every hour, by the end of one day we will have over 16 million bacteria.

End of Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	.. .	24
Bacteria - starting with one	2	4	8	16	32	64	128	256	512	1024	2048	4096	8192	16384	.. .	16777216
Pattern:	2_1	2_2	2_3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}		2^{24}

At the end of 24 hours, there are 16,777,216 bacteria.

By looking at the pattern, we see that the growth in this situation can be represented as a function: $y = 2^x$

Will our formula show this same function? If an amount doubles, the rate of increase is 100%.

Function: $y = a(1+r)^x$	*a* = the initial amount before the growth begins *r* = growth rate *x* = the number of intervals
$y = 1(1+1.00)^x = 2^x$	as *x* ranges from 1 to 24 for this problem

Decay:

Tennis Tournament

Each year the local country club sponsors a tennis tournament. Play starts with 128 participants. During each round, half of the players are eliminated. How many players remain after 5 rounds?

Rounds	1	2	3	4	5
Number of Players left	64	32	16	8	4

There are 4 players remaining after 5 rounds. Function: $y = a(1-r)^x$	a = the initial amount before the decay begins r = decay rate x = the number of intervals
$y = 128(1-.50)^x$	as x ranges from 1 to 5 for this problem

Decay by half-life: The pesticide DDT was widely used in the United States unt ban in 1972. DDT is toxic to a wide range of animals and ac life, and is suspected to cause cancer in humans. The *half-li* DDT can be 15 or more years. *Half-life* is the amount of tim takes for half of the amount of a substance to decay. **Let's mathematically examine the half-life of 100 grams of DD**

By looking at the pattern, we see that this decay can be represented as function: $y = \frac{amount}{2^x}$

Function: $y = a(1-r)^x$	a = the initial amount before the decay begins r = decay rate x = the number of intervals
$y = 100(1-.50)^x$	as x ranges from 1 to 10 for this problem